Cosmology with running parameters

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Abstract. The experimental evidence that the equation of state (EOS) of the dark energy (DE) could be evolving with time/redshift (including the possibility that it might behave phantom-like near our time) suggests that there might be dynamical DE fields that could explain this behavior. We propose, instead, that a variable cosmological term (including perhaps a variable Newton’s gravitational coupling too) may account in a natural way for all these features.

1. Introduction
The accelerated expansion of the universe is nowadays one of the central issues of observational and theoretical cosmology. The usual paradigm assumes the existence of the so-called dark energy (DE)– a mysterious cosmic component with negative pressure. An obvious candidate for the role of DE is the cosmological constant (CC) [1]. Others are the dynamical DE models [2], e.g. quintessence [3], phantom energy [4] etc. Here we generalize the cosmological constant concept allowing the CC term \( \rho_\Lambda \) (with dimensions of energy density) and possibly the gravitational coupling (\( G \)) being variable with the cosmic time, both assumptions compatible with the cosmological principle. The support for such a generalization comes from quantum field theory on the curved space-time [5, 6, 7, 8, 9, 10] and/or quantum gravity approaches [11].

2. Two cosmological pictures: standard DE versus variable cosmological term
The usual dark energy picture assumes the existence of two separately conserved cosmological components, the matter-radiation component and the DE component. Not necessarily so in the variable \( \rho_\Lambda/G \) picture (for short, CC picture). The general Bianchi identity of the Einstein tensor leads to \( \nabla^\mu [G(T_{\mu\nu} + g_{\mu\nu}\rho_\Lambda)] = 0 \), which for FRW metric implies

\[
\frac{d}{dt} [G(\rho + \rho_\Lambda)] + 3GH(\rho + p) = 0.
\]  

This “mixed” conservation law connects the variation of \( \rho_\Lambda \), \( G \) and \( \rho \), and hence the evolution of the matter density \( \rho \) may be noncanonical. A number of variable CC models of various kinds [12], and the renormalization group (RG) models of running \( \rho_\Lambda \) and \( G \) [5, 6, 9, 10, 11] provide the basis for the variability of these cosmological parameters. The general expression for the Hubble parameter in the CC picture in the matter (\( \alpha = 3 \)) or radiation (\( \alpha = 4 \)) epochs is

\[
H^2_{\text{CC}}(z) = H^2_0 \left[ \Omega^0_M f_M(z)(1 + z)^\alpha + \Omega^0_\Lambda f_\Lambda(z) \right].
\]  

Here \( f_M \) and \( f_\Lambda \) are certain functions of redshift (see e.g. the RG cosmological models [10, 13]), with \( f_M(0) = 1 \) and \( f_\Lambda(0) = 1 \), in accordance with the cosmic sum rule \( \Omega^0_M + \Omega^0_\Lambda = 1 \).
3. Matching of pictures and effective dark energy equation of state

The two pictures presented in the preceding section are assumed to be equivalent descriptions of the same cosmological evolution. Their matching requires that the expansion history of the universe is the same in both pictures, i.e. that their Hubble functions are equal: \( H_{\text{DE}} = H_{\text{CC}} \).

The general Bianchi identity \( (\text{II}) \) then leads to an effective EOS parameter \( \omega_e = p_D/\rho_D \) for the CC picture, given by

\[
\omega_e(z) = -1 + \frac{\alpha}{3} \left( 1 - \frac{\xi_A(z)}{\rho_D(z)} \right),
\]

where \( \xi_A(z) \equiv (G(z)/G_0) \rho_A(z) \). The effective DE density in the CC picture can be cast as

\[
\rho_D(z) = \xi_A(z) - (1 + z)^\alpha \int_{z^*}^z \frac{dz'}{(1 + z')^\alpha} \frac{d\xi_A(z')}{dz'}. \tag{4}
\]

Here \( z^* \) is a redshift where \( \xi_A(z^*) = \rho_D(z^*) \). From \( (3) \) it is clear that \( \omega_e \) crosses the \( \omega_e = -1 \) “barrier” just at \( z = z^* \). Quite remarkably, one can show that a value \( z^* \) always exists near our present time: namely in the recent past, immediate future or just at \( z^* = 0 \). The proof of this claim is obtained by straightforward calculation starting from the matching condition and the conditions that the general Bianchi identity \( (\text{II}) \) imposes on functions \( f_A \) and \( f_M \) in \( (\text{II}) \). Let us compute the slope of the \( \rho_D \) function \( (\text{II}) \):

\[
\frac{d\rho_D(z)}{dz} = -\alpha (1 + z)^{\alpha - 1} \int_{z^*}^z \frac{dz'}{(1 + z')^\alpha} \frac{d\xi_A(z')}{dz'}. \tag{5}
\]

