Inflation without inflatons

Reuven Opher and Ana Pelinson

IAG, Universidade de São Paulo, Rua do Matão, 1226
Cidade Universitária, CEP 05508-900. São Paulo, S.P., Brazil

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We present a model which predicts inflation without the presence of inflaton fields, based on the \( \epsilon R^2 \) and Starobinsky models. It links the above models to the reheating epoch with conformally coupled massive particles created at the end of inflation. In the original Starobinsky model, the reheating era was created by massless non-conformally coupled particles. We assume here that non-conformal coupling to gravitation does not exist. In the \( \epsilon R^2 \) model, inflation is produced by the gravitational Lagrangian to which a term \( \epsilon R^2 \) is added, where \( \epsilon \) is a constant and \( R \) is the Ricci scalar. Inflation is created by vacuum fluctuations in the Starobinsky model. Both models have the same late-inflation time-dependence, which is described by a characteristic mass \( M \). There is a free parameter \( H_0 \) on the order of the Planck mass \( M_{\text{Pl}} \) that determines the Hubble parameter near the Planck epoch and which depends upon the number and type of particles creating the vacuum fluctuations in the Starobinsky model. In our model, we assume the existence of particles with a mass \( m \), on the order of \( M \), conformally coupled to gravity, that have a long decay time. Taking \( m \equiv FM \), we investigate values of \( F = 0.5 \) and 0.3. These particles, produced \( \sim 60 \) e-folds before the end of inflation, created the nearly scale invariant scalar density fluctuations which are observed. Gravitational waves (tensor fluctuations) were also produced at this epoch. At \( t_{\text{end}} \), the Hubble parameter begins to oscillate rapidly, gravitationally producing the bulk of the \( m \) particles, which we identify as the origin of the matter content of the Universe today. The time required for the Universe to dissipate its vacuum energy into \( m \) particles is found to be \( t_{\text{dis}} \simeq 6 M_{\text{Pl}}^2 / M^3 F \). We assume that the reheating time \( t_{\text{RH}} \) needed for the \( m \) particles to decay into relativistic particles, is very much greater than that necessary to create the \( m \) particles, \( t_{\text{dis}} \). A particle physics theory of \( m \) can, in principle, predict their decay rate \( \Gamma_{mr} \equiv t_{\text{dis}}^{-1} t_{\text{RH}} ^{-1} \). From the ratio \( f \equiv t_{\text{dis}} / t_{\text{RH}} \), \( F \) and \( g_\ast \) (the total number of degrees of freedom of the relativistic particles) we can, then, evaluate the maximum temperature of the Universe \( T_{\text{max}} \) and the reheat temperature \( T_{\text{RH}} \) at \( t_{\text{RH}} \). From the observed scalar fluctuations at large scales, \( \delta \rho / \rho \sim 10^{-5} \), we have the prediction \( M \simeq 1.15 \times 10^{-6} M_{\text{Pl}} \) and the ratio of the tensor to scalar fluctuations, \( r \simeq 6.8 \times 10^{-4} \). Thus our model predicts \( M, t_{\text{dis}}, t_{\text{end}}, T_{\text{max}}, T_{\text{RH}}, t_{\text{max}} \), and \( t_{\text{RH}} \) as a function of \( f, F \), and \( g_\ast \) (and to a weaker extent the particle content of the vacuum near the Planck epoch). A measured value of \( r \) that is appreciably different from \( r = 6.8 \times 10^{-4} \) would discard our model (as well as the Starobinsky and \( \epsilon R^2 \) models).

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I. INTRODUCTION

The standard model of inflation, based on the existence of a scalar inflaton field, makes the following assumptions:

1. The beginning of inflation occurs at an energy \( \ll M_{\text{Pl}} \) (Planck energy). Its origin is unknown and the state of the Universe before the beginning of inflation is undefined;

2. A large initial displacement of \( \phi \) from the minimum of \( V(\phi) \) is necessary for the onset of inflation (such as in the chaotic inflation model);

3. The potential energy of the inflaton dominates its
kinetic energy during inflation; and

4. The inflaton potential, $V(\phi)$, and its first derivative are defined by observations $\sim 60\,e$-folds before the end of inflation.

In complex inflation theories, there can be more than one inflaton. (See [1] for a recent review of inflation theory with inflatons.)

Here we present a model which links the Starobinsky and $\epsilon R^2$ models of inflation, where $R$ is the Ricci scalar to the reheating era. All three models, the Starobinsky, $\epsilon R^2$ and ours, do not involve inflatons to create inflation. They also avoid most of the above assumptions.

In the Starobinsky model, an $R^2$ term in the effective Lagrangian dominates inflation at late times (see also [2, 3]). There is no sharp boundary between the Starobinsky model and $\epsilon R^2$ model since the latter is the particular case of the former in the limit $M \ll H_0$ (using the notation of Eq.(9), with some small non-local terms (due to non-zero rest masses of conformally coupled quantum fields) omitted). However, the Starobinsky and $\epsilon R^2$ models have the same qualitative behavior at $\sim 60\,e$-folds before the end of inflation, when the presently observed scalar and tensor fluctuations were produced.

