The Standard Model of Particle Physics (SMPP) is an enormously successful description of high energy physics, driving ever more precise measurements to find 'physics beyond the standard model', as well as providing motivation for developing more fundamental ideas that might explain the values of its parameters. Simultaneously, a description of the entire 3-dimensional structure of the present-day Universe is being built up painstakingly. Most of the structure is stochastic in nature, being merely the result of the particular realisation of the 'initial conditions' within our observable Universe patch. However, governing this structure is the Standard Model of Cosmology (SMC), which appears to require only about a dozen parameters. Cosmologists are now determining the values of these quantities with increasing precision in order to search for 'physics beyond the standard model', as well as trying to develop an understanding of the more fundamental ideas which might explain the values of its parameters. Although it is natural to see analogies between the two Standard Models, some intrinsic differences also exist, which are discussed here. Nevertheless, a truly fundamental theory will have to explain both the SMPP and SMC, and this must include an appreciation of which elements are deterministic and which are accidental. Considering different levels of stochasticity within cosmology may make it easier to accept that physical parameters in general might have a non-deterministic aspect.

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1. The Two Standard Models

Probably the most audacious endeavours in modern physical science are: (1) the attempt to understand the laws governing the whole of physical reality down to the smallest imaginable scales; and (2) the attempt to find a quantitative description of the properties of the entire Universe on the largest scales. Since the Universe is known to be expanding and cooling, then these two quests become linked at the earliest epochs, and hence fundamental physics and cosmology are necessarily connected.

We do not yet have a complete theory to describe high energy physics, but at below TeV energies our understanding is on extremely firm ground. The combination of Quantum Chromodynamics with Electroweak Theory is known as the Standard Model of Particle Physics (hereafter SMPP). The SMPP was solidified in the early 1970s and has been incredibly well tested since then (see [16] and earlier editions of the 'Particle Data Book').

The SMPP contains a finite number of parameters, which are unrelated, at least within the context of the theory itself. One imagines that a more complete theory of fundamental physics will explain the relationships among these parameters. The ultimate goal would be to determine the values of the parameters from pure mathematics, once the correct theory is discovered. It may also be that some of the parameters have a stochastic origin, where our Universe is one choice from among an array of possible vacuum states. This used to be called Anthropic reasoning (see e.g. [5, 50, 7]), and received such little respect from many scientists that it became known as ‘the A word’, and would elicit groans at con-

Douglas Scott, Department of Physics & Astronomy, University of British Columbia, Vancouver, B.C., V6T1Z1, Canada. e-mail: dscott@phas.ubc.ca
Table 1. The 26 Parameters of the Standard Model of Particle Physics.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quark masses:</td>
<td>$m_u$, $m_d$, $m_s$, $m_c$, $m_t$, $m_b$</td>
</tr>
<tr>
<td>Quark mixing angles:</td>
<td>$\theta_{12}$, $\theta_{23}$, $\theta_{13}$, $\delta$</td>
</tr>
<tr>
<td>Lepton masses:</td>
<td>$m_e$, $m_\mu$, $m_\tau$, $m_{\nu_e}$, $m_{\nu_\mu}$, $m_{\nu_\tau}$</td>
</tr>
<tr>
<td>Lepton mixing angles:</td>
<td>$\theta'<em>{12}$, $\theta'</em>{23}$, $\theta'_{13}$, $\delta'$</td>
</tr>
<tr>
<td>Electroweak parameters:</td>
<td>$\alpha$, $G_F$, $M_Z$</td>
</tr>
<tr>
<td>Higgs mass:</td>
<td>$m_H$</td>
</tr>
<tr>
<td>Strong CP violating phase:</td>
<td>$\vec{\theta}$</td>
</tr>
<tr>
<td>QCD coupling constant:</td>
<td>$\alpha_S(M_Z)$</td>
</tr>
<tr>
<td>Total parameters</td>
<td>26</td>
</tr>
</tbody>
</table>

The number of parameters within the standard model varies slightly among phenomenologists, depending on precisely how minimal the model under consideration is, and, in particular, how the neutrinos are treated. A popular counting exercise gives 19 parameters in the minimal SMPP, plus 7 additional quantities to describe the neutrino sector. This is shown in Table 1. There are 26 free parameters in this model; if we were to develop the SMPP from scratch, then presumably we would label the parameters as $A$, $B$, $C$, ..., $Z$. Given this proliferation of numbers, one expects that, for the sake of elegance, there must be a more fundamental theory with far fewer parameters.

As is well known, the SMPP has been astonishingly successful, so much so that, for the last 3 decades, the emphasis has been on trying to find inadequacies in it – i.e., searching for ‘physics beyond the standard model’. However, apart from theoretical ideas (some of them admittedly quite appealing), there are still no convincing pieces of evidence for physics beyond the SMPP.

On the other hand, we know that there has to be new physics, beyond the SMPP, due to what we have learned about the properties of the large-scale Universe – particularly cosmological evidence for dark matter, dark energy and inflation.

Cosmology grew from being an arm-chair activity carried out in people’s spare time, to being a dignified scientific pursuit, only in the 1960s. Originally the models were entirely baryonic and involved simple ad hoc initial conditions. In many ways the basic picture has remained the same since then – nearly scale invariant and adiabatic initial conditions, in an almost isotropic and homogeneous Friedmann-Robertson-Walker solution to Einstein’s Field Equations. However, Cold Dark Matter was added to the paradigm in the 1980s (e.g. [43, 6]), leading to the ‘Standard CDM’ picture in which $\Omega_M = 1$. By the end of the 1980s the addition of a cosmological constant $\Lambda$ was known to give better fits to the available data (e.g. [44, 65, 15]).

