Quark-hadron duality in the Rein-Sehgal model

Krzysztof M. Graczyk, Cezary Juszczak, and Jan T. Sobczyk
Institute of Theoretical Physics, University of Wroclaw, pl. M. Borna 9, 50-204, Wroclaw, Poland
(Dated: July 13, 2006)

The quark-hadron duality in CC and NC neutrino interactions is discussed under assumptions that single pion production is described accurately by the Rein-Sehgal model and that it allows reconstruction of the inclusive cross section in the resonance region. The duality is measured by means of integrals of structure functions in the Nachtmann variable for $Q^2 < 3$ GeV$^2$. The results depend on the precision with which contributions from single pion production channels in the overall cross sections are known. Several approaches to evaluate them are compared. The duality is predicted to be seen for proton target reactions and to be absent for neutron and isoscalar targets. Two-component duality between resonant and valence quark contributions to structure functions is also investigated.

PACS numbers: 25.30.Pt, 13.15.+g
Keywords: quark-hadron duality, neutrino-nucleon interactions, Rein-Sehgal model

1. INTRODUCTION

In recent years the quark-hadron (QH) duality has been a subject of many experimental and theoretical studies. The idea of duality comes from Bloom and Gilman [1]. It was based on the comparison of $eN F_2$ resonance structure functions at small values of $Q^2$ with their DIS counterparts at large $Q^2$. Bloom and Gilman observed that in plots in the variable $(2M_\nu + M^2)/Q^2$ resonance peaks are approximately averaged by the DIS structure function. More recent experimental data on duality were reported in [2] where several plots of $F_2$ for $Q^2$ in the range from 0.3 to 3.3 GeV$^2$ as functions of the Nachtmann variable $\xi$ were presented on the same figure. The resonance peaks for $\xi \geq 0.2$ are seen to be averaged by the DIS structure function calculated at $Q^2 = 5$ GeV$^2$. The QH duality can be analyzed in quantitative way by calculating the ratios of integrated strengths over a range in $\xi$ covering a chosen set of resonances and the DIS structure function. The agreement is on the level of 10%.

In the theoretical analysis of the duality [3] one introduces a language of twist expansion of moments of structure functions in powers of $1/Q^2$. The duality then means the suppression of higher twists [4]. A possible explanation how the coherent amplitude in the resonance region can be equal to incoherent sum of amplitudes from the quark constituents of nucleon is proposed in [5] in terms of SU(6) symmetry by means of cancellation of contributions of positive and negative parity. The knowledge of relative strengths of electromagnetic and neutrino induced $N \rightarrow N^*$ transitions leads to theoretical predictions concerning the domain in $W$ in which the duality should hold. The consequences of SU(6) breakdown for ratios of unpolarized and polarized structure functions are also investigated in [6].

The QH duality is usually discussed theoretically and analyzed experimentally in the context of eN interactions. But the subject is relevant also for neutrino physics. Here the exact data is missing and arguments based on duality can be used to provide better estimates of $\nu N$ structure functions and cross sections in the few GeV neutrino energy region where they are known with insufficient accuracy [7]. For these energies it is necessary to consider both quasi-elastic and inelastic channels. Single pion production (SPP) channels are typically treated separately from more inelastic ones which are accounted for by extrapolating DIS formalism as much as necessary. This approach carries a lot of uncertainty and requires a better theoretical understanding in the whole kinematical region. Such understanding and control over the numerical procedures can follow from the QH duality analysis.

When trying to discuss the QH duality in the neutrino interactions the main obstacle is a lack of precise experimental results. The existing SPP data is poor in precision and statistics [8]. In the future the data will hopefully become precise enough to impose more rigid constraints on theoretical models [9] but for a moment the natural strategy is to analyze in detail a generally accepted model. From the point of view of Monte Carlo application this is the Rein-Sehgal (RS) [10] model.
The RS model was developed to describe SPP induced by lepton-nucleon NC and CC interactions. It is currently used in almost all neutrino Monte Carlo generators of events [7]. It is based on the quark model computations of the hadronic current as proposed and developed by Feynman, Kislinger and Ravndal [11]. In the original model the contributions from 18 resonances with masses smaller than 2 GeV are added in the coherent way. The scattering amplitudes contain several form factors. The vector form factors are obtained from the electro-production data by means of standard CVC argument. Axial form factors are less constrained by theoretical arguments (PCAC) and the available data. The RS model contains also an ad hoc prescription for computing the non-resonant background. The extra amplitude is introduced with the quantum numbers of the $P_{31}$ resonance but without the Breit-Wigner term and then added in the incoherent way. Its strength is fine tuned in order to obtain an agreement with the experimental SPP data. The RS model can be supplemented by $m^2$ containing terms $12$ ($m$ is the charged lepton mass) absent in the original paper.

