When comparing recent experimental data from the H1 and ZEUS Collaborations at HERA for diffractive dijet production in deep-inelastic scattering (DIS) and photoproduction with next-to-leading order (NLO) QCD predictions using diffractive parton densities, good agreement is found for DIS. However, the dijet photoproduction data are overestimated by the NLO theory, showing that factorization breaking occurs at this order. While this is expected theoretically for resolved photoproduction, the fact that the data are better described by a global suppression of direct and resolved contribution by about a factor of two comes as a surprise. We therefore discuss in some detail the factorization scheme and scale dependence between direct and resolved contributions and propose a new factorization scheme for diffractive dijet photoproduction.

1. Diffractive $ep$ Scattering

It is well known that in high-energy deep-inelastic $ep$-collisions a large fraction of the observed events are diffractive. These events are defined experimentally by the presence of a forward-going system $Y$ with four-momentum $p_Y$, low mass $M_Y$ (in most cases a single proton and/or low-lying nucleon resonances), small momentum transfer squared $t = (p - p_Y)^2$, and small longitudinal momentum transfer fraction $x_F = q(p - p_Y)/q p$ from the incoming proton with four-momentum $p$ to the system $X$ (see Fig. 1). The presence of a hard scale, as for example the photon virtuality $Q^2 = -q^2$ in deep-inelastic scattering (DIS) or the large transverse jet momentum $p_{T_F}$ in the photon-proton centre-of-momentum frame, should then allow for calculations of the production cross section for the central system $X$ with the known methods of perturbative QCD. Under this assumption, the cross section for the inclusive production of two jets, $e+p \rightarrow e+2\text{jets}+X'+Y$, can be predicted from the well-known formulae for jet production in non-diffractive $ep$ collisions, where in the convolution of the partonic cross section with the
parton distribution functions (PDFs) of the proton, the latter ones are replaced by the diffractive PDFs. In the simplest approximation, they are described by the exchange of a single, factorizable pomeron/Regge-pole.

2. Diffractive Parton Distribution Functions
The diffractive PDFs have been determined by the H1 Collaboration at HERA from high-precision inclusive measurements of the DIS process $ep \to eXY$ using the usual DGLAP evolution equations in leading order (LO) and next-to-leading order (NLO) and the well-known formula for the inclusive cross section as a convolution of the inclusive parton-level cross section with the diffractive PDFs. A similar analysis of inclusive measurements has been published by the ZEUS Collaboration. A longer discussion of the extraction of diffractive PDFs can be found elsewhere.

3. QCD Factorization in Hard Diffraction
For inclusive diffractive DIS it has been proven by Collins that the formula referred to above is applicable without additional corrections and that the inclusive jet production cross section for large $Q^2$ can be calculated in terms of the same diffractive PDFs. The proof of this factorization formula, usually referred to as the validity of QCD factorization in hard diffraction, may be expected to hold for the direct part of photoproduction ($Q^2 \approx 0$) or low-$Q^2$ electroproduction of jets. However, factorization does not hold.
for hard processes in diffractive hadron-hadron scattering. The problem is that soft interactions between the ingoing two hadrons and their remnants occur in both the initial and final state. This agrees with experimental measurements at the Tevatron. Predictions of diffractive dijet cross sections for $p\bar{p}$ collisions as measured by CDF using the same PDFs as determined by H1 overestimate the measured cross section by up to an order of magnitude. This suppression of the CDF cross section can be explained by considering the rescattering of the two incoming hadron beams which, by creating additional hadrons, destroy the rapidity gap.

4. Factorization Breaking in Diffractive Photoproduction
Processes with real photons ($Q^2 \approx 0$) or virtual photons with fixed, but low $Q^2$ involve direct interactions of the photon with quarks from the proton as well as resolved photon contributions, leading to parton-parton interactions and an additional remnant jet coming from the photon (for a review see). As already said, factorization should be valid for direct interactions as in the case of DIS, whereas it is expected to fail for the resolved process similar as in the hadron-hadron scattering process. In a two-channel eikonal model similar to the one used to calculate the suppression factor in hadron-hadron processes, introducing vector-meson dominated photon fluctuations, a suppression by about a factor of three for resolved photoproduction at HERA is predicted. Such a suppression factor has recently been applied to diffractive dijet photoproduction and compared to preliminary data from H1 and ZEUS. While at LO no suppression of the resolved contribution seemed to be necessary, the NLO corrections increase the cross section significantly, showing that factorization breaking occurs at this order at least for resolved photoproduction and that a suppression factor $R$ must be applied to give a reasonable description of the experimental data.

