Lower Neutrino Mass Bound from SN1987A Data and Quantum Geometry

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Abstract

A lower bound on the light neutrino mass $m_{\nu}$ is derived in the framework of a geometrical interpretation of quantum mechanics. Using this model and the time of flight delay data for neutrinos coming from SN1987A, we find that the neutrino masses are bounded from below by $m_{\nu} \gtrsim 10^{-4} - 10^{-3}$eV, in agreement with the upper bound $m_{\nu} \lesssim (\mathcal{O}(0.1) - \mathcal{O}(1))$ eV currently available. When the model is applied to photons with effective mass, we obtain a lower limit on the electron density in intergalactic space that is compatible with recent baryon density measurements.

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I. INTRODUCTION

The detection of neutrinos from SN1987A by the Kamiokande and IMB experiments has confirmed the stellar collapse model, giving new insights into neutrino physics.

In essence, the experimental results are as follows. The first optical record of the supernova SN1987A occurred at $\sim 10:37:55$ UT on 23 February 1987 [1]. About three hours before, neutrino signals were observed, almost simultaneously, at the Kamioka (7:35:35 UT) and IMB (7:35:41 UT) detectors [2].

The small difference between the arrival times of neutrinos and photons (on a distance of $52 \pm 5$ Kpc between SN1987A and Earth) was used to set some constraints on the validity of the equivalence principle [3], by attributing the time delays undergone by photons and neutrinos to different gravity couplings. This analysis rests, however, on the hypothesis that the time difference of the emission of photons and neutrinos during the explosion is not very large.

In the case of massive particles, the signals from SN1987A were also used to evaluate an upper bound on their masses [4]. If, in fact, $d$ is the distance of the neutrino source from Earth, then the time of flight delay of neutrinos relative to that of photons emitted by the same source is

$$\Delta t = \delta t_\nu - \delta t_\gamma \simeq \frac{m_\nu^2}{2E^2} d. \quad (I.1)$$

There is no unanimous agreement on how to fix an upper limit on the duration of neutrino emission. Old analyses of the Kamiokande data use $\sim 4s$ as such a limit and yield an upper bound $m_\nu \lesssim 27eV$ or $m_\nu \lesssim 12eV$, depending on the choice of events used. On the contrary, other models of supernova explosion allow for different limits, of order $\sim 10s$ [5]. Consequently, we will consider a varying emission time limit in the following analysis and derive our results as a function of this parameter.

The aim of this work is to derive a lower bound on the neutrino mass in the framework of a model introduced by Caianiello and co-workers to provide quantum mechanics with
a geometrical framework [6]. An interesting consequence of the model is that the proper acceleration of massive particles has an upper limit $A_m = 2mc^3/h$, where $m$ is the invariant rest mass of the particle. This mass dependent limit, or maximal proper acceleration (MA) $A_m$, can be derived from quantum mechanical considerations [7–9] and the fact that the acceleration is largest in the particle rest frame. The absolute value of the proper acceleration therefore satisfies the inequality $a \leq A_m$. No counterexamples are known to the validity of this inequality that has at times been elevated to the status of principle.

Classical and quantum arguments supporting the existence of a MA have been discussed in the literature [10–12]. MA also appears in the context of Weyl space [13], and of a geometrical analogue of Vigier's stochastic theory [14] and plays a role in several issues. It is invoked as a tool to rid black hole entropy of ultraviolet divergences [15]. MA is at times regarded as a regularization procedure [16] that avoids the introduction of a fundamental length [17], thus preserving the continuity of space-time.

An upper limit on the acceleration also exists in string theory where Jeans-like instabilities occur [18,19] when the acceleration induced by the background gravitational field reaches the critical value $a_c = \lambda^{-1} = (m\alpha)^{-1}$ where $\lambda$, $m$ and $\alpha^{-1}$ are string size, mass and tension. At accelerations larger than $a_c$ the string extremities become casually disconnected. Frolov and Sanchez [20] have found that a universal critical acceleration must be a general property of strings. It is the same cut–off required by Sanchez in order to regularize the entropy and the free energy of quantum strings [21].

Applications of Caianiello’s model include cosmology [22], the dynamics of accelerated strings [23] and neutrino oscillations [24,25]. The model also makes the metric observer–dependent, as conjectured by Gibbons and Hawking [26].

