Supersymmetry Breaking and Moduli Stabilization in AdS

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Abstract

We study the one loop effective potential for the radion superfield in the supersymmetric Randall-Sundrum scenario with detuned brane tensions. At the classical level the distance between the branes is stabilized while the VEV of the fifth component of the graviphoton is a flat direction which breaks supersymmetry. At the quantum level a potential is generated. This leads to a toy model of a supersymmetric compactification with all the moduli stabilized perturbatively.

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1 The Model

One of the main problems facing compactifications of higher-dimensional theories down to four dimensions, is the stabilization of all the moduli of the compactification. In the context of string theory, a possible solution to this problem has been proposed recently where classical effects, such as fluxes, combine with non-perturbative effects to give a potential for all the moduli in a supersymmetric AdS vacuum [1]. By adding supersymmetry breaking effects, it was argued that the cosmological constant could also be lifted to a small positive value, realizing the necessary starting point for phenomenological applications.

The purpose of this note is to study a mechanism of moduli stabilization similar to [1], where perturbative and fully calculable corrections stabilize all the moduli of the compactification. The model in question is the supersymmetric Randall-Sundrum (RS) scenario with detuned brane tensions [2–5]. In the minimal scenario the bosonic part of the action contains, beside the graviton, a $U(1)$ gauge field $B_M$ called the graviphoton. In the $4D$ effective theory the radion has a potential with mass proportional to the $4D$ curvature. The axion partner on the other hand remains a flat direction of the potential as a consequence of the higher dimensional gauge invariance. It was however noticed in [6, 7] that supersymmetry would be broken spontaneously by a non-zero vacuum expectation value (VEV) of the graviphoton modulus. This is due to the fact that the graviphoton gauges a $U(1)$ $R$–symmetry of the $5D$ action, under which the gravitino is charged. The gauging violates explicitly the shift symmetry of $B_5$ and upon compactification contributes to the gravitino bilinears which in turn break supersymmetry by shifting the masses of the fermionic Kaluza-Klein (KK) tower.

With broken supersymmetry, the potential is modified by radiative corrections and the flat directions are lifted, because the fermion and boson contributions do not cancel exactly$^5$. In principle the effective potential could be found by brute force, by computing the Coleman-Weinberg-like potential obtained by integrating out the full KK tower of modes. This computation is highly unwieldy mostly because the $4D$ ground state is curved and the masses of the modes are not known in closed form (see [9]). An interesting feature however is that, since the supersymmetry breaking effect is non-local in the extra-dimension, the final result is guaranteed to be finite and calculable, despite the appearance of divergences at each KK level. In this paper we will follow a different path and rely on the power of supersymmetry to derive the effective potential. Since the superpotential is not renormalized perturbatively all that is needed is the correction to the Kähler function. As we will argue this allows an enormous simplification because it is sufficient to compute the correction in the limit where the $4D$ ground state is flat and supersymmetry is preserved. As expected, a potential for the graviphoton modulus is generated. The correction to the potential is negative with pure gravity in the bulk, so that unbroken supersymmetry corresponds to a global maximum of the potential while the minimum of the potential corresponds to maximal supersymmetry breaking. Nevertheless, because the ground state is AdS, unbroken supersymmetry remains a stable point of the potential. With the addi-

$^5$This mechanism is similar to the one considered in [8] in flat space with the important difference that the supersymmetry breaking parameter is a dynamical variable.
tion of hyper-multiplets in the bulk, the correction can become positive, so that unbroken supersymmetry is the minimum of the potential.

While we focus on a specific model we would like to emphasize that the mechanism presented in this paper could be generic in supersymmetric compactifications. In any supersymmetric model where the ground state is AdS space, half of the moduli have mass simply because the scalars in a chiral multiplet have different masses (split by the curvature of the space). Very generically in a supersymmetric compactification the imaginary components of the chiral fields are axions arising from the form-fields of the higher dimensional theory. The axions are flat directions at tree level due to the shift symmetry inherited from the gauge invariance of the theory. If the flat direction breaks supersymmetry as in the case considered in this paper, the masses of all the moduli will be lifted. The examples that we have in mind are gauged supergravities where the shift symmetry of the axions arising from the gauge fields is violated by the gauging. In this type of models, without invoking non-perturbative contributions, one expects that in an AdS supersymmetric vacuum all the moduli will acquire a mass proportional to the four dimensional curvature. If the minimum of the potential can be lifted without affecting significantly the stabilization (which might not be generic), this would lead to a very peculiar spectrum of masses where half of the scalars have mass at least a loop-factor smaller than the one of the scalar partner.

