Galactic Globular Clusters with Luminous X-Ray Binaries

Joel N. Bregman, Jimmy A. Irwin, Patrick Seitzer, and Matt Flores

Department of Astronomy, University of Michigan, Ann Arbor, MI 48109

jbregman@umich.edu

ABSTRACT

Luminous X-ray binaries ($>10^{34}$ erg s$^{-1}$, LMXBs) have a neutron star or black hole primary, and in globular clusters, most of these close binaries are expected to be have evolved from wider binaries through dynamical interactions with other stars. We sought to find a predictor of this formation rate that is representative of the initial properties of globular clusters rather than of the highly evolved core quantities. Models indicate the half-light quantities best reflect the initial conditions, so we examine whether the associated dynamical interaction rate, proportional to $L^{1.5}/r^{2.5}$, is useful in understanding the presence of luminous LMXBs in the Galactic globular cluster system. We find that while LMXB clusters with large values of $L^{1.5}/r^{2.5}$ preferentially host LMXBs, the systems must also have half-mass relaxation times below $t_{h,\text{relax}} \sim 10^9$ yr. This relaxation time effect probably occurs because several relaxation times are required to modify binary separations, a timescale that must be shorter than cluster ages. The frequency of finding an LMXB cluster is enhanced if the cluster is metal-rich and if it is close to the bulge region. The dependence upon metallicity is most likely due either to differing initial mass functions at the high mass end, or because bulge systems evolve more rapidly from tidal interactions with the bulge. This approach can be used to investigate globular cluster systems in external galaxies, where core properties are unresolved.

Subject headings: globular clusters: general — X-rays: binaries

1. Introduction

X-ray observations can be effective in identifying neutron stars or black holes, since when they are close to a binary companion, Roche lobe overflow from this secondary leads to a radiating accretion disk. In the Milky Way, most of the luminous X-ray binaries lie in the disk and were born to become X-ray binaries. That is, the initial separation of the two stars
was sufficiently close that through the course of stellar evolution, it became a Roche Lobe overflow binary system with a compact primary. When the initial separation is too great, a Roche Lobe overflow X-ray binary will not occur unless gravitational encounters between other stars can redistribute angular momentum, reducing the separation. This mechanism is important only in dense stellar systems, such as globular clusters, where gravitational interactions can occur in less than a Hubble time.

Globular clusters are hundreds of times more likely to host a luminous low mass X-ray binary (LMXB) than an equivalent number of field stars in either the disk or the halo (Hut et al. 1992). One of the challenges has been to understand how LMXBs are produced in globular clusters, for which there has been a great deal of theoretical work. The central region of a globular has the shortest relaxation timescale, so it is this region that contracts most rapidly, ultimately undergoing core collapse, leading to very high central stellar densities. When it was erroneously believed that globular clusters had no binary stars, it was posited that hard binaries would be formed in the central core collapse region. However, in the past decade, it has become clear that globular clusters are born with a significant number of binaries (e.g., Rubenstein & Bailyn 1997), which changed the evolutionary scenario (Fregeau et al. 2003).

Binary stars with orbital velocities less than the velocity dispersion of the globular cluster (soft binaries) are destroyed rather quickly, but binaries with larger orbital velocities (hard binaries) are destroyed more slowly and their destruction slows the process of core collapse. The interaction between stars occurs on the relaxation timescale, which at the half-mass radius, is about $10^8$-$10^9$ yr. The steady destruction of the binaries occurs over about 5-30 relaxation times, about the time for core collapse. The interaction between binaries, and between a binary and a single star or a compact object can lead to close (harder) binaries where mass transfer becomes possible (e.g., Hut et al. 1992; Fregeau et al. 2003). Binaries that are high-luminosity X-ray emitters ($>10^{35}$ erg/s) have neutron star primaries, so one binary component was initially a massive star. Most of the dim sources ($<10^{32}$ erg/s) are CVs (white dwarf primaries), although some may be neutron star transients in their low state, especially in the $10^{32}$ - $10^{34}$ erg/s range (Verbunt 2001). So the nature of the binaries that one finds depends on the evolutionary state of the star cluster as well as the mass of the secondary stars that evolve off the Main Sequence. That is, age plays a role in both the dynamical state of the globular cluster and the stellar evolutionary state of the stars that could become mass-transfer binaries.