Both the Starobinsky and $\epsilon R^2$ cosmologies are characterized by a single mass $M \equiv M_{Pl}/\sqrt{24\epsilon}$ in the $\epsilon R^2$ cosmology and $M_{Pl}/\sqrt{48\pi k_1}$ in the Starobinsky model, where $M_{Pl} \equiv G^{-1/2}$ is the Planck mass and $k_1$ is the coefficient of the term which contains the second derivative of $R$ in the quantum corrected vacuum expectation value of the energy-momentum tensor [Eq.(3)]. The mass $M$ characterizes the end of the inflation period, during which, the Hubble parameter varies slowly. A period then begins, in which $H$ oscillates rapidly as $H \propto (1/t) \cos^2 \omega t$ and the cosmological scale factor varies as $a(t) \propto t^{2/3}[1+\sin(2\omega t)/(3\omega t)]$, where $\omega \simeq M/2$. When averaged over several oscillations, the Universe expands as a classical matter-dominated Universe.

Although the $\epsilon R^2$ model can be considered to be the simplest way to produce inflation, i.e., by means of a simple modification of the gravitational Lagrangian, we concentrate here on the Starobinsky model since it is more complete. It links the beginning of the inflation period to the beginning of the Universe and also describes the end of inflation in detail.

The Starobinsky model suggests that for energy densities and curvatures near the Planck scale, quantum corrections to Einstein’s equations become important (as discussed in detail by Vilenkin [9]). In the Starobinsky model, inflation is driven by one-loop corrections due to quantized matter fields [2] (see also [6, 7, 8, 9, 10, 11]). The model is consistent with a Universe that was spontaneously created, as discussed by Grishchuk and Zel’dovich [12].

The beginning of the Starobinsky inflation period can be associated with the beginning of the Universe due to quantum fluctuations of the vacuum. Tryon [13] was the first to suggest that a closed Universe can be created spontaneously as a result of a quantum fluctuation. Vilenkin [14, 15], Zel’dovich and Starobinsky [16], and Linde [17] were the first to attempt to describe the quantum creation of a Universe in the framework of quantum gravity. The picture that emerges is one of a Universe tunneling quantum mechanically to a de Sitter space time. At the moment of nucleation ($t = 0$), the Universe has a size $a(0) = H_{in}^{-1}$. This is the beginning of time and, from that point on, the Universe evolves along the lines of the inflation scenario.

In the Starobinsky model, inflation is produced by the vacuum energy $\rho_V$, which has negative pressure, $P = -\rho_V$. Inflation in both the Starobinsky and our models can be described by an effective geometric scalar particle $M$. In our model, there is an additional massive particle $m$ produced at the end of inflation, which is freely moving and which produces positive pressure.

Structure in the Universe primarily comes from almost scale-invariant superhorizon curvature perturbations [18, 19]. In our model, a mass $m$ is much less than the Hubble parameter during inflation. The mechanism of $m$ particle production from inflation is based on the observation that particles that are massive in the present-day vacuum, could have been very light during inflation. This implies that fluctuations of a generic scalar field $\chi$ with mass $m \ll H$ during inflation are copiously generated, with an almost scale invariant spectrum [20, 21, 22].
The particles become heavy and non-relativistic at the end of inflation.

The end of the Starobinsky inflation period has been suggested to be due to the masses of the particles in the vacuum fluctuations [14]. Thus, since the mass $M$ describes the end of inflation in both the Starobinsky and our models, $M$ is a natural mass scale for the particles that are created at the end of inflation. In our model, we then have the scenario that particles of mass comparable to the mass $M$ in the vacuum fluctuations first create the inflation, after which, particles of mass $m$ comparable to, but slightly less than $M$, are produced from the vacuum due to the rapid change of the Hubble parameter. The particles $m$ are conformally coupled to gravitation (Ricci scalar). These massive particles create the reheating epoch of the Universe.

Our model can be compared with that of the Starobinsky model, in which massless particles, non-conformally coupled to gravitation, directly create the reheating era. Here we assume that non-conformal coupling does not exist. Previously, gravitational production of massive particles has been investigated in order to explain the observed ultra-high energy cosmic rays, produced as a result of heavy particle decay (masses $> 10^{12}$ GeV) [21]. The gravitational particle production of the heavy particles $m$ can be described assuming a given background metric [24, 25].

The paper is organized as follows. In section II, we give the main results of the Starobinsky model, as discussed by Vilenkin [5]. We discuss the gravitational production of the $m$ particles in section III. In section IV, we derive the scalar density fluctuations produced $\sim 60$ e-folds before the end of inflation. We obtain the ratio $r$ of the tensor to scalar fluctuations in section V. The reheating of the Universe is discussed in section VI. Our conclusions and discussion are presented in section VII.

II. THE STAROBINSKY INFLATIONARY MODEL

In this section, we discuss the Starobinsky model of inflation, following the description and notation of Vilenkin. This model is based on the semiclassical Einstein equations,

$$\nabla R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = -8\pi G < T_{\mu \nu} >,$$

which assume a spatially flat Robertson-Walker metric,

$$ds^2 = a^2(\eta)(d\eta^2 - dx^2),$$

where $dt = d\eta$, $t(\eta)$ is the proper (conformal) time and $a$ is the scale factor.

