Are Neutron-Rich Elements Produced in the Collapse of Strange Dwarfs?

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The structure of strange dwarfs and that of hybrid stars with same baryonic number is compared. There is a critical mass ($M \approx 0.24M_\odot$) in the strange dwarf branch, below which configurations with the same baryonic number in the hybrid star branch are more stable. If a transition occurs between both branches, the collapse releases an energy of about $3 \times 10^{50}$ erg, mostly under the form of neutrinos resulting from the conversion of hadronic matter onto strange quark matter. Only a fraction ($\sim 4\%$) is required to expel the outer neutron-rich layers. These events may contribute significantly to the chemical yield of nuclides with $A \geq 80$ in the Galaxy, if their frequency is of about one per 1500 years.

Keywords: yields of heavy elements; strange matter; strange dwarfs

1. Introduction

It has been suggested already in the 70’s $^{1,2}$ that, because of the extreme densities reached in the core of neutron stars (NS), hadrons can melt, creating a deconfined state dubbed the “quark-gluon plasma”. Stars with a deconfined core surrounded by hadronic matter are called hybrid stars (HS), whereas objects constituted by absolutely stable strange quark matter are christened “strange stars” (SS) $^{3,4}$.

A pure SS is expected to have a sharp edge with a typical scale defined by the range of the strong interaction. It was pointed by Ref. 4 that surface electrons may neutralize the positive charge of strange quark matter, generating a high voltage dipole layer with an extension of several hundred fermis. As a consequence, SS are able to support a crust of nuclear material, since the separation gap created by the strong electric field prevents the conversion of nuclear matter into quark matter. The maximum density of the nuclear crust is essentially limited by the onset of the neutron drip ($\rho_d \approx 4 \times 10^{11}$ gcm$^{-3}$), since above such a value free neutrons fall into the core and are converted into quark matter. Stars with a strange matter core and an outer layer of nuclear material, with dimensions typically of white dwarfs, are dubbed ”strange dwarfs” (SD) $^{5,6}$. The stability of these objects was examined in Ref. 7, where the pulsation frequencies for the two lowest modes $n =$
0, 1, as a function of the central density, were studied. More recently, the formation of deconfined cores has been considered in different astrophysical scenarios. The hadron-quark phase transition induces a mini-collapse of the NS and the subsequent core bounce was already invoked as a possible model for gamma-ray bursts (GRB). A detailed analysis of the bounce energetics \(^8\) has shown that the relativistic \((\Gamma > 40)\) fraction of the ejectum is less than \(10^{46}\) erg, insufficient to explain GRBs. However, as the authors have emphasized, these events could be a significant source of r-process elements. In Ref 9 a different evolutionary path was considered. The point of depart is a HS with a deconfined core constituted only by \(u, d\) quarks. Then, such a core shrinks into a more stable and compact \(u, d, s\) configuration in a timescale shorter than that of the overlying hadronic material, originating a “quark nova” \(^9,10\). The total energy released in the process may reach values as high as \(10^{53}\) erg and \(\sim 10^{-2}\) \(M_\odot\) of neutron-rich material may be ejected in the explosion.

The aforementioned events fail in to explain the GRB phenomenology but could shed some light on the provenance of elements heavier than those of the iron-peak. High densities and temperatures required to produce these elements are usually found in the neutrino-driven wind of type II supernovae \(^11\), although the fine-tuning of the wind parameters necessary to explain the observed abundance pattern is still an unsolved issue \(^12,13\).

In the present work an alternative possibility is explored by considering a binary system in which one of the components is a ”strange dwarf”. If this star accretes mass what will be its new equilibrium state? The present investigation indicates that the star can either evolve in SD branch by increasing its radius or make a jump to the HS (or SS) branch by undergoing a collapse in which the strange core mass increases at the expense of the hadronic layer. We argue that there is a critical mass \((\sim 0.24M_\odot)\) below which a jump to the HS branch is energetically more favorable. In this case, the released energy emitted mostly under the form of neutrinos, is enough to eject a substantial fraction (or almost completely) of the outer neutron rich layers, whose masses are typically of the order of \((2 - 5) \times 10^{-3} M_\odot\). This paper is organized as follows: in Section II, strange dwarf models and energetics are presented, in Section III, the ejection of the envelope and abundances are discussed and, finally, in Section IV the main conclusions are given.