5. Factorization Scale Dependence for Real Photons
As already mentioned elsewhere, describing the factorization breaking in hard photoproduction as well as in electroproduction at very low $Q^2$ by suppressing the resolved contribution only may be problematic. An indication for this is the fact that the separation between the direct and the resolved process is uniquely defined only in LO. In NLO these two processes are related. The separation depends on the factorization scheme and the factorization scale $M_\gamma$. The sum of both cross sections is the only
physically relevant cross section, which is approximately independent of the factorization scheme and scale \( R \). As demonstrated in Fig. 2, multiplying the resolved cross section with the suppression factor \( R = 0.34 \) destroys the correlation of the \( M_{\gamma} \)-dependence between the direct and resolved part \( R \) and the sum of both parts has a stronger \( M_{\gamma} \)-dependence than for the unsuppressed case \( (R = 1) \), where the \( M_{\gamma} \)-dependence of the NLO direct cross section is compensated to a high degree by the \( M_{\gamma} \)-dependence of the LO resolved part.

The introduction of the resolved cross section is dictated by perturbation theory. At NLO, collinear singularities arise from the photon initial state, which are absorbed at the factorization scale into the photon PDFs. This way the photon PDFs become \( M_{\gamma} \)-dependent. The equivalent \( M_{\gamma} \)-

![Figure 2](image-url). Photon factorization scale dependence of resolved (dashed) and direct (dotted) contributions to the diffractive dijet photoproduction cross section (full curve). Also shown is the sum of the direct and suppressed resolved contribution (dot-dashed curve).
dependence, just with the opposite sign, is left in the NLO corrections to the direct contribution. With this knowledge, it is obvious that we can obtain a physical cross section at NLO, i.e. the superposition of the NLO direct and LO resolved cross section, with a suppression factor $R < 1$ and no $M_\gamma$-dependence left, if we also multiply the $\ln M_\gamma$-dependent term of the NLO correction to the direct contribution with the same suppression factor as the resolved cross section. We are thus led to the theoretical conclusion that, contrary to what one may expect, not all parts of the direct contribution factorize. Instead, the initial state singular part appearing beyond LO breaks factorization even in direct photoproduction, presumably through soft gluon attachments between the proton and the collinear quark-antiquark pair emerging from the photon splitting. This would be in agreement with the non-cancellation of initial state singularities in diffractive hadron-hadron scattering.

6. The Transition Region of Virtual Photoproduction

We now present the special form of the $\ln M_\gamma$-term in the NLO direct contribution and demonstrate that the $M_\gamma$-dependence of the physical cross section cancels to a large extent in the same way as in the unsuppressed case ($R = 1$). These studies can be done for photoproduction ($Q^2 \simeq 0$) as well as for electroproduction with fixed, small $Q^2$. Since in electroproduction the initial-state singularity in the limit $Q^2 \to 0$ is more directly apparent than for the photoproduction case, we shall consider in this contribution the low-$Q^2$ electroproduction case just for demonstration. This diffractive dijet cross section has been calculated recently. A consistent factorization scheme for low-$Q^2$ virtual photoproduction has been defined and the full (direct and resolved) NLO corrections for inclusive dijet production have been calculated in. In this work we adapt this inclusive NLO calculational framework to diffractive dijet production at low-$Q^2$ in the same way as in, except that we multiply the $\ln M_\gamma$-dependent terms as well as the resolved contributions with the same suppression factor $R = 0.34$, as an example, as in our earlier work. The exact value of this suppression factor may change in the future, when better data for photoproduction and low-$Q^2$ electroproduction have been analyzed. We present the $\ln M_\gamma$-dependence of the partly suppressed NLO direct and the fully suppressed NLO resolved cross section $d\sigma/dQ^2$ and their sum for the lowest $Q^2$ bin.
The NLO corrections for virtual jet photoproduction have been implemented in the NLO Monte Carlo program JETVIP\textsuperscript{18} and adapted to diffractive dijet production in\textsuperscript{15}. The subtraction term, which is absorbed into the PDFs of the virtual photon $f_{\gamma/\gamma}(x, M_{\gamma})$, can be found in\textsuperscript{19}. The main term is proportional to $\ln(M_{\gamma}^2/Q^2)$ times the splitting function