The discussion of the QH duality for neutrino scattering based on the RS model is in a very important point different from the electron scattering analysis based on the experimental data. The aim of the RS model is to describe SPP channels only. In the electron scattering studies the experimental data is that of the inclusive cross section. In order to perform the analogous analysis for the RS model it is necessary to extract and add contributions from more inelastic channels in the kinematical region of invariant hadronic mass up to 2 GeV.

Our analysis is based on two basic assumptions. The first is that the RS model predictions for SPP cross sections are fairly close to what will come out from future precise cross section measurements. The second is that we know the probability that at a given point in the kinematically allowed region the final state is that of SPP. Throughout this paper we will call the probabilities 1-pion functions. We obtained these functions numerically $13$ using the LUND algorithm $14$. The functions (one for each exclusive SPP channel) were discussed also in $12$ and they turn out to depend only on the invariant hadronic mass. For the value of invariant mass $W \sim 1.6$ GeV the contribution from more inelastic channels amounts to about 50%. Using these functions one can rescale SPP contributions to the structure functions and evaluate the overall structure functions in the region $W \leq 2$ GeV. In section $2.3$ we discuss the precision with which the 1-pion functions are reconstructed. We present the experimental data for the proton photoproduction 1-pion function $17$. We present also an alternative approach to calculate the 1-pion function for available hadron multiplicity data and KNO model $16$.

The aim of our paper is to investigate the question if the duality holds also in $\nu N$ reactions. Our methodology is to repeat the analysis done for $e N$ scattering. We calculate structure functions as they are defined by the RS model and compare with the DIS structure functions based on GRV94 PDFs and evaluated at $Q^2_{\text{DIS}} = 10$ GeV$^2$. We perform also a quantitative comparison of integrated strengths over resonances. All the comparisons are done first for proton and neutron structure function and then for their average i.e. for isoscalar (deuteron-like) target. Both charged current (CC) and neutral current (NC) reactions are discussed.

It is well known that CC SPP channels on proton and neutron have distinct properties. The strength of $\Delta$ resonance for proton reaction is three times as big as its neutron reaction counterpart due to isospin rules. In the neutrino-proton reaction there is no need to introduce non-resonant background which gives significant contribution to the neutrino-neutron SPP channels. The way in which non-resonant background is treated in the RS model in not completely satisfactory. For this reason we address the idea of two-component duality proposed by Harari and Freund $15$. They suggested that resonance and non-resonance contributions to the low energy $\pi N$ scattering amplitude (s-channel) correspond to contributions given by the high energy amplitudes (t-channel) due to Reggeon and Pomeron exchange respectively. Using the modern language it can be expressed as existence of a relation between resonance/valence quark and non-resonant/sea quark contributions to the structure functions. A confirmation of this idea in $eN$ interactions was found in $10$: the $F_2$ structure function averaged over resonances at low values of the Nachtmann variable ($\xi \leq 0.3$) behaves in the way which strongly resembles the behavior of valence quark contribution to DIS scaling curve. There is also a striking similarity between the above mentioned averaged $F_2$ rescaled by a factor of $\frac{3}{5}$ and the $xF_2$ $\nu N$ data. If resonance contribution to structure function is dual to the valence DIS contribution and if the overall duality is satisfied then also non-resonant background should be dual to the sea quark contribution. The last duality could be then used to provide a model for non-resonant background.

The question of the QH duality in $\nu N$ interactions was discussed already by Matsui, Sato and Lee $20$ and by Lalakulich, Paschos and Piranishvili $21$. In $20$ the Lee-Sato model $22$ for $\Delta$ production in $eN$ and $\nu N$ scattering was analyzed. It was shown that in the vicinity of $\Delta$ excitation peak the local duality holds for both proton and iso-scalar structure functions in CC and NC neutrino reactions. The resonance model investigated in $21$ includes four resonances $P_{33}$, $P_{11}$, $D_{13}$ and $S_{11}$ of $W < 1.6$ GeV. The model (unlike the RS model) contains correction for the non-zero charged lepton mass. The conclusions are in agreement with those contained in $20$. Some qualitative elements of the present analysis of the RS model can be found in $23$. In $24$ several theoretical and practical issues related to the problem of how to combine smoothly the RS and DIS contributions in Monte Carlo generators are addressed.
Contribution of SPP

FIG. 1: Elasticities of resonances and the 1-pion functions for neutrino CC reactions.

FIG. 2: Comparison of the 1-pion functions for neutrino-proton CC scattering (solid line) and proton photoproduction (dashed line). The 1-pion function for the photoproduction was extracted based on the data from [17].