2. Strange dwarf models

A sequence of equilibrium (non-rotating and non-magnetic) models were calculated by solving numerically the Tolman-Oppenheimer-Volkoff equations\(^14,15\) \((G = c = 1)\), e.g.,

\[
\frac{dp}{dr} = -\frac{[p(r) + \epsilon(r)] [m(r) + 4\pi r^3 p(r)]}{r(r - 2m(r))} \tag{1}
\]

and

\[
m(r) = 4\pi \int_0^r \epsilon(r)r^2 dr , \tag{2}
\]
The deconfined core is described by the well known MIT bag model \(^{16}\), from which one obtains respectively for the pressure and energy density

\[
p = -B + \frac{1}{4\pi^2} \sum_f \left[ \mu_f k_f (\mu_f^2 - \frac{5}{2} m_f^2) + \frac{3}{2} m_f^4 \ln \left( \frac{\mu_f + k_f}{m_f} \right) \right]
\]

and

\[
\epsilon = B + \frac{3}{4\pi^2} \sum_f \left[ \mu_f k_f (\mu_f^2 - \frac{1}{2} m_f^2) - \frac{1}{2} m_f^4 \ln \left( \frac{\mu_f + k_f}{m_f} \right) \right],
\]

where \(B\) is the bag constant, here taken to be equal to \(60\) \(\text{MeV} \, \text{fm}^{-3}\), \(k_f\) is the Fermi momentum of particles of mass \(m_f\) and \(\mu_f = \sqrt{k_f^2 + m_f^2}\). The sum is performed over the flavors \(f = u, d, s\), whose masses were taken respectively equal to \(m_u = 5\) \(\text{MeV}\), \(m_d = 7\) \(\text{MeV}\) and \(m_s = 150\) \(\text{MeV}\).

The hadronic layer begins when the pressure at the core radius reach the value corresponding to the density \(\rho_d\) of the neutron drip. The equation of state used for this region is that calculated in Ref. 25. Notice that the bottom of hadronic layer does not represent a true phase transition since Gibbs criteria are not satisfied. The strange matter core may absorb hadrons of the upper layers if they get in contact, which is precluded by the strong electric field, as already mentioned. The overall equation of state is shown in Fig 1. At this point, it is important to emphasize the following point. The transition between the two phases (deconfined and hadronic) occurs when the Gibbs conditions (equality between the chemical potential and pressure of both phases) are satisfied, case of a first order phase transition. A mixed phase has also been proposed \(^{17}\) but the allowance for the local surface and Coulomb energies may render this possibility energetically less favorable \(^{18}\). In the literature, hybrid stars are those with a deconfined core whose transition to the hadronic crust is of first order or have a mixed phase. In the present context, we also call hybrid stars compact configurations (radii of few km) with a quark core and an outer hadronic layer, separated by a strong electric field, as in the case of strange dwarfs, but this is clearly an abuse of language.

The sequence of strange dwarfs and hybrid models was calculated by varying the central energy density. In Fig 2 we show the derived mass-radius (M-R) relation for our models. The solid curve represents the SD branch, whereas the dashed curves represent respectively the branches of pure white dwarfs (on the right) and of strange & hybrid stars (on the left).

The point C in the M-R diagram corresponds to the stability edge characterized by a mass \(M=0.80M_\odot\) and a radius \(R=1397\) km. The point A indicates the position of the minimum mass of a stable strange dwarf and coincides with the minimum mass model for hybrid configurations. It corresponds to a mass of \(0.0237\) \(M_\odot\) and a radius of \(341.82\) km. As the central density is increased, the star moves along the segment A \(\rightarrow\) B in the M-R diagram. In this range, strange dwarfs have a gravitational mass slightly higher than that of hybrid stars of same baryonic number, allowing the possibility for a transition from the SD branch to the HS branch. This
Fig. 1. The adopted equation of state describing the deconfined core and the hadronic crust. Both regions are connected at the neutron drip density.

is not the case for strange dwarfs above point B, corresponding to a mass of 0.23 $M_\odot$, since their gravitational masses are smaller than those of HS stars having the same baryonic number. The position of the fiducial points A, B and C in the M-R diagram as well as the curve AC itself depend on the adopted value for the bag constant. A higher value would reduce the mass and radius corresponding to point A and similarly, points B and C would be displaced toward smaller masses. This occurs because the role of the strong forces increase, leading to more compact core configurations.