All-sky surveys, as well as pointed observations with several instruments have discovered luminous point sources ($>10^{35}$ erg/s) in 12 Galactic globular clusters (e.g., Hut et al. 1992); henceforth, we only consider these luminous LMXBs. The radial distribution of these LMXBs within the clusters are consistent with being neutron stars rather than massive black holes.
(e.g., Grindlay 1993; Grindlay et al. 2001). It has also been pointed out that these often occur in clusters designated as core collapse systems, supporting the general picture that enhanced stellar interactions lead to LMXBs. Also, it was noted that the metal-rich systems are more likely to host an LMXB (Grindlay 1993), an effect that is seen in globular cluster systems in early-type galaxies (Kundu, Maccarone, & Zepf 2002).

An important advantage of analyzing LMXBs in Galactic globular clusters is that there is a vast database available for the globular cluster system (Harris 1996), including structural parameters, metallicities, velocities, and often, ages (Salaris and Weiss 2002). With the availability of this extensive data set, we examine the LMXB host systems in detail, where we test basic model predictions and quantify various correlations in the data.

2. Model Expectations and Sample Selection

Our goal is to estimate a quantity representative of the rate of stellar interactions that lead to LMXBs over the lifetime of the globular cluster. This quantity can be compared to the properties of globular clusters with luminous LMXBs to determine if it is a useful predictor. As is well-known, the relevant quantities are the density of single stars, the density of binary stars, and the velocity dispersion of the globular cluster. Equivalently, a stellar density can be determined from the globular cluster luminosity and half-light radius. In principle, one would like to know these quantities at the time that the globular clusters were formed, rather than today, as these can evolve with time. In practice, models show that the half-mass radius of the majority of the single stars does not evolve strongly with time until the final destruction of the cluster (see Fig. 8 in Fregeau et al. 2003), unlike the core radius, which can change considerably and is a poor indicator of the initial properties of the cluster. Also, although the number of stars decreases with time as the cluster evolves, the number of stars changes by only about 30% at the time of the first core collapse. These evolutionary variations are smaller than the range of values between globular clusters in the Milky Way: more than two orders of magnitude for \( L \); and a factor of 30 for \( r_h \). Therefore, \( r_h \) and \( L \) should be extremely useful indicators of the initial size and mass of a globular cluster. There may be some globular clusters presently in the final stages of destruction (e.g., NGC 6712; de Marchi et al. 1999; Andreuzzi et al. 2001; Paltrinieri et al. 2001), in which case there are no good measures of their original properties, but such clusters should comprise only a small fraction of the total cluster population. The above relationship for the interaction rate has the advantage that it can be applied to globular clusters in nearby galaxies as well as in the Milky Way, since the half-light radius can be fit for galaxies as distant as the Virgo cluster (using the Hubble Space Telescope; Jordan et al. 2005).
If binaries are transformed into close mass-transfer systems by interactions with passing stars, the formation rate is proportional to the gravitational interaction rate, which is proportional to \( v n_b n \sigma_b \) (\( \sigma_b \) is the interaction cross section, \( v \) is the interaction velocity, \( n \) is the density of single stars, and \( n_b \) is the density of binary stars). Most of the interactions that produce the close binaries are between single stars and binary stars and where the differential velocity is less than the initial binary orbital velocity (Hut & Bahcall 1983; Fregeau et al. 2004), in which case \( \sigma_b \propto v^{-2} \). If, at early time, the binary fraction \( f \) was similar between globular clusters (an assumption that we examine again later), then \( n_b = \frac{f}{1-f} n \), and we can express the interaction rate with a \( n^2 \) term and an unknown binary fraction. One can integrate the interaction rate over the volume, and for a constant mass-to-light ratio, the total rate is proportional to \( L^2 v^{-1} r_h^{-3} \frac{1}{1-f} \) where \( L \) is the optical luminosity of the globular cluster (proportional to the number of stars) and \( r_h \) is the half-mass radius of the globular cluster. We can identify the stellar velocity \( v \) with the velocity dispersion of the cluster \( \sigma_* \), which is related to \( r_h \) and \( L \) through the fundamental plane relationships (Djorgovski 1995; McLaughlin 2000), \( r_h^{-0.7} \propto \sigma_*^{1.45\pm0.2} L^{-0.85\pm0.1} \), which, to within the uncertainties, is the same as the scaling law expected from the Virial Theorem for a constant mass-to-light ratio, \( r_h \propto L \sigma_*^{-2} \). Using the scaling from the Virial Theorem, we find that the total interaction rate is proportional to \( L^{1.5} r_h^{-2.5} \frac{1}{1-f} \).

