Dual superconducting properties of the QCD vacuum

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A consistent description of the confining QCD vacuum as a dual superconductor requires a determination of fundamental parameters such as the superconductor correlation length $\xi$ and the field penetration depth $\lambda$, which determine whether the superconductor is of type I or type II. We illustrate preliminary results of a lattice determination of $\xi$ for the case of pure Yang-Mills with two colors, obtained by measuring the temporal correlator of a disorder parameter detecting dual superconductivity.

1. Introduction

Color confinement is an absolute property of strongly interacting matter which is still not explained from QCD first principles. Models exist, which relate confinement to some property of the QCD fundamental state. One of those models, proposed by ‘t Hooft and Mandelstam, is based on dual superconductivity of the vacuum \cite{1,2}: confinement is related to the spontaneous breaking of a magnetic symmetry produced by the condensation of some magnetically charged Higgs field. The magnetic condensate filling the QCD vacuum repels electric fields out of the medium (dual Meissner effect), thus leading to the formation of flux tubes between colored charges, to the linearly rising potential, and to confinement. The broken magnetic group is chosen by a procedure known as Abelian projection \cite{3}: a local operator $\phi(x)$ transforming in the adjoint representation is diagonalized, leaving a residual $U(1)^{N_c - 1}$ gauge symmetry.

The description of the QCD vacuum as a dual superconductor requires a determination of some fundamental parameters which characterize it. These are the correlation length $\xi$ of the Higgs condensate, and the field penetration depth $\lambda$. They determine whether the superconductor is of type I ($\xi > \lambda$) or type II ($\xi < \lambda$). In a superconductor of type I an external field $B$ is always expelled from the medium till a critical value $B_c$ beyond which superconductivity disappears. In a superconductor of type II there are instead two different critical values $B_{c1}$ and $B_{c2}$, and for $B_{c1} < B < B_{c2}$ the external field can penetrate the medium in the form of Abrikosov flux tubes, without disrupting superconductivity. In a type II superconductor there is repulsive interaction between two parallel flux tubes, which is instead attractive for type I.

Whether the QCD vacuum behaves as a type I or type II dual superconductor is a question which can be answered by numerical lattice simulations. A direct way to determine $\lambda$ is a lattice analysis of the flux tube which is formed between two static color charges: this is done by measuring the chromoelectric field correlated to a temporal Wilson loop. The longitudinal field is expected to decay exponentially at large distances from the flux tube axis, as dictated by the field penetration depth $\lambda$. Several studies have been done \cite{4,5,6,7,8,9}, giving $\lambda \sim 0.16$ fm for the $SU(2)$ pure gauge theory.

Determinations of $\xi$ instead have usually been based on an indirect analysis of the chromoelectric field distribution in the flux tube: $\xi$ is determined either through an analysis of violations of the London equation close to the center of the flux tube \cite{5} or through some global fit to the whole set of Ginzburg-Landau equations \cite{7,8}. Some assumption is needed anyway and the determinations are not always consistent. An approximate picture emerges placing the $SU(2)$ QCD vacuum roughly at the boundary between a type I and type II dual superconductor. A determination of $\xi$ through some observable directly related to the Higgs field would probably clarify the is-
sue: this has been the subject of our study. The observable chosen in our work is the operator \( \mu \) developed by the Pisa group which creates a magnetic monopole \([10]\). Its v.e.v. \( \langle \mu \rangle \) is a good disorder parameter detecting dual superconductivity (\( \langle \mu \rangle \neq 0 \)) and the transition to the deconfined - normal conducting phase (\( \langle \mu \rangle = 0 \)) both in pure gauge theory \([11,12,13]\) and in full QCD \([14,15]\). Another approach, also aimed at a direct determination of \( \xi \) but by using a different observable, has been followed in Ref. \([16]\), which has appeared at the stage of writing these proceedings up.