Using the idea of duality one can combine experimental and theoretical arguments and find suitable modification of the structure functions which average over resonances for small values of $Q^2$. One expects that duality should hold for $Q^2 \geq 0.5 \text{ GeV}^2$. For $Q^2$ approaching zero $F_2$ structure function for electro-production should behave like $F_2 \sim Q^2$ due to gauge invariance. The presence of axial current modifies this behavior. An important piece of information is provided by the Adler sum rule. It is argued that vector and axial parts of the structure functions should be modified in a different way.

Our paper is organized as follows. In Section 2 the necessary theoretical introduction is given. The recipe to obtain structure functions from the Rein-Sehgal model is presented and the 1-pion functions are introduced. The functions $R_{2,3}$ which measure how well the duality holds are also defined and ambiguities in their definition are discussed. In Section 3 we present the results of the numerical analysis and their discussion. Section 4 contains the conclusions.

2. THEORETICAL FOUNDATIONS OF THE NUMERICAL ANALYSIS

2.1. Rein-Sehgal structure functions

We consider the following SPP charged current and neutral current reactions:

\[
\begin{align*}
\nu(k) + N(p) &\rightarrow l(k') + N^*(p') \\
\nu(k) + N(p) &\rightarrow \nu(k') + N^*(p') \\
\end{align*}
\]

In the LAB frame the momentum transfer is:

\[
q^\mu = k^\nu - k'^\nu = (\nu, 0, 0, q), \quad q_{\mu}q^{\mu} = \nu^2 - q^2 = -Q^2.
\]
The leptonic current is defined as:
\[ J_{\text{lepton}}^\mu = \bar{u}(k')\gamma^\mu (1 - \gamma_5) u(k). \]  

In the RS model the leptonic mass is set to be zero. In this limit
\[ q_\mu J_{\text{lepton}}^\mu = 0. \]

One can introduce the basis of three vectors of length ±1 orthogonal to \( q^\mu \):
\[ e^\mu_L = \frac{1}{\sqrt{2}}(0, 1, -i, 0), \]
\[ e^\mu_R = \frac{1}{\sqrt{2}}(0, -1, -i, 0), \]
\[ e^\mu_S = \frac{1}{\sqrt{Q^2}}(q, 0, 0, \nu). \]

Correspondingly, the leptonic tensor can be decomposed as:
\[ L^{\mu\nu} = k^\mu k'^\nu + k^\mu k' - g^{\mu\nu} k \cdot k' - i\varepsilon^{\mu
u\alpha\lambda} k_\alpha k'_\lambda = \sum_{\alpha,\beta \in (S, L, R)} M^{\alpha\beta} e^\mu_{\alpha} (e^\nu_{\beta})^*. \]

When we calculate the contraction of the leptonic tensor with the hadronic tensor
\[ W_{\mu\nu} = \left( -g_{\mu\nu} W_1 + \frac{p_\mu p_\nu}{M^2} W_2 - \frac{i\varepsilon_{\mu\nu\sigma\beta} p^\sigma q^\beta}{2M^2} W_3 \right), \]

\((M\text{ is the nucleon mass})\) we find that
\[ L^{\mu\nu} W_{\mu\nu} = L^{\mu\nu}_{\text{diag}} W_{\mu\nu}, \]

where
\[ L^{\mu\nu}_{\text{diag}} = A^2 e^\mu_S (e^\nu_S)^* + B^2 e^\mu_L (e^\nu_L)^* + C^2 e^\mu_R (e^\nu_R)^*. \]

\(A^2, B^2, C^2\) are Lorentz scalars which can be evaluated in the LAB frame:
\[ A^2 = L_{\mu\nu} e^\mu_S (e^\nu_S)^* = \frac{Q^2}{2q^2} ((2E - \nu)^2 - q^2), \]
\[ B^2 = L_{\mu\nu} e^\mu_L (e^\nu_L)^* = \frac{Q^2}{4q^2} (2E - \nu + q)^2, \]
\[ C^2 = L_{\mu\nu} e^\mu_R (e^\nu_R)^* = \frac{Q^2}{4q^2} (2E - \nu - q)^2. \]

FIG. 3: Dependence of the Nachtmann variable \( \xi \) on hadronic invariant mass calculated at \( Q^2 = 0.4, 1, 3 \) and 10 GeV\(^2\).
Decomposition of the leptonic tensor entails the decomposition of the cross section into three contributions $\sigma_L, \sigma_R, \sigma_S$ which are interpreted as cross sections of intermediate boson in given polarization states scattered off nucleon. In its final form the Rein-Sehgal formula for the cross section reads:

$$d^2\sigma = \frac{G_F^2 \cos^2 \theta_C}{4\pi^2} \left( \frac{Q^2}{q^2} \right) \kappa \left( u^2 \sigma_L + v^2 \sigma_R + 2uv \sigma_S \right) dQ^2 \, d\nu,$$