$$P_{q_i \rightarrow \gamma}(z) = 2N_c Q_i^2 z^2 + \frac{(1-z)^2}{2},$$

(1)

where $z = p_1 p_2 / p_0 q \in [x; 1]$ and $Q_i$ is the fractional charge of the quark $q_i$. $p_1$ and $p_2$ are the momenta of the two outgoing jets, and $p_0$ and $q$ are the momenta of the ingoing parton and virtual photon, respectively. Since $Q^2 = -q^2 \ll M_{\gamma}^2$, the subtraction term is large and is therefore resummed by the DGLAP evolution equations for the virtual photon PDFs. After this subtraction, the finite term $M(Q^2)_{\overline{MS}}$, which remains in the matrix element for the NLO correction to the direct process\textsuperscript{17}, has the same $M_{\gamma}$-dependence as the subtraction term, i.e. $\ln M_{\gamma}$ is multiplied with the same factor. As already mentioned, this yields the $M_{\gamma}$-dependence before the evolution is turned on. In the usual non-diffractive dijet photoproduction these two $M_{\gamma}$-dependences cancel, when the NLO correction to the direct part is added to the LO resolved cross section\textsuperscript{16}. Then it is obvious that the approximate $M_{\gamma}$-independence is destroyed, if the resolved cross section is multiplied by a suppression factor $R$ to account for the factorization breaking in the experimental data. To remedy this deficiency, we propose to multiply the $\ln M_{\gamma}$-dependent term in $M(Q^2)_{\overline{MS}}$ with the same suppression factor as the resolved cross section. This is done in the following way: we split $M(Q^2)_{\overline{MS}}$ into two terms using the scale $p_T^* \gamma$ in such a way that the term containing the slicing parameter $y_s$, which was used to separate the initial-state singular contribution, remains unsuppressed. In particular, we replace the finite term after the subtraction by

$$M(Q^2, R)_{\overline{MS}} = \left[ -\frac{1}{2N_c} P_{q_i \rightarrow \gamma}(z) \ln \left( \frac{M_{\gamma}^2 z}{p_T^2 (1-z)} \right) + \frac{Q_i^2}{2} \right] R \left[ -\frac{1}{2N_c} P_{q_i \rightarrow \gamma}(z) \ln \left( \frac{p_T^2}{z Q^2 + y_s^2} \right) \right]$$

(2)

where $R$ is the suppression factor. This expression coincides with the finite term after subtraction (see Ref.\textsuperscript{19}) for $R = 1$, as it should, and leaves the second term in Eq. (2) unsuppressed. In Eq. (2) we have suppressed in addition to $\ln(M_{\gamma}^2/p_T^2)$ also the $z$-dependent term $\ln(z/(1-z))$, which is specific to the $\overline{MS}$ subtraction scheme as defined in\textsuperscript{17}. The second term in
Eq. (2) must be left in its original form, i.e. being unsuppressed, in order to achieve the cancellation of the slicing parameter \( y_s \) dependence of the complete NLO correction in the limit of very small \( Q^2 \) or equivalently very large \( s \). It is clear that the suppression of this part of the NLO correction to the direct cross section will change the full cross section only very little as long as we choose \( M_\gamma \simeq p_T^* \). The first term in Eq. (2), which has the suppression factor \( R \), will be denoted by DIRs in the following.

To study the left-over \( M_\gamma \)-dependence of the physical cross section, we have calculated the diffractive dijet cross section with the same kinematic constraints as in the H1 experiment\(^{20}\). Jets are defined by the CDF cone algorithm with jet radius equal to one and asymmetric cuts for the transverse momenta of the two jets required for infrared stable comparisons with the NLO calculations\(^{21}\). The original H1 analysis actually used a symmetric cut of 4 GeV on the transverse momenta of both jets\(^{22}\). The data have, however, been reanalyzed for asymmetric cuts\(^{21}\).