2. The method

The operator \( \mu \) is defined in the continuum as

\[
\mu^a(\vec{x}, t) = e^{i \int d\vec{y} \text{Tr}(\phi^a(\vec{y}, t) \vec{E}(\vec{y}, t))} \tilde{b}_\perp(\vec{x} - \vec{y})
\]

where \( \phi^a(\vec{y}, t) \) is the adjoint field defining the abelian projection, \( \tilde{b}_\perp \) is the field of the monopole sitting at \( \vec{x} \). On the lattice correlation functions of \( \mu(\vec{x}, t) \) are written as

\[
\langle \bar{\mu}(t', \vec{x}')\mu(t, \vec{x}) \rangle = \bar{Z}/Z
\]

where \( Z \) is the usual QCD partition function and \( \bar{Z} \) is obtained by changing the usual pure gauge action \( S \rightarrow \bar{S} \) by insertion of the monopole field in temporal plaquettes at slices \( t \) and \( t' \). Instead of \( \langle \bar{\mu}\mu \rangle \) the following quantity is measured

\[
\rho = \frac{d}{d\beta} \ln\langle \bar{\mu}\mu \rangle = \langle S \rangle_S - \langle \bar{S} \rangle_{\bar{S}} .
\]

The correlation length of \( \mu, \xi_\mu \), can be obtained by measuring the mass \( M \) of the lowest state coupled to \( \mu, \xi_\mu = M^{-1} \). This can be done by studying the asymptotic behaviour of the temporal correlator \([10]\)

\[
\langle \bar{\mu}(t, \vec{x})\mu(0, \vec{x}) \rangle \simeq \langle \mu \rangle^2 + \gamma e^{-Mt} .
\]

In the confined phase \( \langle \mu \rangle \neq 0 \), so for large \( t \) the exponential decaying term is small and we obtain (the hat denotes adimensional lattice quantities)

\[
\rho(t) \simeq \frac{d}{d\beta} \left( \ln(\langle \mu \rangle^2) + \frac{\gamma}{(\langle \mu \rangle)^2} e^{-Mt} \right) = A + (B + Ct)e^{-Mt}
\]

\[
A = \frac{d\ln(\langle \mu \rangle^2)}{d\beta}; \quad B = \frac{d}{d\beta} \frac{\gamma}{(\langle \mu \rangle)^2}; \quad C = -\frac{\gamma}{(\langle \mu \rangle)^2} \frac{dM}{d\beta}
\]

Since

\[
\rho(t) = \langle S \rangle_S - \langle \bar{S}(\bar{t}) \rangle_{\bar{S}(\bar{t})}
\]

and \( \langle S \rangle_S \) is independent of \( \bar{t}, \) we get

\[
\langle \bar{S}(\bar{t}) \rangle_{\bar{S}(\bar{t})} = A' + (B + C\bar{t})e^{-\bar{t}/\xi_\mu}
\]

We can thus obtain \( \xi_\mu = M^{-1} \) through the measurement of the expectation value \( \langle \bar{S}(\bar{t}) \rangle_{\bar{S}(\bar{t})} \) as a function of \( \bar{t} \).

Before showing our numerical results, it is necessary to discuss some relevant questions. How is related the so measured \( \xi_\mu \) to the actual correlation length of the Higgs field, \( \xi \)? We do not really know which is the Higgs field which condenses in the QCD vacuum, but we know that \( \langle \mu \rangle \) is a good disorder parameter detecting dual superconductivity, therefore it is surely coupled to the Higgs field. \( M \) is the mass of the lowest state coupling to \( \mu \), we have therefore \( \xi_\mu = M^{-1} \geq \xi \).

Our investigation can then give a definite answer on the type of dual superconductivity at work in the QCD vacuum only if we get \( \xi_\mu < \lambda \), which implies \( \xi < \lambda \), i.e. superconductivity of type II. The result \( \xi_\mu > \lambda \) would not lead to a definite conclusion.

The next issue to be discussed regards the possible dependence of \( \xi_\mu \) on the abelian projection chosen to define \( \mu \). The natural physical expectation is that one only coherence length characterize the QCD vacuum. This is consistent with ’t Hooft ansatz that all abelian projections are equivalent to each other: that equivalence also emerges clearly from numerical determinations of \( \langle \mu \rangle \) \([11,12,13]\). A possible theoretical argument is the following: the operator \( \mu \) defined in one particular abelian projection creates magnetic charge in every other abelian projection \([14,15]\): this implies that the lowest mass state coupled to \( \mu \) should be universal, i.e. \( \xi_\mu \) should be independent of the abelian projection chosen. We will use a definition of \( \mu \) which averages over different abelian projections \([13]\).
3. Numerical Results

We first show results obtained for $SU(2)$ pure gauge theory using the standard plaquette action at $\beta = 2.5115$: this is the case which has been mostly studied in previous literature. The determination is made quite difficult by the fact that the signal/background ratio is very small: a precision of order $10^{-6}$ is necessary in order to resolve the exponentially decaying term in Eq. (7). Some technical improvements, like link integration, have been used in order to improve the precision on $\langle \tilde{S}(\hat{t}) \rangle_{\tilde{S}(\hat{t})}$.