(12)

where

$$u^2 = B^2 \frac{q^2}{E^2 Q^2}, \quad v^2 = C^2 \frac{q^2}{E^2 Q^2}, \quad uv = A^2 \frac{q^2}{E^2 Q^2}, \quad \kappa = \nu - \frac{Q^2}{2M}.$$  

Thus we can write:

$$d^2\sigma = \frac{G_F^2 \cos^2 \theta_C}{4\pi^2 E^2} \kappa \left( B^2 \sigma_L + C^2 \sigma_R + A^2 \sigma_S \right) dQ^2 \, d\nu.$$

(13)

$\sigma_{L,R,S}$ are then calculated within the quark model and are given in the explicit way for each SPP channel separately.

In order to identify $\sigma_{L,R,S}$ as linear combinations of the structure functions we calculate:

$$L_{\mu\nu} W^{\mu\nu} = A^2 \left( W_2 \frac{q^2}{Q^2} - W_1 \right) + B^2 \left( W_1 + W_3 \frac{q}{2M} \right) + C^2 \left( W_1 - W_3 \frac{q}{2M} \right)$$

and

$$\frac{d^2\sigma}{d\nu \, dQ^2} = \frac{G_F^2 \cos^2 \theta_C}{4\pi^2 E^2} \left[ A^2 \left( W_2 \frac{q^2}{Q^2} - W_1 \right) + B^2 \left( W_1 + W_3 \frac{q}{2M} \right) + C^2 \left( W_1 - W_3 \frac{q}{2M} \right) \right].$$

(14)

A simple comparison gives the following identification of the $\sigma'$s in terms of $W$'s:

$$-W_1 + W_2 \frac{q^2}{Q^2} = \frac{\kappa}{\pi} \sigma_S,$$  

(15)

$$W_1 + W_3 \frac{q}{2M} = \frac{\kappa}{\pi} \sigma_L,$$  

(16)

$$W_1 - W_3 \frac{q}{2M} = \frac{\kappa}{\pi} \sigma_R.$$  

(17)

Using the definition of the structure functions $F'$s we write down the final expressions for the RS model structure functions:

$$F_1^{RS} = MW_1 = M \frac{\kappa}{2\pi} (\sigma_L + \sigma_R),$$  

(18)

$$F_2^{RS} = \nu W_2 = \nu \frac{\kappa}{2\pi} \frac{Q^2}{q^2} (2\sigma_S + \sigma_L + \sigma_R),$$  

(19)

$$F_3^{RS} = \nu W_3 = \nu \frac{\kappa}{\pi} \frac{M}{q} (\sigma_L - \sigma_R).$$  

(20)

### 2.2. DIS structure functions

We apply the simple model for the DIS structure functions: they are given by appropriate linear combinations of the parton distribution functions (PDFs). In the kinematical region we are interested in the charm contribution can be neglected. For CC $\nu N$ interaction the DIS structure functions are $^{27}$:

$$F_2^{CC} (\nu p) = 2x \left( d \cos^2 \theta_c + s \sin^2 \theta_c + \bar{u} \right),$$  

(21)

$$xF_2^{CC} (\nu p) = 2x \left( d \cos^2 \theta_c + s \sin^2 \theta_c - \bar{u} \right),$$  

(22)

$$F_2^{CC} (\nu n) = 2x \left( u \cos^2 \theta_c + s \sin^2 \theta_c + \bar{d} \right),$$  

(23)

$$xF_2^{CC} (\nu n) = 2x \left( u \cos^2 \theta_c + s \sin^2 \theta_c - \bar{d} \right).$$  

(24)
For $\nu N$ NC interaction the DIS structure functions are:

\[ F_N^2(\nu p) = 2x \left( (g_L^2 + g_R^2) (u + \bar{u}) + (g_L^2 + g_R^2) (d + \bar{d} + 2s) \right), \tag{25} \]
\[ xF_N^3(\nu p) = 2x \left( (g_L^2 - g_R^2) (u - \bar{u}) + (g_L^2 - g_R^2) (d - \bar{d}) \right), \tag{26} \]
\[ F_2^NC(\nu n) = 2x \left( (g_L^2 + g_R^2) (d + \bar{d}) + (g_L^2 + g_R^2) (u + \bar{u} + 2s) \right), \tag{27} \]
\[ xF_3^NC(\nu n) = 2x \left( (g_L^2 - g_R^2) (d - \bar{d}) + (g_L^2 - g_R^2) (u - \bar{u}) \right). \tag{28} \]

with

\[ g_L = \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W, \quad g_R = -\frac{2}{3} \sin^2 \theta_W, \tag{29} \]
\[ g'_L = -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W, \quad g'_R = \frac{1}{3} \sin^2 \theta_W. \tag{30} \]

Wherever we present the plots of $F_1$ we use the Callan-Gross relation:

\[ F_2 = 2xF_1. \tag{31} \]

In the quantitative analysis we restrict ourselves to $F_2$ and $xF_3$ only.

