A possible mechanism for QPOs modulation in neutron star sources

Jiří Horáč

Astronomical Institute, The Czech Academy of Sciences, Boční II, CZ-140 31 Prague, Czech Republic

Received; accepted; published online

Abstract. It was pointed out by Paczyński (1987) that the X-ray luminosity of accreting neutron stars is very sensitive to the physical properties of the accretion flow close to the innermost stable circular orbit. The X-ray radiation is dominated by that emitted in the boundary layer, where accreted matter hits a star surface. The X-ray luminosity of the boundary layer is proportional to the local accretion rate. In this note, we estimate local accretion rate variations from the disk that undergoes non-stationary axisymmetric perturbations. The perturbations are given by the poloidal-velocity potential. We obtain a simple formula describing the modulation of the accretion rate for the particular case of global vertical disk oscillations that have been recently studied by Abramowicz et al. (2005).

Key words: neutron stars, QPOs, accretion, strong gravity

1. Introduction

The quasi-periodic oscillations (QPOs) appear in the light curves of more than 20 bright low-mass X-ray-binaries (LMXBs) with accreting neutron stars (van der Klis 2000). Much attention is attracted to the kilohertz QPOs because their frequencies are comparable with orbital frequencies in the innermost parts of the accretion disks. The orbital frequency of a particle orbiting the neutron star of mass $M$ at the innermost stable circular orbit (ISCO) is $\nu_{\text{ISCO}} = 1580(1 + 0.75j)\, \text{Hz} \times 1.4M_\odot/M_{\text{ISCO}}$ (Kluźniak et al. 1990).

Many models have been proposed to explain the excitation mechanism of QPOs and subsequent modulation of the X-ray signal (see van der Klis 2000 and McClintock & Remillard 2003 for a detailed discussion of observations and models). Recently, it has been suggested that the high frequency QPOs arise from a resonance between two oscillation modes of the innermost part of the accretion disk (Kluźniak & Abramowicz 2001; Abramowicz & Kluźniak 2001). In the case of the neutron-stars sources, the modulation of the X-ray radiation may originate in the modulation of the local accretion rate (Kluźniak & Abramowicz 2004).

In LMXBs that are not pulsars, the magnetic field of the neutron star is sufficiently weak, allowing the accretion disk to extend down to ISCO. The strongest X-ray radiation then originates in the boundary layer, where accreted material hits the star surface. Depending on the star radius $R_*$, the amount of energy released in the boundary layer exceeds that radiated by the whole disk. It gives about 69% of the total luminosity if $R_* = 3R_\odot$, or even 86% if $R_* = 1.5R_\odot$.

In this context, Paczyński (1987) pointed out that a variability of X-ray luminosity of accreting neutron stars may be governed by physical properties of the accretion flow close to ISCO. In Einstein gravity, the inner edge of the pressure supported thick accretion disks is slightly below ISCO (Abramowicz 1985). The material is accreted from the disk through a narrow potential nozzle onto the neutron star. Obviously, if the innermost part of the disk is not stationary but is a subject to some oscillations then the fine structure of the flow at the inner disk edge is significantly changed. This strongly affects the accretion rate through the nozzle and the resulting X-ray luminosity of the boundary layer. This scenario is in agreement with the recent observations of Gilfanov et al. (2003) and more recently Revnivtsev & Gilfanov (2005) that strongly point to the fact that neutron-star QPOs are modulated in the boundary layer.

In sections 2 and 3 we briefly summarize equations important for the disk structure close to ISCO and reproduce the calculations of the accretion rate through the inner edge of the stationary disk. Then in section 4 we calculate the accretion rate from the disk that is subject to nonstationary ax-
isymmetric perturbations. We derive a simple formula for the accretion rate modulation of a vertically oscillating disk.

2. Disk structure close to ISCO

We consider an axisymmetric thick disk made of a perfect fluid surrounding a neutron star of mass $M$. The dynamics of the fluid is governed by Euler equation, poloidal component of which takes the form

$$\frac{\partial v}{\partial t} + v \cdot \nabla v = \ell \frac{\ell^2}{r^2} \mathbf{e}_r + \nabla p - \nabla \Phi = 0,$$

where the bold-face letters refer to the poloidal part of the vectors, $\alpha \equiv (a, a^*)$, $\Phi$ is a gravitational potential, $r$ denotes radial coordinate (we employ the cylindrical coordinates $\{r, \phi, z\}$, with the origin coinciding with the center of the star) and $p, \rho$ and $\ell$ are the pressure, density and the angular momentum of the orbiting flow respectively (in general all depend on $r$ and $z$). The azimuthal component of the Euler equation gives conservation of angular momentum,

$$\frac{\partial \ell}{\partial t} + v \cdot \nabla \ell = 0. \tag{2}$$

We assume that the angular momentum is constant in the whole volume of the disk, $\ell(r, z) = \ell_0$, and that the fluid obeys the polytropic equation of state,

$$P = \frac{\kappa}{\kappa - 1} \rho^{\kappa + 1}/n,$$

where the bold-face letters refer to the poloidal part of the potentials are determined by the distribution of the fluid angular momentum. Here we consider the simplest case of the constant distribution, $\ell(r, z) = \ell_0$. The plot shows projections of the equipotential surfaces to the poloidal plane (solid lines) and the distribution of the fluid (shaded region). The matter that overflows the Roche lobe (the equipotential surface that crosses itself) is accreted onto the neutron star.