Physical properties of some SD and HS models are given in Table I. Models in both branches are characterized by a given baryonic mass, shown in the first column. The gravitational mass (in solar unit) and radius (in km) for strange dwarfs are given respectively in the second and third columns, whereas the same parameters for hybrid stars are given in columns four and five. The last column of Table I, gives the energy difference $\Delta E = (M_{G}^{SD} - M_{G}^{HS})c^2$ between both branches. It is worth mentioning that $\Delta E$ is the maximum amount of energy which could be released in the process. The variation of the gravitational energy is higher but it covers essentially the cost of the hadronic matter conversion onto strange quark matter.
Notice that $\Delta E > 0$ for masses lower than $\sim 0.23 M_\odot$ and $\Delta E < 0$ for masses higher than the considered limit, as mentioned above. The maximum energy difference occurs around $\sim 0.15 M_\odot$, corresponding to $\Delta E \sim 2.9 \times 10^{50}$ erg. Strange dwarfs above point B in the M-R diagram, if they accrete mass, will evolve along the segment B $\rightarrow$ C, decreasing slightly the mass and the radius of the deconfined core, but increasing slightly the extension of the hadronic layer. The core properties for the same models are shown in Table II. Inspection of this table indicates that SD along the segment A $\rightarrow$ C in the M-R diagram have slightly decreasing deconfined core masses and radii. On the contrary, in the HS branch, the deconfined core develops more and more as the stellar mass increases.

A comparison between energy density profiles for SD, HS and SS configurations is shown in Fig.3. All stars have the same baryonic mass ($M_B = 0.10696 M_\odot$). It is also interesting to compare our results with those of Ref 6, who have also performed similar calculations, but with a slight different equation of state for the quark matter. For their model sequence using the same bag constant and the same density for the core-envelope transition point, they have obtained comparable values for the fiducial points defining the SD branch, e.g., a mass of $0.017 M_\odot$ and a radius...
Fig. 3. Energy density distribution for a strange dwarf (solid line), hybrid star (dotted line) and a pure strange star (dashed line). All configurations have the same baryonic mass, $M_B = 0.10696 M_\odot$. 

Table 1. Properties of Strange Dwarfs and Hybrid Stars. The last model corresponds to the minimum mass star and, consequently, has only one possible configuration.

<table>
<thead>
<tr>
<th>$M_B/M_\odot$</th>
<th>$M^{NP}<em>G/M</em>\odot$</th>
<th>$R_{SD}$</th>
<th>$M^{NS}<em>G/M</em>\odot$</th>
<th>$R_{HS}$</th>
<th>$\Delta E$ (x10^49 erg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40022</td>
<td>0.36407</td>
<td>3582</td>
<td>0.36577</td>
<td>8.873</td>
<td>-30.5</td>
</tr>
<tr>
<td>0.30543</td>
<td>0.27782</td>
<td>3547</td>
<td>0.27878</td>
<td>8.105</td>
<td>-17.2</td>
</tr>
<tr>
<td>0.25669</td>
<td>0.23165</td>
<td>3505</td>
<td>0.23179</td>
<td>7.721</td>
<td>-2.50</td>
</tr>
<tr>
<td>0.20258</td>
<td>0.18423</td>
<td>3442</td>
<td>0.18411</td>
<td>7.396</td>
<td>+2.22</td>
</tr>
<tr>
<td>0.16808</td>
<td>0.15282</td>
<td>3357</td>
<td>0.15265</td>
<td>7.211</td>
<td>+2.93</td>
</tr>
<tr>
<td>0.10696</td>
<td>0.09723</td>
<td>3119</td>
<td>0.09711</td>
<td>7.249</td>
<td>+2.04</td>
</tr>
<tr>
<td>0.05185</td>
<td>0.04709</td>
<td>2556</td>
<td>0.04699</td>
<td>9.324</td>
<td>+1.74</td>
</tr>
<tr>
<td>0.03626</td>
<td>0.03291</td>
<td>2309</td>
<td>0.03285</td>
<td>15.02</td>
<td>+1.06</td>
</tr>
<tr>
<td>0.03000</td>
<td>0.02718</td>
<td>1756</td>
<td>0.02718</td>
<td>28.77</td>
<td>+0.02</td>
</tr>
<tr>
<td>0.02613</td>
<td>0.02367</td>
<td>341.8</td>
<td>0.02367</td>
<td>341.8</td>
<td>0</td>
</tr>
</tbody>
</table>

