Casimir forces in Bose-Einstein condensates: finite size effects in three-dimensional rectangular cavities

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Abstract

The Casimir force due to thermal fluctuations (or pseudo-Casimir force) was previously calculated for the perfect Bose gas in the slab geometry for various boundary conditions. The Casimir pressure due to quantum fluctuations in a weakly-interacting dilute Bose-Einstein condensate (BEC) confined to a parallel plate geometry was recently calculated for Dirichlet boundary conditions. In this paper we calculate the Casimir energy and pressure due to quantum fluctuations in a zero-temperature homogeneous weakly-interacting dilute BEC confined to a parallel plate geometry with periodic boundary conditions and include higher-order corrections which we refer to as Bogoliubov corrections. The leading order term is identified as the Casimir energy of a massless scalar field moving with wave velocity equal to the speed of sound in the BEC. We then obtain the leading order Casimir pressure in a general three-dimensional rectangular cavity of arbitrary lengths and obtain the finite-size correction to the parallel plate scenario.
1 Introduction

After nearly 50 years since its prediction in 1948 by Casimir [1], the Casimir force has now been successfully measured by a modern series of experiments starting with Lamoreaux’s 1997 landmark experiment [2] with a torsion pendulum which reduced errors dramatically compared to the early 1958 experiment by Spaarnay [3]. The force was subsequently measured more precisely in 1998 using an atomic force microscope [4] and the measurements agreed with theoretical predictions to within 1% after finite conductivity, roughness and temperature corrections were taken into account. The Casimir force in the more difficult parallel plate geometry was then successfully measured to 15% accuracy [5]. Thus the modern era of precise Casimir measurements was born. All the measurements of the Casimir force to date have been limited to the case of the electromagnetic field. However, experiments may soon measure the Casimir force for a massless scalar field via quantum fluctuations in Bose-Einstein condensates (BEC). It has been noted (see [6]) that the quasiparticle vacuum in a zero-temperature dilute weakly-interacting BEC should give rise to a measurable Casimir force. The fact that a scalar field propagates at the speed of sound in a BEC medium in contrast to the speed of light in Minkowski spacetime does not change anything fundamental in relation to the Casimir energy. If the speed of propagation is constant in a given medium, the Casimir energy in units of this speed will be the same value regardless of whether the medium is spacetime or a BEC. Moreover, a generally covariant action analogous to what we see in General Relativity exists for scalar fields propagating in a particular fluid. The Lagrangian is similar to that of a massless Klein-Gordon field with the Minkowski metric $\eta_{\mu \nu}$ of spacetime replaced by an effective or acoustic metric $g_{\mu \nu}$ [8]. Quoting directly from [9], “at low momenta linearized excitations of the phase of the condensate wavefunction obey a (3+1)-dimensional d’Alembertian equation coupling to a (3+1)-dimensional Lorentzian-signature ‘effective metric’ that is generic, and depends algebraically on the background field.”.

The Casimir force due to thermal fluctuations in a perfect Bose gas confined to a slab geometry was recently calculated for Dirichlet, Neumann and periodic boundary conditions [10] (see also comment [11] on the work of [10]). The Casimir pressure due to quantum fluctuations for a weakly-interacting dilute Bose-Einstein condensate confined to a parallel plate geometry was also recently calculated [7]. In this paper, we extend the work of [7] on Casimir forces in BEC’s to include a general three-dimensional cavity and consider periodic instead of Dirichlet boundary conditions. We first calcu-
late the Casimir pressure due to quantum fluctuations of the quasiparticle vacuum in a zero-temperature homogeneous dilute weakly-interacting BEC confined to a “parallel plate” geometry with periodic instead of Dirichlet boundary conditions. We show that the leading order term for the Casimir energy is equal to that of a massless scalar field moving with wave velocity equal to the speed of sound in the BEC. We also obtain the much smaller ‘Bogoliubov’ corrections due to the nonlinearity of the Bogoliubov dispersion relation. We then generalize the results to a three-dimensional “rectangular” cavity of arbitrary lengths and obtain the leading order finite size corrections to the parallel plate scenario for periodic boundary conditions. The quotes around “rectangular” or “parallel plate” are simply to remind the reader that with periodic boundary conditions we are of course dealing with a hypertoroidal geometry. We drop the quotes from now on.

To maintain a homogeneous gas in a slab geometry requires periodic boundary conditions. Though such boundary conditions constitute an idealization, it enables one to obtain relevant analytical results for the Casimir pressure of the BEC prior to a numerical analysis of the Casimir force in a non-homogeneous gas confined to a disk-like geometry via an anisotropic harmonic potential.