We use GRV94 (LO) PDF’s [28] which are defined for $Q^2 > 0.23$ GeV$^2$ and $x \geq 10^{-5}$ and distinguish valence and sea quark contributions. GRV94 PDF’s are used in many Monte Carlo generators of events [7].

### 2.3. 1-pion functions

The 1-pion functions are defined for each SPP channel separately as probabilities that at a given value of $W$ the final hadronic state is that of SPP:

\[ f_1 \pi (W) \equiv \frac{d\sigma^{SPP}}{dW} \frac{d\sigma^{DIS}}{dW}. \tag{32} \]

The 1-pion functions used in this paper were obtained from the Monte Carlo simulation based on the LUND algorithm. Therefore they are defined by fragmentation and hadronization routines implemented there. The comparison of the 1-pion functions with elasticity factors of resonances included in the RS model is shown in Fig. 1. The agreement is satisfactory. It was also checked that the simulations based on LUND give rise to charged hadron multiplicities consistent with the experimental data [33].

We assume that the following relation holds between the structure functions of the RS model and the overall structure functions $F_j^{RES}$ for the inclusive cross section:

\[ F_j^{RES}(x, Q^2) = \frac{F_j^{RS}(x, Q^2)}{f_1 \pi (W(x, Q^2))}, \tag{33} \]

where $j = 1, 2, 3$. In the case of neutron (CC reactions) and NC structure functions in the above formula we apply the sum of the 1-pion functions for two SPP channels.

We will see that our results are sensitive to the details of the 1-pion functions in the region of $W \in (1.5, 2)$ GeV where the rescaling effects are most important. We investigated this point in more detail:

i) We extracted the 1-pion function from $\gamma p$ photo-production data [17], see Fig. 2. We conclude that it is rather similar to the function we used in our numerical computations.

ii) We tried to evaluate the 1-pion functions from available hadron multiplicities data in neutrino reactions [16] extrapolating the predictions to the region $W \in (2, 3)$ GeV.

We know the average charged hadron multiplicities
\[ \langle n_{ch} \rangle_{\nu p} = -0.05 \pm 0.11 + (1.43 \pm 0.04) \ln(W^2), \]

\[ \langle n_{ch} \rangle_{\nu n} = -0.2 \pm 0.07 + (1.42 \pm 0.03) \ln(W^2), \]

and the neutral pion multiplicity measured in \( \nu p \) reactions:

\[ \langle n_{\pi^0} \rangle_{\nu p} = 0.14 \pm 0.26 + (0.5 \pm 0.08) \ln(W^2). \]

We assume KNO distribution of multiplicities and that in the first approximation there are only nucleons and pions in the final state. For \( \nu p \) reaction it is also necessary to make some assumptions about charged pions. We expect that the fraction of charged pions is the same as in the \( \mu p \) reaction for which the experimental data is available.

We obtained the following values for the 1-pion functions at \( W = 2 \) GeV: 0.14 for \( \nu p \) and 0.38 for \( \nu n \). The obtained values must be reduced by \( \sim 10 \% \) due to the presence of other exclusive channels. We conclude that the KNO results seems to be in general agreement with basic properties of our 1-pion functions: the fraction of SPP channels on neutron is bigger than on the proton SPP and the orders of magnitude are quite similar.

### 2.4. Kinematics

In the QH duality analysis different kinematical regions are simultaneously involved in the discussion. A common presentation of the duality is done by means of the comparison of plots of structure functions in the Nachtmann variable:

\[ \xi(x, Q^2) = \frac{2x}{1 + \sqrt{1 + 4x^2 M^2/Q^2}} \]  

which takes into account target mass corrections.

The resonance region is defined in terms of invariant hadronic mass as \( W \in (M + m_\pi, 2 \) GeV) which is the natural choice for the Rein-Sehgal model. Other options (\( W_{\text{max}} < 2 \) GeV) for the definition of the resonance region were considered in [29].