The COBE satellite detection of large-scale Cosmic Microwave Background (CMB) anisotropies in 1992 [58] brought an end to many wilder proposals which had been floated in the era of continually improving CMB upper limits (see [36] for a discussion). It became clear that the CMB normalization, together with galaxy clustering data, pointed to the ‘$\Lambda$CDM’ variant of the CDM paradigm ([14, 31]), despite the reluctance of many theorists to let the elegance of Standard CDM slip away (e.g. [67]). The cosmological constant became an accepted part of the model by the mid-to-late 1990s, following the results from distant supernova surveys and degree-scale CMB experiments. Soon the concept of $\Lambda$ was generalised to that of Dark Energy. As the CMB anisotropy measurements grew increasingly precise,

1 And it has become known as ‘the other L word’.

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Table 2. The 12 Parameters of the Standard Model of Cosmology.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>$T_0$</td>
</tr>
<tr>
<td>Timescale</td>
<td>$H_0$</td>
</tr>
<tr>
<td>Densities</td>
<td>$\Omega_A$, $\Omega_{CDM}$, $\Omega_B$, $\Omega_\nu$</td>
</tr>
<tr>
<td>Pressure</td>
<td>$w \equiv p/\rho$</td>
</tr>
<tr>
<td>Mean free path</td>
<td>$\tau_{\text{reion}}$</td>
</tr>
<tr>
<td>Fluctuation descriptors</td>
<td>$A$, $n$, $n' \equiv d\ln n/d\ln k$, $r \equiv T/S$</td>
</tr>
<tr>
<td>Total parameters</td>
<td>12</td>
</tr>
</tbody>
</table>

it became clear that (at least in principle) several parameters could be measured which would constrain the inflaton potential. But to do this carefully, one had to take into account other astrophysical effects on the CMB anisotropies, particularly anisotropy suppression in the period since the Universe became reionized – hence another parameter needed to be added.

We have thus ended up with a Standard Model of Cosmology (hereafter SMC), which is based on ideas as old as the SMPP, but which solidified only about a decade ago. Determining the precise date when the SMC was in place is a little murky (to say the least). The late 1980s and early 1990s were a time of increasing tension among different pieces of observational data, which (at least in hindsight) was because the SMC was about to fall into place. There were also a few false leads, such as the early supernova results apparently suggesting deceleration, increased interest in models with a significant hot dark matter (i.e. high $\Omega_\nu$) component, and arguments over the naturalness of open inflationary models. But despite all of this, the SMC was clearly in place by 1995 [34, 42].

The number of parameters required to describe this model varies to some extent depending on the tastes of individual cosmologists. However, a typical count gives the number of required parameters as 12, which are listed in Table 2. This is not a complete set of possible parameters, but there is currently no evidence that we need any more. If we were to develop the SMC from scratch, then presumably we would choose a simpler set of symbols, for example: $A, E, H, I, K, L, M, N, O, P, U, W$. The parameters are also not all on an equal footing. For some of them, there is no indication at the moment that they differ from their default values (e.g. $\sum \Omega_i = 1$ or $n' = 0$), and hence the final SMC may actually have fewer parameters.

There are several assumptions that underlie the SMC. We certainly assume that physics is the same everywhere in the observable Universe (but see Section 5), and that General Relativity fully describes gravity on large scales. The SMC also relies on the hot Big Bang picture being correct, and that something akin to inflation created the density perturbations. The astonishing thing about modern cosmology is that most of these assumptions are testable (or at least falsifiable), and that for the reality in which we find ourselves living there are ways of determining the values of the quantities that describe the nature of the entire observable Universe.

2. The Miracle of the CMB Sky

Many different observable quantities can be used to constrain the cosmological parameters. Traditionally these have involved trying to estimate distances of very distant objects (which is hard), estimating masses of large amounts of matter (which requires the distance), measuring the clustering of galaxies (which is related in a complicated way to the clustering of mass), and determining primordial abundances (which is fraught with systematic effects). While each of these approaches have been useful, they all rely on using tracers that are well into the non-linear regime, i.e. objects with density.
**Fig. 1.** Theoretical CMB anisotropy power spectrum. This is the expected variance per logarithmic interval in $\ell$ for the SMC (together with an arbitrarily normalised tensor component). The physics of the CMB anisotropies splits into 4 regions, separated approximately into decades of $\ell$ (see [53] for more details).

![Image of CMB anisotropy power spectrum]

contrasts $\delta \gg 1$.

Fortunately the Universe has given us another opportunity to learn about its large scale properties – anisotropies in the Cosmic Microwave Background (see e.g. [66, 27, 25, 53] for reviews). These are essentially a projection of the 3-dimensional structure at early times (redshift $z \sim 1000$, or $t \sim 300,000$ years), when densities were still very much in the linear regime, $\delta \sim 10^{-5}$.

Apart from the dipole (see later), the CMB is extremely isotropic. This ‘horizon problem’ is often taken to be evidence that the initial conditions were acausal, or that something happened to make them appear so. But more important information comes from the fact that the CMB actually contains minute anisotropies, first detected by the COBE satellite, and confirmed and extended by about 20 separate experiments in the last 13 years. The particular anisotropies that we observe are thought to be stochastic, in the sense that different volumes of the Universe will have different CMB skies. But the power spectrum of anisotropies, from which our sky’s temperatures are drawn, is considered as a fundamental quantity, which depends on the underlying cosmological model.

If the density perturbations are Gaussian, then the power spectrum describes *all* of the statistical information. Non-Gaussianities may certainly exist and are worthy of study, but they are weak in all inflationary-type models, and hence the power spectrum contains the vast majority of the information and has therefore become the main focus of all modern CMB experiments.