of 450 km for point A and a mass of 0.96 $M_\odot$ and a radius of 2400 km for point C. These differences, on the average 20% in the mass and 30% in the radius, are probably due to differences in the treatment of the quark matter since the equation
Table 2. Core properties of hybrid stars

<table>
<thead>
<tr>
<th>$M_B/M_\odot$</th>
<th>$M_{SD}^{0.01}/M_\odot$</th>
<th>$R_{SD}^{0.01}$ (km)</th>
<th>$M_{HS}^{0.01}/M_\odot$</th>
<th>$R_{HS}^{0.01}$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40022</td>
<td>0.01972</td>
<td>2.663</td>
<td>0.36560</td>
<td>8.288</td>
</tr>
<tr>
<td>0.30543</td>
<td>0.02036</td>
<td>2.711</td>
<td>0.25832</td>
<td>7.135</td>
</tr>
<tr>
<td>0.25469</td>
<td>0.02117</td>
<td>2.750</td>
<td>0.23167</td>
<td>6.471</td>
</tr>
<tr>
<td>0.20258</td>
<td>0.02260</td>
<td>2.801</td>
<td>0.18401</td>
<td>5.923</td>
</tr>
<tr>
<td>0.16808</td>
<td>0.02270</td>
<td>2.825</td>
<td>0.15257</td>
<td>5.202</td>
</tr>
<tr>
<td>0.10696</td>
<td>0.02277</td>
<td>2.842</td>
<td>0.09675</td>
<td>4.543</td>
</tr>
<tr>
<td>0.05185</td>
<td>0.02290</td>
<td>2.844</td>
<td>0.04676</td>
<td>3.599</td>
</tr>
<tr>
<td>0.03626</td>
<td>0.02296</td>
<td>2.849</td>
<td>0.03187</td>
<td>3.169</td>
</tr>
<tr>
<td>0.03000</td>
<td>0.02298</td>
<td>2.851</td>
<td>0.02687</td>
<td>3.006</td>
</tr>
<tr>
<td>0.02013</td>
<td>0.02323</td>
<td>2.868</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3. Strange stars properties.

<table>
<thead>
<tr>
<th>$M_B/M_\odot$</th>
<th>$M_{SS}^{0.01}/M_\odot$</th>
<th>$R_{SS}^{0.01}$ (km)</th>
<th>$\Delta E$ ($\times 10^{50}$ erg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40022</td>
<td>0.39087</td>
<td>7.024</td>
<td>-451</td>
</tr>
<tr>
<td>0.30543</td>
<td>0.27672</td>
<td>6.424</td>
<td>-320</td>
</tr>
<tr>
<td>0.25469</td>
<td>0.23543</td>
<td>6.058</td>
<td>-67.3</td>
</tr>
<tr>
<td>0.20258</td>
<td>0.18521</td>
<td>5.608</td>
<td>-21.4</td>
</tr>
<tr>
<td>0.16808</td>
<td>0.15278</td>
<td>5.305</td>
<td>-0.57</td>
</tr>
<tr>
<td>0.10696</td>
<td>0.09707</td>
<td>4.552</td>
<td>0.80</td>
</tr>
<tr>
<td>0.05185</td>
<td>0.04699</td>
<td>3.609</td>
<td>1.84</td>
</tr>
<tr>
<td>0.03626</td>
<td>0.03284</td>
<td>3.207</td>
<td>1.23</td>
</tr>
<tr>
<td>0.03000</td>
<td>0.02711</td>
<td>3.031</td>
<td>1.18</td>
</tr>
<tr>
<td>0.02013</td>
<td>0.02367</td>
<td>2.876</td>
<td>0.15</td>
</tr>
</tbody>
</table>

