Coulomb deexcitation of pionic hydrogen within close-coupling method

V. N. Pomerantsev, V. P. Popov

Institute of Nuclear Physics, Moscow State University

The Coulomb deexcitation of \((\pi p)\) atom in collisions with the hydrogen atom has been studied in the fully quantum mechanical close-coupling approach for the first time. The total Coulomb deexcitation cross-sections of \(n\ell \rightarrow n'\ell'\) transitions and \(l\)-averaged cross-sections are calculated for \(n = 3-8\) and at the relative energies \(E = 0.01 - 100\) eV. The strong interaction and vacuum polarization shifts of \(ns\)-states are taken into account. It is shown that the \(\Delta n > 1\) transitions are very important and make up a substantial fraction of the Coulomb deexcitation cross-section (up to \(\sim 47\%\)).

Exotic hydrogen atoms are formed in highly excited states and the evolution of their distributions on the quantum numbers and kinetic energy is defined by the competition of both collisional and radiative deexcitation processes during the atomic cascade. The interest in the problem attracted special attention after the new experiments [5, 6, 7, 8] on the time-of-flight (NTOF) spectra in the experiments are attributed to the Coulomb deexcitation (CD) process, where the energy of the transition is shared between the colliding objects. In order to analyze the above study of the CD process in muonic hydrogen [13], where such as elastic scattering (\(n'n = n, l' = l\)), Stark transitions (\(n'n < n, l' \neq l\)), and CD (\(n'n > n, l' = l\)) are calculated in unified manner. The method has been recently employed for the study of the CD process in muonic hydrogen [14], where substantially new results in comparison with the known in the literature [9, 12] (see references therein) have been obtained. Unless otherwise stated the atomic units (\(\hbar = e = m_e m_p/(m_e + m_p) = 1\)) will be used throughout the paper.

\[
H = \frac{1}{2m} \Delta R + h_{ex}(\rho) + h_H(\mathbf{r}) + V(r, \rho, \mathbf{R},) \tag{2}
\]

where \(m\) is the reduced mass of the system, \(\mathbf{R}\) is the radius vector between the c.m. of the colliding atoms, \(\rho\) and \(\mathbf{r}\) are their inner coordinates. The interaction potential, \(V(\mathbf{r}, \rho, \mathbf{R})\), is a sum of four Coulomb pair interactions between the projectile atom and the target atom particles. \(h_{ex}(\rho)\) and \(h_H(\mathbf{r})\) are the hydrogen-like Hamiltonians of the free exotic and hydrogen atom, whose eigenfunctions together with the angular wave function \(Y_{\ell \Delta}(\mathbf{R})\) of the relative motion form the basis states, \((1s, n\ell, L : J M)\), with the conserving total angular momentum \((J M)\) and parity \(\pi = (-1)^{l + L}\). The total wave function of the system can be expanded as follows

\[
\Psi_{E, J \pi}(r, \rho, \mathbf{R}) = R^{-1} \sum_{n\ell L} G_{n\ell L}^{J \pi}(1s, n\ell, L : J M). \tag{3}
\]

The expansion (3) leads to the close-coupling second order differential equations for the radial functions of the relative motion, \(G_{n\ell L}^{J \pi}(R)\),

\[
\left( \frac{d^2}{dR^2} + \frac{k_n^2}{R^2} - \frac{L(L + 1)}{R^2} \right) G_{n\ell L}^{J \pi}(R) = 2m \sum_{n'\ell'L'} W_{n'n'L', n\ell L}^{J}(R) G_{n'n'L'}^{J}(R). \tag{4}
\]

The channel wave number is defined as \(k_n^2 = 2m(E_{cm} + E_{n\ell\ell'} - E_{n\ell})\), where \(E_{cm}\) and \(E_{n\ell}\) are the energy of the relative motion and the exotic atom quantum numbers in the entrance channel, respectively. The bound energy of \(\pi p\) atom, \(E_{n\ell} = \varepsilon_{n\ell} + \varepsilon_{n\ell}\), includes the eigenvalue of \(h_{ex}(\rho)\), \(\varepsilon_{n\ell}\), and the energy shift, \(\Delta \varepsilon_{n\ell} = \Delta \varepsilon_{n\ell}^{pp} + \Delta \varepsilon_{n\ell}^{str}\), due to the vacuum polarization and strong interaction. Hereafter, the energy \(E_{cm}\) will be referred to \(\varepsilon_{n\ell\ell'\neq 0}\) in the entrance channel (we assume that \(\Delta \varepsilon_{n\ell\ell'\neq 0} = 0\)).