3. Stationary flow

The stationary accretion rate for Roche overflow was first calculated by Kozlowski et al. (1978), who used Einstein's theory. Here we closely follow the Newtonian calculations of Abramowicz et al. (1983). We consider a small overflow, so that all quantities can be expanded to the second order in the vicinity of the Lagrange point $L$. Particularly, the vertical profile of the enthalpy can be expressed as

$$h(r_{\text{in}}, z) = h^* - \frac{1}{2} \kappa z^2, \quad \kappa^2 \equiv - \left( \frac{\partial^2 h}{\partial z^2} \right)_L, \tag{4}$$

where $h^* \equiv h(r_{\text{in}}, 0)$ denotes a maximal value of the enthalpy on the cylinder $r = r_{\text{in}}$. The linear order does not contribute because the flow is symmetric with respect to the equatorial plane. The thickness of the inner edge is $H = \sqrt{2h^*/\kappa}$. Close to $r = r_{\text{in}}$ the accretion flow becomes transonic. After Abramowicz (1983), we assume that the radial velocity of the flow equals to the local sound speed and that the vertical component of the velocity is negligible compared to the radial one. This significantly simplifies the solution because it allows us to express the poloidal velocity using the enthalpy,

$$v = \sqrt{\frac{h}{\kappa}} \mathbf{e}_r. \tag{5}$$

![Fig.1. Accretion from a thick stationary accretion disk. The position of the disk inner edge and the shape of the equipotentials are determined by the distribution of the fluid angular momentum. Here we consider the simplest case of the constant distribution, $\ell(r, z) = \ell_0$. The plot shows projections of the equipotential surfaces to the poloidal plane (solid lines) and the distribution of the fluid (shaded region). The matter that overflows the Roche lobe (the equipotential surface that crosses itself) is accreted onto the neutron star.](image-url)
The parameter potential. This introduces the vertical epicyclic frequency
profile can be expressed using a derivative of the effective
where

\[ M = \int_0^{2\pi} r_1 \, d\phi \int_{-H}^{H} \, \dot{m} \, dz \]

\[ = \left(2\pi\right)^{3/2} \frac{r_{in}}{n^{1/2}} \frac{1}{K(n+1)} \frac{\Gamma(n+3/2)}{\Gamma(n+2)} \frac{(h^*)^{n+1}}{\kappa}, \]

where \( \Gamma(x) \) is the Euler gamma function.

In the Bernoulli equation (5) we keep the term \( v^2/2 \) and
neglect only the time derivative because of stationarity of the
flow. We obtain

\[ \frac{v^2}{2} + h + \mathcal{U} = \left(1 + \frac{1}{2n}\right) h + \mathcal{U} = \mathcal{U}_S. \]

The parameter \( \kappa \) that determines the shape of the enthalpy
profile can be expressed using a derivative of the effective
potential. This introduces the vertical epicyclic frequency \( \omega_z \)
to the problem. From equation (7) we obtain

\[ \kappa^2 = \left(\frac{n}{n+1/2}\right) \omega_z^2, \quad \omega_z = \left(\frac{\partial^2 \mathcal{U}}{\partial z^2}\right)_L. \]

By substituting the equations (7) and (8) and introducing
\( \Delta \mathcal{U} \equiv \mathcal{U}_0 - \mathcal{U}_S \) we finally recover the result obtained by
Abramowicz (1985)

\[ \dot{M} = A(n) \frac{r_{in}}{\omega_z} \Delta \mathcal{U}^{n+1}, \]

\[ A(n) \equiv \left(2\pi\right)^{3/2} \frac{1}{K(n+1)} \frac{1}{n+1/2} \frac{\Gamma(n+3/2)}{\Gamma(n+2)} \times \]

\[ \frac{1}{n+1/2} \left(1 + \frac{1}{2n}\right) \frac{\Gamma(n+3/2)}{\Gamma(n+2)} \frac{(h^*)^{n+1}}{\kappa}, \]

(9)

4. A perturbed flow

Now, we suppose that the disk is disturbed and oscillates.
In that case, the accretion flow will not be stationary any-
more and in order to describe the flow we must use the
Bernoulli equation (3) in the full form. The presence of the
‘non-stationary’ term \( \partial \chi/\partial t \) breaks however the correspond-
ence between the enthalpy and the effective potential.
The equipotential surfaces and the surfaces of constant enthalpy
will not coincide anymore. If the oscillations are a small per-
turbation, we can expand the Bernoulli equation in the vicin-
ity of the stationary flow considered above.