Recently, the model has been applied to particles falling in the gravitational field of a spherically symmetric collapsing object [27]. In this problem MA manifests itself through a spherical shell external to the Schwarzschild horizon and impenetrable to classical and quantum particles [28]. The shell is not a sheer product of the coordinate system, but survives, for instance, in isotropic coordinates. It is also present in the Reissner-Nordström
and Kerr [30] cases. In the model, the end product of stellar collapse is therefore represented by compact, impenetrable astrophysical objects whose radiation characteristics are similar to those of known bursters [31].

Caianiello’s model is based on an embedding procedure [27] that stipulates that the line element experienced by an accelerating particle is represented by

\[ d\tau^2 = \left(1 + \frac{g_{\mu\nu} \dddot{x}^\mu \dddot{x}^\nu}{\mathcal{A}_m^2}\right) g_{\alpha\beta} dx^\alpha dx^\beta = \left(1 + \frac{a^2(x)}{\mathcal{A}_m^2}\right) ds^2 \equiv \sigma^2(x) ds^2, \tag{I.2} \]

where \( g_{\alpha\beta} \) is a background gravitational field. The effective space-time geometry given by (I.2) therefore exhibits mass-dependent corrections that in general induce curvature and violations of the equivalence principle. The MA corrections appear in the conformal factor in (I.2) and can not therefore modify null geodesics, hence the dynamics of massless particles, as stated below.

The four–acceleration \( \dddot{x}^\mu = \frac{d^2 x^\mu}{ds^2} \) appearing in (I.2) is a rigorously covariant quantity only for linear coordinate transformations. Its transformation properties are however known and allow the exchange of information among observers. Lack of covariance for \( \dddot{x}^\mu \) in \( \sigma^2(x) \) is not therefore fatal in the model. The justification for this choice lies primarily with the quantum mechanical derivation of MA which applies to \( \dddot{x}^\mu \), requires the notion of force, is therefore Newtonian in spirit and is fully compatible with special relativity. The choice of \( \dddot{x}^\mu \) in (I.2) is, of course, supported by the weak field approximation to \( g_{\mu\nu} \) which is, to first order, Minkowskian. On the other hand, Einstein’s equivalence principle does not carry through to the quantum level readily [32,33], and the same may be expected of its consequences, like the principle of general covariance [34]. Complete covariance is, of course, restored in the limit \( \hbar \to 0 \), whereby all quantum corrections, including those due to MA, vanish.

As shown below, the existence of a lower bound on the neutrino mass follows, in this framework, from the fact that MA corrections depend inversely on the mass of the particle (as well as directly on their energy). In particular, the gravitational time delay undergone by neutrinos in the interaction with the gravitational field of the supernova is affected by these corrections.
In Section II, we consider the data registered by the Kamiokande experiment, and perform an analysis similar to that used in [4] and briefly mentioned above. Moreover, we assume, as in [4], that all events are due to neutrinos of the same mass and we do not consider the possibility of neutrino oscillations.

Even though our method can also be applied to the photon-neutrino delay, the comparison between the arrival times of neutrinos of different energies provides a more stringent bound.

The time delays are calculated in the Section III, which is then followed by the actual determination of the lower mass bounds and a short discussion.

**II. TIME DELAYS AND MAXIMAL ACCELERATION**

It is sufficient, for our purposes, to consider particles that escape radially from the SN core. The MA effects induced by the gravitational field of our Galaxy are completely negligible, even though the time delay caused by our Galaxy is relevant in testing the equivalence principle [3]. In the following we will use units $\hbar = c = 1$.

The time delay for a massive particle is given by [11]

$$\delta t = \left(1 + \frac{m^2}{2E^2}\right)(d - r) + \frac{1}{2} \int_r^d h_{\mu\nu}(r')k^\mu k'^\nu dr',$$

(II.1)

where the metric deviation $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ for a weak, spherically symmetric gravitational field characterized by the Newtonian potential $\phi(r)$, is given, in General Relativity, by $h_{00} = 2\phi(r), h_{ij} = -2\phi(r)\delta_{ij}$. In (II.1) the particle momentum is $k^\mu = (1, \hat{k})$ ($\hat{k} = k/E$), so that $k^\mu k_\mu = m^2/E^2$. The point where the particles are generated is $r \ll d$. The reference frame is located at the source center.