The quantum corrections to the expectation value of the energy-momentum tensor for massless particles in curved space time are

$$< T_{\mu \nu} > = k_1 (1) H_{\mu \nu} + k_3 (3) H_{\mu \nu},$$

where

$$(1) H_{\mu \nu} = 2 R_{\mu \nu} - 2 g_{\mu \nu} R = 2RR_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R^2,$$

$$(3) H_{\mu \nu} = R_{\mu \nu} - \frac{3}{2} R R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R^\sigma R_{\sigma \tau} + \frac{1}{4} g_{\mu \nu} R^2,$$

and $k_1$, $k_3$ are constants. The value of the coefficient $k_1$ can be determined by observations. However, the value of $k_3$ is fixed by the condition

$$k_3 = \frac{1}{1440 \pi^2} \left( N_0 + \frac{11}{2} N_{1/2} + 31 N_1 \right),$$

where $N_0$, $N_{1/2}$, and $N_1$ are the numbers of quantum matter fields with spins 0, 1/2, and 1, respectively.

It is convenient to define two new parameters, $H_0$ and $M$, in terms of $k_1$, $k_3$, and the Planck mass, $M_{Pl}$:

$$H_0 = \frac{M_{Pl}}{\sqrt{8\pi k_3}}$$

and

$$M = \frac{M_{Pl}}{\sqrt{48\pi k_1}}.$$

The Planck constant $h$ and the speed of light $c$ are given in natural units, $h = c = 1$, and $M_{Pl} = G^{-1/2} \approx 1.22 \times 10^{19}$ GeV. To evaluate $H_0$, a minimal SU(5) model, where $N_0 = 34$, $N_{1/2} = 45$, $N_1 = 24$, is assumed, giving

$$H_0 \cong 0.74 M_{Pl}.$$
From the above, the energy-momentum tensor in Eq. (9) is

\[ T_{\mu \nu} = \frac{1}{8\pi} \left[ \frac{1}{6} H^2 \left( \frac{M^2}{M} \right)^2 + \left( \frac{H}{M} \right)^2 \right]. \]  

Whereas \( T_{\nu}^{\nu} = 0 \) for classical conformally invariant fields, a trace anomaly arises from the process of regularization (see [26, 27, 28]),

\[ \left< T_{\nu}^{\nu} \right> = \frac{M^4}{8\pi H_0^2} \left[ \frac{1}{3} R^2 - R_{\mu\nu} R^{\mu\nu} - \left( \frac{H}{M} \right)^2 R_{\mu\nu}^{\mu\nu} \right]. \]  

The evolution equation for the Hubble parameter \( H(t) = \dot{a}/a \) in a flat Universe is

\[ H^2 (H^2 - H_0^2) = \left( \frac{H}{M} \right)^2 (2H \ddot{H} + 6H^2 \dot{H} - \dot{H}^2). \]  

Assuming that \( H \) is slowly varying during inflation, \( \dot{H} \ll H^2 \), and that \( \ddot{H} \ll H \dot{H} \), the solution of Eq. (11) is

\[ H = H_0 \tanh \left[ \gamma \frac{M^2 t}{6H_0} \right], \]  

where

\[ \gamma = 1/2 \ln[2/\delta_0], \]  

\[ \delta_0 = |H_{in} - H_0|/H_0, \]  

and \( H_{in} \) is the initial value of the Hubble parameter in the inflation era.

We see from Eq. (12) that \( H \) changes on a time scale,

\[ \tau \sim 6H_0/M^2. \]  

A long period of inflation occurs when \( M^2 \ll 6H_0^2 \). The solution of Eq. (12) in terms of \( a \), valid up to the time at the end of inflation, when \( H \sim M \), is

\[ t_{end} = \frac{6\gamma H_0}{M^2}, \]  

is

\[ a(t) = H_0^{-1} \left( \frac{\cosh \gamma}{\cosh[\gamma - t/\tau]} \right)^{H_0 \tau}, \]  

where \( \tau \) is given by Eq. (15).

Assuming that \( H \ll H_0 \) near the end of inflation, Eq. (11) simplifies to

\[ 2H \dddot{H} + 6H^2 \dot{H} - \dot{H}^2 + M^2 H^2 = 0, \]  

which has the solution

\[ H(t) \approx \frac{4}{3t} \cos^2 \left( \frac{Mt}{2} \right) \left( 1 - \sin \left( \frac{Mt}{M} \right) \right) \]  

and

\[ a(t) \propto t^{2/3} \left[ 1 + \frac{2}{3M^2t_0} \right] \sin \left[ \frac{Mt}{M} \right]. \]  

Thus, from Eqs. (14) and (20), \( H(t) \) and \( a(t) \) are in an oscillating phase at the end of inflation. At a time \( t_0 \gg M^{-1} \), the period of oscillation is much shorter than the average Hubble time, \( 2/3t_0 \). For a time interval \( t_0 \gg \Delta t \gg M^{-1} \), we can neglect the power law expansion in Eq. (20), so that

\[ a(t) \approx 1 + \left( \frac{2}{3M^2t_0} \right) \sin \left[ \frac{Mt}{M} \right]. \]  

In the Starobinsky model, massless particles are gravitationally produced at the end of inflation. For a scalar field of mass \( m \) that satisfies the equation

\[ \Box \phi + (m^2 + \xi R)\phi = 0, \]  

the field is conformally coupled if \( \xi = 1/6 \) and non-conformally coupled if \( \xi \neq 1/6 \). For \( m > M/2 \), the particle production is exponentially depressed. In a conformally flat spacetime, massless conformally coupled particles cannot be gravitationally produced [5, 29]. Therefore, these particles must be non-conformally coupled.

