Comments on ”Oriental magic mirrors and the Laplacian image”

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Abstract


Berry just presented a comprehensive explanation of the physical part for the optics of the magic mirror [1]. The story related to this strange metal mirror could be traced in [2-4]. The characteristic is: the raised images appeared in a screen are on the opposite side of an opaque mirror but not the reflecting face [5-6]! This amazing optical effect for many centuries made a tremendous impression on observers, from which such mirrors became known as magical. Master craftsmen passed down the secret of their fabrication from generation to generation. The ancients were unable to give the phenomenon a convincing explanation, but it was reflected in the old saying "the truth always comes out in the sun." [5-6].

In fact, archeological finds contain extremely scanty information on the magic mirrors of China. One of the first such mirrors was discovered during excavations of the tomb of the exalted grandee Yu in the province of Uhan in the south of China and has been dated to approximately 500 B.C. These years belong to the Han period of history, when, in the fabrication of mirrors, special significance was attached to the purity of the metal. The necessity for such careful fabrication of the mirrors was caused by ritual-magical purposes; these were often reflected in inscriptions and images on the mirrors, which were supposed to keep evil forces away from their owners (cf. Fig. 2 in [6])! Faith in the power and magical properties of these symbols in Han China was very widespread. Other legend is: the reverse side of the mirrors bore raised images and inscriptions of lyrical, symbolic, and edifying content: "a wise man uses his mind like a mirror" [6].

In China, attempts were made from ancient times to explain the production of the remarkable optical effect created by magic mirrors. The first version that has come down to us apparently belongs to the scholar Kua Shen (1031-1095)! [4]. He explained this phenomenon by saying that, in casting, the thinner part cools faster than the thick part, and this causes the formation of small deformations of the profile, not observable by the naked eye. At the same time, he pointed out that very thin mirrors did not manifest this property, and gave credit to the high skill of the ancients. At the end of the thirteenth century, Dzhou Mi observed a mirror that reproduced in reflected light the finest details of the image on the back side.

The archeologist Uchkhi Ien gives another explanation of the effect, associating it with the use of bimetals of different density. Thus, if an image of a dragon was created on the back side by casting, the image was engraved on the front side and was then filled with bronze of another density, after which the surface was carefully polished. He saw such a mirror that had been cut
apart and was absolutely certain that his explanation was correct [6]. Other attempts could be traced in [1-2].

In such an interesting regime of geometrical optics, the image intensity could be given simply by the Laplacian of the height function of the relief as Berry demonstrated in [1]. For instance, Berry used the error function (cf. Eqn. (14) in [1]) to model smoothly and approximately the single step (the \( l \)-smoothed step, with height \( h_0 \)). The present author, however, likes to propose another way: use \( \text{Tanh} \) (hyperbolic tangent function) to model the sharp step. It reads

\[
(1) \quad h(x) = C_0 h_0 \frac{e^{x/l_t} - e^{-x/l_t}}{e^{x/l_t} + e^{-x/l_t}}, \quad l_t = a_0 l,
\]

\( C_0 \) is a normalization constant and \( a_0 \) is a constant for adjusting the sharpness. The result and comparison is shown in Figure 1. There is no doubt that our proposal could be either smooth (enough for the Laplace operator) or sharp enough (to approximate the step). The obtained function could be easily implemented in

\[
I_{\text{Laplacian}} = 1 + Z \nabla^2 h(r), \quad Z = 2D/M,
\]

where \( D \) is the distance of the screen from the reference plane, \( M \) is the magnification, \( Z \) is the reduced distance, and \( r \) is the demagnified observation position [1].

Meanwhile, as commented by Berry in [1]: "It is possible that there are different types of magic mirror, where for example the relief is etched directly onto the reflecting surface and protected by a transparent film [7], but these do not seem to be common. Sometimes, the pattern reflected onto a screen is different from that on the back, but this is probably a trick, achieved by attaching a second layer of bronze, differently embossed, to the back of the mirror.". Berry only briefly discussed the manner in which the pattern embossed on the back gets reproduced on the front at the end of [1]: "Referring to (11), this involves the sign of the coefficient \( a \) in the relation between \( h_{\text{back}} \) and \( h \). There have been several speculations about the formation of the relief. One is that the relief is generated while the mirror is cooling, by unequal contraction of the thick and thin parts of the pattern [8]; it is not clear what sign of \( a \) this leads to. Another [9] is that cooling generates stresses, and that during vigorous grinding and polishing the thin parts yield more than the thick parts, leading to the thick parts being worn down more; this leads to \( a < 0 \). However, this seems to contradict the observations, which point firmly to \( a > 0 \): bright (dark) lines on the image, indicating low (high) sides of the steps on the reflecting face, are associated with the low (high) sides of the steps on the back (cf. figure 7(a) in [1]), not the reverse (cf. figure 7(b) in [1]).". Here, the present author would like to mention the significance of the material which made the strange mirror.

As reported in [6], a mirror of the same size and thickness as the Han mirror was fabricated in the University of modern China. They used a mirror from the Shanghai museum as a model for the casting. The copy was fabricated from an alloy containing 73% copper, 23% tin, and 4% lead. After cooling, the mirror was ground to a thickness of 0.5mm in the thin places. When the copy was illuminated, it behaved exactly like the original. This could be compared to those larger magic mirrors, now found in various collections, belong to the period of government of the Ming dynasty (1368-1644). They are distinguished by the size and character of the images. The
reflective surface was made somewhat convex and was carefully polished by means of a mercury amalgam. The back side often had intricate images of birds, flowers, or dragons or scenes from mythology. The spread in height of the relief is about 25%. The production technique was casting, using the lost-wax technique. One of the largest of such mirrors, fabricated later in China, in 1875, is 52 cm in diameter and 1.3 cm thick and weighs more than 12 kg. As with other Chinese mirrors, it is fabricated from bronze, being an alloy of copper (80%), tin (15%), and lead (5%) [6]. Finally, from all previous information, it is evident that, in the image obtained on a screen, dark zones appear where the light is deflected by convex microsections of the surface corresponding to the thinner regions of the mirror, while bright zones are formed by flat microsections corresponding to the thicker regions. Parts of the latter were illustrated in [1] clearly.

References

Fig. 1 Comparison of the smoothing of the single step: the $l$-smoothed step, with height $h_0$ (cf. Berry’s proposal [1]). We adopt the hyperbolic tangent (Tanh) function and the result is much more close to the sharp step compared to that by Berry (cf. Eqn. (14) in [1]).