Figure 1 shows a calculation of the power spectrum of CMB anisotropies, where $C_\ell \equiv |a_{\ell m}|^2$ and the $a_{\ell m}$s are the amplitudes of the spherical harmonic expansion of the temperature field. The structure
Fig. 2. CMB band-powers from the WMAP [23], BOOMERANG [29], VSA [13], CBI [49] and ACBAR [35] experiments. Some of the higher and lower $\ell$ band-powers with the poorest error bars have been omitted. There are also some other experiments with data of comparable quality.

- WMAP
- BOOMERANG
- VSA
- CBI
- ACBAR

CMB anisotropies can be determined with exquisite precision for specific cosmological models [26, 55]. This is because the physics generating the anisotropies is well understood, requiring just linear perturbation theory for gravity-driven acoustic oscillations, together with simple Thomson scattering of the CMB photons off free electrons.

Experimental results are usually quantified in terms of ‘band-powers’ on the anisotropy power spectrum. A compilation of some of the highest quality data-sets is shown in Figure 2 (note, there is a linear $x$-axis here, unlike in Figure 1).

Clearly one does not need a model to be plotted to guide the eye on this figure. The first three acoustic peaks are now convincingly detected. So the general paradigm, of a hot early Universe, containing roughly scale-invariant and adiabatic initial perturbations over a wide range of scales, is strongly supported. It is easy to see how considerable cosmological information is encoded in the rich structure of the power spectrum. In fact a 6 parameter model space is very tightly constrained by the current CMB data [60, 38], and an extended set of parameters can be constrained using other astrophysical
measurements.

The most precise quantity measured from CMB anisotropies is the angular scale of the acoustic peak structure in the power spectrum. This is often parameterized by \( \theta \) [33], defined to be 100 times the ratio of the sound horizon to the angular diameter distance at last scattering. It turns out that, defined this way, the value is remarkably close to unity, with the best value somewhere around 1.045 ± 0.004 [38]. This quantity is related to the underlying cosmological parameters – for example, it depends strongly on the overall curvature of space.

The CMB does not of course measure everything. Several degeneracies in fitting the SMC parameters are easily broken by including other data, e.g. from supernovae, galaxy clustering or direct estimates of \( \Omega_M \) or \( H_0 \). These data can be used as priors when fitting parameters to CMB data. Hence, one must be careful when using particular results, to consider what priors were used, as well as whether restricted parameter spaces or ranges of parameters were considered.

The CMB is about 10% polarized, since Thomson scattering of an anisotropic radiation field generates linear polarization (see e.g. [28, 68]). Measurements of the polarization power spectrum and the cross-power spectrum of polarization with temperature are also now being measured. So far such measurements have only served to confirm the basic paradigm of the SMC (but see 3.5), although it is expected that more precise polarization measurements will help break degeneracies and place quite different constraints on parameters.

Another particularly promising cosmological measurement technique now being developed is weak gravitational lensing (also called ‘cosmic shear’, see e.g. [63]). This approach to measuring cosmological fluctuations shares some of the benefits (and much of the mathematics) of CMB anisotropies. Like CMB polarization, the results so far have confirmed the SMC picture, but have not set very strong additional constraints. That will likely change with the more ambitious lensing projects now underway.

3. The Values of the Parameters in the SMC

Now that the SMC has been outlined and some of its observational probes described, let us examine each of the parameters in turn.

3.1. The temperature

The CMB was discovered by Penzias & Wilson in 1965 after a convoluted history of false starts and neglected theoretical predictions. It was eventually found to have a very accurately blackbody spectrum, with deviations severely constrained over 3 decades in wavelength (see [59] and references therein). The CMB temperature is measured to be

\[ T_0 = 2.725 \pm 0.001 \text{ K}, \]

where the error bar represents 1σ, but is dominated by systematics [39]. The fact that this is such a good blackbody is one of the strongest pieces of evidence for the hot Big Bang picture. It is extremely difficult, if not impossible, to contrive to make such a smooth thermal spectrum through some local process, but it is easy to achieve in a hot early period of the Universe, since the thermal equilibrium timescale is naturally very much shorter than the expansion time.

This temperature corresponds to a photon number density of \( n_\gamma \simeq 411 \text{ cm}^{-3} \) and an energy density of \( \rho_\gamma \simeq 0.260 \text{ eV cm}^{-3} \) for a thermal spectrum. One can also derive other quantities, such that the intensity peak is \( I_\nu \simeq 385 \text{ MJy sr}^{-1} \) at \( \nu \simeq 160.2 \text{ GHz} \), that the r.m.s. magnetic field in the CMB corresponds to \( B_\gamma \simeq 3.24 \mu \text{G} \), etc.

Another quantity we can determine from observations of the CMB is the size of the dipole, i.e. the amplitude of a \( \cos \theta \) function fit to the whole sky once the overall monopole amplitude \( T_0 \) has been removed. This tells us our ‘local’ velocity through the sea of CMB photons. This velocity is generated
by the gravity field of lumps on the scale of the local Supercluster, and hence varies between different observers in the Universe. In other words this is a quantity that is established by the particular realisation of the observable Universe around us.

The observed dipole implies that the Solar System is moving at \( v \approx 370 \text{ km s}^{-1} \) towards a particular direction (just above the equatorial plane at about 11\(^\circ\)), or, correcting for estimated velocity vectors, one can find a speed for the Local Group of galaxies corresponding to about 630 km s\(^{-1}\) \cite{18}.