The optical catalog of (Harris 1996) contains 147 clusters, but several do not have some of the structural properties that we need (or they are poorly defined), such as \( r_h \) or the core radius, \( r_c \). Removing these objects reduces the number of clusters to 141. The 12 globular clusters with luminous LMXB have been discussed by (e.g., Hut et al. 1992; Verbunt 2001, and references therein). They are not constant brightness sources, and the time history of their luminosity is not extensive, so we do not include the X-ray luminosity in our statistical analysis. However, many of these sources are transients and visible in X-rays only a fraction of the time, which needs to be considered in the analysis. Luminous X-ray sources (>10^{36} erg/s) were detectable at considerable distances with the ROSAT all-sky survey (to 90 kpc for the survey limit; Voges et al. 2000), so most sources would have been detected. The detection rate of LMXBs is not greatly diminished by gas in the Galactic disk, since most of the luminosity of LMXBs emerges above 1 keV, where absorption is not significant for most sight lines.

3. Analysis of Globular Cluster Properties

Many of the optical structural properties of globular clusters have been discussed in the literature (e.g., Ashman & Zepf 1998), so what we add is a quantitative analysis of
whether the globular clusters that host LMXBs are distinguished in some way. One of the expectations is that as clusters evolve, their tidal radius is determined by environmental factors, such as whether the cluster has passed through the disk or passed by the inner bulge of the Galaxy. The tidal radius should be largely decoupled from the inner part of the cluster, where most of the LMXBs would be formed, so we would not expect a good relationship between the presence of LMXBs and $r_t$ (or $r_t$ normalized by $r_h$, the ratio $r_t/r_h$). This expectation is confirmed in that the presence of an LMXB is randomly distributed as a function of $r_t/r_h$, as seen in the histogram (Figure 1). Henceforth, we concentrate on quantities other than $r_t$ in this investigation.

For unevolved clusters, we would expect a linear correlation between $r_h$ and $r_c$, but as the clusters evolve dynamically, $r_c$ is predicted to change much more than $r_h$. Therefore, the smallest clusters (as measured by $r_h$) often have the shortest relaxation times, so the departure from the original relationship between $r_h$ and $r_c$ should be greatest, and this is evident in Figure 2. At the larger values of $r_h$, there is a narrow and well-defined relationship with $r_c$, with a dispersion of less than a factor of two. However, at the smaller values of $r_h$, there is a very large range in $r_c$, with a range of about an order of magnitude and extending only toward small values of $r_c$. Only one of the LMXBs occurs in a cluster that lies on the unevolved $r_c - r_h$ relationship, with the other 11 having values of $r_c$ that are 3-30 times smaller than $r_c$ for the unevolved relationship. This one outlier is NGC 6712, which as mentioned above, is probably in the process of being disrupted (Paltrinieri et al. 2001), so its value of $r_h$ is likely not representative of its value at formation.