In Fig. 1 we report the data for $\langle \tilde{S}(\hat{t}) \rangle_{\tilde{S}(\hat{t})}/6V$ as a function of $\hat{t}$ measured on a $16^3 \times 20$ lattice.

![Figure 1](image1.png)

Figure 1. $\langle \tilde{S}(\hat{t}) \rangle_{\tilde{S}(\hat{t})}/6V$ as a function of $\hat{t}$ at $\beta = 2.5115$ on a $16^3 \times 20$ lattice. The continuous line is the result of a fit to Eq. (7) where the first point at $\hat{t} = 1$ has been discarded.

A fit to the form of Eq. (7) gives $C \approx 0$, i.e. the linear term is not visible. In Fig. 2 we show the best fit result for $\xi_{\mu}/a$ as the starting point $\hat{t}_{i}$ is varied: the stability of the results is the signal that the asymptotic large $\hat{t}$ behaviour has been reached. A good and stable fit is obtained for $\hat{t}_{i} \geq 2$ ($\chi^2$/d.o.f. = 1.3 at $\hat{t}_{i} = 2$). From our fits we infer $\xi_{\mu}/a = 1.03 \pm 0.10$. Using $a(\beta = 2.5115) \approx 0.091$ fm, as inferred from the non-perturbative determination of the $\beta$-function for $SU(2)$ reported in Ref. [18], we obtain $\xi_{\mu} = 0.094 \pm 0.009$ fm. As discussed above $\xi \leq \xi_{\mu}$, therefore this result, when compared with the value $\lambda \sim 0.16$ fm reported in previous literature, gives indication for dual superconductivity of type II. Our result for $\xi$ is consistent with those reported in Ref. [18].

Using $a(\beta = 2.4) \approx 0.132$ fm, we have obtained $\xi_{\mu}/a = 1.09 \pm 0.13$: no evident difference is observed, thus excluding relevant finite size effects.

In order to check the scaling to the continuum of our results, we have also repeated our determination for a different value of $\beta$, $\beta = 2.4$. We have used a lattice size $12^3 \times 20$, obtaining $\xi_{\mu}/a = 1.09 \pm 0.13$: no evident difference is observed, thus excluding relevant finite size effects.

The creation operator $\mu$ brings in the magnetic field of a monopole. This is a long range field and finite lattice size effects could then be important. We have therefore repeated our measurement on a $12^3 \times 20$ lattice, obtaining $\xi_{\mu}/a = 1.09 \pm 0.13$: no evident difference is observed, thus excluding relevant finite size effects.

In order to check the scaling to the continuum of our results, we have also repeated our determination for a different value of $\beta$, $\beta = 2.4$. We have used a lattice size $12^3 \times 16$, which taking into account $a(\beta = 2.4) \approx 0.132$ fm is comparable in physical units to the largest one used at $\beta = 2.5115$. We have obtained $\xi_{\mu} = 0.11 \pm 0.01$ fm,
which is compatible with the value at $\beta = 2.5115$.

4. Conclusions

We have investigated the nature of the dual superconducting vacuum in $SU(2)$ pure gauge theory by determining the superconductor correlation length $\xi$ through a measurement of the temporal correlator $\langle \bar{\mu}(t, \vec{x}) \mu(0, \vec{x}) \rangle$ of a disorder parameter detecting dual superconductivity.

Our determination, when compared with previous determinations of the dual field penetration depth $\lambda$, give indication of weak type II dual superconductivity for the QCD vacuum.

This result must be confirmed both by an increment of statistics and by a wider study of the scaling to the continuum; a direct test of the abelian projection independence should be also performed.

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REFERENCES