2 Gravitational Multiplet

The 4D low energy effective theory for the supersymmetric RS model with general brane tensions was computed in [6], at the classical level. The low energy dynamics is described by an \( N = 1 \) supersymmetric sigma-model coupled to supergravity, with the following Kähler potential and superpotential,

\[
\begin{align*}
K(T, \bar{T}) &= -3M_5^2 \log \left( 1 - e^{-k\pi(T+\bar{T})} \right) \\
W(T) &= \sqrt{1 - e^{-2k\pi r_0}} \frac{M_5^2}{L} \left( 1 - e^{i\phi} e^{k\pi r_0} e^{-3\pi k T} \right),
\end{align*}
\]

(1)
to leading order in the 4D curvature (see discussion below). Here \( T \) is the radion superfield, whose scalar component is \( r + ib \), where \( r \) is the radion and \( b \) is the zero mode of \( B_5 \). The parameterization of \( K \) and \( W \) is chosen so that the minimum of the potential is located at \( r = r_0 \), and \( L \) is the radius of the 4D AdS ground state while \( k \) is the curvature of the bulk AdS\(_5\).

An interesting feature of the low energy effective action (1), is that it is entirely determined by the shift symmetry of \( b \) up to the phase \( \phi \) which can be set to zero without loss of generality (it amounts to changing the origin of \( b \)) [6]. Since the bosonic part of the action is invariant under the shift of \( B_5 \), there is no potential for \( b \) at tree level\(^6\). This condition, together with the fact that the ground state is AdS, fixes the form of \( K \) and \( W \) as in (1). Even though there is no potential for \( b \) at tree level, the physics does depend on its VEV, because the shift symmetry is violated in the fermionic sector by the superpotential. In particular, computing the covariant derivative \( D_T W \), one can see that supersymmetry is

\(^6\)This symmetry is indeed violated by the Chern-Simons term but this is irrelevant in perturbation theory.
broken unless $b = 2n/(3k)$. All this has a beautiful explanation in terms of the dual CFT description of the model [10] (see also [11]). From the holographic point of view, the radion is the Goldstone boson for the spontaneous breaking of conformal invariance. In fact the Kähler potential and superpotential are dictated by conformal invariance. The constant piece in $W$ arises due to the explicit breaking of conformal invariance in the ultraviolet which also induces the coupling to 4D gravity. It is very useful to consider the physics of $b$ in the CFT picture. The photon gauges a $U(1)$ subgroup of the $R-$symmetry which is broken by the boundary conditions. According to the AdS/CFT dictionary, gauge symmetries on the AdS side are dual to global symmetries of the CFT, so $b$ is the Goldstone boson for the breaking of the $U(1)$ $R-$symmetry of the CFT. Being a Goldstone boson there cannot be any potential. The constant term in the superpotential however breaks the $R-$symmetry and therefore a potential is generated at one loop.

Let us now turn to the explicit computation of the quantum effects. To any order in perturbation theory the superpotential is not renormalized and is given by the tree-level result above. Since the fermionic part of the action does not respect the shift symmetry of $b$, at one loop there will be corrections to $K$ which do not respect the structure of eq. (1) and generate a potential for $b$. The effective action is an expansion in derivatives and in powers of the curvature $1/L^2$. As explained in detail in [6], working consistently to two derivatives requires that we also work to leading order in $1/L^2$, because terms such as $R^n$ are not included. This observation simplifies the computation enormously. Since the superpotential is already of order $1/L$, it follows that to work to two derivatives we just need to compute the Kähler potential to zero order in $1/L^2$. What this means is that to calculate the one loop potential, all we need is the correction to the Kähler potential in the supersymmetric RS model with tuned brane tensions. In fact, the very same arguments could be used for any theory with unbroken supersymmetry in AdS background.

In the flat supersymmetric limit the shift of $b$ becomes an exact symmetry so that $K$ is a function of $T + \bar{T}$. The correction can be derived from the remarkably simple formula [11],

$$\Delta \Omega_{\text{gravity}} = \sum_n \int \frac{d^4 p}{(2\pi)^4} \frac{2}{p^2} \log(p^2 + m_n^2)$$

where the sum runs over the masses of the KK tower and $\Omega = \Omega_{\text{tree}} + \Delta \Omega$ is related to the Kähler potential by

$$K = -\frac{3M_5^2}{4\pi^2} \log[k\Omega/(3M_5^3)]$$

where $M_5$ is the 5D Planck mass (we follow the normalizations in [6]). Using dimensional regularization one finds,

$$\Delta \Omega_{\text{gravity}} = -2 \frac{\Gamma(1 - d/2)}{(4\pi)^{d/2}} \sum_n m_n^{d-2} ,$$