The oscillation term in Eq. (21) is small and can be considered to be a perturbation in the calculation of particle production. A perturbative technique for calculating the production of very low mass \( m \ll M \) particles was developed by Zel’dovich and Starobinsky [30] and Birrel and Davies [32], treating \( |\xi - 1/6| \) as a very small parameter. They showed that for a massless non-conformally coupled field \( (m = 0, \xi \neq 1/6) \), the particle production rate is

\[ \dot{n} = \frac{1}{16\pi} (\xi - 1/6)^2 R^2, \]  

where

\[ R \approx 6 \ddot{a} = - \left( \frac{4M}{t_0} \right) \sin \left[ \frac{Mt}{M} \right]. \]  

Taking the average of the particle production over the period of oscillation and using the fact that the particles
are produced in pairs with energy $m/2$ per particle, the average rate of energy loss is ($m = FM$)
\[
\bar{\rho} = \frac{FM}{2} \bar{n} = \frac{FM^3}{4\pi t_0^4} \left( \xi - \frac{1}{6} \right)^2 ,
\]  
(25)

The rate at which the vacuum energy is dissipated into particles is
\[
\Gamma \equiv \bar{\rho}/\rho = \frac{3FM^3}{2M_{Pl}^2} \left( \xi - \frac{1}{6} \right)^2 ,
\]  
(26)

where
\[
\bar{\rho} = \frac{t_0^2}{6\pi G}
\]  
(27)
is the energy density of the particles.

Since $\bar{\rho} \propto t^{2/3}$, we have
\[
\bar{\rho} = \frac{2}{3t_0} ,
\]  
(28)
so that Eq. (24) becomes
\[
\bar{\rho} = \frac{3}{8\pi G} \bar{H}^2
\]  
(29)
(the Friedmann relation).

### III. Gravitational Production of Particles

As discussed in section II and in detail by Vilenkin, massless particles ($\ll M$) are non-conformally produced at the end of inflation in the Starobinsky model [2]. These massless particles are assumed to reheat the Universe.

Although the main purpose of the analysis of Vilenkin was to evaluate the gravitational production of massless non-conformally coupled particles in the Starobinsky model [Eq. (20)], an expression for the conformally coupled gravitational production of massive $m$ particles was also obtained. We use this expression in our analysis of massive $m$ particles.

The created $\phi$ field [Eq. (22)] can be expanded in terms of creation and annihilation operators,
\[
\phi(x) = \int d^3k \left[ a_k u_k(x) + a_k^\dagger u_k^*(x) \right] ,
\]  
(30)

where
\[
u_k(x) = \frac{1}{(2\pi)^{3/2}} a^{-1}(t) e^{i\vec{k} \times \vec{x}} \chi_k(t)
\]  
(31)
and the functions $\chi_k(t)$ satisfy the equation for the field $\chi$ [26],
\[
\ddot{\chi}_k + k^2 \chi_k + \left[ m^2 + \left( \xi - \frac{1}{6} \right) R \right] a^2 \chi_k = 0 .
\]  
(32)

Linearizing Eq. (32) and using Eqs. (21) and (23), we have
\[
\ddot{\chi}_k + \omega_k^2 \chi_k - \left[ \frac{4}{3Mt_0} \right] \bar{m}^2 \sin [Mt] \chi_k = 0 ,
\]  
(33)
where
\[
\omega_k = (k^2 + m^2)^{1/2}
\]  
(34)
and
\[
m^2 = m^2 - 3 \left( \xi - \frac{1}{6} \right) M^2 .
\]  
(35)

If the main contribution to the particle production comes from the modes with $k \sim M/2$ and the mass $m \ll M/2$, we can replace $\omega_k^2$ with $k^2$ in Eq. (34). The expression of the production of $m$ particles for $\xi = 1/6$ (conformal production),
\[
\Gamma = \frac{G\bar{m}^4}{6M}
\]  
(36)
was presented in the original paper [2].

As noted by Vilenkin, although Eq. (36) was derived assuming $m \ll M$, it also gives a correct order of magnitude for the conformally coupled decay rate for $m \sim M$, ($m = FM$, $F = 0.3, 0.5$)
\[
\Gamma = \frac{F^4}{6} \frac{M^3}{M_{Pl}^2}
\]  
(37)

We can compare the time it takes to dissipate the vacuum energy due to the gravitational production of massless particles, non-conformally coupled, using Eq. (20),
\[
t_{\text{dis}}(\equiv \Gamma^{-1}) = \frac{2}{3} \frac{M_{Pl}^2}{F^3 M^3} \frac{1}{(\xi - 1/6)^2} ,
\]  
(38)
with that of massive particles $M$, conformally coupled, from Eq. (37),
\[
t_{\text{dis}} = \frac{6}{F^4} \frac{M_{Pl}^2}{M^3}
\]  
(39)
The ratio of the two times is
\[
\frac{t_{\text{dis}}} {t_{\text{dis}}^\xi} = \frac{F}{9 (\xi - 1/6)^2} .
\]  
(40)
Since it is generally assumed that \( (\xi - 1/6) \ll 1 \), the vacuum energy is dissipated more rapidly in the case of the emission of the massive \( m \) conformally coupled particles [Eq. (10)].