In order to compute the MA corrections to the time delay we start with the effective metric (I.2) experienced by a massive particle and write

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + \frac{a^2}{A_m^2} \eta_{\mu\nu} + O\left(\frac{a^2}{A_m^2} h_{\mu\nu}\right) = \eta_{\mu\nu} + h_{\mu\nu} + \tilde{h}_{\mu\nu}.$$  

(II.2)

In the ultra-relativistic approximation $E \gg m$ [11,27,28] we find
\[ \tilde{h}_{\mu\nu}(r')k^\mu k^\nu = \frac{m_\nu^2}{A_m^2E^2} \tilde{h}_{\mu\nu} \tilde{x}^\mu \tilde{x}^\nu \simeq \frac{4}{A_m^2} \left( \frac{E}{m_\nu} \right)^2 \phi'^2(r) = \frac{4}{A_m^2} \left( \frac{E}{m_\nu} \right)^2 \left( \frac{GM_r}{r^2} \right)^2, \]  

where \( M_r \) is the mass enclosed in the sphere of radius \( r \) concentric to the gravitational source.

The corresponding result for rigorously massless photons cannot be obtained from (II.3) in the limit \( m \to 0 \). It is shown in the Appendix that for photons \( \tilde{h}_{\mu\nu}k^\mu k^\nu = 0 \), and that, therefore, their MA corrections do not contribute to the present problem.

With the exclusion of massless particles, the MA corrections (II.3) scale as \( 1/m^4 \). In fact, in the weak field and ultrarelativistic approximations, the proper acceleration \( a \) is related to the acceleration in the lab frame \( \ddot{r} \) by \( |a|^2 = \gamma^2|\ddot{r}|^2 \simeq \gamma^4 \phi'^2(r) \), where \( \gamma = E/m \). The \( m_\nu^{-4} \) dependence is then produced by the factor \( (m/E)^2 \) due to the definition of \( k^\mu \) and the factor \( 1/m^2 \) due to \( A_m^{-2} \).

Eq.(II.1) becomes

\[ \delta t_{A_m} = \left( 1 + \frac{m_\nu^2}{2E^2} \right) (d - r) + \frac{1}{2} \int_r^d \tilde{h}_{\mu\nu}(r')k^\mu k^\nu dr' = \delta t + \frac{2}{A_m^2} \left( \frac{E}{m_\nu} \right)^2 \int_r^d \phi'^2(r')dr'. \]  

In order to perform the integral in (II.1) and obtain the MA correction to be compared with the experimental data, we need a model of the supernova and of its gravitational field. For the purpose of estimating a lower bound for the neutrino mass, very detailed models are not particularly useful. To give significance to our results, it is nevertheless necessary to show that they are not too dependent on the model used to describe the collapsing star.

We therefore use in the following a simple model and show that the results are sufficiently robust against reasonable variations of the parameters. The model we refer to is intended to describe the density profile of the supernova a few milliseconds after the implosion, when, according to our current theoretical understanding, neutrinos are emitted [35]. We consider a time-independent mass distribution, implicitly assuming that the time scale of the emission is so rapid that the evolution of the profile is negligible. We choose a density profile

\[ \rho(r) = \begin{cases} 
\rho_c & r < r_c \\
\rho_c \left( \frac{r}{r_c} \right)^{-n} & r > r_c,
\end{cases} \]  

(II.5)
which describes a SN with a hard core and a halo of matter of decreasing density. As typical parameters we take \( \rho_c = 10^{14} \text{ g cm}^{-3} \), \( r_c = 15 \text{ Km} \), \( 3.5 \leq n \leq 10 \). Their influence on the final results is discussed later. An elementary integration gives

\[
M_r = \begin{cases} 
M_c \left( \frac{r}{r_c} \right)^3 & r < r_c \\
M_c \left[ \frac{n-2}{n-3} - \frac{1}{n-3} \left( \frac{r_c}{r} \right)^{n-3} \right] & r > r_c,
\end{cases}
\]

(II.6)

where \( M_c \simeq 1.4 \times 10^{30} \text{ Kg} \simeq 0.7 M_{\odot} \) is the core mass. Our model satisfies the weak field approximation to the gravitational field and the perturbation approach required to calculate the MA corrections. As reference values, we have, for \( r = r_c \),

\[
\phi(r_c) = \frac{n-1}{n-2} \frac{GM_c}{r_c} \sim 0.15 < 1
\]

(II.7)

\[
g_c = \frac{GM_c}{r_c^2} \simeq 4.15 \times 10^{11} \frac{m}{s^2} \ll A_m \quad \text{for} \quad m_\nu \gg 10^{-12} \text{eV}.
\]

Both approximations therefore hold true.

