On the information-splitting essence of two types of quantum key distribution protocols

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With the help of a simple quantum key distribution (QKD) scheme, we discuss the relation between BB84-type protocols and two-step-type ones. It is shown that they have the same essence, i.e., information-splitting. More specifically, the similarity between them includes (1) the carrier state is split into two parts which will be sent one by one; (2) the possible states of each quantum part are indistinguishable; (3) anyone who obtains both parts can recover the initial carrier state and then distinguish it from several possible states. This result is useful for related scheme designing and security analyzing.

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The goal of cryptography is to ensure that a secret message is transmitted between two users, traditionally called Alice and Bob, in a way that any eavesdropper (such as Eve) cannot read it. Till now, it is generally accepted that the only proven secure cryptosystem is the scheme of one-time pad, which utilizes a previously shared secret key to encrypt the message transmitted in the public channel. However, it is difficult for all existing classical cryptosystems to establish a random key with unconditional security between Alice and Bob. Fortunately, quantum key distribution (QKD) [1−5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20], the approach using quantum mechanics principles for the distribution of secret key, can accomplish this task skillfully.

The first QKD scheme, i.e., BB84 protocol, was proposed in 1984 by Bennett and Brassard [1]. Here we give a brief introduction of a well-known variation of it, i.e., the delayed choice BB84 protocol. In this scheme Alice randomly generates single photons polarized along one of four possible directions, 0°, 45°, 90° or 135°, and sends them to Bob. Note that these four states form two conjugate bases {0°, 90°} and {45°, 135°}. After Bob received these photons, Alice tells Bob which basis she used for each one. Afterwards, Bob measures every photon in the same basis as that of Alice and it follows that he can know what states Alice sent from the measurement results. As the different states can be encoded into 0 and 1, Alice and Bob will share a sequence of key bits. Eve’s presence can be detected by publicly comparing a sequence of these bits. Obviously, Eve cannot elicit correct information without introducing errors because she does not know, when these photons are transmitted in the quantum channel, which of the two conjugate bases was used to prepare each of them. If Eve tries to measure one certain photon to acquire information, she will bring disturbance once she uses a wrong basis (the announcement of bases from Alice is too late for Eve). Nevertheless Bob will obtain all of the key bits (noise is not concerned here) sent by Alice because he can perform his measurement after Alice’s announcement. Several other schemes utilize similar means to distribute key (for example, see Refs. [3, 4]). We customarily call them BB84-type protocols. The main feature of these protocols is that the communicators use nonorthogonal states so that Eve cannot distinguish them completely, on which the security is based.

Since Goldenberg and Vaidman came up with a QKD protocol based on orthogonal states in 1995 [8], much attention has been focused on this type of quantum cryptography [3,12,13,21]. In these schemes the information carrier is composed of a set of orthogonal states. As we know, orthogonal states can be easily distinguished with no disturbance and, therefore, as the carrier, they cannot be transmitted in the public quantum channel directly. The above proposals give a smart resolvent, that is, splitting the information unit, which denotes the information encoded in one complete unit of carrier, into two parts, e.g., the two parts of a photon wave packet [8] or two correlated particles [12,13,21], and sending them one by one (see Fig. 1). Note that one can never read the total information transmitted without introducing disturbance by measuring only one of the two parts. In this condition, Eve cannot access the two parts simultaneously and it follows that her eavesdropping is doomed to failure. In this Letter we ignore the case that Eve intercepts legal wave packets (or particles) and sends some fake ones to Bob, where Eve will introduce errors though she can obtain both two parts at the same time. Because the information unit is always transmitted by two steps, we call this type of schemes two-step-type protocols.

In this Letter we focus our attention on the relation between BB84-type protocols and two-step-type ones. In fact, Peres and the authors of Ref. [3] discussed the similar topic long time ago, but they did not make an agreement about it [22, 23]. As far as the great influence of both types of QKD on quantum cryptography is concerned,
If the classical part is 1, Bob exchanges the positions of particles $b_1$ and $a_2$, that is, rearranging $[a_1, b_1, a_2, b_2]$ into $[a_1, a_2, b_1, b_2]$, which can be easily realized by storage rings. After that these particles form the quantum part. While the classical part is 0 or 1, corresponding to the operation (1) or (2) respectively.

(iii) Alice sends the two parts to Bob by two steps as in the previous two-step-type protocols. More concretely, the quantum part is sent firstly and then the classical part follows. The delayed time is brought by the storage rings $SR_1$. That is, when the quantum part is transmitted in the upper channel, the classical part is still in $SR_1$. At the same time, the quantum part is already in $SR_2$ when the classical part is transmitted in the lower channel. As a result, Eve cannot access both parts simultaneously.