In general, the vacuum can lose energy, both by the gravitational production of \( m \) particles that are conformally coupled or by non-conformal gravitational production of massless relativistic particles. For simplicity, we assume that \( \xi = 1/6 \) and, thus, that only conformal production of \( m \) particles exist. From Eqs. (89) and (70) (below), we have

\[
 t_{\text{dis}} \simeq \frac{1}{F^4} 6.8 \times 10^{13} r^{-3/2} t_{Pl}, \tag{41}
\]

where \( r \) is the ratio of tensor to scalar fluctuations and \( t_{Pl} = M_{Pl}^2 \simeq 5.39 \times 10^{-44}\text{sec} \) is the Planck time.

In this paper, we assume the possible existence of elementary particles of mass \( m \equiv FM \), which are conformally produced during or at the end of inflation. From their lifetime, we obtain the maximum temperature of the Universe \( T_{\text{max}} \) the reheat temperature \( T_{RH} \) and their respective times, \( t_{\text{max}} \) and \( t_{RH} \). From the gravitational production of the \( m \) particles at \( \sim 60 \) e-folds before the end of inflation, we obtain the scalar density fluctuations (section IV).

**IV. SCALAR DENSITY FLUCTUATIONS**

Structure in the Universe primarily comes from nearly scale-invariant superhorizon curvature perturbations. These perturbations originate from the vacuum fluctuations during the nearly exponential inflation. In our model, structure is due to a scalar field \( \chi \) of mass \( m \ll H_{60} \), where \( H_{60} \) is the Hubble parameter \( \sim 60 \) e-folds before the end of inflation. The scalar field \( \chi \) of mass \( m \ll H_{60} \) in a quasi-de Sitter phase was shown to produce quantum fluctuations, whose power spectrum is scale invariant if they are superhorizon \[20, 21\].

During the inflationary epoch, the fluctuations of the field \( \chi \) of mass \( m \) obey the equation

\[
\delta \ddot{\chi}_k + 3H\dot{\delta \chi}_k + \left( \frac{k}{a} \right)^2 \delta \chi_k = 0 \tag{42}
\]

The fluctuations \( \delta \chi \) are described in terms of the variance,

\[
< \chi^2 > = \int \frac{d^3k}{(2\pi)^3} |\delta \chi|^2, \tag{43}
\]

which obeys the equation

\[
\frac{d< \chi^2 >}{dt} = \frac{H^3}{4\pi^2} \tag{44}
\]

during inflation \[20\]. (The formula Eq.(44) was first independently obtained in \[36, 37, 38\].) In our model,

\[
H \simeq H_0 \left[ \gamma - \frac{M^2 t}{6H_0} \right] \tag{45}
\]

until the end of inflation, before oscillations set in. Substituting Eq.(45) into Eq.(44), we obtain

\[
\frac{d< \chi^2 >}{dt} = \frac{H_0^3 \gamma^3}{4\pi^2} \left[ 1 - \frac{t}{t_{end}} \right]^{-3}, \tag{46}
\]

where \( t_{end} \) is given by Eq.(19).

We note that the major contribution to the variance comes from \( t \ll t_{end} \), while very little comes from \( t \sim t_{end} \). Integrating Eq.(46) from \( (1/60)t_{end} \) to \( t_{end} \), we estimate the variance of the fluctuations that were created \( \sim 60 \) e-folds before the end of inflation,

\[
< \chi^2 > \simeq \frac{3H_0^2 \gamma^4}{8\pi^2 M^2}. \tag{47}
\]

From Eq. (19), the Hubble parameter is

\[
H_{\text{end}} \simeq \frac{4}{3t_{end}} \tag{48}
\]

at the end of inflation. Substituting Eq.(47) into Eq.(48), we find

\[
H_{\text{end}} \simeq \frac{2}{9} \frac{M^2}{\gamma H_0}. \tag{49}
\]

In a flat Universe, we have

\[
H_{\text{end}}^2 = \frac{8\pi}{3M_{Pl}^2} [\rho_{V_{\text{end}}} + \rho_{M_{\text{end}}}], \tag{50}
\]

where \( \rho_{V_{\text{end}}} \) is the vacuum energy density at the end of inflation. From Eqs. (49) and (50), \( \rho_{V_{\text{end}}} \) is given by

\[
\rho_{V_{\text{end}}} = \frac{1}{54\pi} \frac{M^4 M_{Pl}^2}{\gamma^2 H_0^2}, \tag{51}
\]

assuming that \( \rho_{V_{\text{end}}} \) is very much greater than \( \rho_M \) at the end of inflation.

The long wavelength \( \chi \) modes satisfy the equation

\[
\delta \ddot{\chi}_k + 3H\dot{\delta \chi}_k + m^2 \delta \chi_k = 0 \tag{52}
\]
From Eq. (47), we then have
geneous field. Their number density is given by
\[ n_{m_{\text{end}}} = \frac{\rho_{m_{\text{end}}}}{m} = m < \chi^2 > . \]  
(54)

From Eq. (54), we then have
\[ \rho_{m_{\text{end}}} = \frac{3H_0^4}{16\pi^2} \]  
(55)

We assume that the m particles decay into relativistic particles with a decay rate, \( \Gamma_{mr} < \Gamma_{V m} = t_{\text{dis}}^{-1} \). Thus, we can separate the decay of the vacuum into m particles (\( \Gamma_{V m} \)) from the decay into relativistic particles.