The matrix elements of the interaction potential,

\[
W_{n'n'L', n\ell L}^{J}(R) = \langle 1s, n'n', L' : J M | V | 1s, n\ell, L : J M \rangle. \tag{5}
\]

\(^1\) The strong interaction shifts are calculated according to \(\Delta \varepsilon_{n\ell}^{pp} = \Delta \varepsilon_{n\ell}^{str}/n_3\), where \(\Delta \varepsilon_{1s} = -7.11\) eV from [13]. The next values \((-3.24\) eV \((1s), -0.37\) eV \((2s), \) and \(-0.11\) eV \((3s)\)) for the vacuum polarization shifts are used in the calculations. For \(n > 3\) we assume \(\Delta \varepsilon_{n\ell}^{pp} = \Delta \varepsilon_{n-1\ell}^{pp}((n - 1)/n)^3\).
are obtained by averaging it over the electron wave function of the 1s-state and then applying the multipole expansion. The integration over \((\rho, \hat{R})\) reduces the matrix elements (5) to the multiple finite sum.

At fixed \(E_{cm}\) the coupled differential equations (4) for the given \(J\) and \(\pi\) values are solved numerically by Numerov method with the standing-wave boundary conditions involving the real symmetrical \(K\)-matrix related to \(T\)-matrix by the equation \(T = 2K/(I - iK)\). All exotic atom states with \(n\) values from 1 up to \(n_0\) have been included in the close-coupling calculations. The following cross sections will be discussed:

- partial cross section

\[
\sigma_{nl\rightarrow n\ell'}^J(E) = \frac{\pi}{k_d^2} \frac{2J + 1}{2L + 1} \sum_{\ell\ell'} |T_{nl\pi\rightarrow n\ell'\pi}|^2, \quad (6)
\]

- total cross section of the transition \(nl \rightarrow n\ell'\)

\[
\sigma_{nl\rightarrow n\ell'}(E) = \sum_J \sigma_{nl\rightarrow n\ell'}^J(E), \quad (7)
\]

- total cross section from \(ns\)-state

\[
\sigma_{ns\rightarrow n\ell'}(E) = \sum_{\ell'} \sigma_{ns\rightarrow n\ell'}(E), \quad (8)
\]

and the \(l\)-averaged cross section \((l > 0)\)

\[
\sigma_{nl\rightarrow n\ell'}^{l>0}(E) = \frac{1}{l^2 - 1} \sum_{l>0, \ell'} (2l + 1) \sigma_{nl\rightarrow n\ell'}(E). \quad (9)
\]

We assume the pionic atom states with \(l \neq 0\) are degenerated and so the \(l\)-averaged cross section (9) can be used to illustrate the basic features of the CD process.

The CC calculations of the CD cross sections Eqs. (6-9) were carried out for \((\pi^-p)_{nl} + H\) collisions for \(n = 3-8\) and at energy range \(E_{cm} = (0.01 - 100)\) eV. Summation over the partial waves in Eq. (6) was done up to a value \(J_{max}\) until an accuracy better than 0.1% was reached at all energies. The analysis of the \(J\) dependence of the partial cross sections \(\sigma_{nl\rightarrow n\ell'}^J\) shows that most the CD cross sections give the partial waves with rather low \(J\) values as compared with the elastic scattering and Stark transition processes. Besides, our study shows that a significant underestimation of the CD cross sections (up to one order of the magnitude) can be obtained if one neglects the higher multipole terms in the expansion of the coupling matrix (5). Such a strong effect together with the relatively low \(J\) values involved in the inelastic transitions are related with the fact that the CD process is determined by the comparably more short-range interaction.

In order to illustrate the influence of the \(ns\) state energy shifts on the CD cross sections, the calculations were performed both with and without taking energy shifts into account. The effect is the most pronounced for the low-lying states and some of the results for \(n = 3, 4\) are shown in Figs. 1 and 2 and presented in Table I.