2 Casimir pressure for BEC in parallel plate geometry and massless scalar fields

Consider a weakly interacting BEC characterized by an interparticle contact pseudopotential, $8\pi a \delta^{(3)}(r)$, where $a$ is the 2-particle positive scattering length (we work in units of $\hbar = 1$ and $2m = 1$). For this delta function potential, the contribution $E$ of the depletion to the ground state energy due to quantum fluctuations in a zero-temperature untrapped homogeneous dilute weakly-interacting BEC is given by \cite{12, 13}

$$E = \frac{1}{2} \sum_{k \neq 0} E(k) = \frac{1}{2} \sum_{k \neq 0} (k \sqrt{k^2 + 2\mu} - k^2 - \mu)$$  \hspace{1cm} (1)

where $\mu \equiv 8\pi a \rho$, $N$ is the number of atoms, $\rho$ is the density $N/V$ and

$$E(k) \equiv E_B - k^2 - \mu = k \sqrt{k^2 + 2\mu} - k^2 - \mu.$$  \hspace{1cm} (2)

$E_B \equiv k \sqrt{k^2 + 2\mu}$ is the Bogoliubov dispersion relation.
With periodic boundary conditions, the homogeneous gas is confined to a parallel plate geometry with a trapping potential which is zero everywhere. It is worth noting that for quantum fluctuations to manifest themselves, the plate separation $d$ must be much greater than the healing length i.e. $d > > \mu^{-1/2}$. In an $L_1 \times L_2 \times d$ hypertoroidal space, $k^2 = (2 n \pi / d)^2 + (2 n_1 \pi / L_1)^2 + (2 n_2 \pi / L_2)^2$ where $(n, n_1, n_2) \neq (0, 0, 0)$ and $n, n_1$ and $n_2$ are integers that run from $-\infty$ to $+\infty$. Both the volume $V = L_1 L_2 d$ and the number of atoms $N$ are assumed large with the density $\rho = \frac{N}{V}$ low enough that the dilute condition $\sqrt{\rho a^3} << 1$ is satisfied. The triple sum for the ground state energy (1) can be broken up in the following convenient fashion:

$$\sum_{k \neq 0} E(k) = \sum_{n=-\infty}^{\infty} \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} E(k) + \mu$$

$$= 8 \sum_{n=1}^{\infty} \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} E(k) + 8 \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} E(k) + 8 \mu . \quad (3)$$

For parallel plates, $L_1$ and $L_2$ are very large (infinite limit) and the sums over $n_1$ and $n_2$ become integrals. The double sum in (3) becomes a double integral and does not contribute to the Casimir energy being purely a continuous term. The constant $8\mu$ also does not contribute to the Casimir energy. The relevant term for the Casimir energy, the triple sum in (3), becomes a sum over a double integral and $k^2$ reduces to $(2 n \pi / d)^2 + r^2$. The relevant energy $E$ per unit area is then given by

$$\frac{1}{L_1 L_2} E = \sum_{n=1}^{\infty} f(n) \quad (4)$$

where

$$f(n) \equiv \frac{1}{2 \pi} \int_{0}^{\infty} \left( \left( \frac{4 n^2 \pi^2}{d^2} + r^2 \right)^{1/2} + 2 \mu \left( \frac{4 n^2 \pi^2}{d^2} + r^2 \right) - \frac{4 n^2 \pi^2}{d^2} - r^2 - \mu \right) \frac{r}{d^2} dr$$

$$= \frac{\mu^2}{\pi} \int_{\frac{2 n^2 \pi^2}{d^2 \mu}}^{\Lambda} \left( \sqrt{u^2 + u - u - \frac{1}{2}} \right) du . \quad (5)$$

The integral can be evaluated but this is not necessary since only the derivatives of $f(n)$ will be needed to determine the Casimir energy. The parameter
\( \Lambda \) is an ultraviolet cut-off introduced because the energy \( E \) given by (1) and hence \( f(n) \) are formally divergent. There is nothing physical about this divergence. It simply reflects that the delta function contact pseudopotential cannot be naively extrapolated to very high momentum where the wavelength is comparable or smaller than the interparticle spacing. This simple cut-off regularization scheme cannot be used to calculate the actual finite energy \( E \) of the depletion because the result is clearly cut-off dependent. One must either extract the low energy physics from the formally infinite expression by making use of effective field theory and dimensional regularization [14] or use the finite expression for \( E \) derived in [12, 13] via a modified pseudopotential. However, to evaluate the Casimir energy, it is perfectly fine to use (5) and in fact this is what was done in [7]. The reason is that the Casimir energy is the difference between two energies – the discrete and the continuum – and this difference turns out to be independent of the cut-off \( \Lambda \). The Casimir energy in the BEC picks out the long wavelength behavior near \( k = 0 \) and is therefore oblivious to the ultraviolet cut-off. We will show explicitly that the terms in the Casimir energy correspond to terms in the series expansion of \( E(k) \) about \( k = 0 \).

