Landau Quantization of Neutral Particles in an External Field

Claudio Furtado\textsuperscript{1*}, J. R. Nascimento and L. R. Ribeiro
\textsuperscript{1}Departamento de Física, CCEN, Universidade Federal da Paraíba, Cidade Universitária, 58051-970 João Pessoa, PB, Brazil

The quantum dynamics of an induced electric dipole in the presence of a configuration of crossed electric and magnetic fields is analyzed. This field configuration confines the dipole in a plane and produces a coupling similar to the coupling of a charged particle in the presence of external magnetic field. In this work we investigate the analog of Landau levels in induced electric dipoles in a system of neutral particles. The energy levels and eigenfunctions are obtained exactly.

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The quantum dynamics of cold atoms in the presence of an electromagnetic field promises new possibilities to study the quantum properties of many body systems. This system can be easily controlled and manipulated by electromagnetic fields. The cold atoms make them ideal candidates for the study of several intrinsically quantum phenomena. In recent years the advance of cold atom technology made it possible to simulate several solid state effects employing neutral atoms and techniques from quantum optics\textsuperscript{[1, 2, 3]}. Recently, the neutral atoms techniques to simulate the behavior of a charged particle in this systems\textsuperscript{[3, 4, 5, 6]} has been developed. In this way, the study of systems that simulate the strongly interacting system in a cold atom has attract much attention in recent years\textsuperscript{[7, 8]}.

Electric dipole moment of atoms in the presence of an appropriate electromagnetic field configuration has been utilized in literature to inquire above some physical effects\textsuperscript{[7, 8, 9, 10]}. The study of quantum phases in the quantum dynamics of the electromagnetic dipole has its origin in the Aharonov-Casher effect\textsuperscript{[11]} for the magnetic dipole in the presence of an electric field, that is reciprocal effect of the well known Aharonov-Bohm\textsuperscript{[12]} effect. The dual effect of Aharonov-Casher effect was studied independently by He and McKellar\textsuperscript{[13, 14]} and Wilkens\textsuperscript{[15]}, which investigated the quantum dynamics of the electric dipole in the presence of magnetic field. In fact, electric dipoles can give rise to a variety of different phenomena that generate geometric phases\textsuperscript{[12, 13, 14, 15, 16]} in different field-dipole configurations. Wei, Han and Wei\textsuperscript{[17]} have proposed a more realistic field configuration that generate a He-McKellar-Wilkens effect. In fact, all quantum effects in the dynamics of the dipole occur due to the coupling of electric/magnetic dipole with external magnetic/electric field make a mimic of minimal coupling of charged particle with an external magnetic field. In this work we use this configuration to study a possibility of an analog Landau quantization due to induced electric dipoles in cold atom systems.

The interaction of the electromagnetic field with a charged particle, plays an important role in the generation of collective phenomena, such as, fractional statistics\textsuperscript{[22]} and the quantum Hall effect. The quantum motion of a charged particle in the presence of a constant magnetic field is described by the theory of Landau\textsuperscript{[23]}. The Landau quantization in two dimensions makes the energy levels coalesce into a discrete spectrum. The Landau levels present a remarkable interest in several areas of the physics among them the quantum Hall effect\textsuperscript{[22]}. On the order hand, the Landau levels were studied for different curved surfaces\textsuperscript{[24, 25]} with the interest in several areas of physics. The idea of analogs of Landau quantization was proposed initially by Ericsson and Sjöqvist\textsuperscript{[26]} inspired in the the work of Paredes et al. that studied the possibility of an analog of the Hall effect in Bose-Einstein condensates. The idea of Ericsson and Sjöqvist\textsuperscript{[27]} was that the Aharonov-Casher interaction can be used to generate an analog of Landau levels in systems of neutral atoms. In a recent work we extend this idea to systems of electric dipoles in the presence of a magnetic field and demonstrated also a similar effect of Landau quantization to electric dipoles\textsuperscript{25}. In the present work we analyze this possibility in a system of induced dipoles in the presence of electric and magnetic fields. The aim this work is to demonstrate that the Wei, Han Wei configuration for topological phase can be used to investigate the landau quantization in a similar scheme that was treated in the Landau-Aharonov-Casher effect by Ericsson and Sjöqvist\textsuperscript{[24]} and in the Landau-He-McKellar-Wilkens effect by us\textsuperscript{[25]}. The central idea is that a system of cold atom in this condition exhibits Landau quantization and can be used in future study of atomic quantum Hall in the condition presented here. The cold atom is treated as structureless induced dipole moment. We consider that our system was submitted the following electric field

$$E = \frac{\rho}{2} \hat{e}_r,$$  \hspace{1cm} (1)

where $\rho$ is the charge density. The system is also submitted to the external magnetic field

$$B = B_0 \hat{e}_z.$$  \hspace{1cm} (2)

The presence of electric field induces an electric dipole in the cold atom given by

$$d = \alpha (E + \mathbf{v} \times \mathbf{B})$$  \hspace{1cm} (3)