As our current knowledge of SN dynamics suggests, we assume that neutrinos are generated outside the core, a distance \( R \) from the centre. Because \( d \gg R \), we can also replace the upper limit of integration in (II.4) with \(+\infty\). We then have

\[
\int_{R}^{\infty} \phi^2(r')dr' = \frac{g_c^2 r_c}{(n-3)^2} \int_{x}^{\infty} \frac{(n-2-x^{3-n})^2}{z^4} dz
\]

(II.8)

\[
= \frac{g_c^2 r_c}{(n-3)^2} \left[ \frac{(n-2)^2}{3x^3} - \frac{2(n-2)}{nx^n} + \frac{1}{(2n-3)x^{2n-3}} \right]
\]

\[
\simeq g_c^2 r_c F(x, n),
\]

where \( x = R/r_c \). Considering the substantial agreement among the current SN models on the values of \( M_c \) and \( r_c \), the function \( F(x, n) \) is the really model-dependent part of our calculation. It could in principle be very sensitive to the choice of the parameters and must therefore be analyzed carefully. According to current astrophysical models, the choice \( 1.5 \leq x \leq 5.5 \), i.e. \( 20 \text{ Km} \leq R \leq 80 \text{ Km} \), is plausible. To have an idea of the magnitude of \( F(x, n) \) and of its variation, we plot it in Fig.1 for values of the parameters in the range considered. We see that \( 0.0025 \lesssim F(x, n) \lesssim 0.25 \). For the sake of clarity, we
write $F(x, n) = 0.025 \Delta$, with $1/10 \leq \Delta \leq 10$. The uncertainty in $F(x, n)$ extends then by two orders of magnitude. We will see, however, that this results only in a very small error in the estimate of the neutrino mass.

Finally, the time delay for a massive particle with MA corrections can be written in the form

$$\delta t_{A_m} = \delta t + 2r_c \left( \frac{g_c}{A_m} \right)^2 \left( \frac{E}{m_\nu} \right)^2 F(x, n),$$

or, numerically, as

$$\delta t_{A_m} \simeq \delta t + 0.8 \times 10^{-16} \Delta \left( \frac{E}{\text{MeV}} \right)^2 \left( \frac{m_\nu}{\text{eV}} \right)^{-4} \text{s.} \quad (\text{II.10})$$

If we re-write the last expression as

$$\delta t_{A_m} \simeq \delta t + 0.8 \times 10^{-16} \left( \frac{E}{\text{MeV}} \right)^2 \left( \frac{1}{\sqrt{\Delta}} \frac{m_\nu}{\text{eV}} \right)^{-4} \text{s}, \quad (\text{II.11})$$

we see that an error of one order of magnitude in $F(x, n)$ only results in a factor $\sqrt{10} \simeq 1.8$ in our estimate of the neutrino mass. This clearly shows the stability of our predictions against a reasonable variation of the parameters used.

Starting from this result, we derive in the next section a lower bound on the neutrino mass, using the experimental results on the relative time delay between neutrinos of different energies.

III. LOWER BOUNDS ON THE NEUTRINO MASS

The arrival time of neutrinos of different energies, recorded by the Kamioka experiment, can be now analyzed. We closely follow the standard analysis that leads to an upper bound on the mass of the neutrinos [4].

If $t_0 \simeq 5.3 \times 10^{12}$ s is the light travel time from SN1987A to Earth for a neutrino of energy $E$, then the relationship between observation and emission times is

$$t_{obs} - t_{em} = t_0 \left( 1 + \frac{m_\nu^2}{2E^2} \right) + \delta t_{A_m}. \quad (\text{III.1})$$
As above, we assume that $t_0 m^2_\nu / 2E^2$ is negligible with respect to $\delta t_{A_m}$ in the mass range of interest, and discuss this assumption at the end of our analysis. We then obtain

$$t_{em} = t_{obs} - \left[0.8 \times 10^{-4} \left(\frac{E}{\text{MeV}}\right)^2 \left(\frac{m_\nu}{\sqrt{\Delta}10^{-3}\text{eV}}\right)^{-4}\right] \text{s}.$$  (III.2)

As normally done in the literature [4], we consider only events 1-5 and 7-9 of the Kamioka experiment (see Table I). We exclude event 6 because it received fewer than 20 photomultiplier hits, and events 10-12, probably associated with a late burst of the SN or with a long tail of the emission time distribution.