The value of \( T_\gamma \) changes with cosmological epoch so that \( T_\gamma(z) = T_0(1 + z) \). There is some evidence that its value was higher in the past, consistent with cooling through expansion (e.g. \cite{40}). But it would take a Hubble time to observe a substantial change in its value (and hence extraordinary precision to detect a change over a human lifetime). Therefore, we should regard \( T_0 \) as a fundamental constant, which fixes the energy density of radiation today.

Although Alpher, Gamow & Herman \cite{2} made an order of magnitude prediction of the CMB temperature (based on assuming that while nuclei were undergoing nuclear reactions in the early Universe they were in thermal equilibrium with photons), there is no ab initio calculation for the CMB temperature today.\(^3\) Hence \( T_0 \) should be regarded as an empirical parameter of the SMC. Its value may be related to the details of: (a) reheating at the end of inflation; (b) the SMPP and its extensions, defining the particles which annihilate in the early Universe to give most of today’s CMB photons; and (c) the particular time that we happen to exist and make measurements.

### 3.2. The timescale

The derivative of the cosmological scale factor measured today, \( H_0 \equiv (\dot{a}/a)|_{t=t_0} \), is referred to as the Hubble constant. Locally it provides the calibration of the Hubble expansion law, \( v = H_0 r \), and hence is measured in units of \( \text{km s}^{-1}\text{Mpc}^{-1} \) (and sometimes defined as \( h \equiv H_0/100 \) in these units). The best direct estimate comes from the Hubble Space Telescope Key Project \cite{19}. This is often used as a prior for the determination of parameters from CMB anisotropies. The recent multi-parameter fit by the BOOMERANG team gives \( H_0 = 75.8^{+5.6}_{-5.1} \), and with the addition of further constraints from large-scale structure they find \cite{38}

\[
H_0 = 69.6 \pm 2.4 \text{ km s}^{-1}\text{Mpc}^{-1}.
\] (2)

The Hubble constant evolves according to \( H(z) = H_0 \sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda} \) for flat cosmologies (i.e. \( \Omega_M + \Omega_\Lambda = 1 \), and for recent epochs where we can ignore radiation). So we have to wait of order the Hubble time to observe its value change dramatically. In fact, in a purely cosmological constant dominated model, the value approaches a constant as the Universe approaches the de Sitter limit: \( H(z) \rightarrow \sqrt{\Omega_\Lambda} H_0 \) as \( z \rightarrow -1 \), giving a final value a little below 60 for the currently preferred SMC parameters.\(^4\)

But even although \( H_0 \) is varying with time, we can regard it as a constant for all practical purposes. Since the Hubble constant is a rate, then its reciprocal defines a timescale. For the parameter values of the current SMC it turns out (through a coincidence, because of the combined effects of deceleration and acceleration) that the age of the Universe is \( t_0 \approx 1/H_0 \) to within 10%.

Once one has a specific set of parameters for the SMC (actually just the \( \Omega_M \) and \( H_0 \), one can derive the present-day age of the Universe precisely. Hence \( t_0 \) is a derived quantity, with current estimates in the range 13–14 Gyr.

Since \( H_0 \) changes with time, then its value would have been different if we happened to live, say, 5 billion years in the past or the future. But in a \( \Lambda \)-dominated universe a final value of \( H_0 \) is eventually reached. However, even this value, \( \sqrt{\Lambda/3} \), is not independent of the other parameters, since it just depends on the cosmological constant.

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\(^3\) For some numerological ideas see http://www.astro.ubc.ca/people/scott/whochosetemp.html

\(^4\) So one could say that Sandage was right, he was just billions of years ahead of his time!
3.3. The densities

The amounts of each of the components that make up the Universe are usually defined in terms of their contributions to the critical mass (or energy) density. Here $\rho_{\text{crit}} = \frac{3H_0^2}{8\pi G}$ is the density required to make the Universe spatially flat. The total matter density $\Omega_M \equiv \Omega_{\text{CDM}} + \Omega_B$ can be estimated fairly directly using various dynamical methods, which typically give values of 0.2–0.3. The baryonic contribution (which traditionally includes electrons) can be separately constrained by Big Bang Nucleosynthesis [17].

The CMB places a strong constraint on the overall curvature of space, essentially through the $\theta$ parameter, i.e. the $\ell$ scaling of the acoustic peaks. This means that $\Omega_{\text{tot}} \equiv \sum \Omega_i$ (which is basically $\Omega_M + \Omega_A$) is strongly constrained to be close to unity.\(^5\) Other than this geometric effect, the CMB anisotropies depend on the physical densities of baryons and CDM, i.e. $\rho_B \propto \Omega_B h^2$ and $\rho_{\text{CDM}} \propto \Omega_{\text{CDM}} h^2$.

Useful additional constraints, with different parameter dependencies, come by adding data from the clustering of galaxies, e.g. from the Sloan Digital Sky Survey [48] or 2 Degree Field Galaxy Redshift Survey [11]. These large-scale structure (LSS) data depend mainly on the combinations $\Omega_M h$ and $\Omega_B / \Omega_M$, as well as the normalization (called ‘bias’) of the galaxy clustering relative to that of the dark matter.

The newest BOOMERANG results, in combination with WMAP and other CMB experiments, and using some LSS information, yield [38]

$$\Omega_B h^2 = 0.0227 \pm 0.0008 \quad (3)$$

and

$$\Omega_{\text{CDM}} h^2 = 0.120 \pm 0.005. \quad (4)$$

The precise values depend on how much freedom is allowed on the shape of the primordial power spectrum and which LSS data are used.

The constraint on the cosmological constant (or Dark Energy) using CMB data plus LSS data is

$$\Omega_A = 0.67 \pm 0.05, \quad (5)$$

and tighter constraints are possible if one restricts the Universe to be flat. The best current limits show that $\Omega_{\text{tot}}$ is within a few per cent of unity.