We now evaluate the Casimir energy density \( E_{\text{casimir}} \). This is equal to the difference between the energy density of the discrete and continuum modes (both bulk and surface terms). As in [7], we evaluate the Casimir energy via the Euler-Maclaurin formula [15]:

\[
E_{\text{casimir}} = \sum_{n=1}^{\infty} f(n) - \int_0^\infty f(n) + \frac{1}{2} f(0)
\]

\[
= - \sum_{p=1}^{\infty} \frac{B_{2p}}{(2p)!} f^{2p-1}(0) = - \frac{B_2}{2!} f^1(0) - \frac{B_4}{4!} f^3(0) - \frac{B_6}{6!} f^5(0) + \cdots
\]

\[
= - \frac{\pi^2 (2 \mu)^{1/2}}{90 \, d^3} + \frac{2 \pi^4}{315 (2 \mu)^{1/2} \, d^5} + O(\mu^{-3/2} \, d^{-7})
\]

\[
= - \frac{\pi^2 \, v}{90 \, d^3} + \frac{2 \pi^4}{315 \, v \, d^5} + O(\mu^{-3/2} \, d^{-7})
\]

where \( v = (2 \mu)^{1/2} \) is the speed of sound in the BEC and \( f^{2p-1}(0) \) are odd derivatives evaluated at zero. The leading order result, \( E_{\text{scalar}} = -\pi^2 \, v / (90 \, d^3) \), is exactly equal to the Casimir energy density of a massless scalar field obeying a linear dispersion relation and confined to parallel plates with periodic boundary conditions [16, 17]. The massless scalar field propagates with wave velocity \( v \) equal to the speed of sound in
the BEC instead of the speed of light. The next term is a ‘Bogoliubov’ correction that arises because the Bogoliubov dispersion relation is nonlinear and contains higher powers of $k$ when expanded about $k = 0$. The magnitude of the ratio of the correction $E_{\text{Bogo}} = 2\pi^4/(315v d^5)$ to the leading order result $E_{\text{scalar}}$ is much smaller than unity since the plate separation $d$ is assumed to be much greater than the healing length $\mu^{-1/2}$ i.e.

$$\frac{E_{\text{Bogo}}}{|E_{\text{scalar}}|} = \frac{2.82}{\mu d^2} << 1.$$  \hspace{1cm} (7)

Therefore, to a very good approximation, the Casimir energy of the BEC corresponds to quantum fluctuations of a massless scalar field propagating at the speed of sound (i.e. acoustic phonons). This correspondence is not only significant conceptually but also computationally. Results on the Casimir energy of massless scalar fields can be applied to the BEC to obtain leading order terms. We apply this principle in the next section to obtain finite-size corrections to the parallel plate scenario.

It is worth noting that the terms in the Casimir energy correspond to terms in the series expansion of $E(k)$ about $k = 0$ i.e.

$$E(k) = k \sqrt{k^2 + 2\mu - k^2 - \mu} = -\frac{v^2}{2} + v k - k^2 + \frac{k^3}{2v} - \frac{1}{8v^3} k^5 + O(k^7).$$  \hspace{1cm} (8)

In other words, the Casimir energy picks out the long wavelength behavior near $k = 0$ term by term. The first term in the above expansion is a constant and does not contribute to the Casimir energy. The next term proportional to $k$ is the linear dispersion relation of a massless scalar field and is responsible for the leading order Casimir energy $E_{\text{scalar}}$. The next term, proportional to $k^2$, does not contribute to the Casimir energy. This can be seen from the fact that $k^2 = (2n\pi/d)^2 + r^2$ contains a term proportional to $n^2$ whose odd-derivatives evaluated at zero are zero or alternatively, from zeta function regularization we obtain $\zeta(-2) = 0$. The $k^3$ term is responsible for the correction $E_{\text{Bogo}}$. Let us show this explicitly. The function $f(n)$ for $k^3 = (4n^2\pi^2/d^2 + r^2)^{3/2}$ is $f(n) = 1/(2\pi) \int_0^\Lambda (4n^2\pi^2/d^2 + r^2)^{3/2} r dr$. The odd-derivative terms $B_{2p} f^{2p-1}(0)/(2p)!$ are all zero except $-B_6 f^5(0)/6! = 4\pi^4/(315 d^5)$. Multiplying by the $k^3$ coefficient $1/(2v)$ yields $2\pi^4/(315v d^5)$ which is equal to $E_{\text{Bogo}}$.