3. The neutron-rich envelope

3.1. The ejection mechanism

The astrophysical environment in which slow and rapid neutron capture reactions take place is still a matter of debate. The ejecta of type II supernovae and binary
neutron star mergers are possible sites in which favorable conditions may develop. Difficulties with the electron fraction $Y_e$ in the neutrino-driven ejecta were recently reviewed in Ref. 21. For instance, a high neutron-to-seed ratio, required for a successful r-process, is obtained if the leptonic fraction $Y_e$ is small, a condition not generally met in the supernova envelope. Non orthodox issues based on neutrino oscillations between active and sterile species, able to decrease $Y_e$, have been explored 21 and here another alternative scenario is examined.

It is supposed a binary system in which one of the components is a strange dwarf. The evolutionary path leading to such a configuration does not concern the present work. As it was shown in the previous section, strange dwarfs with masses in the range $0.024 < M/M_\odot < 0.24$, in a state of accretion, may jump to the HS branch since this transition is energetically favorable. These considerations are based on binding energies calculated for equilibrium configurations and future dynamical models are necessary to investigate in more detail this possibility.

The jump SD $\rightarrow$ HS is likely to occur within the free-fall timescale, e.g., $t_d \sim 1/\sqrt{G\rho}$, which is of the order of a fraction of millisecond. During the transition, hadronic matter is converted onto strange quark matter. This conversion leads to an important neutrino emission, via weak interaction reactions, consisting the bulk of the energy released in the process, which amounts to about $3 \times 10^{50}$ erg. The typical energy of the emitted neutrino pair is about 15-17 MeV, corresponding approximately to the difference between the energy per particle of the $ud$ and the $uds$ quark plasma. These neutrinos diffuse out the core in a timescale of the order of $\sim 0.1$ s (see, for instance, a discussion in Ref 10) through the remaining hadronic layers, placed above the high voltage gap, providing a mechanism able to eject the outer parts of the envelope. Masses of the hadronic crust are around $2 \times 10^{-4} M_\odot$ and their ejection requires a minimum energy of about $8 \times 10^{48}$ erg, corresponding to about 4% of the available energy.

Neutrinos interact with the crust material through different processes: scattering by electrons and nucleons and capture by nucleons. Cross sections for these different interactions can be found, for instance, in Ref 22. The dominant process in the crust is by far the neutrino-nucleon scattering, whose cross section is

$$\sigma_{\nu-n} = 4 \times 10^{-43} N^2 \left( \frac{E_\nu}{10 \ MeV} \right)^2 \ cm^2 \quad (5)$$

where $E_\nu$ is the neutrino energy and $N$ is the number of neutrons in the nucleus. As we shall see below, nuclei in the crust have typically $N \sim 35$ and $A \sim 60$. Therefore, the “optical depth” for neutrinos is

$$\tau_\nu = \int \sigma_{\nu-n} \left( \frac{\rho}{A m_N} \right) ds = 3.3 \times 10^{-18} \int \rho \ ds \quad (6)$$

The outer hadronic layers have column densities typically of the order of $(1.3 - 3.0) \times 10^{16} g cm^{-2}$, leading to optical depths of the order of $\tau_\nu \sim 0.043$-0.10, corresponding to a fraction of scattered neutrinos of about 4.2-9.5 %. Thus, the momentum imparted to nuclei is able to transfer enough energy to expel the
envelope. However, a firm conclusion must be based on a detailed analysis of the momentum transfer by neutrinos, coupled to hydrodynamic calculations.

3.2. Ejected abundances

The equation of state and the chemical composition of the external hadronic matter for densities below the neutron drip were calculated by different authors. Nuclei present in the hadronic crust are stabilized against \( \beta \)-decay by the filled electron Fermi levels, becoming more and more neutron-rich as the matter density increases. The dominant nuclide present at a given density is calculated by minimizing the total energy density, including terms due to the lattice energy of nuclei, the energy of isolated nuclei and the contribution of degenerate electrons, with respect to the atomic number \( Z \) and the nucleon number \( A \).