A minor (but not negligible) contribution to the overall energy density comes from massive neutrinos. They have the effect of partially erasing fluctuations on small scales. This process depends on the sum of the neutrino masses, assuming they are Fermi-Dirac distributions with zero chemical potential and the temperature expected in the Big Bang picture, i.e. $T_\nu \simeq \left(\frac{4}{11}\right)^{1/3} T_\gamma$. The derived constraint on $\Omega_\nu$ depends sensitively on the data which are used for smaller scale clustering, as well as the freedom allowed on the primordial spectrum and how one treats galaxy bias. A fairly robust limit is

$$\Omega_\nu < 0.02, \quad (6)$$

corresponding to $\sum m_\nu < 1 \text{ eV}$, although tighter constraints have also been claimed (see e.g. [47]).

As a fraction of the critical density, we find (using $T_\nu$) that $\Omega_\gamma h^2 = 2.471 \times 10^{-5}$ for the photon contribution. This value is small enough that it is negligible in today’s Universe, although one needs to properly account for it when considering evolution at early times. Apart from components we have already mentioned, the only other contribution to the energy density is from gravity waves, but this makes a fraction if $\rho_{\text{crit}}$ which is several orders of magnitude lower still.

The $\Omega$s change with time, and hence they cannot be regarded as ‘fundamental’ (although, like $T_0$ and $H_0$ they vary negligibly over a human lifetime). Relative to each other, the matter contributions do

\(^5\) And so the non-Euclidean isotropic spaces which used to form an important part of all basic cosmology courses can now be relegated to a footnote.
not evolve, but the Dark Energy contribution is growing with time (largely because the matter density is dropping, while Dark Energy is roughly constant per unit volume).

We can also consider the underlying physics which determines each of the individual density parameters: \( \Omega_\Lambda \) today is determined if we already know the vacuum (or Dark Energy) density \( \Lambda \) plus \( H_0 \); \( \Omega_\nu \) depends on the neutrino masses, as well as \( T_0 \) and \( H_0 \); \( \Omega_{\text{CDM}} \) is presumably set by the physics of the dark matter particles (a mass and a cross-section); and \( \Omega_B \) is set by the physics of baryogenesis (for a given \( T_0 \)).

### 3.4. The pressure

Pure vacuum energy (a.k.a. the cosmological constant) is fixed per unit volume, and hence evolves as \( \rho \propto (1 + z)^0 \), while ‘curvature’ scales as \( (1 + z)^2 \). Vacuum has pressure \( p = -\rho \), while curvature behaves like a fluid with \( p = -(1/3)\rho \). Hence the generalisation of \( \Lambda \) is to a ‘Dark Energy’ (see e.g. [9]) with an equation of state parameter \( w \equiv p/\rho \) in the range \(-1 < w < -1/3\). For empirical purposes, this range should be extended to \( w < -1 \), although there are certainly theoretical difficulties with such ‘phantom energy’.

The CMB alone is not very sensitive to the Dark Energy content, apart from its contribution to \( \Omega_{\text{tot}} \). But additional constraints, particularly from supernovae data (e.g. [52, 3]) give upper limits around

\[
-0.7 < w,
\]

(see e.g. [38, 56]). This is entirely consistent with a pure cosmological constant. If it were discovered that \( w \neq -1 \), then that would be extremely exciting, since from its behaviour we could hope to understand what the Dark Energy actually is. Although there are plenty of alternative names that have been dreamed up by theorists, there are no well motivated theoretical models for Dark Energy. Hence there is no calculation that can tell us how small to expect \( w + 1 \) to be.

### 3.5. The mean free path

We know (from the spectra of distant quasars) that the Universe reionized at some redshift higher than 6 (see [4] for a review). Presumably this was achieved by the ionizing photons generated in the earliest stars. Understanding the ‘End of the Cosmic Dark Ages’ is one of the most exciting issues in modern astrophysics. Fortunately the CMB can help, since the extra Thomson scattering from \( z = 0 \) out to \( z = z_{\text{reion}} \) partially erases the anisotropies. This effect is hard to separate from an overall change in amplitude and/or slope, except that it also generates a large-angle polarization pattern.

The CMB fluctuations are expected to be weakly polarized and with a pattern that is correlated with the anisotropies. The WMAP experiment was able to measure the temperature-polarization correlations on the largest angular scales (above 10\(^\circ\)), where a signal was detected that appeared to be consistent with that expected from reionization, except somewhat stronger than would have been guessed [32]. The estimated value for the Thomson optical depth (\( \int n_e \sigma_T dr \)) is

\[
\tau = 0.17 \pm 0.04.
\]

This result can be recast into a mean free path or a redshift, given the other background cosmology parameters, e.g. it corresponds to \( 9 < z_{\text{reion}} < 30 \). The error bar is still quite large, however, and so somewhat different values can be obtained using other choices of data, priors, etc.

Clearly \( t_{\text{reion}} \) is calculable in principle, since it depends on small scale density perturbations going non-linear and turning into stars. However, this requires knowing details of the inflationary power spectrum at smaller scales than are otherwise probed, plus understanding complexities of ‘gastrophysics’, which are currently well beyond the capabilities of numerical simulations. So for the foreseeable future it should be regarded as an empirical parameter which has to be taken into account when fitting the CMB (including polarization) anisotropies. One may regard \( t_{\text{reion}} \) as non-fundamental, but in the

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sense that its value is an outcome of the (unknown) underlying theory, it is really no different than, say, $T_0$.