for the gravity multiplet. The finite, radion dependent contribution arising from the sum can then be rewritten as [12, 13],

$$\Delta \Omega_{\text{gravity}} = \frac{k^2 a_\pi^2}{4\pi^2} \int_0^\infty dy \ y \log \left(1 - \frac{I_1(ya_\pi)K_1(y)}{K_1(ya_\pi)I_1(y)} \right) ,$$

$$a_\pi^2 = e^{-k\pi(T + \bar{T})} .$$

\textsuperscript{7} The only potential for the radion compatible with conformal invariance is $\phi^4$ which is precisely what follows (in the rigid limit and upon canonical normalization) from the exponential term in $W$. 
The above result is exact at one loop for any value of the bulk curvature $k$. The formulas simplify when the warping is large so we focus on this case. In this limit one obtains [11, 13],

$$K = -3 M_4^2 \log \left[ (1 + \alpha) - (1 + \beta) e^{-k \pi (T + \bar{T})} + \gamma e^{-2k \pi (T + \bar{T})} + \ldots \right], \quad (5)$$

where the dots stand for higher terms in the exponential expansion. The last term is the finite calculable contribution obtained from (4). The precise numerical coefficient is given by,

$$\gamma = - \frac{c_G k^2}{12 \pi^2 M_4^2}, \quad (6)$$

where $c_G = \frac{1}{2} \int_0^\infty dx x^3 K_1(x) \approx 1.165$. In this formula we have used the relation $M_4^2 = M_5^2/k$ which is valid to leading order in the exponential expansion. This term does not respect the tree level structure of $K$ so as we will show it gives rise to a $b$ dependent potential.

The parameters $\alpha$ and $\beta$ parameterize renormalizations of tree-level terms. Unlike $\gamma$ they do not generate a $b$-dependent potential. If we are only interested in the $b$ dependent correction to the potential, we could just ignore the corrections $\alpha$ and $\beta$ which will only enter at two-loops. Still, it is instructive to consider these divergent parameters. As shown in [11], $\alpha$ and $\beta$ correspond to divergent brane terms generated radiatively on the ultraviolet (UV) and infrared (IR) branes respectively. In particular, the brane terms contain the Ricci scalar, which has a non-zero VEV in AdS, and therefore contributes to the effective brane tensions. Hence $\alpha$ corresponds to a correction of the UV brane tension. Since the detuning of the UV brane compared to the bulk cosmological constant determines the $4D$ curvature $1/L$ (we use the parameterization of [6]), a non-zero $\alpha$ modifies the value of $L$. Indeed, from the point of view of the $4D$ theory, a non-zero $\alpha$ in (5) changes the overall scale of the potential, and therefore shifts the value of the $4D$ curvature $1/L$ with respect to the $4D$ Planck scale. Similarly, $\beta$ corrects the IR brane tension, which determines the radius. Thus the only effect of the divergent parameters $\alpha$ and $\beta$ is to modify the radius and curvature of the $4D$ theory.

From the discussion above it follows that $\alpha$ and $\beta$ should be fixed by matching to the $5D$ theory. Consider the contribution of some KK supermultiplet to the vacuum energy. With unbroken supersymmetry the contribution vanishes identically. Therefore, a natural matching condition is that the one-loop correction to the potential vanishes when supersymmetry is unbroken,

$$\Delta V(r, b) = 0 \quad \text{for} \quad r, b \text{ such that } D_T W = 0, \quad (7)$$

where $\Delta V$ is the one-loop correction to the potential obtained from the Kähler potential (5) and superpotential (1) through the standard supergravity formula,

$$V = e^{\frac{K}{M_4^2}} \left( K^{T \bar{T}} D_T W D_{\bar{T}} W - \frac{3}{M_4^2} W W \right). \quad (8)$$
The second condition that we require is that the value of the radion at the minimum does not change when supersymmetry is unbroken. These two conditions determine,

\[\alpha = \gamma e^{-4k\pi r_0}, \quad \beta = 2\gamma e^{-2k\pi r_0}.\]  

(9)

to leading order in \(e^{-2k\pi r_0}\).