Because $r_c$ can change by very large factors during the evolution of the cluster, it is a poor indicator of the original state of the cluster, and its value can be uncertain due to the small number of stars involved in its determination. The best indicator of the original size of the cluster is $r_h$, while the best indicator of the total number of stars is $M(V)$. Neither are perfect indicators, as they can differ significantly from their original values when the cluster is being tidally torn apart. Nevertheless, we can use these two properties to estimate the interaction rate (discussed above), where we find that clusters hosting LMXBs tend to be small and luminous, preferentially with high interaction rates, given by $L^{1.5}/r_h^{2.5}$ (Figure 3; we have dropped the factor due to binary fraction and will return to that issue). A t-test shows that the distribution of interaction rates differs at greater than the 99% level between clusters hosting LMXBs and those without luminous LMXBs. Closer inspection of this figure shows that the lines of constant interaction rates are not ideal predictors of the presence of an LMXB. At constant interaction rate, the smaller clusters appear to be more likely than the larger ones to host an LMXB, so some modification of the nominal prediction is implied.

Although the core radius is a current property rather than an original property of the
cluster, we see the strong preponderance for clusters with small $r_c$ to host LMXBs, as has been noted previously (Figure 4; note that the range in radius is much larger than when using $n_h$). Many of the systems with very small $r_c$ have undergone core collapse, or may have undergone core collapse, as this can be difficult to identify if there has been a core “bounce” as most models predict. Also, for the less luminous systems, few stars comprise the core so it can be difficult to obtain a reliable value of $r_c$. Finally, a number of systems have power-law density distributions into the smallest observable radii, so the concept of a core radius becomes less useful (Noyola & Gebhardt 2003).

Another approach to examining these clusters is by using the relaxation time defined at the half-mass radius. With the relaxation time $t_{\text{h,relax}} \propto L^{0.5} r_h^{1.5}$, we can form a stellar interaction rate for the number of stars in the cluster, $N_0 / t_{\text{h,relax}} \propto L^{0.5} / r_h^{1.5}$, which has a steeper relationship between $L$ and $n_h$ at a constant rate ($L \propto n_h^3$) than does the two-body interaction rate ($L \propto n_h^{5/3}$ at constant rate), due to the inclusion of the binary star cross section in the second rate. When inspecting the relationship between $t_{\text{h,relax}}$ and $M(V)$ (Figure 5) there appears to be a relaxation time, at about $2 \times 10^9$ yr, above which there is only one LMXB, even though about half of the clusters have larger relaxation times. This suggests that at least five relaxation times are required to form enough close binaries that luminous X-ray sources are seen.

A particularly striking relationship is the success of a cluster hosting an LMXB as a function of central density (Figure 6), which is a current cluster quantity. Of the optically luminous systems ($M(V) < -7$), $8/9$ have the highest central densities of the sample, with $5 < \log \rho < 5.5$. For these optically luminous clusters, at these high densities, the number of clusters with LMXBs is greater than those without LMXBs! Only one cluster hosting an LMXB has a central density that is near the average value for clusters, the system NGC 6712, which is probably in the process of being disrupted (as discussed above). For the next two fainter clusters ($-7 < M(V) < -5$), the LMXB-bearing systems have central densities that are well above average for the sample, although not quite as dense as the more luminous systems. The very faintest cluster with an LMXB has a density near the sample average, but there are very few clusters in this region and this cluster may be evolving rapidly now, undergoing destruction.

There are two other properties, involving metallicity and position, that affect the likelihood of a cluster hosting an LMXB. The cluster metallicity distribution is bimodal, with most clusters being low metallicity (the modal value of the low metallicity component is log[Fe] = -1.6 and we use log[Fe] = -1.1 as the division between high and low metallicity systems; Ashman & Zepf 1998), yet most of the clusters with LMXBs are high-metallicity systems (Figure 7), an effect that has been noted previously in the Galaxy and in early-type
galaxies (e.g., Grindlay 1993; Kundu, Maccarone, & Zepf 2002). Also, there is a preponderance for clusters hosting LMXBs to lie within 4 kpc of the center of the Milky Way (Figure 8). This may be due to the tidal influence of the bulge causing the bulge population of clusters to evolve more rapidly. Alternatively, this bulge cluster population may simply have formed more compact systems initially.