### 3.6. The fluctuation descriptors

The adiabatic nature of the CMB anisotropies at angular scales above those corresponding to the Hubble length at last scattering suggests there were ‘synchronized initial conditions’ at apparently acausal scales. More direct evidence for this comes from the temperature-polarization anti-correlation seen by WMAP above degree scales [32], which gives a clearer indication for an early period of cosmic acceleration than the observed near flatness, Gaussianity and scale-invariance. Although there is undoubtedly convincing evidence for something here,⁶ we need more direct probes of the physics of inflation in order to understand whether it really happened (see [37] for a review).

There are 4 basic observables which can help probe the initial perturbations – 2 amplitudes, a slope and a curvature. The 2 amplitudes are for scalar modes (density perturbations) and tensor modes (gravity waves). They are usually parameterised with $A$ for the variance of the scalars at a fixed wavenumber (e.g. $0.05 \; \text{Mpc}^{-1}$), together with the tensor to scalar ratio $r$ at even larger scales. Then there is the slope of the scalar power spectrum, i.e. $P(k) \propto k^n$. This spectral index $n$ can also have a ‘running’, i.e. one can potentially measure the curvature through $n' \equiv dn/d \ln k$. Higher derivatives seem destined to be unobservable, as does the tensor spectral index (although in most models it is fixed by the value of $r$ in any case).

The results from the WMAP experiment first year data are [60]

\begin{align}
A &= 2.7(\pm 0.3) \times 10^{-9}, \\
N &= 0.97 \pm 0.03 \\
n &= 0.97 \pm 0.03
\end{align}

Again, slightly different constraints can be obtained by combining with different data-sets and by restricting the parameter space being considered (particularly whether one allows for $n' \neq 0$).

For the running of the spectral index there appears to be weak evidence that the slope changes from blue to red as one moves from small $k$ to large $k$, with estimates around

\begin{align}
n' &= -0.06 \pm 0.03
\end{align}

However, the precise value depends on the data-sets and priors used [56, 38]. The result is currently $2\sigma$ at best, and hence there is no strong evidence for non-zero running. However, a convincing detection would be quite constraining for inflationary models [45].

In slow roll inflation $A \propto V^3/(V')^2$, where $V(\phi)$ is the inflaton potential. The slope $n$ also involves $V''$, while $n'$, etc. involves higher derivatives. The nice thing is that the tensor amplitude is proportional to $V$ itself. Hence if one could measure the gravity wave contribution to the CMB anisotropies (see Figure 1), then one would have an estimate of the energy scale of inflation. And if one could measure $A$ and $n$ precisely, then one would be constraining a Taylor expansion of the potential. But extracting a low amplitude tensor component from the temperature $C/\ell$s is clearly difficult.

One of the most exciting prospects for CMB polarization measurements is that they possibility provides a clean way of extracting the tensor signal. The polarization ‘B-mode’ (a geometrical component of the pattern which contains a curl) cannot be sourced by scalars, since they have no handedness, while primordial gravity waves do generate B-modes. Hence there is a great quest underway to improve the sensitivity of polarization experiments to try to detect this signature, if it is there. This will not be easy, but the pay-off makes it worth the effort.

⁶ A defensible statement is ‘Something like inflation is something like proven.’
Table 3. Confirmed Predictions of the Standard Model of Cosmology.

<table>
<thead>
<tr>
<th>Prediction</th>
<th>Year of Confirmation</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMB acoustic peaks</td>
<td>1994</td>
</tr>
<tr>
<td>Acceleration from SNe</td>
<td>1998</td>
</tr>
<tr>
<td>Cosmic shear</td>
<td>2000</td>
</tr>
<tr>
<td>CMB polarization</td>
<td>2001</td>
</tr>
<tr>
<td>High-z cosmic jerk in SNe</td>
<td>2001</td>
</tr>
<tr>
<td>ISW-LSS correlations</td>
<td>2003</td>
</tr>
<tr>
<td>Baryon oscillations</td>
<td>2005</td>
</tr>
</tbody>
</table>

Dates are for the first clear indication of the predicted effect; in most cases much more significant confirmations soon followed.

Presumably these 4 inflationary parameters come from some more fundamental theory. At the phenomenological level they are related to the inflaton potential, although eventually one imagines there will be a theory with specific fields and couplings, rather than simple mathematical forms. Whether there can be different values for these inflationary parameters in different volumes (or universes for that matter) is a topic of debate.

### 3.7. Additional parameters

Since the SMC is still being developed, and does not come with any sense of completeness, then the adding of extra ingredients is limited only by the imaginations of theorists. At the moment, there is no compelling evidence for adding any additional degrees of freedom when fitting the SMC to data. However, it is certainly fruitful to continue to try, since this is the only way that missing ingredients will be uncovered. Among the less crazy ideas for adding features are: a component of isocurvature perturbations; a free parameter for the primordial helium abundance; features in the initial power spectrum; additional phenomenological reionization parameters; parameters describing the evolution of $w$ in some basis; or perturbations in the Dark Energy itself.

### 4. The SMPP vs the SMC

Although it is hard to resist the temptation to compare the two Standard Models, the SMPP and the SMC are clearly quite different. Probably the most basic difference is that the SMPP, although it is called a ‘model’, is actually a ‘theory’ – in fact one of the best tested and most successful theories ever devised – while the SMC is very much just a model. The SMPP predicted the existence of the W and Z bosons, as well as a host of other features which have now been precisely measured. The SMC is certainly less mature, but in fact there are a number of confirmed predictions which can already be claimed. This is in addition to generic attributes of the observable Universe, which simply relate to the Big Bang or hierarchical structure formation frameworks. At least 7 such predictions could be advanced as successes of the SMC, starting with the emergence of the CMB acoustic peaks (a process which began in 1994 [54], and was certainly definitive by 2000 [46]). These tests of the SMC are listed in Table 3.