Given \(K\) and \(W\) we can now compute the potential from (8). In the large warping limit we find,

\[\delta V = -\frac{c_G}{4\pi^2} \frac{(e^{-2k\pi r_0})^k}{L^2} \left[ 3 - 4e^{-2\pi k(r-r_0)} + e^{-4\pi k(r-r_0)} \right.\]

\[\left. - 4e^{-5\pi k(r-r_0)} + 4e^{-6\pi k(r-r_0)} + 8e^{-5\pi k(r-r_0)} \sin^2 \left( \frac{3}{2} \pi kb \right) \right].\]  

(10)

Using the fact that the mass of the first KK mode is roughly \(\pi ke^{-k\pi r_0}\), one can write the \(b\)-dependent piece in the more suggestive form,

\[\delta V = -\frac{2c_G}{\pi^4} \frac{m_{KK}^2 e^{-2\pi k r_0}}{L^2} \sin^2 \left( \frac{3\pi k b}{2} \right),\]  

(11)

with \(m_{KK} \equiv \pi ke^{-k\pi r_0}\). This is essentially the result that one would guess from effective field theory considerations alone. Since the supersymmetry breaking scale obtained from (11) is proportional to \(1/L\) and the low energy effective theory is cut-off at the KK scale \(m_{KK}\), the correction computed within the effective theory must be proportional to \(m_{KK}^2/L^2\). The extra suppression is related to the fact that the supersymmetry breaking parameter is suppressed in comparison to \(1/L\). This can be seen from (11) because even for maximal supersymmetry breaking the shift of the gravitino mass is only,

\[\delta m_3^2 \approx \frac{e^{-2k\pi r_0}}{L}.\]  

(12)

The correction (11) is always negative, so the supersymmetric vacuum \(b = 0\) is a maximum of the potential. The global minimum corresponds to maximal supersymmetry breaking. At the supersymmetric stable point the mass of \(b\) is given by

\[m_b^2 = -\frac{3c_G}{2\pi^4} \frac{1}{L^2} \frac{m_{KK}^2}{M_4^2}.\]  

(13)

Even though \(b = 0\) is a maximum of the potential, it remains a stable point by virtue of the fact that the ground state is AdS. In other words, radiative corrections cannot destabilize the supersymmetric vacuum. In fact supersymmetry automatically guarantees that all the masses are above the stability bound for the scalars in AdS, \(m^2 \geq -9/(4L^2)\). The radion mass is also corrected as,

\[m_r^2 = \frac{4}{L^2} - \frac{5c_G}{2\pi^4} \frac{1}{L^2} \frac{m_{KK}^2}{M_4^2}.\]  

(14)

Together the masses of the scalars correspond to a supersymmetric multiplet labeled by the Casimir,

\[E = \frac{3}{L^2} - \frac{c_G}{2\pi^4} \frac{1}{L^2} \frac{m_{KK}^2}{M_4^2}.\]  

(15)
3 Vector and Hyper Multiplets

In the presence of vector and hyper multiplets in the bulk there will be additional corrections to the quantum potential. While the contribution of vector multiplets is expected to have the same sign as the gravity multiplet, hyper-multiplets should give the opposite sign. To see this, recall that from the five-dimensional point of view, the physics of these corrections is the following: the action in flat space is invariant when $B_5$ shifts by a constant. The AdS theory is obtained by gauging a $U(1)$ subgroup of the $SU(2)$ $R$–symmetry. This breaks explicitly the shift symmetry so one expects perturbative corrections to the potential for $B_5$. The fields transforming under the $R$–symmetry are the gravitinos, the gauginos, and the hyper-scalars. Since a VEV of $B_5$ shifts the fermion masses for the gravity and vector multiplets, and the boson masses for the hypers, they will contribute with opposite sign.

The full supersymmetric 5D action coupled to branes has not yet been constructed. Still, as in the gravity case, we can circumvent this obstruction by computing the correction to the Kähler potential since this can be obtained in the tuned limit and depends only on the KK spectrum. The formula (4) generalizes to,

$$\Delta \Omega = N \frac{k^2 a_\pi^2}{8 \pi^2} \int_0^{\infty} dy \log Z(y).$$

(16)

where $N$ is the number of vector multiplets $N_V$, or minus the number of hypermultiplets $N_H$ respectively. $Z(y)$ is a function whose zeros on the positive imaginary axis are the masses of the KK particles which is given respectively by [14],

$$Z(y)_\text{hyper} = 1 - \frac{I_{|c+1/2|}(ya_\pi)K_{|c+1/2|}(y)}{K_{|c+1/2|}(ya_\pi)I_{|c+1/2|}(y)}.$$  

$$Z(y)_\text{vector} = 1 - \frac{I_0(ya_\pi)K_0(y)}{K_0(ya_\pi)I_0(y)}.$$  

(17)

The contribution of the hyper-multiplets depends on the parameter $c$ which is related to the bulk mass. Since for any $c$ the KK reduction of a hyper-multiplet always produces a massless chiral multiplet and a tower with masses starting at $\pi ke^{-\pi kr_0}$, the hyper-multiplets do not decouple for large mass. What depends on $c$ is instead the localization of the zero mode. Expanding the result in the limit of large warping one finds,

$$\Delta \Omega_{\text{hyper}} = N_H \frac{c_H}{8 \pi^2} \frac{k^2 e^{-(|c+\frac{1}{2}|+1)+1}k\pi(T+\bar{T})}{},$$  