4. Discussion and Interpretation

These results are interpreted within the context of models, which have advanced greatly in the last decade, but are not yet complete in the sense of having a “standard model” to use. Consequently, we will interpret our results in terms of the generic features of present models, relying most heavily on those of Fregeau et al. (2003), and in particular, those with King profiles ($W_0 = 7$). These models include binaries with different amounts of initial binary fractions from 2-20% and they calculate the time variation of various binary properties, such as their destruction and their hardness distribution. These models show that as the cluster evolves dynamically, the binaries are hardened (and destroyed), a process that delays core collapse. The binary fraction decreases (but hardens) by about half in 10-20 $t_{h,\text{relax}}$ times, which we consider a characteristic time. This implies that when $t_{h,\text{relax}} \lesssim 10^9$ yr, the cluster has significantly modified the initial binary population, possibly causing close binaries that form into LMXBs, in general agreement with our findings.

The initial predictor of close binary formation from the rate of binary collisions should then be modified by the time needed for a cluster to begin evolving, relative to its age. The original rate (proportional to $L^{1.5}/r_h^{2.5}$ might be modified by a term such as $(1-\exp(t_{\text{age}}/\eta t_{h,\text{relax}}))$, where $\eta \approx 5-10$.

Provided that the cluster does not become disrupted, the number of binaries eventually decreases to the level at which it can no longer prevent core collapse (about 0.2% binary fraction). The core collapses and oscillates even as $r_h$ hardly changes (e.g., the 5% binary fraction model of Fregeau). During this process, some binaries still exist and are extremely hard and therefore likely to become mass transfer systems during the course of stellar evolution. At these high densities, direct collisions between neutron stars and red giants can produce LMXBs as well (Ivanova et al. 2005).

Half of the LMXB clusters are designated as core collapse systems (compare to one-fifth for the non-LMXB clusters), where the definition of a core collapse object is that it has a power-law optical surface brightness distribution into the center with no apparent core radius (for objects labeled as “possible core collapse”, it is difficult to be certain of the power-law
distribution into the center). We can take a different evolutionary definition on the basis of the models of Fregeau et al. (2003). In their models, the ratio \( r_c/r_h \) begins near 0.2-0.3 and slowly decrease to about 0.07, after which a core-collapse event causes \( r_c/r_h \) to fall to 0.01 or less. This deep collapse is short-lived and the core reexpands, spending most of its time at \( r_c/r_h \approx 0.05-0.1 \) before a subsequent recollapse. The quantitative values of this model cannot apply to all clusters since about half of the clusters have values of \( r_c/r_h > 0.3 \), so we must take caution in making conclusions based on these models. If we consider objects with \( r_c/r_h < 0.1 \) to have undergone or about to undergo core collapse, then 4/12 (33\%) LMXB-clusters are core collapse candidates, compared to 16/129 (12.4\%) for the non-LMXB clusters. If this identification with core collapse is correct, having passed through this stage is helpful for the formation of LMXBs, but not essential. The more important influence in forming LMXBs may be the many stellar interactions that occur prior to core collapse.

The preference of finding LMXB clusters toward the inner part of the Galaxy may reflect the role of the bulge on the evolution of clusters. Tidal influences are greater in the inner part of the Galaxy, and this removes the “hotter” stars, effectively cooling the cluster and allowing it to dynamically evolve more rapidly, which can in turn, can produce more close binaries.