The two Standard Models are certainly distinct, since none of the parameters of the SMC can be derived using the SMPP – for example, one needs to know at least the value of $H_0$ (and then the full SMPP would give $\Omega_w$) or the addition of $T_0$ and information about baryogenesis (to get $\Omega_B$). It is also

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7 The first two are often regarded as part of the basis of the SMC, rather than being confirmed predictions, but this is not really the case, since they came after the SMC was already essentially in place.
the case that very few of the SMPP parameters have much bearing on the SMC, since only the physics of photons, protons, neutrons and electrons is of primary importance for physical cosmology, with the neutrino sector being less important, and the rest being largely irrelevant (except indirectly, through the relationship with higher energy physics beyond the SMPP).

Because the sets of parameters are essentially disjoint, then any fundamental answer to the question of ‘Life, the Universe and Everything’ will have to explain the parameters of both Standard Models.

Another basic difference between the two Standard Models comes from how they were developed. The SMPP may be an incomplete theory (in the sense that one would like to solve the ‘hierarchy problems’, unify with gravity and include the extra physics implied by the SMC), but at least it has a well-defined mathematical structure. The SMC, on the other hand is not something that one might have dreamed up without peering at the Universe. Words like ‘unsatisfactory’, ‘baroque’ and ‘preposterous’ are used often invoked to describe the nature of the SMC. However, as we have seen, the SMC has to be taken quite seriously, since it has made several testable predictions which have subsequently been confirmed, thus conforming broadly to the scientific method. While the more fundamental theory of high energy physics is is expected to still contain the SMPP as its lower energy limit, the same may not be true for the SMC within the ultimate Cosmological Model. But given the SMC’s decade of success, it is hard to see at this point how the entire edifice can be askew.

Another apparent difference is the sense in which the parameters of the SMPP are more fundamental than those of the SMC, although the distinction is perhaps becoming less clear. This is connected with the traditional view that in physics everything is exact, while in astronomy everything is approximate.

In high energy physics one measures cross-sections, decay rates, branching ratios, etc. and relates these to the parameters of the SMPP. Every calculation is imagined to be correct, up to a next-order term which has been neglected. In practice things are often more complicated, with some calculations seeming to require non-perturbative approaches, but still, there is an implicit assumption that there are exact values waiting to be measured.

In astronomy generally, the philosophy is quite different. Instead of measuring the properties of sub-atomic particles imbued with a few quantum numbers, one is trying to measure the properties of macroscopic objects over a huge range of mass scales, and hence approximate answers are often sufficient (and intrinsic scatter in properties is expected). But physical cosmology has become more precise than most other branches of astrophysics, since one is attempting to get at a description of the entire observable Universe by measuring astrophysical quantities, which are related in a complicated way to the underlying parameters of the SMC.

One accepts that there is a general level of uncertainty here, sometimes referred to as ‘cosmic variance’, meaning that the properties within our Hubble volume may be slightly different from the expectation value averaged over a large number of such volumes. This idea is not too far from those of the ‘Landscape’, and hence it may be that practitioners of sub-atomic physics may also have to get used to the idea of ‘cosmic variance’.

5. Three Levels of Stochasticity

Discussion of the origin of the SMC parameters may help illuminate how parameters are fixed in general. One can imagine that some parameters are entirely deterministic, i.e. they are fixed by the mathematical structure of the fundamental theory. Other parameters may be more ‘accidental’ in nature, and it may be that if we can figure out where the stochasticity comes in, that may help distinguish among theories (see e.g. [24]). But what does ‘stochastic’ mean in the context of the properties of the Universe?

There are 3 different levels of randomness that enter into this discussion (see Table 4). At the weakest level, some SMC parameters are stochastic in the sense that we happen to measure them at a particular epoch. There is no obvious property which would have prevented us from asking about the
Table 4. The 3 Levels of Stochasticity.

<table>
<thead>
<tr>
<th>Level</th>
<th>Affected quantities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observer epoch</td>
<td>$T_0, H_0, \Omega_s, \ldots$</td>
</tr>
<tr>
<td>Cosmic variance</td>
<td>Dipole, other $a_{\ell m}$, local $H_0, \Omega_{tot}, \ldots$</td>
</tr>
<tr>
<td>Different vacua</td>
<td>$\Lambda$, inflation, SMPP, \ldots</td>
</tr>
</tbody>
</table>

Universe if we lived in a civilisation several billion years into the past or the future. If we define some parameters to be simply the quantities at the time we measure them\(^8\), then their values will be partly accidental in the sense that the epoch we are living at contains some randomness. This may seem like a fairly ineffectual kind of stochasticity, but it has the virtue of being easy to understand, and undeniably real.

A stronger level of stochasticity comes from the ‘cosmic variance’ idea. There is a certain volume in which we can make measurements, defined by the region within which light can have reached us in the age of the Universe. We know that each Hubble patch has an overall density fluctuation of order $10^{-5}$, since the power spectrum gives roughly that amplitude at ‘Horizon crossing’. The density within our patch could easily be a several $\sigma$ excursion from the average value, without it seeming terribly unlikely. This means that the global value of $\Omega_{tot}$ can never be determined to better than 4 decimal places. And the same applies to all the other $\Omega$s. A more extreme example of this same principle comes from the cosmic dipole – one can measure its value, but since its relative variance is large between observers, then the value we measure is almost entirely a random number. In general, all of the detailed 3-dimensional structure of the observable Universe falls into this category. Some initial fields chose particular realisations consistent with some expectation values, and this ultimately led to objects such as the Great Wall, the Local Supercluster, the Milky Way galaxy, and the detailed configuration of matter within them.