For the NLO resolved virtual photon predictions, we have used the PDFs \( \text{SaS1D} \)\(^{23}\) and transformed them from the DIS to the MS scheme as in Ref.\(^{17}\). If not stated otherwise, the renormalization and factorization scales at the pomeron and the photon vertex are equal and fixed to \( p_T^* = p_{T,jet}^* \). We include four flavors, i.e. \( n_f = 4 \) in the formula for \( \alpha_s \) and in the PDFs of the pomeron and the photon. With these assumptions we have calculated the same cross section as in our previous work\(^{15}\). First we investigated how the cross section \( d\sigma/dQ^2 \) depends on the factorization scheme of the PDFs for the virtual photon, i.e. \( d\sigma/dQ^2 \) is calculated for the choice \( \text{SaS1D} \) and \( \text{SaS1M} \). Here \( d\sigma/dQ^2 \) is the full cross section (sum of direct and resolved) integrated over the momentum and rapidity ranges as in the H1 analysis. The results, shown in Fig. 2 of Ref.\(^{19}\), demonstrate that the choice of the factorization scheme of the virtual photon PDFs has negligible influence on \( d\sigma/dQ^2 \) for all considered \( Q^2 \). The predictions agree reasonably well with the preliminary H1 data\(^{20}\).

We now turn to the \( M_\gamma \)-dependence of the cross section with a suppression factor for DIRs. To show this dependence for the two suppression mechanisms, (i) suppression of the resolved cross section only and (ii) additional suppression of the DIRs term as defined in Eq. (2) in the NLO correction of the direct cross section, we consider \( d\sigma/dQ^2 \) for the lowest \( Q^2 \)-bin, \( Q^2 \in [4,6] \text{ GeV}^2 \). In Fig. 3 this cross section is plotted as a function of \( \xi = M_\gamma/p_T^* \) in the range \( \xi \in [0.25;4] \) for the cases (i) (light full curve) and (ii) (full curve). We see that the cross section for case (i) has an appreciable \( \xi \)-dependence in the considered \( \xi \) range of the order of 40%,
Figure 3. Photon factorization scale dependence of resolved and direct contributions to $d\sigma/dQ^2$ together with their weighted sums for (i) suppression of the resolved cross section and for (ii) additional suppression of DIR$_{IS}$, using SaS1D virtual photon PDFs.

which is caused by the suppression of the resolved contribution only. With the additional suppression of the DIR$_{IS}$ term in the direct NLO correction, the $\xi$-dependence of $d\sigma/dQ^2$ is reduced to approximately less than 20%, if we compare the maximal and the minimal value of $d\sigma/dQ^2$ in the considered $\xi$ range. The remaining $\xi$-dependence is caused by the NLO corrections to the suppressed resolved cross section and the evolution of the virtual photon PDFs. How the compensation of the $M_{\gamma}$-dependence between the suppressed resolved contribution and the suppressed direct NLO term works in detail is exhibited by the dotted and dashed-dotted curves in Fig. 3. The suppressed resolved term increases and the suppressed direct NLO term decreases by approximately the same amount with increasing $\xi$.

In addition we show also $d\sigma/dQ^2$ in the DIS theory, i.e. without subtraction of any $\ln Q^2$ terms (dashed line). Of course, this cross section must be independent of $\xi$. This prediction agrees very well with the experimental
point, whereas the result for the subtracted and suppressed theory (full curve) lies slightly below. We notice, that for $M_\gamma = p_T^* T$ the additional suppression of $D_{\text{ISR}}$ has only a small effect. It increases $d\sigma/dQ^2$ by 5% only.

7. Conclusion

When comparing experimental data from the H1 and ZEUS Collaborations at HERA for diffractive dijet production in DIS and photoproduction with NLO QCD predictions using diffractive parton densities from H1 and ZEUS, good agreement is found for DIS assuming the H1 diffractive PDFs. However, the dijet photoproduction data are overestimated by the NLO theory, showing that factorization breaking occurs at this order. While this is expected theoretically for resolved photoproduction, the fact that the data are better described by a global suppression of direct and resolved contribution by about a factor of two comes as a surprise. We have therefore discussed in some detail the factorization scheme and scale dependence between direct and resolved contributions and proposed a new factorization scheme for diffractive dijet photoproduction.

Acknowledgments

The author thanks the organizers of the Ringberg workshop on New Trends in HERA Physics 2005 for the kind invitation, G. Kramer for his continuing collaboration, and the Comité de Financement des Projets de Physique Théorique de l’IN2P3 for financial support.

References