The Casimir pressure is readily obtained by taking the derivative of the
Casimir energy density:

\[ P_{\text{casimir}} = \left( -\frac{\partial E_{\text{casimir}}}{\partial d} \right)_N = -\frac{7\pi^2}{180} \frac{v}{d^4} + \frac{\pi^4}{35v^6} + O\left(v^{-3}d^{-8}\right). \]  

(9)

The leading order Casimir pressure is

\[ P_{\text{scalar}} = -\frac{7\pi^2}{180} \frac{v}{d^4}. \]  

(10)

It is negative (attractive) and inversely proportional to the fourth power of the distance as in Casimir’s original calculation for the electromagnetic field \([1]\). Casimir obtained \(-\pi^2 c/(240d^4)\) for the attractive force per unit area between parallel conductors in vacuum. The Casimir pressure in the BEC is considerably weaker than in the electromagnetic case because the speed of sound \(v\) is orders of magnitude smaller than the speed of light \(c\). Although Casimir’s original calculation was performed in 1948, one had to wait nearly 50 years before the Casimir force between two conductors was conclusively confirmed by experiments starting with Lamoreaux’s 1997 landmark experiment with a torsion pendulum \([2]\) and then by Mohideen et al. 1998 experiment with an atomic force microscope \([3]\). Here also, one can expect that theory is considerably ahead of experiments and that measuring the much smaller Casimir force in a BEC is something for the next generation of BEC experiments.

3 Leading order Casimir pressure in three-dimensional cavity and finite-size corrections

In this section we obtain expressions for the leading order Casimir pressure in a zero-temperature homogeneous dilute weakly-interacting BEC confined to a three-dimensional rectangular cavity of arbitrary dimensions \(L_1 \times L_2 \times L_3\) with periodic boundary conditions (all three lengths are assumed to be much greater than the healing length). We then determine finite-size corrections to the parallel plate result \(E_{\text{scalar}}\) and \(P_{\text{scalar}}\) obtained in the last section.

As in the previous section, the only parameter of the BEC that enters in our calculations is the speed of sound \(v = (2\mu)^{1/2}\). We take advantage of

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1Note that the derivative of the speed of sound \(v\) with respect to \(d\) (by definition keeping the number of atoms \(N\) constant) is not zero but equal to \(-v/(2d)\). If the derivative had been assumed to be zero, the numerical factor of the leading term in (9) would have been be \(-6/180\) instead of \(-7/180\).
the equality between the Casimir energy of a massless scalar field and the leading order Casimir energy of a BEC by making use of recently derived formulas for the Casimir energy of massless scalar fields propagating with speed $v$ in a $d$-dimensional rectangular cavity of arbitrary lengths $L_1, L_2, \ldots, L_d$ [17]. The speed $v$ in this context is the speed of sound in the BEC. The Casimir energy in [17] was conveniently expressed as a compact analytical part plus remainder. For periodic boundary conditions (inserting $\bar{\hbar}$) the Casimir energy is given by [17]:

$$E_{p,L_1,\ldots,L_d}(d) = -\bar{\hbar} \pi v \sum_{j=0}^{d-1} \frac{L_1 \ldots L_j}{(L_{j+1})^{d+1}} \left( \Gamma\left(\frac{j+2}{2}\right) \pi^{-\frac{j+4}{2}} \zeta(j+2) + R_j \right). \quad (11)$$

$R_j$ is a remainder term expressed as an exponentially fast converging sum:

$$R_j = \sum_{n=1}^{\infty} \sum_{l_i=-\infty}^{\infty} \frac{2(n L_{j+1})^{\frac{j+1}{2}}}{\pi \left[ (\ell_1 L_1)^2 + \cdots + (\ell_j L_j)^2 \right]^{\frac{j+1}{4}}} \left( K_{\frac{j+1}{2}} \left( \frac{2\pi n L_{j+1}}{\ell_{j+1}} \sqrt{(\ell_1 L_1)^2 + \cdots + (\ell_j L_j)^2} \right) \right). \quad (12)$$