(18)

with the numerical coefficient given by

$$c_H = \frac{21^{-|2c+1|}}{\Gamma(|c+\frac{1}{2}|)\Gamma(1+|c+\frac{1}{2}|)} \int_0^{\infty} dy y^{2|c+\frac{1}{2}|+1} \frac{K_{|c+1/2|}(y)}{I_{|c+1/2|}(y)}.$$  

Note that the functional dependence of the correction to $\Omega$ depends on $c$, that is on the localization of the zero mode. For the special value $c = 1/2$, the “conformal hypermultiplet”, the contribution is minus a half the one of gravity. The potential for $N_H$ such multiplets is then,

$$\delta V = \left( \frac{N_H}{2} - 1 \right) 2c_G m^2_{KK} e^{-2\pi kr_0} \sin^2 \left( \frac{3k\pi b}{2} \right).$$  

(19)
For $N_H > 2$ the correction is positive and the unbroken supersymmetry point $b = 0$ becomes the minimum of the potential.

For completeness the result for the vector multiplet is given by,

$$\Delta \Omega_{\text{vector}} = -N_v \frac{c_v}{8\pi^2} \frac{k}{\pi(T + \bar{T})} e^{-k\pi(T + \bar{T})},$$

with the numerical coefficient $c_v = \int_0^\infty dx xK_0(x)/I_0(x) \approx .631$.

**4 Outlook**

In this brief note we computed the one loop effective potential for the radion superfield in the supersymmetric detuned RS model. At tree level, the zero mode of $B_5$ is an exactly flat direction of the potential, with supersymmetry spontaneously broken for $b \neq 0$. The scalar partner of $b$, the radion, is already stabilized at tree level. Due to supersymmetry breaking effects, $b$ develops a periodic potential at one-loop. This potential is finite because the supersymmetry breaking effect is non-local and therefore does not depend on the ultraviolet completion of the theory. We derived the correction to the potential using the powerful supersymmetric approach which only requires to compute the correction to the Kähler potential in the tuned limit of the model.

The model analyzed in this paper shares some of the basic features of the flux compactifications of string theory which have attracted a lot of attention recently, starting with [1]. In these constructions one considers a compactification of 10D supergravity on a Calabi-Yau manifold to four dimensions. By adding fluxes, branes and including non-perturbative effects, it is possible to stabilize all the moduli of the theory. The ground state is then a supersymmetric AdS background with the masses of the volume modulus proportional to the curvature of AdS space. The strategy for constructing semi-realistic models is then to start with a vacuum with large negative cosmological constant, and add supersymmetry-breaking effects such as anti-$D$ branes that lift the vacuum energy to a tiny positive value as required by observations. Under the assumption that this last step does not jeopardize the stabilization of the scalars, one finally obtains a compactification with all the moduli stabilized and small positive cosmological constant.

Let us now cast our results in light of the discussion of the previous paragraph. The stabilization of the radial modulus is similar to the one in [1]. In fact the superpotential responsible for the stabilization of the radion has the same form as the one in [1] but while in our case this superpotential is induced at the classical level, in [1] it is a non-perturbative effect$^8$. Due to the fact that the ground state is AdS, the masses of the scalars within the same multiplet are different. This guarantees that in any AdS compactification at least half of the moduli will have a potential (of course with the caveat that in AdS the masses of the scalars could be negative). The main difference in the detuned RS case is that one of the scalars is massless at tree level but as we have seen it acquires a mass radiatively. It would be interesting then to build a toy model of flux compactification based on the detuned RS model which would allow to test the consistency of the construction in a simple model. In some of the examples we considered here, the negative tree-level potential was indeed

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$^8$Note however that the Kähler potentials are different at least in the large warping limit.
modified by a positive supersymmetry-breaking contribution. However, the resulting new maximum of the potential was stable because the total cosmological constant was still negative, so that the space was AdS$_4$. Clearly, such a maximum would no longer be stable if the correction leads to a net zero or positive cosmological constant.

Finally let us mention that the computation presented in this paper may be of interest from the point of view of the AdS/CFT correspondence. We computed a finite loop effect in a weakly coupled gravity theory with AdS$_4$ background. This corresponds to a subleading $1/N$ effect in the dual large $N$ three dimensional conformal field theory. It would be interesting to understand our results from the point of view of the corresponding $3D$ field theory.

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