We suppose that the velocity potential can be expressed as

\[ \chi(r, t) = \chi_0(r) + \epsilon \chi_1(r, t), \]

where the subscript ‘(0)’ refers to the stationary flow and the
dimensionless parameter \( \epsilon \) characterizes strength of the
perturbation. We assume \( \epsilon \ll 1 \). Then, using the definition
\( \mathbf{v}(r, t) = \nabla \chi(r, t) \) we find

\[ \mathbf{v} = \mathbf{v}_0 + 2 \epsilon \mathbf{v}_1 + \mathbf{v}_1 + \epsilon^2 \mathbf{v}_2 \]

\[ = \epsilon \mathbf{v}_0 + 2 \epsilon \mathbf{v}_1 + \epsilon^2 \mathbf{v}_2 \]

\[ = \epsilon \mathbf{v}_0 + 2 \epsilon \mathbf{v}_1 + \epsilon^2 \left(\frac{\partial \chi_1}{\partial r} + \frac{\partial \chi_1}{\partial z}\right)^2. \]

(11)

The enthalpy is also affected by the perturbation. The new
value can be approximated by an expansion in the parameter
\( \epsilon \)

\[ h = h_0 + \epsilon h_1 + \epsilon^2 h_2 + \mathcal{O}(\epsilon^3). \]

(12)

By substituting into the Bernoulli equation (3) and equatin g
equations of same powers of \( \epsilon \), we get

\[ h_1 = -\frac{\partial \chi_1}{\partial t} - \left(\frac{h_0}{n}\right)^{1/2} \frac{\partial \chi_1}{\partial r}, \]

\[ h_2 = -\frac{1}{2} \left[\left(\frac{\partial \chi_1}{\partial r}\right)^2 + \left(\frac{\partial \chi_1}{\partial z}\right)^2\right]. \]

(13)

This way all thermodynamic quantities are expressed using
the poloidal-velocity potential.

To progress further, we need a particular form of the pertur-
bation \( \chi_1(r, t) \). This is a difficult global problem that
often involves numerical calculations. Several authors stud-
yed it under different simplifications. For example, Blaes (1985)
gives all possible modes (e.g. eigenfrequencies and
eigenfunctions) of slender-torus oscillations. In this limit the
size of the stationary torus is small enough that the enthal-
py can be approximated by a quadratic function in the whole
torus. Blaes (1985) considered the problem in general relativity
potential and pointed to the existence of a particular mode
when the torus moves rigidly up and down across the equatorial plane (see also
Abramowicz et al. (2005) for more detailed calculations).
The eigenfrequency of this mode is equal to the vertical epicyclic
frequency \( \omega_z \). The presence of rigid modes in a torus oscil-
lations has been found also in recent numerical simulations
(e.g. Lee et al. 2004; Rubio-Herrera & Lee 2005).

In the following, we model vertical disk oscillations by a
simple ansatz for the poloidal-velocity potential

\[ \chi_1 = z v_z \cos \omega t, \]

where \( \omega \) is the frequency of the oscillations. Calculating ve-
locity perturbation, we find

\[ \mathbf{v}_1 = v_z \mathbf{e}_z \cos \omega t. \]

Hence, \( \epsilon v_z = \text{const} \) can be interpreted as the amplitude of
the vertical velocity. Equations (13) and (15) give

\[ h_1 = z v_z \omega \sin \omega t, \quad h_2 = -\frac{1}{2} \epsilon^2 v_z^2 \cos^2 \omega t. \]

(14)

The vertical profile of the enthalpy at \( r = r_{in} \) reads

\[ h(r_{in}, z, t) = h^* - \kappa \omega^2 z^2 + \epsilon v_z \omega \sin \omega t - \]

\[ -\frac{1}{2} \epsilon^2 v_z^2 \cos^2 \omega t + \mathcal{O}(\epsilon^3). \]

(15)

that is quadratic in the variable \( z \). The position of the enthalpy
maximum on the cylinder \( r = r_{in} \) is shifted from \( z = 0 \) to
height \( \delta z(t) \) given as

\[ \delta z(t) = \frac{\delta Z}{\omega} \sin \omega t, \quad \delta Z = \frac{\epsilon v_z \omega z}{\kappa^2}. \]

(16)