The prime means that the case where all $\ell$’s are zero is excluded. There is no remainder for $j = 0$ (it starts at $j = 1$). As discussed in [17], the remainder term is small if we label the longest length $L_1$, the next greatest length $L_2$, etc. In three dimensions, we therefore label the lengths such that $L_1 \geq L_2 \geq L_3$. There is a clear physical interpretation to the analytical and remainder part in (11), which is discussed in section 3 of [17]. The analytical part is the sum of individual parallel plate energies out of which the rectangle is constructed while the remainder is a small contribution due to the nonlinearity of the energy. For a cube with periodic boundary conditions the remainder is 1.5% of the Casimir energy (a table of numerical results can be found in section 5 of [17]). For three arbitrary lengths, the remainder is even smaller than in the case of the cube.

From (11) we easily obtain the Casimir energy in three dimensions ($d = 3$) for periodic boundary conditions:

$$E_p = \hbar v \left( -\frac{\pi}{6 L_1} \zeta(3) \frac{L_1}{L_2^2} - \frac{\pi^2}{90} \frac{L_1 L_2}{L_3^2} - \frac{\pi L_1}{L_2} R_1(L_1, L_2) - \frac{\pi L_1}{L_3} R_2(L_1, L_2, L_3) \right) \quad (13)$$

The remainders $R_1$ and $R_2$ are sums over modified Bessel functions given
by (12) i.e.

\[
R_1(L_1, L_2) = \sum_{n=1}^{\infty} \sum_{\ell=1}^{\infty} \frac{4n}{\pi \ell} \frac{L_2}{L_1} K_1 \left( 2\pi n \ell \frac{L_1}{L_2} \right)
\]

\[
R_2(L_1, L_2, L_3) = \sum_{n=1}^{\infty} \sum_{\ell_1, \ell_2=-\infty}^{\infty} \frac{2n^{3/2}}{\pi} K_{3/2} \left( 2\pi n \sqrt{\left( \frac{\ell_1 L_1}{L_3} \right)^2 + \left( \frac{\ell_2 L_2}{L_3} \right)^2} \right).
\]

Note that only the ratios of lengths appear in the remainders and that \(L_1/L_3 \geq 1, L_2/L_3 \geq 1\) and \(L_1/L_2 \geq 1\) since the lengths are labeled such that \(L_1 \geq L_2 \geq L_3\). The Bessel function \(K_1(2\pi n \ell L_1/L_2)\) has a maximum value of \(9.87 \times 10^{-4}\) which occurs when \(n = \ell = 1\) and \(L_1/L_2 = 1\). As the ratio of lengths increases, the Bessel function decreases exponentially fast and becomes tiny quickly. For example, if \(L_1/L_2 = 10\), then \(K_1(2\pi 10)\) is of order \(10^{-28}\). One obtains the same order of magnitude for \(K_{3/2}(2\pi 10)\).

In the parallel plate scenario, the plate separation is the smallest of the three lengths and is therefore \(L_3\). The Casimir energy per unit area is then

\[
\frac{1}{L_1 L_2} E_p = \frac{\hbar v}{L_3^3} \left( -\frac{\pi^2}{90} - \frac{\zeta(3)}{2\pi} \left( \frac{L_3}{L_2} \right)^3 - \frac{\pi}{6} \left( \frac{L_3}{L_1} \right)^2 \frac{L_3}{L_2} + R \right).
\]

where \(R\) is the remainder contribution given by

\[
R = -\pi \left( \frac{L_3}{L_2} \right)^3 R_1(L_1, L_2) - \pi R_2(L_1, L_2, L_3).
\]

We recognize the leading term as the parallel plate result

\[
E_{\text{scalar}} = -\pi^2 \frac{\hbar v}{(90 d^3)}
\]

with \(d = L_3\). The other three terms are the finite size corrections to the Casimir energy. Let us now calculate the pressure along the plate separation \(L_3\). This is given by the negative derivative of the Casimir density with respect to \(L_3\). Keep in mind that the velocity \(v\) has a dependence on \(L_3\) and that \(\partial v/\partial L_3 = -v/(2L_3)\). The Casimir pressure on the plates is given by:

\[
P = -\frac{1}{L_1 L_2} \frac{\partial E_p}{\partial L_3}
\]

\[
= \frac{\hbar v}{L_3^4} \left( -\frac{7\pi^2}{180} - \frac{\zeta(3)}{4\pi} \left( \frac{L_3}{L_2} \right)^3 - \frac{\pi}{12} \left( \frac{L_3}{L_1} \right)^2 \left( \frac{L_3}{L_2} \right) + P_R \right).
\]
$P_R$ is the contribution of the remainder given by
\[ P_R = -\frac{\pi}{2} \left( \frac{L_3}{L_2} \right)^3 R_1 - \frac{7\pi}{2} R_2 + \pi L_3 R_2' \]  
(18)

where the prime is a derivative with respect to $L_3$. These derivatives are trivial to evaluate using the expressions (14) and yield again Bessel functions.