(iv) When Bob has received both the two parts, he combines them and tries to recover the complete information. If the classical part is 1, Bob exchanges the positions of the second and the third particles. Otherwise, he does nothing.

(v) Bob measures this carrier unit to obtain the key. Bob performs two Bell measurements on the first two particles and the rest two particles. As a result, he can get four bits of key from his measurement result (according to the above coding rule).

In the above description, for simplicity, we only considered the first carrier unit, i.e., the first two EPR pairs. The other EPR pairs generated by Alice can be used in a similar way. At last, Alice and Bob publicly compare a subsequence of the bits sent by Alice with those received by Bob to detect Eve’s presence. If there is no eavesdropping, Alice and Bob will obtain a sequence of secure key after error correction and privacy amplification. The operation of order rearrangement is inspired by the work of Deng and Long. It should be emphasized that our main aim is using our scheme to discuss the relation between BB84-type protocols and two-step-type ones instead of the QKD scheme itself.

It can be seen that the mid protocol possesses the same features of two-step-type protocols such as using orthogonal states as carrier, splitting the information unit into two parts and sending them one by one, etc. The only difference between the mid protocol and two-step-type protocols is that the former splits the information unit into a quantum part and a classical one, while the latter splits it into two quantum parts. From this perspective, the mid protocol is a variation of two-step-type protocols.

Now let us observe the relation between the mid protocol and BB84-type protocols. Obviously, if we interpret the delayed time as the effect of storage rings, the delayed choice BB84-type protocols can be depicted by Fig. 1 too. Equivalently, Alice sends the photons to Bob through the upper channel and, after Bob has received them, tells Bob the classical bits (i.e., the bases information) through the lower channel. Therefore, the mid protocol and BB84-type protocols have very similar feature. Intuitively, the only difference between them is that the mid protocol uses orthogonal states as carrier while BB84-type protocols are based on nonorthogonal

\[
\begin{align*}
(\Phi^+) = 1/\sqrt{2}((00) \pm (11)) \\
(\Psi^+) = 1/\sqrt{2}((01) \pm (10))
\end{align*}
\]

represent bit values 00, 01, 10, and 11, respectively.
states. However, it is just a superficial phenomenon. In fact, if we consider a certain complete carrier unit such as $|a_1, b_1, a_2, b_2\rangle$ in the mid protocol, its possible states (i.e., the two possible states after Alice’s information-splitting operation) are nonorthogonal indeed. For example, suppose both the two EPR pairs are in state of $|\Phi^+\rangle$, that is, $|\varphi\rangle_{a_1, b_1} = |\varphi\rangle_{a_2, b_2} = |\Phi^+\rangle$. In this condition the state of this carrier unit can be written as

$$|\varphi\rangle_{a_1, b_1, a_2, b_2} = |\Phi^+\rangle \otimes |\Phi^+\rangle \quad (1)$$

$$= \frac{1}{2}(|0000\rangle + |0011\rangle + |1100\rangle + |1111\rangle).$$

After Alice’s information-splitting operation, this carrier unit will be changed into one of two possible states $|\varphi\rangle_{a_1, b_1, a_2, b_2}$ and $|\varphi\rangle_{a_1, a_2, b_1, b_2}$, where

$$|\varphi\rangle_{a_1, a_2, b_1, b_2} = \frac{1}{2}(|0000\rangle + |0101\rangle + |1010\rangle + |1111\rangle). \quad (2)$$

From Eqs. (1) and (2) we can calculate the inner product of these two states

$$a_1, b_1, a_2, b_2 \langle \varphi |\varphi\rangle_{a_1, a_2, b_1, b_2} \quad (3)$$

$$= \frac{1}{2}(|0000\rangle + |0011\rangle + |1100\rangle + |1111\rangle)$$

$$= \frac{1}{2}(|0000\rangle + |0101\rangle + |1010\rangle + |1111\rangle) = \frac{1}{2}.$$

Obviously, the two possible states are nonorthogonal and Eve can never distinguish them determinately without Alice’s classical information. When the two EPR pairs are originally in other Bell states, we can obtain the same conclusion by similar deduction. Therefore, in substance, the states transmitted in the quantum channel are nonorthogonal and the mid protocol has the same essence as that of BB84-type protocols.