The values of $t_{em}$ obtained from the data are shown in Fig.2 as a function of $(m_\nu/\sqrt{\Delta})^{-4}$. For each event two lines are shown, corresponding to energies at the upper and lower limits of the declared error in the energy measurements.

We now calculate the minimum time interval over which neutrinos could have been emitted. The result is shown in Fig.3, where this quantity is plotted as a function of $(m_\nu/\sqrt{\Delta})^{-4}$. By requiring that the minimum time interval is less than a fixed value (depending on the specific supernova model), we obtain a significant lower bound on the neutrino mass.

As two relevant examples, we choose $\Delta t \leq 4s$ and $\Delta t \leq 10s$. We obtain

$$\Delta t \leq 4s \quad \left(\frac{m_\nu}{\sqrt{\Delta}10^{-3}\text{eV}}\right) \lesssim 91.4 \quad \rightarrow \quad m_\nu \gtrsim (0.17 - 0.54)10^{-3}\text{eV} \quad \text{(III.3)}$$

$$\Delta t \leq 10s \quad \left(\frac{m_\nu}{\sqrt{\Delta}10^{-3}\text{eV}}\right) \lesssim 196.1 \quad \rightarrow \quad m_\nu \gtrsim (0.15 - 0.48)10^{-3}\text{eV}. $$

We note that our conclusions are very robust against a variation of the supernova explosion model.

We can verify that the kinematical time delay is not relevant for a mass of this order of magnitude. Recalling that the light travel time is $t_0 \simeq 5.3 \times 10^{12}$ s, the kinematical contribution to $\delta t$ is

$$\delta t_k = t_0 \frac{m^2_\nu}{2E^2} \approx 10^{-8}\text{s}$$  (III.4)

for $E \approx 10\text{MeV}$, which is completely negligible.
In other approaches, an improvement in the bound is obtained by considering only events 1-5 as neutrinos emitted in an initial pulse of less than 1s duration. This type of analysis does not lead to a sizable improvement in our case and we do not pursue it here.

It has been pointed out that photons acquire an effective mass \( m_{\text{eff}} \) while travelling in the intergalactic medium and that (II.4) may therefore be applied to this physical situation with the substitution \( m_\nu \to m_{\text{eff}} \). This follows from the fact that the dispersion relation of the photon propagating in a medium becomes \( \omega^2 - k^2 = \Pi_a(\omega, k) \), where \( \Pi_a \) are functions that represent the medium response to the electromagnetic field. The effective mass is then defined by \( m_{\text{eff}}^2 = \Pi_a(\omega, k) \) and differs for different polarizations and wave vectors \([5]\).

It is difficult, in the general case, to think of \( m_{\text{eff}} \) as an object capable of a single, unified response to mechanical and gravitational solicitations. If, however, \( m_{\text{eff}} \) becomes a constant independent of wave number and frequency as in the case of an interstellar plasma at \( T = 0 \), then \( m_{\text{eff}} = \omega_p = \sqrt{4\pi\alpha\frac{m_e}{n_e}} \), where \( \alpha \) is the fine structure constant and \( m_e \) and \( n_e \) electron mass and density respectively. We take \( n_e \approx n_{\text{baryons}} \). Under these conditions \( m_{\text{eff}} \) is the only information regarding the mass in the particle’s wave equation. Accordingly, we may assume that \( m_{\text{eff}} \) behaves mechanically as a true mass and tentatively apply Caianiello’s model to it. Then the weak field approximation condition \( |\tilde{h}_{\mu\nu}| < 1 \) gives

\[
r > \sqrt{\frac{2E^2GM}{m_{\text{eff}}^3}} \equiv r_\gamma.
\]

Assuming that \( m_{\text{eff}} \) makes a contribution when the photon is well in intergalactic space, but before arrival so that \( d > r > r_\gamma \), we find

\[
n_e > \frac{m_e}{4\pi\alpha} \left( \frac{2E^2GM}{d^2} \right)^{2/3}. \tag{III.5}
\]