For the production of the m particles, we have the relation
\[ \dot{\rho}_m + 3H \rho_m = \dot{\nu} \]  
(56)

where the second term on the left describes the dilution of the m particles due to the expansion of the Universe and the term on the right, the production of m particles due to the decay of the vacuum.

The production of m particles starts at the end of the inflation period, when
\[ \rho_V(t = t_{\text{end}}) = \rho_{V_{\text{end}}} \]  
(57)

and
\[ \rho_m(t = t_{\text{end}}) = \rho_{m_{\text{end}}} \ll \rho_{V_{\text{end}}} . \]  
(58)

During the time \( m^{-1} \ll t \ll t_0 \), when the m particles were produced, the average Hubble value was \( H = 2/3t_0 \). The vacuum energy decays in a time \( t_{\text{dis}} = \Gamma_{V m}^{-1} \), during which, the m particles are produced:
\[ \rho_V(t) = \rho_{V_{\text{end}}} \exp \left[ -\Gamma_{V m} t \right] . \]  
Let us take \( t_0 \approx t_{\text{dis}} \). We can then describe the production of the m particles by Eq. (56) in the form
\[ \dot{\rho}_m + 3H \rho_m = \rho\dot{V}_{\text{end}} / t_{\text{dis}} . \]  
(59)

The solution of the homogeneous form of Eq. (59) is
\[ \rho_m(t) = \rho_{m_{\text{end}}} e^{-3H(t-t_{\text{end}})} \]  
(60)

and that of the inhomogeneous form,
\[ \rho_m(t) = \rho_{m_{\text{end}}} e^{-3H(t-t_{\text{end}})} + \frac{\rho_{V_{\text{end}}}}{t_{\text{dis}}} (t - t_{\text{end}}) . \]  
(61)

We take the initial time for the decay of the m particles into relativistic particles to be
\[ t_{m_{\text{ri}}} \equiv t_{\text{dis}} \gg t_{\text{end}} . \]  
(62)

From Eq. (61), we have
\[ \rho_m(t = t_{m_{\text{ri}}}) \equiv \rho_{mi} \approx \rho_{V_{\text{end}}} . \]  
(63)

Since the Universe had an average cosmic scale factor \( H \propto t^{2/3} \) from \( t_{\text{end}} \) to \( t_{\text{dis}} \), it expanded by a factor \( (t_{\text{dis}}/t_{\text{end}})^{2/3} \) and diluted \( \rho_{m_{\text{end}}} \) by a factor \( (t_{\text{end}}/t_{\text{dis}})^2 \) during this time interval. Observations show that the ratio of the energy density of the mass fluctuations at \( t_{\text{dis}} \), \( \rho_{m_{\text{end}}} (t_{\text{end}}/t_{\text{dis}})^2 \), to that of matter, \( \rho_{mi} \), is \( \sim 10^{-5} \).

From Mukhanov and Chibisov (1981) [8], we have an approximately flat scalar fluctuation spectrum for the Starobinsky model with an amplitude
\[ \delta_{ST} \sim 3 \left( \frac{M}{M_{Pl}} \right) \left( \frac{8\pi}{3} \right)^{1/2} . \]  
(64)

From Eqs. (62) and (63), we then have
\[ \frac{(t_{\text{end}}/t_{\text{dis}})^2 \rho_{m_{\text{end}}}}{\rho_{V_{\text{end}}}} \sim 10^{-5} \sim \delta_{ST} . \]  
(65)

Although \( \delta_{ST} \) was evaluated for massless particles, similarly to Vilenkin [8], as discussed in Sec.II, we assume that this is also the amplitude for the production of massive particles on the order of \( M \). From Eqs. (64) and (65),
\[ M \simeq 1.15 \times 10^{-6} M_{Pl} . \]  
(66)

V. RATIO OF TENSOR TO SCALAR FLUCTUATIONS

From Vilenkin, the tensor power spectrum is
\[ P_T(k) = |h_k|^2 = \frac{GM^2}{2\pi^2} \frac{1}{k^3} \]  
(67)

and the scalar power spectrum is
\[ P_S(k) = \frac{\delta^2}{k^3} = \frac{(\delta \rho/\rho_{\text{hor}})^2}{k^3} , \]  
(68)
where $k$ is the wavenumber of the fluctuations and $(\delta \rho/\rho)_{\text{hor}}^2$ are the density fluctuations on the order of the horizon. The ratio $r$ of tensor to scalar fluctuations is, then,

$$ r \equiv \frac{P_T(k)}{P_S(k)} = \frac{GM^2}{2\pi^2} \frac{1}{(\delta \rho/\rho)^2_{\text{hor}}}. \quad (69) $$

We used the observed value of the scalar fluctuations, $(\delta \rho/\rho)_{\text{hor}} \approx 10^{-5}$, to obtain the value of $M$ in terms of $r$,

$$ M \approx 4.4 \times 10^{-5} \sqrt{r} M_{\text{Pl}}. \quad (70) $$

From Eq. (69) we obtain $t_{\text{dis}}$. We find that for $F = 0.5$, $t_{\text{dis}} \approx 6.28 \times 10^{19} t_{\text{Pl}}$ and that for $F = 0.3$, $t_{\text{dis}} \approx 4.85 \times 10^{20} t_{\text{Pl}}$.