Fig. 3 illustrates the dependence of the Nachtmann variable on hadronic invariant mass at fixed values of \( Q^2 \). For typical \( Q^2 \) for the resonance region \( Q^2 \in (0.5, 3) \) GeV\(^2 \) one obtains the following domain in the Nachtmann variable: \( \xi \in (0.13, 0.76) \). This region in \( \xi \) when combined with \( Q^2_{\text{DIS}} = 5, 10, 20 \) GeV\(^2 \) corresponds to:

\[ Q^2 = 5 \text{ GeV}^2 \quad \rightarrow \quad W \in (1.3, 5.8) \text{ GeV}, \]  
\[ Q^2 = 10 \text{ GeV}^2 \quad \rightarrow \quad W \in (1.8, 8.2) \text{ GeV}, \]  
\[ Q^2 = 20 \text{ GeV}^2 \quad \rightarrow \quad W \in (2.5, 11.6) \text{ GeV}. \]  

In our numerical analysis we use \( Q^2_{\text{DIS}} = 10 \) GeV\(^2 \).

### 2.5. Quark-Hadron Duality

The QH duality is said to be present on the quantitative level if the following relation between resonance and scaling structure functions holds:

\[ \int_{\xi_{\text{min}}}^{\xi_{\text{max}}} d\xi F_1(R) \approx \int_{\xi_{\text{min}}}^{\xi_{\text{max}}} d\xi F_1(D). \]  

The above equation should hold for different values of \( Q^2_{\text{RES}} \) characteristic for the resonance production and for a fixed value of \( Q^2_{\text{DIS}} \). The region of integration – RES region – is defined to be identical with the resonance region of the RS model: \( W_{\text{min}} = M + m_\pi \) and \( W_{\text{max}} = 2 \) GeV, which is then translated into appropriate region in \( \xi \):

\[ \xi_{\text{min}} = \xi(W_{\text{max}}, Q^2_{\text{RES}}), \quad \xi_{\text{max}} = \xi(W_{\text{min}}, Q^2_{\text{RES}}). \]
FIG. 4: Uncertainties in due to different definitions of $Q_{DIS}^2$. Solid line corresponds to and dashed line to.

FIG. 5: Comparison of the Rein-Sehgal structure functions at $Q^2 = 0.4, 1$ and $2$ GeV$^2$ with the appropriate scaling functions at $Q_{DIS}^2=10$ GeV$^2$. In the first row $x_{F_1}$, $F_2$ and $x_{F_3}$ structure functions for CC neutrino-proton scattering are plotted. In the second row the structure functions for CC neutrino-neutron scattering are shown.

In the quantitative analysis we define ratios of two integrals over the resonance region:

$$
\mathcal{R} \left( f, Q_{RH}^2; g, Q_{DH}^2 \right) = \frac{\int_{\xi_{\text{min}}}^{\xi_{\text{max}}} d\xi f(\xi, Q_{RH}^2)}{\int_{\xi_{\text{min}}}^{\xi_{\text{max}}} d\xi g(\xi, Q_{DH}^2)}
$$

(40)
Using the above quantity, we define the function:

\[ R_2(Q_{RES}^2, Q_{DIS}^2) \equiv R(F_{RES}^{2}, Q_{RES}^2; F_{DIS}^{2}, Q_{DIS}^2). \] (41)

and

\[ R_3(Q_{RES}^2, Q_{DIS}^2) \equiv R(xF_{RES}^{3}, Q_{RES}^2; xF_{DIS}^{3}, Q_{DIS}^2). \] (42)

There is an ambiguity in the quantitative analysis of duality because the above functions depend on the arbitrarily chosen value of \( Q_{DIS}^2 \). The differences between scaling curves calculated at different \( Q^2 \) are not relevant for making qualitative statements about the duality but do matter in quantitative analysis. To illustrate this we calculate

\[ R_{10/5} \equiv R(F_{DIS}^{2}, Q_{DIS}^2 = 10; F_{DIS}^{2}, Q_{DIS}^2 = 5), \] (43)

and

\[ R_{10/20} \equiv R(F_{DIS}^{2}, Q_{DIS}^2 = 10; F_{DIS}^{2}, Q_{DIS}^2 = 20), \] (44)

the integration region of the above integrals is defined by \( Q_{RES}^2 \). The results are shown in Fig. 4. We see that the ambiguity is of the order of 7% and is largest for \( Q_{RES}^2 \sim 1.5 \text{ GeV}^2 \). This limits the precision of the quantitative statements about the QH duality.

In the investigation of two-component duality we single out resonant and non-resonant contributions to the RS model structure functions

\[ F_{j}^{RES} = F_{j, res} + F_{j, nonres}. \] (45)
and we also separate valence and sea quark contributions to the DIS structure functions:

\[ F_{j}^{\text{DIS}} = F_{j,\text{sea}} + F_{j,\text{val}}. \]  

We calculate the following functions:

\[ R_{2}^{\text{val}}(Q_{\text{RES}}^{2}, Q_{\text{DIS}}^{2}) \equiv R(F_{2,\text{res}}, Q_{\text{RES}}^{2}; F_{2,\text{val}}, Q_{\text{DIS}}^{2}). \]  

and

\[ R_{3}^{\text{val}}(Q_{\text{RES}}^{2}, Q_{\text{DIS}}^{2}) \equiv R(xF_{3,\text{res}}, Q_{\text{RES}}^{2}; xF_{3,\text{val}}, Q_{\text{DIS}}^{2}). \]

3. NUMERICAL RESULTS AND DISCUSSION

In the numerical analysis we confine ourselves to the case of neutrino interactions and leave out the antineutrino ones.