$^*$Electronic Address: furtado@fisica.ufpb.br
where \( \alpha \) is the dielectric polarizability of the cold atoms and \( v \) is the velocity of the atom. The Lagrangian that describes the electric dipole in the presence of an electromagnetic field is given by

\[
L = \frac{1}{2}Mv^2 + \frac{1}{2} \mathbf{d} \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B}).
\]  

(4)

Considering that the dipole is induced in cold atoms, the Lagrangian is written as

\[
L = \frac{1}{2}(M + \alpha B^2)v^2 + \frac{1}{2} \alpha E^2 + \alpha \mathbf{v} \cdot \mathbf{B} \times \mathbf{E}.
\]  

(5)

The term \( \alpha \mathbf{v} \cdot \mathbf{B} \times \mathbf{E} \) is known as the energy of Rötpgen and is responsible for the modification in the canonical momentum of the system that differs from the mechanical momentum of the atom. The study of this term and its similarity with the Chern-Simons term has been widely investigated recently. Baxter has showed that the systems of cold Rydberg atoms and Chern-Simons theory are analog to each other. In a similar system, Zhang has analyzed the possibility of testing spatial noncommutativity. The study of this term and its similarity with the Chern-Simons theory gives the precise condition under which the Landau quantization occurs in the cold atom system. Note also that, the Wei, Han and Wei field configuration used in this work constrained the motion of the dipole to be planar and to obey these conditions. Now, the problem is reduced to the solution of the Schrödinger equation associated to \( (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \). In this way, we write the Schrödinger equation for this system, in cylindrical coordinates, in the following form

\[
-\frac{1}{2m^*} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \phi^2} \right] - \frac{i \omega}{2} \frac{\partial \psi}{\partial \phi} + \frac{m^* \omega^2}{8} r^2 \psi + \frac{m^* \omega^2}{8 \alpha B_0} r^2 \psi = \mathcal{E} \psi,
\]  

(10)

where \( \omega = \frac{\alpha B^2}{m^*} \). We use the following Ansatz to the solution of Schrödinger equation

\[
\psi = e^{i \ell \phi} R(r),
\]  

(11)

where \( \ell \) is an integer number. Using the Eq. (11), the Eq. (10) assumes the following form:

\[
\frac{1}{2M} \left( R'' + \frac{1}{r} R' - \frac{m^*}{r^2} R \right) + \left( \mathcal{E} - \frac{M \omega^2}{8} r^2 - \frac{\ell \omega}{2} - \frac{m^* \omega^2}{8 \alpha B_0} r^2 \right) R = 0.
\]  

(12)

We use the following change of variables

\[
\xi = \frac{m^* \omega}{2} r^2.
\]  

(13)

In this way, Eq. (12) is transformed into

\[
\xi R'' + R' + \left( -\frac{\delta \xi}{4} + \beta - \frac{\ell^2}{4 \xi} \right) R = 0,
\]  

(14)

where \( \beta = \frac{\xi}{\omega} - \frac{m^*}{\omega B_0} \) and \( \delta = 1 + \frac{m^*}{\alpha B_0} \). Assuming for the radial eigenfunction the form

\[
R(\xi) = e^{-\delta \xi/2} \xi^{(\ell/2)} \zeta(\xi),
\]  

(15)

which satisfies the usual asymptotic requirements and the finiteness at the origin for the bound state, we have

\[
\xi \frac{d^2 \zeta}{d \xi^2} + \left( \left| \ell \right| + 1 \right) \delta \xi \frac{d \zeta}{d \xi} - \gamma \zeta = 0,
\]  

(16)

where \( \gamma = \beta - \frac{2(\ell + 1)}{\delta \xi} \). We find that the solution of equation (16) is the degenerated hypergeometric function

\[
\zeta(\xi) = F \left( -\gamma, \left| \ell \right| + 1, \delta \xi \right).
\]  

(17)
In order to have normalization of the wave function, the series in (17) must be a polynomial of degree \( \nu \), therefore,

\[
\gamma \delta = \frac{1}{\delta} \beta - \frac{(|\ell| + 1)}{2} = n. \tag{18}
\]

Where \( n \) is an integer number. With this condition, we obtain discrete values for the energy, given by

\[
E_{n,\ell} = \left( n + \frac{|\ell|}{2} + \frac{1}{2} \right) \omega + \frac{\ell}{2} \omega, \tag{19}
\]

where \( \omega = \delta \omega \). In the limit that \( \delta \rightarrow 1 \) the eigenvalues are given by

\[
E_{n,\ell} = \left( n + \frac{|\ell|}{2} + \frac{1}{2} + \frac{\ell}{2} \right) \omega \tag{20}
\]

Note that, this limit is characterized by a high magnetic field. In this case the system is similar to Landau levels. The radial eigenfunction is then given by

\[
R_{n,\ell} = \frac{1}{a^{1+|\ell|}} \left[ \frac{|(|\ell| + n)|!}{2^{|\ell|} n! |\ell|!} \right]^{1/2} \exp \left( -\frac{\delta r^2}{4a^2} \right) \times r^{|\ell|} F \left[ -n, |\ell| + 1, \frac{\delta r^2}{2a^2} \right]. \tag{21}
\]

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