Regarding the role of metallicity, our initial expectation was that these are younger systems and thereby different in their stellar population than the older systems. The best uniform study of cluster ages is by Salaris and Weiss (2002), where they produce a uniform set of ages for 55 globular clusters. While they find a mild dependence for some metal-rich clusters being younger, they also find that the systems within the Solar Circle have the same ages, regardless of metallicity. Their sample contains five of our LMXB clusters (NGC 1851, NGC 6624, NGC 6652, NGC 6712, and NGC 7078), all of which have ages in the range 9.2-11.7 Gyr (±1.1 Gyr) that are typical for the rest of the sample. Since this study rules out age as the important parameter for producing LMXBs in Galactic globular clusters, there must be some other mechanism at work, such as the initial binary fraction, number of high mass stars that become neutron stars (a difference in the IMF), or the degree of tidal influence on these systems by the bulge.

The separation by metallicity and location of LMXB clusters may reflect the difference of two types of globular cluster populations, those that formed with the bulge and those that are halo objects. The more metal-rich bulge clusters spend more time close to the inner part of the Galaxy, hence they experience more tidal forces, leading to more rapid evolution and destruction. Many if not most of the halo clusters did not form with the bulge and they occupy a larger volume and have a larger velocity dispersion. Although they are found at all radii, including R < 4 kpc, they spend relatively less time in this region due to their orbits
having larger semi-major axis.

A difference in the binary fraction between high and low metallicity systems should be observable, but ideally, one wants to measure the binary fraction in relatively unevolved clusters, since significant cluster evolution is expected to destroy binaries but leave the remaining ones with relatively small separations (i.e., only hard binaries remain). The current set of data for binary fractions is not uniform, favors low metallicity systems, and has several evolved systems. A cluster listed as a core collapse system, NGC 6397 ([Fe/H] = -1.95) has a low binary fraction, with an upper limit of about 5% (Cool & Bolton 2002). This is to be expected since the large majority of the initial binaries should have been destroyed. However, another core collapse system is NGC 6752([Fe/H] = -1.56, rc/rh = 0.07), yet it has a much larger binary fraction of 15-38% in the core region (Rubenstein & Bailyn 1997). A less evolved system is NGC 6121 (M4; [Fe/H] = −1.20, the most metal rich of this group of four clusters), with properties that make its evolution about average relative to the entire globular cluster sample (rc/rh = 0.23 and t_{h,relax} = 6.6×10^8 yr) but its binary fraction is only 1-2% (Richer et al. 2004). The least evolved system for which there has been a binary study is NGC 288 ([Fe/H] = -1.24), just to the low metallicity side of our low/high metallicity dividing line, with rc/rh = 1.0 and t_{h,relax} = 6.5×10^9 yr. In NGC 288, Bellazzini et al. (2002) find a binary fraction of 10-20% within rh. Although considerably more observations need to be made, especially of relatively unevolved systems, there is no evidence that the lower metallicity systems are lacking in binaries (e.g., NGC 288 and NGC 6752), suggesting that the high metallicity systems had a shallower IMF at the high mass end or that they evolve more due to Galactic tidal influences. We note that it might be possible to use blue stragglers to gain insight into the binary population (e.g., Piotto et al. 2003), but these are binaries that have undergone dynamical encounters and are not good tracers of the initial binary population.

The relationship between the nature of globular cluster densities and the number of compact objects has been studied using the more sensitive Chandra data, that permit observers to study sources fainter than 10^{34} erg s^{-1} (summary in Heinke et al. 2003). In an analysis of these sources for a dozen clusters, Pooley et al. (2003) found that the number of X-ray sources per cluster (L_x > 4×10^{30} erg s^{-1}) was correlated with the total number of stars in the cluster, but more tightly correlated with the cluster encounter rate. Their encounter rate is the volume integral of the local encounter rate, \rho^2/v, which is different from our rate in that it emphasizes the present day conditions, particularly in the core, which can dominate the integral. Once many clusters are observed to low luminosities, this approach certainly will be superior to the study presented here as the statistical measure of hard binaries is better quantified.
Some of the correlations discussed in our study occur in other galaxies, such as the correlation between the presence of an LMXB and the metallicity (color is the proxy for metallicity in most extragalactic studies, aside from Local Group galaxies; e.g., Kundu, Maccarone, & Zepf 2002). Yet other galaxies will permit the studies of the LMXB rate with age, since some galaxies are younger than others or have globular clusters known to be young (e.g., M31, LMC). With Galactic globular clusters, it might be possible to measure the binary fraction rate for high and low metallicity systems, testing our conclusion. Also, the number of globular clusters with LMXBs is a modest 12 systems, and the study of certain elliptical galaxies offers the opportunity of increasing the statistics by an order of magnitude (e.g., Angelini, Loewenstein, & Mushotzky 2001; Irwin, Athey, & Bregman 2003), since $r_h$ can be measured in external galaxies at the distance of the Virgo Cluster (Jordan et al. 2005). Finally, we look forward to the improvement in models, which in principle should be able to predict the frequency of LMXBs for globular clusters of various initial properties and evolutionary states.