In Figs. 5 - 7 we present a comparison of the scaling structure function with the RS structure functions calculated at \( Q_{\text{RES}}^{2} = 0.4, 1 \) and 2 GeV\(^2\). The Figs. 5 and 6 correspond to CC and NC reactions respectively with proton structure functions in the upper row and neutron structure functions below.

In the case of the RS model for neutrino-proton CC reaction the \( \Delta \) resonance contribution dominates overwhelmingly over other resonances. One can see the typical manifestation of local duality: the sliding of the \( \Delta \) peaks (calculated at different \( Q_{\text{RES}}^{2} \)) along the scaling function.

For neutrino-neutron CC reaction the resonance structure is much richer. The contributions from the \( \Delta \) are usually dominant but those from more massive resonances are also significant. In the figure with the \( F_{2} \) structure function three peaks of comparable size are seen. The DIS contributions dominate over the RS ones in this case.
Our plots are comparable with those obtained in Ref. [20]. However in our paper the strengths of $\Delta$ peaks for all the structure functions are proportionally lower. The difference between structure functions based on CTEQ6 [20] and GRV94 PDF’s is very small.

It is seen that it is virtually impossible to have simultaneously duality in CC reactions on proton and neutron targets. The strength of $\Delta$ excitation on proton is approximately three times as big as for neutron and DIS cross section on neutron is much bigger than on the proton.

For the NC reactions the dominant $\Delta$ peaks also slide along scaling function. The structure functions for the proton and neutron are almost the same.

In Fig. 7, the analysis is performed for the isoscalar target. The plots of the structure functions for CC (upper row) and NC (lower row) reactions are presented. In each plot the $\Delta$ peak slides along the scaling function and the local duality is seen.

Apparently (with the exception for CC reaction on proton) there is little hope for the QH duality in the whole resonance region: the scaling structure functions are on average larger than the RS ones. Only the local duality is present after a suitable region in $W$ around $M_{\Delta} = 1.23$ GeV is chosen. But in the Figs. 5–7 the rescaling of RS structure functions by means of the 1-pion functions (see Eq. (33)) is not yet included. The rescaling procedure increases the RES structure functions making the duality more likely to appear. We notice also that the rescaling cannot spoil the statements about the local duality around the $\Delta$ resonance because the values of the 1-pion function for $W$ in the vicinity of $M_{\Delta}$ are close to 1 (see Fig. 1).

In Fig. 8 we show how the resonance structure functions are modified by means of the 1-pion functions. The modifications apply mainly to hadronic invariant mass close to 2 GeV.

In order to perform a quantitative analysis of the duality we make use of the functions $R_i$ defined in Eqs. (41–48). We restrict our plots to the values of $Q^2_{RES} \leq 3$ GeV$^2$ characteristic for the resonance production.

In what follows we use the RES structure functions rescaled by means of the 1-pion functions. We have checked that introduction of the 1-pion function improves the duality significantly. For example for CC reaction on neutron $R_2$ is
increased by a factor of $\sim 1.55$ and for proton by $\sim 1.39$. The difference is caused by the overwhelming dominance of the $\Delta$ excitation in the case of proton.

A characteristic feature of most of the plots of $R_j(Q^2_{\text{RES}})$ is a presence of two qualitatively distinct behaviors. For $Q^2_{\text{RES}}$ smaller then $\sim 0.5$ GeV$^2$ the functions $R_j$ vary quickly while for larger values of $Q^2_{\text{RES}}$ they become slowly changing. This seems to correspond to predictions done in [5]. Our statements about the duality will apply only to the region of $Q^2_{\text{RES}} \geq 0.5$ GeV$^2$.

In Figs. 9 and 10 the plots of $R_2$ and $R_3$ for proton, neutron and isoscalar targets are presented. In the case of CC interaction the duality is seen on the proton target (accuracy $\leq 20\%$) but for the neutron and isoscalar targets the duality is absent. In both cases the average strength of resonance structure functions amounts to only about a half of the strength of DIS structure functions. The plots for the NC interactions are almost independent on the target and in all the cases the DIS contributions are approximately two times as big as resonance ones. A different choice of $Q^2_{\text{DIS}}$, namely $Q^2_{\text{DIS}} = 20$ GeV$^2$ makes the values of $R_{2,3}$ even lower (see Fig. 11).