For a given model, once the crust structure is calculated from the equilibrium equations (see Section 2), the mass under the form of a given nuclide \((Z,A)\) can be calculated from

\[
M = 4\pi \int_{R_1}^{R_2} \rho(r, Z, A)r^2 dr
\]  

and the integral limits correspond to the density (or pressure) range where the considered nuclide is dominant. These nuclides and their respective density range were taken from tables given by Refs 25 and 26. Both set of computations have used similar mass formulas but slightly different energy minimization procedures. As a consequence, some differences in the abundance pattern can be noticed. In particular, \( ^{26}\text{Fe} \) is the dominant nuclide at densities \( \sim 0.4\rho_d \) according to Ref 25, whereas in the calculations by Ref 26 the dominant nuclide is \( ^{40}\text{Zr} \).

When the envelope is ejected, the neutron-rich nuclei are no more stabilized and decay into more stable configurations. Notice that the cross section ratio between neutrino capture and scattering is \( \sim \sigma_\alpha/\sigma_s \approx 0.008 \), indicating that neutrinos will not affect significantly the original abundance pattern. Nuclei stability were investigated using a modified Bethe-Weizsacker mass formula given in Ref 19, more adequate for neutron-rich nuclei, and nuclide tables given in Ref 20.

The resulting masses in the crust for different nuclides are given in Table IV and Table V, corresponding to the dominant nuclide data by Ref 25 and Ref 26 respectively. For both cases, the envelope mass is \( 3.6 \times 10^{-4}M_\odot \). In the first column are given the nuclides present in the crust at high pressures, stabilized against \( \beta \)-decay by the presence of the degenerate electron sea. In the second column are given the stable nuclides originated from the decay of the unstable neutron-rich nuclides. The corresponding masses in the envelope are given in the third column and abundances by number relative to \( ^{26}\text{Fe} \) are given in the fourth column. The last column gives an indication of the expected origin of these (stable) nuclides in nature: \( s \)- and/or \( r \)-process and SE for stellar evolution processes in general, including explosive nucleosynthesis.
Inspection of table IV reveals a peak around nuclides in the mass range 56-64 (Fe-Ni peak) also found in ejecta of type Ia supernovae \(^{27}\). However, the contribution to the iron yield in the Galaxy by one of these events is about \(10^4\) times less than a single type Ia supernova. Nevertheless, in spite of the small mass of the ejected envelope, these events could contribute to the chemical yields of some nuclides like Se, Kr and Sn, which are usually supposed to be originated from \(s\) and \(r\) processes. Here their origin is completely diverse, since they are the result of the decay of neutron-rich nuclides stabilized by a degenerate electron sea present in the hybrid star.

The required frequency of these events, in order that they could contribute significantly to the chemical yield of the Galaxy, can be estimated by using the procedure by Ref 28. Assuming that all iron in the Galaxy was produced essentially by type Ia supernovae and adopting for Se, Kr and Sn, nuclides which are here supposed to be produced by the collapse of a SD, the present abundances given by Ref 29, then the required frequency of these events in the Galaxy is about one each 1500 yr.
4. Conclusions

Gravitational masses for a sequence of models in the strange dwarf, hybrid and strange star branches were computed. Results of these calculations indicate that there is a critical mass in the strange dwarf branch, $M = 0.24 M_\odot$, below which a configuration of same baryonic number in the hybrid branch has a smaller energy, allowing a transition between both branches.

If a transition occurs, the envelope radius shrinks typically from a dimension of about $\sim 3200$ km to about $\sim 7$ km, with conversion of hadronic matter onto strange quark matter. In this collapse, the released energy is about $3 \times 10^{50}$ erg carried out essentially by $\nu_e \bar{\nu}_e$ pairs with energies typically of the order of 15-17 MeV. This value corresponds approximately to the energy per particle difference between $ud$ and $uds$ quark matter. Our estimates indicate that neutrino-nucleon scattering can transfer about 4-9 % of the released energy to nucleons, which is enough to expel partially or completely the hadronic crust, having masses typically of about of $(2 - 5) \times 10^{-4} M_\odot$.

The ejecta of these events is rich in nuclides of high mass number and could be the major source for the chemical yields of elements like Se, Kr, Sn, if the frequency of these events in the Galaxy is about one per 1500 yr.

5. Acknowledgements

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References