This compact expression reveals some counterintuitive and general aspects of the effective DE density evolution for a variable CC model in which \( \xi_A(z) \) is a monotonous function of \( z \). Thus, for \( \xi_A(z) \) decreasing with \( z \), \( \rho_D \) behaves like quintessence for \( z > z^* \), whereas for \( z < z^* \) it behaves phantom-like! (For a concrete framework, see Section \( (\text{III}) \) and Fig. \( (\text{II}) \).) Especially interesting results are obtained when in the variable CC model the matter component \( \rho \) is separately conserved. In this case we have \( d\xi_A/dt = -(\rho/G_0) dG/dt \), which results in the following expression for the slope of \( \rho_D \):

\[
\frac{d\rho_D}{dz} = \alpha (1 + z)^{\alpha - 1} \rho(0)/G_0 \left[ G(z) - G(z^*) \right]. \tag{6}
\]

Thus in this case the properties of \( \rho_D \) depend only on the scaling of \( G \) with redshift, e.g. if \( G \) is asymptotically free and \( z^* > z \), then \( \rho_D \) behaves effectively as quintessence.

4. Effective dark energy picture of a running \( G \) and \( \rho_A \) model

As an illustration of the aforementioned procedure for obtaining the effective dark energy properties of a variable CC model, in this section we present the analysis of the renormalization group model of \( (\text{III}) \) characterized by \( G = \text{const} \) and the CC evolution law \( \rho_A = C_1 + C_2 H^2 \).

Here \( C_1 = \rho_{\Lambda,0} - (3\nu H_0^2)/(8\pi G) \) and \( C_2 = (3\nu)/(8\pi G) \), where \( \nu \) is the single free parameter of the model — a typical value is \( |\nu| = \nu_0 \equiv 1/12\pi \). In this particular model \( \xi_A(z) = \rho_A(z) \). Therefore for the flat universe case the effective parameter of EOS of this running \( \rho_A(z) \) model one finds

\[
\omega_e(z) |_{\Delta\Omega \neq 0} = -1 + (1 - \nu) \frac{\Omega_M^0 (1 + z)^{3(1 - \nu)} - \tilde{\Omega}_M^0 (1 + z)^3}{\Omega_M^0 [(1 + z)^{3(1 - \nu)} - 1] - (1 - \nu) [\tilde{\Omega}_M^0 (1 + z)^3 - 1]}. \tag{7}
\]

Here \( \Delta\Omega_M \equiv \Omega_M^0 - \tilde{\Omega}_M^0 \neq 0 \) is the difference of parameters in the two pictures, corresponding to two different fits of the same data. For \( |\nu| \ll 1 \) we may expand the previous result in first order in \( \nu \). Assuming (conservatively) that \( \Delta\Omega_M = 0 \) we find

\[
\omega_e(z) \simeq -1 - 3 \nu \frac{\Omega_M^0 (1 + z)^3 \ln(1 + z)}{\Omega_A^0 (1 + z)^3}. \tag{8}
\]
This result reflects the essential qualitative features of the analysis presented in the previous section, where in this case $z^* = 0$. For $\nu > 0$, Eq. (5) shows that we can get an (effective) phantom-like behavior ($\omega_e < -1$), and for $\nu < 0$ we can have (effective) quintessence behavior. We see that this variable CC model can give rise to two types of very different behaviors by just changing the sign of a single parameter. In Fig.1 we show a more general case where $z^* \gtrsim 0$, corresponding to $\Delta \Omega_M \neq 0$. In contrast to the previous situation, here the variable CC model may exhibit phantom behavior for $\nu < 0$ (if $\Delta \Omega_M < 0$), and manifests itself through the existence of a transition point $z^*$ in our recent past – marked explicitly in the figure.

5. Conclusions
We have shown that a model with variable $\rho_\Lambda$ (and may be also with variable $G$) generally leads to a non-trivial effective EOS; the model can effectively appear as quintessence, and even as phantom energy, without need of invoking any combination of fundamental quintessence and phantom fields. This possibility should be taken into account in the next generation of high precision cosmology experiments aiming to determine the effective EOS of the DE.

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References
[1] For a review, see e.g. Weinberg S 1989 Rev. Mod. Phys. 61 1.