The remaining plots address the question of two component duality. We concentrate on the case of the possible duality between the resonance and valence quark contributions.

In Fig. 11 the plot of $R^\text{val}_2$ for the CC interactions is shown. We notice the good duality picture in the case of proton target but a huge departure from duality in the case of neutron and isoscalar targets. It is worth noting that this discrepancy is larger than one shown in Fig. 9 where the general (not two component) notion of duality was discussed. The novel feature is the apparently singular behavior at low $Q^2_{\text{RES}}$: $R^\text{val}_2$ rises quickly in contrast with $R_2$ falling down when $Q^2_{\text{RES}}$ approaches zero.

The explanation of this follows from the Fig. 12 where the region of small $Q^2_{\text{RES}}$ was analyzed in more detail. We notice that for $Q^2_{\text{RES}}$ approaching zero the valence quarks scaling function tends to zero while the resonance strengths remains virtually unchanged.

Finally in Fig. 13 the analogous two-component duality analysis is done for $R^\text{val}_3$. The discussion of $xF_3$ seems to be favorable for the two-component duality because in the DIS contribution on the isoscalar target there is no sea quark contribution. We remind also that for the CC reaction on the proton the non-resonant contribution is absent.
In Fig. 13 we see that two component duality is satisfied within $\sim 30\%$ for the proton target but it is absent for neutron and isoscalar targets. We notice also that contrary to what we have seen in the plots for $R_{2}^{val}$ now at low $Q_{RES}^2$ all the curves tend to zero.

The explanation of this behavior follows from the Fig. 14. One can see that in the case of $xF_3$ both the resonance and valence quark structure functions fall down for $Q^2$ approaching zero. The behavior of $xF_3$ is the same as that discussed in [19].

We do not present plots exploring the duality between the non-resonant part of the resonance model and the sea quark contribution. No sign of two component duality is seen in this case.
4. CONCLUSIONS

The quark-hadron duality in the $\nu N$ reactions has been investigated by comparing the structure functions obtained from the Rein-Sehgal model and those from the deep inelastic formalism. The 1-pion functions were used to construct comparable quantities. The qualitative analysis was based on the plots of the RS and the DIS structure functions at several values of $Q^2_{RES}$ while the quantitative one was based on the functions $R_j$ defined as the ratios of the corresponding integrals.

We are aware that our model of resonance structure functions is a subject of several uncertainties. Here is a list of them:
FIG. 14: Comparison of the isoscalar CC Rein-Sehgal $x F_3$ structure functions at $Q^2 = 0.05, 0.1, 0.2$ and $0.4$ GeV$^2$ with appropriate valence quark contribution to scaling function (solid line) calculated at $Q^2_{DIS} = 10$ GeV$^2$.

FIG. 15: The ratios presented in Fig. 9 for isoscalar target are calculated without 1-pion functions but with all the elasticities equal to 1 with amplitudes added in either coherent or incoherent way.

a) It is possible that the structure functions extracted from the RS model are underestimated for $W \geq 1.7$ GeV where tails of heavier resonances are not included [24].

b) In the original RS model the non-resonant background is treated in not a satisfactory way. In particular the shape of the non-resonant background does not seem to agree with the precise electro-production fits presented in Ref. [31].

c) Our conclusions depend on the precision with which we reconstructed 1-pion functions. Therefore we decided also to investigate the duality with no assumptions about the 1-pion functions but rather with overall cross sections for the resonance production (i.e. under the assumption that all the elasticities are equal 1). We considered two cases: (i) the resonances are added incoherently and (ii) the interference patterns are the same as in SPP channels. The typical results are shown in Fig. 15: predictions of all three models turn out to be similar.
Our main conclusions can be summarized as follows:

1) for $Q^2 \geq 0.5$ GeV$^2$ the duality is present in the whole resonance region $W \in (M + m_\pi, 2$ GeV) with an accuracy of $\sim 20\%$ only for CC proton target reaction. We remind that the way in which the duality is defined carries an uncertainty of $\sim 5\%$;

2) from the Figs. 5 - 7 it follows that there is also a local duality for isoscalar target in the case of CC reaction and for all the targets in the case of NC reactions in a suitable chosen vicinity of the $\Delta$ resonance.

The results obtained in this paper can be useful for the investigation of the question how to modify DIS structure functions in the low $Q^2$ resonance region so that they provide a good average description $^2$. Such modifications should be confronted with available neutrino scattering data from CHORUS, NOMAD and NuTeV experiments $^3$.

**Acknowledgements**

The authors (supported by the KBN grant: 105/E-344/SPB/ICARUS/P-03/DZ211/2003-2005) thank Jaroslaw Nowak and Olga Lalakulich for helpful conversations.