We would like to acknowledge support for this work from NASA through grant NAG5-10765. Also, we would like to thank Mario Mateo, Renato Dupke, Edward Lloyd-Davies, and an anonymous referee for their valuable advice.

REFERENCES


Noyola, E., & Gebhardt, K. 2003, Revista Mexicana de Astronomia y Astrofisica Conference Series, 18, 78
Fig. 1.— The ratio of the tidal radius to half-mass radius for the globular clusters in the sample. The hatched regions are clusters without LMXBs while the solid regions are clusters with LMXBs.
Fig. 2.— The distribution of globular clusters by core radius and half-mass radius, where the solid diamonds are systems with luminous LMXBs and the crosses are those without LMXBs. The line, of slope unity, may indicate the unevolved population of globular clusters, since \( r_c \) can evolve, becoming much smaller, while \( r_h \) changes relatively little until disruption occurs.
Fig. 3.— The distribution of $r_h$ and $M(V)$ for the globular cluster sample, where the solid diamonds are systems with LMXBs. Lines of constant binary collision rate are shown ($L^{1.5}/r_h^{2.5}$), with a difference of 0.5 dex between the lines. There are two outliers ($M(V) > -6$), which are small systems that may be in the process of disruption. Note that relatively few LMXB clusters have $r_h > 3$ pc, suggesting that the success rate of having an LMXB depends more strongly on $r_h$ than predicted from the collision rate.
Fig. 4.— The distribution of $r_c$ and $M(V)$ for the globular cluster sample, with the same symbols and lines as in Figure 3. Note that the range of $r_c$ is nearly three orders of magnitude, compared to one order of magnitude for $r_h$. 
Fig. 5.— The half-mass relaxation time as a function of $M(V)$ for the LMXB clusters (diamonds) and non-LMXB clusters (crosses). The dotted lines are loci of constant $L/t_{h,\text{relax}}$, with a difference of 0.5 dex between the lines. Most of the LMXB hosts have relatively small values of $L/t_{h,\text{relax}}$, which is similar to, but not the same as the two-body collision rate.
Fig. 6.— The logarithm of central luminosity density ($L_\odot$ pc$^{-3}$) as a function of absolute magnitude shows that most clusters that host a luminous LMXB have extreme values of the central density.
Fig. 7.— The histogram of metallicities (LMXB clusters are solid) with the familiar double-peaked distribution of high and low metallicity systems shows that most of the LMXB clusters are of moderate to high metallicity.
Fig. 8.— The globular cluster sample as a function of $t_{h,\text{relax}}$ and distance from the center of the Galaxy (R). Metal-rich clusters with LMXBs are solid diamonds, metal-poor LMXB clusters are half-filled diamonds, metal-rich clusters without LMXBs are open circles and metal-poor clusters without LMXBs are crosses. Clusters with $R > 12$ kpc or $M(V) > -2$ are not shown. Most of the LMXB clusters lie in the region $R < 4$ kpc and $t_{h,\text{relax}} < 10^9$ yr (dashed lines), all of which are metal-rich clusters.