The interpretation of (17) is straightforward. When $L_1 \to \infty$ and $L_2 \to \infty$, only the first term survives and one recovers the leading order parallel plate result $P_{scalar} = -7\pi^2 h v/(180 d^4)$ with $d = L_3$. The other three terms in (17) are the finite size corrections and depend on the ratios $L_3/L_1$, $L_3/L_2$ and their inverse. $P_R$ is orders of magnitude smaller than the other two correction terms because, as we have seen, the Bessel functions and their derivatives are tiny even in the case where the lengths $L_1$ and $L_2$ are equal to $L_3$. It is therefore an excellent first approximation to drop $P_R$ to determine the finite size corrections. Let us therefore make a few quick numerical calculations. We quote results in units of $\hbar v/L_3^4$. In these units $P_{scalar} = -7\pi^2/180 = -0.383818$. Let us begin with the extreme case where $L_1$ and $L_2$ are equal to $L_3$ i.e. a “cube”. Then $P = -7\pi^2/180 - \zeta(3)/(4\pi) - \pi/12 = -0.741274$. The force remains attractive but is much stronger. It constitutes a 93% difference from $P_{scalar}$. Clearly, forces depend on the finite sizes of $L_1$ and $L_2$. We are however interested in finite-size corrections to the parallel plate scenario where $L_1$ and $L_2$ are at least a few times longer than $L_3$. Consider then the case where $L_1$ and $L_2$ are 5 times longer than $L_3$. Then $P = -0.386678$. This constitutes only a 0.75% change from $P_{scalar}$. If $L_1$ and $L_2$ are 10 times longer than $L_3$, the pressure reduces to $P = -0.384175$, which constitutes only a 0.09% change from $P_{scalar}$. Therefore, for any realistic parallel plate scenario where $L_1$ and $L_2$ are at least a few times longer than $L_3$, the Casimir pressure is dominated by $P_{scalar}$.

The result for the pressure given by (17), including the remainders, is a leading order result. It does not include the next order, the Bogoliubov corrections which arise from the $k^3$ term in the series expansion of $E(k)$ given by (8). We saw last section that these Bogoliubov corrections are small. Nonetheless, if we wish to include them, one needs to obtain the analog formulas to (11) for higher power dispersion relations and this is work for the future. However, we can already include the dominant Bogoliubov correction. There is a separate Bogoliubov correction associated with each of the four terms in (17). If we label the four terms $P_1, P_2, P_3$ and $P_4$ then there exists a Bogoliubov correction for each term which we label $P_{1Bogo}, P_{2Bogo}, P_{3Bogo}$ and $P_{4Bogo}$. The dominant Bogoliubov correction,
$P_{1\text{Bogo}}$, is associated with the leading term $P_1 = -7\pi^2 \hbar v/(180 L_3^4)$. Its value has already been calculated in (9) i.e. $P_{1\text{Bogo}} = \pi^4/(35 v d^6)$. To match the units and notation of this section, we insert $\hbar$ and $m$ and let $d = L_3$. This yields $P_{1\text{Bogo}} = \hbar^3 \pi^4/(140 m^2 v L_3^6)$. So we can already include the most important Bogoliubov correction into the result (17) by replacing $P$ with $P + P_{1\text{Bogo}}$. $P_{1\text{Bogo}}$ is clearly not a significant correction to $P_1$, but it can still compete with the finite-size corrections $P_2$ and $P_3$ when $L_1$ and $L_2$ are sufficiently larger than $L_3$. Of course, both corrections are then small. To summarize, in the parallel plate scenario $P_1 = P_{\text{scalar}}$ is the dominant Casimir pressure in the BEC and corresponds to the parallel plate result for a massless scalar field. There are then small finite-size corrections to $P_{\text{scalar}}$. If one is interested in keeping track of tiny corrections, Bogoliubov corrections become comparable to finite-size corrections when $L_1$ and $L_2$ are sufficiently larger than $L_3$.

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