We have

$$ r \approx 6.8 \times 10^{-4} \quad (71) $$

from Eqs. (70) and (66).

A small value for $r$, bounded from below by $r > 3 \times 10^{-6}$ (unless $V''/V$ in the inflaton potential is unreasonably large), was previously indicated in Eq. (31).

From Eqs. (58), (59), $\rho_{\text{end}}$ and $\rho_{\text{end}}$ [Eqs. (51) and (55) respectively], $t_{\text{end}}$ [Eq. (16)], and $t_{\text{dis}}$ [Eq. (39)], we obtain

$$ \gamma \approx \frac{6.7 \times 10^{-3}}{H_0/M_{\text{Pl}}} \quad (72) $$

From Eqs. (72), (13), and (14) for the SU(5) model, the initial value of the Hubble parameter $H_{\text{in}}$ of the inflation era for $F = 0.5$ and 0.3 is

$$ H_{\text{in}} \approx H_0 \approx M_{\text{Pl}}. \quad (73) $$

VI. REHEATING OF THE UNIVERSE

In this section, the reheating of the Universe in our model is discussed. It is based on the discussion of this epoch in Eq. (32).

The particles $m$ have a decay rate into relativistic particles,

$$ \Gamma_{mr} = f \Gamma_{Vm}, \quad (74) $$

where we assume that

$$ f \ll 1. \quad (75) $$

The equation describing the decay of the $m$ particles is

$$ \dot{\rho}_m + 3H \rho_m = -\rho_m \Gamma_{mr}, \quad (76) $$

which has the analytical solution,

$$ \rho_m = \rho_{m0} \left( \frac{a_I}{a} \right)^3 e^{-\Gamma_{mr}(t-t_{mri})}, \quad (77) $$

where $a_I$ is the cosmic scale factor at $t = t_{mri}$. For the radiation energy density, $\rho_r$, the evolution equation is

$$ \dot{\rho}_r + 4H \rho_r = \rho_m \Gamma_{mr}, \quad (78) $$

where $\rho_r$ is the energy density of the relativistic decay products. In order to obtain the energy density of the relativistic particles as a function of time, we also need the Friedmann equation,

$$ H^2 = \frac{8\pi G}{3} (\rho_m + \rho_r). \quad (79) $$

From $t = t_{mri}(\approx t_{\text{dis}})$ until $t = t_{RH}(\equiv \Gamma_{mr}^{-1})$, the $m$ particles dominate the mass density and the Universe is matter dominated, $a(t) \propto t^{2/3}$, with $\rho_r \sim 0$ at $t_{mri}$. During the $m$-dominated epoch, an approximate solution for $\rho_r$ is given by

$$ \rho_r(t) \approx \frac{\rho_{mi} \left( \Gamma_{mr} t_{mri} \right)}{10\pi t} \left[ 1 - \left( \frac{t}{t_{mri}} \right)^{-5/3} \right] $$

$$ \approx \frac{\sqrt{6/\pi}}{30} \rho_{mi} \left( \Gamma_{mr} t_{mri} \right) \left( \frac{a}{a_I} \right)^{-3/2} \left[ 1 - \left( \frac{a}{a_I} \right)^{-5/2} \right] \quad (80) $$

Thus, $\rho_r$ rapidly increases from $\approx 0$ at $t_{mri}$ to a value of $\approx \rho_{mi} \left( \Gamma_{mr} t_{mri} \right)$ at $t_{RH}$, decreasing thereafter as $a^{-3/2}$.

Once the relativistic decay products interact sufficiently, they thermalize and we have

$$ \rho_r = g_s \pi^2 T^4/30, \quad (81) $$

where $g_s$ is the number of relativistic degrees of freedom and is generally estimated to be $100 \lesssim g_s \lesssim 1000$. What is commonly called the reheat temperature, $T_{RH}$, is not
the maximum temperature of the Universe, $T_{\text{max}}$, which is given by

$$T_{\text{max}} \simeq 0.8 \left( \rho_{\text{mr}} \Gamma_{\text{mr}} t_{\text{mr}} \right)^{1/4} g_*^{-1/4}. \quad (82)$$

The reheat temperature at the beginning of the radiation dominated epoch, $T_{\text{RH}} \equiv T(t_{\text{RH}} = \Gamma_{\text{mr}}^{-1})$, is given by

$$T_{\text{RH}} \simeq 0.55 \left( \frac{1}{2} \Gamma_{\text{mr}} \rho_{\text{mr}} t_{\text{mr}} \right)^{1/2} g_*^{-1/4} \quad (83)$$

and the ratio of $T_{\text{max}}$ to $T_{\text{RH}}$ is

$$\frac{T_{\text{max}}}{T_{\text{RH}}} = \sqrt{\frac{\Gamma_{\text{V}} m}{\Gamma_{\text{mr}}}} \simeq 1.45 f^{1/4} \quad (84)$$

since $\Gamma_{\text{mr}} = f \Gamma_{\text{V} m}$, where $f \ll 1$. In Table I we show the values of $T_{\text{max}}$ [Eq. (82)] and $T_{\text{RH}}$ [Eq. (83)] for $r = 6.8 \times 10^{-4}$, $f = 10^{-5}$ and $10^{-10}$, and $F \equiv m/M = 0.5$ and 0.3.

