Comment on “Surprises in threshold antikaon-nucleon physics”

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It has recently been claimed \cite{oller} that the DEAR kaonic hydrogen data can be reconciled with $K^-p$ scattering data in a chiral unitary approach. In this comment we demonstrate that the proposed solution in \cite{oller} violates fundamental principles of scattering theory.

Now that new accurate results for the strong-interaction shift and width of kaonic hydrogen from the DEAR experiment are available \cite{dear}, there is renewed interest in an improved analysis of the $K^-p$ system. In general, non-perturbative coupled-channel techniques based on driving terms from the chiral SU(3) effective Lagrangian have proved successful in the strangeness $S = -1$ sector. However, a thorough investigation \cite{oller} of low-energy $K^-p$ interactions within such approaches pointed to questions of consistency of the DEAR experiment with previously measured sets of $K^-p$ scattering data. In contrast, it has been claimed very recently by Oller et al. \cite{oller} that within a chiral unitary approach both the scattering and the DEAR data can be accommodated. In the following we demonstrate that the proposed solution \cite{oller} to this problem violates basic principles of scattering theory and is rather an artifact of the model.

We have first reproduced the $A_I^+$ fit of \cite{oller}. The $s$-wave interaction kernel of the coupled-channels approach is derived from the Weinberg-Tomozawa term, contact interactions of next-to-leading chiral order as well as direct and crossed Born terms. The $s$-wave contribution from the crossed Born term, however, leads to unphysical subthreshold cuts which are an artifact of the on-shell formalism used in the coupled-channels approach. These unphysical cuts would not be present in a full field theoretical calculation as discussed in some detail in \cite{oller}. The subthreshold singularities induce imaginary pieces in the interaction kernel and hence spoil exact unitarity of the approach. As a test case we have explicitly checked the $S$-matrix element for $\pi^0\Lambda \to \pi^0\Lambda$, the channel with the lowest threshold. Its modulus deviates from 1 in the region below the first inelastic threshold ($\pi\Sigma$). Such unitarity violations can in fact be sizeable.

Nonetheless, we adopt the approach of \cite{oller} in order to investigate their results in more detail. A study of the analytical continuation of the $T$-matrix into the complex $\sqrt{s}$-plane reveals an isospin $I = 1$ pole at $\sqrt{s} = (1431 + 1.3i)$ MeV, i.e., right at the $K^-p$ threshold and above the physical region. Besides the fact that such a pronounced pole with a width of about 3 MeV on the real axis is not seen empirically (e.g. in the $\pi\Sigma$ invariant mass spectrum), it certainly violates fundamental principles of scattering theory. An unphysical $T$-matrix pole close to the physical region leads in its vicinity to a strong phase shift variation with energy as shown in Fig. \ref{fig:oller}. Such a steep decrease of a phase shift is in sharp contradiction to the Wigner condition which—based on a general causality principle for any finite-range interaction—imposes a certain bound on the rate at which the phase shift can change with energy \cite{wigner}. The limit is given by the range of the underlying interaction, i.e. roughly 1 fm in the case of strong interactions, whereas the steep fall-off in Fig. \ref{fig:oller} would require an interaction range of more than 300 fm in order to fulfill the Wigner condition. The appearance of the $I = 1$ pole in the upper half plane therefore violates causality and the postulate of maximal analyticity in $S$-matrix theory.

We conclude by emphasizing that the proposed solution of \cite{oller} and the artificial pole arise for certain highly fine-tuned combinations of the effective Lagrangian parameters due to some shortcomings of the applied coupled-channels approach, but should disappear in a full field theoretical calculation. This is further substantiated by the observation that tiny variations in the parameter values of fit $A_I^+$ lead to sizeable changes in the $K^-p$ scattering length, since the unphysical pole shifts slightly its position, bringing the fit into disagreement with the DEAR measurement.

We thank J. A. Oller for comparing the results of \cite{oller} with ours.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{oller.png}
\caption{$\pi^0\Lambda \to \pi^0\Lambda$ phase shift for the fit $A_I^+$ of \cite{oller}.}
\end{figure}

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