The CD cross sections for a number of transitions \(3l \rightarrow 2\ell'\) with or without the energy shifts of the \(ns\)-states are shown in Fig. 1. In the case \(\Delta \varepsilon_{ns} \neq 0\) the energy dependence of the cross sections changes crucially. The cross sections corresponding to the \(3p(3d) \rightarrow 2s, 2p\) transitions at energies above and below the threshold of the Stark transitions \(3s\rightarrow 3p, 3d\) are strongly suppressed. The effect is more clearly observed at energies in the vicinity of 0.1 eV (the suppression is more or about one order of the magnitude). At \(E_{cm} \gg |\Delta \varepsilon_{ns}|\) the differences between the corresponding pair of the curves (see Fig. 1) are negligible at \(E_{cm} \geq 15\) eV for \(n = 3\) and at \(E_{cm} \geq 1\) eV for \(n = 4\). On the contrary, the transitions \(3s \rightarrow 2s, 2p\) are enhanced at the threshold by a factor about 3 – 4. The main reason of such behavior of the CD cross section is related with the inelastic nature of the Stark transitions between the splitted upper states.

![Fig. 1: The CD cross sections \(\sigma_{nl\rightarrow n\ell'}\) for \((\pi p)_{nl} + H\) collisions calculated both with (thick lines) and without (thin lines) taking the \(ns\)-state energy shifts into account.](image1)

![Fig. 2: The \(l\)-averaged CD cross sections \(\sigma_{nl\rightarrow n\ell'}^{l>0}\) (thick lines) and \(\sigma_{nl\rightarrow n\ell'}^{l>0}\) (thin lines) for \((\pi p)_n + H\) collisions calculated with (solid lines) and without (dashed lines) taking the \(s\)-states energy shifts into account.](image2)

The similar regularities are also observed in the \(l\)-averaged CD cross sections. In Fig. 2 (see also Table 1) the \(l\)-averaged CD cross sections for the \(4 \rightarrow 3\) and \(4 \rightarrow 2\) transitions and total cross sections from 4s-state are presented. One can see that the maximal suppression due to
the energy shift of 4s state is weaker and less or about two times at very low energy both for $4 \rightarrow 3$ and $4 \rightarrow 2$ transitions, while at $E_{cm} > 1$ eV does not exceed 15%. The important observation follows from Fig. 2: the CD cross sections have such a scale factor depending on $\gamma > 1$ nearly to $1/\sqrt{E_{cm}}$. Concerning the power $n$-dependence nearly to $n^3$ with $\gamma > 2$, the present consideration doesn’t confirm that the CD cross sections have such a scale factor depending on $n$ (see Fig. 3 and Table 1). Moreover, for $n = 4 \div 6$ the non-monotone behaviour of the $\sigma^{CD}_n$ as a function of $n$ is revealed.

Beginning from the paper [9], it is commonly believed that the CD cross sections for $\Delta n = 1$ transitions behave like $\sim n^3/E_{cm}$. The $n$ and $E_{cm}$ dependences of the $l$-averaged CD cross sections ($l > 0$) for $n = 5 \div 8$ are shown in Fig. 3 for the $\Delta n = 1$ transitions. The straight lines show the results of the parameterization [11] for $n = 5$ and 7 (thin lines).

Beginning from the paper [9], it is commonly believed that the CD cross sections for $\Delta n = 1$ transitions behave like $\sim n^3/E_{cm}$. The $n$ and $E_{cm}$ dependences of the $l$-averaged CD cross sections ($l > 0$) for $n = 5 \div 8$ are shown in Fig. 3 for the $\Delta n = 1$ transitions. The straight lines show the results of the parameterization [11] for $n = 5$ and 7 (thin lines).
FIG. 5: The CD cross sections from $s$-states $\sigma_{n\rightarrow n-1}$ for $n = 5 - 8$. The notations of curves are the same as in Figs. 3 and 4.

2 We thank Dr. T. Jensen for the preliminary results of the analysis with the present CD cross sections of the NTOF spectra. Our results explain the high energy component around 105 eV due to the $5 \rightarrow 3$ transition and lead to a very good agreement with the experimental weight of the $3 \rightarrow 2$ component at 209 eV.

We are grateful to Prof. G. Korenman for fruitful discussions. This work was partially supported by Russian Foundation for Basic Research, grant No. 03-02-16616.