The final level of randomness comes when one considers that there may be multiple vacuum states into which different volumes fell. Each of these volumes would then have inflated in a different way and may have had different values of the parameters of the SMPP. Whether we consider these as separate ‘universes’ or as different volumes within a single space-time may be simply semantics.

A useful outcome of considering the origin of the parameters of the SMC may be that one is naturally led to the idea that several of the parameter values contain a non-deterministic element, and that there are different levels of randomness. It may then be more reasonable to imagine that quantities such as the vacuum energy could be stochastic in nature. And so more generally, the idea of different volumes having different SMPP parameters may seem like less of a conceptual leap. Seen in this light the details of the SMPP may just be extensions of the SMP, as observed in our particular universe.

Whether the probability distributions of any quantities can be calculated (or even defined) is altogether a different question. A great deal has already been said, and debated, about the reality of the Multiverse\(^9\), and principles of selection, mediocrity, fine tuning, biophilia, anthropy, Copernicanism and the like [10, 8, 64, 62, 30, 12, 22, 57, 1, 20], and little can be added here.

The main point to make is that our current picture of the Universe contains elements of happenstance, and that learning more about the cosmological model may help us to understand this randomness, and hence the bigger picture.\(^{10}\) An extension of the idea that stochasticity plays some role in

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\(^{8}\) A fairly natural choice, at least for our present purpose.

\(^{9}\) A term which certainly grabbed the imagination of science fiction writers, probably even being used by them before it was adopted by physicists [41].

\(^{10}\) Or then again, it may not.
establishing parameter values is that we may be able to lump the parameters of the SMC and the SMPP together as part of the set of observables of our corner of the multiverse.

6. The Future

There are many ongoing and planned surveys which will improve the determination of the parameters of the SMC. These include new CMB experiments, particularly the next generation satellite mission \textit{Planck}, scheduled for launch in about 2 years. The capabilities of \textit{Planck} mean that it may be the experiment which takes us beyond the SMC. There are several new experiments designed to measure CMB polarization from either the ground or balloons, and there are also ambitious plans for another satellite mission in the more distant future to try to detect the inflationary B-modes. Additionally, there are aggressive plans for measuring weak lensing, supernova distances and galaxy cluster abundances, all of which should particularly help improve the uncertainties on \( w \).

If the SMC is already in good shape, then why should we bother placing more stringent bounds on its parameters? The main reason to determine these quantities more precisely is that these are the numbers that describe the whole Universe, and so they should just be measured better simply because they’re there.\(^\text{11}\) The secondary reason is that one can only uncover inadequacies in the framework by making more precise measurements. Since the SMC is not self-contained in any way, then there may well be major features which are currently missing.

Certainly there are many mysteries suggested by the SMC. The obvious ones are: What are the Dark Matter and Dark Energy? How did baryogenesis work? Did inflation really happen?

The first 3 of these issues may have resolutions at energy scales which might be accessible at accelerators. But the inflationary parameters are probably probing energies \( \sim 10^{16}\)GeV, far beyond the reach of any terrestrial experiment.

The quantum fluctuations of the inflaton field generically have the right properties to be the initial conditions for the CMB anisotropies, and indeed to make all structures in the Universe today through gravitational instability. Inflation also solves some conceptual problems with the Big Bang framework\(^\text{12}\), including answering the ‘why is the Universe expanding today’ problem – the old answer used to be ‘because it was expanding yesterday’, but now there is the slightly more satisfactory answer ‘because inflation started it off’. However, there is still the question of why the Universe inflated. Or more succinctly, ‘where did everything come from?’

Then there are the coincidence problems: Why is the Dark Energy starting to dominate now? And why is \( \Omega_{\text{CDM}} \approx 5 \Omega_B \)? The first of these questions suggests some selection in time, if not among vacuum states. The second may be a clue about physics beyond the SMPP, but whether it needs to have a stochastic element is unclear. Of course some coincidences may not need an underlying explanation, e.g. why is the density in stars \( \Omega_\ast \approx \Omega_\nu \) and why is \( H_0 t_0 \) so close to 1? But other apparent coincidences exist where it is not obvious if an explanation is required or not, e.g. why is the redshift of matter-radiation equality so close to that of recombination of the hydrogen plasma?

As well as tackling these ‘why’-type issues, astrophysical cosmologists are busy trying to understand how galaxies form and evolve, when and how the very first stars formed, and what the relationship is between the formation of galaxies and their central black holes. This may seem like an entirely different endeavour from determining the parameters of the SMC, but in fact it cannot be done in isolation, since one needs to understand details of the non-linear tracers, foregrounds and astrophysical processing of all cosmological signals in order to measure the SMC parameters.

Given that the SMC raises more questions than it resolves, it seems clear that there must be a bigger picture which remains to be uncovered. It is truly surprising that the SMC has remained essentially

\(^{11}\) And the Universe surely deserves at least as much attention paid to its statistics as the average sports team.

\(^{12}\) Although admittedly most people ignored them until there was a proposed solution!
unchanged for a whole decade, but it seems inconceivable that our current model is the final word. It is, of course, possible that some of today’s list of 12 parameters will be found to be fixed at their default values, at the same time that new degrees of freedom need to be added in order to fit future data-sets. There is no way to predict how the final Cosmological Model will be different from the current SMC.

With the rich pickings of cosmological data out there, the SMC will continue to be probed with increasing precision. One can see that the next decade looks every bit as exciting as the last. Eventually one can imagine a day when we know precisely how many parameters are required and have a better idea of how they are determined (including the level of randomness).

One can also take comfort in the fact that if we lived substantially earlier or later in the history of the Universe it would be considerably more difficult to make precise cosmological measurements from which to quantify the Universe in which we live. We do indeed live in interesting times.

References