In Fig. 1, we show $\rho_\tau$ [Eq. (80)] and the temperature as a function of time [Eq. (81)] for $f = 10^{-5}$ and $F = 0.5$ and 0.3 during the time interval $t_{\text{mr}} < t < t_{\text{RH}}$. The times $t_{\text{mr}}$, $t_{\text{RH}}$, and $t_{\text{max}}$ are shown in Table I.

VII. CONCLUSIONS AND DISCUSSIONS

We presented here a model which relates the Starobinsky and $\epsilon R^2$ models, both of which predict inflation, to the reheating era by a massive conformally coupled particle. In the original Starobinsky model, the coupling to the reheating era was made by massless non-conformally coupled particles. Here we assumed that non-conformally coupling to gravitation does not exist.

In the Starobinsky model, inflation is due to vacuum fluctuations. Inflation is due to a modification of the gravitational Lagrangian in the $\epsilon R^2$ model. In both models, inflation takes place without the need for a scalar inflaton field.

The end of the inflation period predicted by the Starobinsky and $\epsilon R^2$ models is characterized by a parameter $M (M = M_{\text{Pl}}/\sqrt{48 \pi k_1})$ in the Starobinsky model, where $k_1$ is the coefficient of the $^{(1)}H$ tensor (Eqs. 31) of the expectation value of the vacuum energy-momentum...
tensor due to vacuum fluctuations and $M = M_{Pl}/\sqrt{24\epsilon}$ in the $\epsilon R^2$ model).

In our model, we assumed the existence of particles of mass $m$ and linked their conformal gravitational production during inflation to the observed scale invariant density fluctuations. Their gravitational production at the end of inflation was linked to the matter density of the Universe.

Our model of two massive scalar particles $M$ and $m$ has a certain resemblance to the double inflation model of Gottl"ober, M"uller, and Starobinsky [39]. Gottl"ober, M"uck and Starobinsky discussed the confrontation of the model with observations [40]. In the simple chaotic inflation model, the scalar potential, $(1/2)\mu^2\phi^2$, is characterized by the mass $\mu$. The gravitational term, $R^2/M^2$, is characterized by the mass $M$ in the late inflation in the Starobinsky model. In the double inflation model of Gottl"ober, M"uller and Starobinsky, both the scalar field potential, $(1/2)m^2\phi^2$, and the gravitational term, $R^2/M^2$, are present. Thus this double inflation model is described by two masses $M$ and $m$. It is to be noted that in our model, described by the two masses $M$ and $m$, only the mass $M$ describes the inflation era, whereas the mass $m$ is a free particle, produced at the end of inflation, linking the inflation era to the reheating era. The $m$ particles in our model, have a decay time into relativistic particles $t_{RH}$ very much greater than $t_{dis}$, the time for the production of the $m$ particles at the end of inflation.

Our model depends upon the parameter $H_0$, the normalization of the Starobinsky inflation solution for the Hubble parameter near the Planck era [Eq.(12)]. The parameter $H_0$ is on the order of $M_{Pl}$ and depends on $N_0$, $N_{1/2}$, and $N_1$, the number of quantum matter fields in the vacuum of spin 0, 1/2, and 1, respectively. To evaluate $H_0$, we assume a minimal SU(5) particle content and obtain $H_0 \cong 0.74 M_{Pl}$ [Eq.(3)]. The time at the end of inflation depends on $H_0$ and $M$. For $r \approx 6.8 \times 10^{-4}$ and $M \approx 1.15 \times 10^{-6} M_{Pl} \approx 1.4 \times 10^{13}$ GeV, as predicted by the Starobinsky model for $F = 0.3$, for example, we find $t_{end} \approx 1.37 \times 10^{11} t_{Pl} \approx 10^{-32}$ sec and $t_{dis} \approx 4.8 \times 10^{20} t_{Pl} \approx 10^{-23}$ sec, respectively.

In the future, the particle $m$ could be incorporated into a particle physics theory that would define the decay time into relativistic particles $t_{RH}$, the reheating time. From $f \equiv t_{dis}/t_{RH}$, we evaluate the maximum temperature of the Universe $T_{max}$ and the reheat temperature $T_{RH}$ as a function of $g_*$ (the number of degrees of freedom of the relativistic particles) and $F \equiv m/M$. The times $t_{str}, t_{max}$, and $t_{RH}$ can, then, also be evaluated. A measured value of $r$, the ratio of tensor to scalar fluctuations, that is appreciably different from $r \approx 6.8 \times 10^{-4}$ [Eq.(72)] would discard our model (as well as the Starobinsky and $\epsilon R^2$ models).

In order not to overproduce gravitinos, it is frequently suggested that $T_{RH} \lesssim 10^9$ GeV [32]. If this is, indeed, a true upper limit for $T_{RH}$, it puts limits on the possible values for $f$ and $F$ from the above analysis. However, it is to be noted that, for our model, $10^9$ GeV is not a strong upper limit for $T_{RH}$ since supersymmetry is still not a well developed theory.

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(1986).