We can interpret \( \delta Z \) as the amplitude of the oscillations. Also
the value of enthalpy in the maximum differs from the sta-
tionary case by

\[ \delta h^* \equiv h(r_{in}, \delta z) - h^* \]

\[ = \frac{1}{2} \kappa^2 \left[\delta z^2 - \frac{\kappa^2}{\omega^2} (\delta Z^2 - \delta z^2)\right] + \mathcal{O}(\epsilon^3). \]

(17)
profiles with the enthalpy maxima at $\delta z$ shown (thick line). Right: The modulated accretion rate from the oscillating disk (thin solid line). The accretion rate for the unperturbed stationary disk is also shown (dotted line). The accretion rate is modulated with twice the frequency of oscillations. The amplitude of oscillations is $\delta Z / H$. The vertical axisymmetric oscillations. The oscillations were modelled by a simple ansatz for the perturbation of poloidal-velocity field. We believe, however, that several features would be present also in more sophisticated (perhaps numerical) solutions: (1) the first correction to the stationary accretion rate is of the quadratic order in both the actual perturbation $\delta z$ and the amplitude $\delta Z$. This is probably because of the symmetry of the stationary flow with respect to the equatorial plane. Hence, the frequency of the modulation must be twice the oscillation frequency. (2) The accretion rate is maximal when the disk reaches the maximal amplitude $\delta z = \delta Z$. (3) The averaged accretion rate from the periodically perturbed flow is greater than the that of the stationary flow.

Fig. 2. Left: The vertical profiles of the enthalpy $h(r_{in}, z)$ on the cylinder $r = r_{in}$ during vertical disk oscillations (thin lines). The amplitude of oscillations is $\delta Z = 0.2H$ and we chose the polytropic index of the fluid $n = 1.5$. The figure captures profiles with the enthalpy maxima at $\delta z = 0, 0.1H$ and $0.2H$. The enthalpy profile for the unperturbed stationary disk is also shown (thick line). Right: The modulated accretion rate from the oscillating disk (thin solid line). The accretion rate for the stationary disk is plotted by thick line. Time is rescaled by the period of oscillations. For reference we plot also the phase of disk oscillations (dotted line). The accretion rate is modulated with twice the frequency of oscillations.

According to equation (6), the actual accretion rate depends on the maximal enthalpy as $\dot{M} \propto (h^*)^{n+1}$. This relation can be applied also in the case of vertical oscillations because the $z$-dependence of enthalpy on the cylinder $r = r_{in}$ can be approximated by a quadratic function also in this case and the oscillations do not contribute to the radial velocity of accreted matter. Hence, using equations (24), (8) and (21) and assuming that the frequency of oscillations equals to the local vertical epicyclic frequency, $\omega = \omega_z$, we arrive at our final result

$$\frac{\delta \dot{M}}{\dot{M}(0)} = (n+1) \frac{\delta h^*}{h^*},$$

$$= \frac{2 - p}{2 - 2p} \left[ 1 + p \right] \frac{\delta z^2}{H^2} - \frac{p \delta Z^2}{H^2},$$

where $\delta M \equiv \dot{M} - \dot{M}(0)$ and $p = n/(n+1/2)$.

Figure 2 shows the result. The enthalpy profiles $h(r_{in}, z)$ are shown for several values of $\delta z$ in the left panel. The amplitude of oscillations is $\delta Z / H = 0.3$. The right panel shows the modulation of the accretion rate from the oscillating disk. The time is rescaled by the oscillation period, $T = 2\pi / \omega_z$. Finally, the time-averaged accretion rate is given by

$$\langle \delta \dot{M} \rangle = \frac{1}{4} (2 - p) \frac{\delta Z^2}{H^2},$$

that is positive for reasonable values of $n$.

5. Conclusions

In this note we studied the accretion rate from a non-stationary pressure supported accretion disk that undergoes the vertical axisymmetric oscillations. The oscillations were modelled by a simple ansatz for the perturbation of poloidal-velocity field. We believe, however, that several features would be present also in more sophisticated (perhaps numerical) solutions: (1) the first correction to the stationary accretion rate is of the quadratic order in both the actual perturbation $\delta z$ and the amplitude $\delta Z$. This is probably because of the symmetry of the stationary flow with respect to the equatorial plane. Hence, the frequency of the modulation must be twice the oscillation frequency. (2) The accretion rate is maximal when the disk reaches the maximal amplitude $\delta z = \delta Z$. (3) The averaged accretion rate from the periodically perturbed flow is greater than the that of the stationary flow.

Acknowledgements. The author is grateful to Marek Abramowicz for the invitation to participate in Nordita days on QPOs conference and also to Wlodek Kluzniak and Vladimir Karas for helpful discussion about the subject. This work was supported by the GAAV grant IAA 300030510.

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