From another point of view, we can recognize the similarity between the mid protocol and BB84-type protocols more clearly by reinterpreting the latter. That is, Alice randomly prepares single photons polarized along one of two possible directions, 0° or 90° (Note that they are orthogonal states). Afterwards, Alice makes an information-splitting operation which can be described as follows. Alice performs, at random, one of the following two operations on each photon: (1) doing nothing; (2) rotating its polarized direction by 45° clockwise, that is, changing its state into the corresponding one in the other basis \{45°, 135°\}. After that the photon forms the quantum part. While the classical part is 0 or 1, corresponding to the operation (1) or (2) respectively. At the destination, when Bob received both the two parts, he can recover and obtain the original information just by a reverse operation and a measurement in the basis \{0°, 90°\}. It can be seen that the carrier states are orthogonal (nonorthogonal) before (after) the information-splitting operation, which is quite similar with that of the mid protocol. In a word, the mid protocol is actually equivalent to BB84-type protocols (the difference between them is their material carriers).

From above analysis we can draw a conclusion that BB84-type protocols and two-step-type protocols are not entirely opposite. Contrarily, they have the same essence, i.e., information splitting. An orthogonal carrier or a nonorthogonal carrier, which looks like the main difference between these two types of protocols, is not an important, even not a very explicit matter. Let us take the mid protocol as our example (BB84-type protocols have the same feature). As far as Eve is concerned, the possible states of the carrier unit, which is the smallest system she has to distinguish, are nonorthogonal. But for Bob, the possible states that need to be distinguished are indeed orthogonal because Bob knows which two particles were initially in a Bell state (or which basis was used when Alice prepared each photon in BB84-type protocols) with the help of Alice’s classical information. One may argue that the possible states of the separate quantum parts in some two-step-type protocols are nonorthogonal. (For example, the quantum part can be in mixed state, mostly in maximally mixed state \(\rho = \frac{1}{4}(|0\rangle\langle 0| + |1\rangle\langle 1|) \quad [10, 12, 21]\). Non-maximally mixed state is subtly employed to remove the need for random timing tests in Ref. [12].) However, we can find that each quantum part has a feature that Eve cannot elicit the key information (i.e., the complete information) without introducing disturbance, which is same as that of the above so-called nonorthogonal states. Here we generally call both of them “indistinguishable states”.

Before we conclude, it is worthwhile to inspect our so-called “the same essence” of BB84-type protocols and two-step-type protocols, which includes (1) the carrier state, which contains the complete key information, is split into two parts (two quantum parts or one quantum part and one classical one) and they are sent one by one to prevent Eve from simultaneously taking control of both of them. As described in Fig. 4 if Eve wants to access both two parts at the same time, she must send fake particles or wave packets to Bob, which will be detected by the legal communicators; (2) the possible states of each quantum part are indistinguishable states, which means that Eve cannot elicit useful information without introducing errors. Of course, Eve cannot extract key information from the classical part, either; (3) anyone who obtains both parts can recover the initial carrier state and then distinguish it from several possible (orthogonal or distinguishable) states. Namely, when Bob received both two parts he can obtain the key information Alice sent. In fact, the security of BB84-type protocols and two-step-type protocols is just based on the above essence.

Finally, we have to confess that there are still some differences between BB84-type protocols and two-step-type protocols. The main one is that the former splits the information unit into a quantum part and a classical one, while the latter splits it into two quantum parts. As a consequence, Bob can guess the value of the classical part (i.e., the one comes later) instead of waiting until its arriving in BB84-type protocols. Equivalently, Bob ran-
domly selects a basis to measure each photon and at last the communicators discard the bits from those photons that are prepared and measured in different bases, which is just the idea of the original BB84 protocol \footnote{C. H. Bennett, and G. Brassard, in Proceedings of IEEE International Conference on Computers, Systems and Signal Processing, Bangalore, India (IEEE, New York, 1984), p.175.} (this feature also applies to the mid protocol). On the contrary, in two-step-type protocols the later part is a quantum one and Bob cannot do such guesswork. It should be emphasized that, however, this difference between these two types of QKD protocols cannot cover up their same essence described above.

To summarize, we have mainly discussed the relation between BB84-type protocols and two-step-type ones. By presenting a simple QKD scheme (i.e., the mid protocol) we take cognizance of some connections between these two types of protocols and draw a conclusion that they have the same essence as described above (see Fig. \ref{fig:relation}). This result can help us to make clear the base of some QKD protocols’ security. Furthermore, it is useful for related scheme designing and security analyzing.

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