Recent measurements of the baryon density by the WAMP collaboration \([36]\) place upper limits on \( n_e \) \([37]\). Choosing for illustrative purposes the values \( E \sim 1.4 \text{ eV} \) (for frequencies \( \sim 6 \times 10^{14} \text{Hz} \)), \( M \sim M_\odot \), and \( d \sim 50 \text{ kpc} \), we find \( n_e > 9.2 \times 10^{-10} \text{cm}^{-3} \), in agreement with the upper limit \( n_e < 2.7 \times 10^{-7} \text{cm}^{-3} \) from WMAP measurements. This limit gives \( m_{\text{eff}} \sim 1.93 \times 10^{-14} \text{eV} \), \( r_\gamma \sim 1.5 \times 10^{-2}d \). Caianiello’s model therefore yields the lower bound for \( n_e \) given by Eq. (III.5) when applied to a photon of non-vanishing effective mass.
As for the time delay produced by $m_{\text{eff}}$, one finds the results

$$
\delta t_{\gamma} \leq \frac{1}{3r_{\gamma}^3} \left( \frac{2 E_{\text{GM}}}{m_{\text{eff}}^2} \right)^2 \simeq 1.5 \times 10^{-18} s \tag{III.6}
$$

and

$$
\delta t_{\gamma k} \simeq t_o \frac{m_{\text{eff}}^2}{2E^2} \sim 1.6 \times 10^{-16} s , \tag{III.7}
$$

that are negligible relative to the corresponding neutrino values.

**IV. CONCLUSIONS**

In conclusion, we have shown that working in the framework of Caianiello’s quantum geometry, the time of flight delay of neutrinos from SN1987A leads to a lower bound on the neutrino mass

$$
m_{\nu} \gtrsim (10^{-4} - 10^{-3}) \text{eV}.
$$

This bound is very close to upper bounds coming from cosmological constraints. Actually, neutrinos with mass $m_{\nu} \gg kT_{\text{CMB}} \sim 3 \times 10^{-4} \text{eV}$, where $T_{\text{CMB}} \sim 3 \text{ K}$ is the present CMB temperature, contribute to the known mass density of the Universe $\Omega_m = \rho_m/\rho_c$, where $\rho_c = 3H_0^2/8\pi G_N$ is the critical density and $H_0 = 100h \text{ km/s Mpc}$ is the Hubble constant ($h \sim 0.7$) relative to non-relativistic matter. The neutrino density energy $\Omega_{\nu}$ is given by $\Omega_{\nu} h^2 = \sum m_i/93\text{eV}$ [38]. Here $\Omega_m h^2 \sim 0.15$ and $\Omega_{\nu} < \Omega_m$. Experimental data about the cosmological parameters lead to the upper bound on the neutrino mass sum $\sum m_i \lesssim 3 \text{ eV}$ [38], hence $m_{\nu}$ should have the upper bound $m_{\nu} \lesssim 1 \text{ eV}$. Future data from MAP/PLANCK CMB and the high precision galaxy survey (Sloan Digital Sky Survey [39]) might relax the bound to $\sum m_i < 0.3 \text{ eV}$ [40], or to $\sum m_i < 0.12 \text{ eV}$ [41] at 95% C.L.

More stringent limits on the light neutrino mass follow from data on neutrino oscillations, which fix $\Delta m_{21}^2$ and $\Delta m_{32}^2$, and from the usual relations that link $m_2$ and $m_3$ to $m_1$

$$
m_2 = \sqrt{m_1^2 + \Delta m_{21}^2} , \quad m_3 = \sqrt{m_1^2 + \Delta m_{21}^2 + \Delta m_{32}^2} . \tag{IV.1}
$$
In the case of normal mass hierarchy $m_1 \ll m_2 \ll m_3$ ($\Delta m_{12}^2 = \Delta m_{\text{sol}}^2$ and $\Delta m_{32}^2 = \Delta m_{\text{atm}}^2$), one finds $m_2 \simeq \sqrt{\Delta m_{\text{sol}}^2} \sim 7 \times 10^{-3}\text{eV}$ and $m_3 \simeq \sqrt{\Delta m_{\text{atm}}^2} \sim 5 \times 10^{-2}$, thus $m_1 \ll 10^{-3}\text{eV}$.

Using the oscillation parameters, the neutrino mass can be expressed in the form [42]

$$m_{\nu}^2 = m_1^2 + (\sin^2 \theta_{\text{sol}} + \cos^2 \theta_{\text{sol}}|U_{e3}(\theta)|^2) \Delta m_{\text{sol}}^2 + |U_{e3}(\theta)|^2 \Delta m_{\text{atm}}^2,$$

(IV.2)

where, with obvious meaning of the symbols, the matrix element $U_{e3}(\theta)$ is related to the mixing angle $\theta_{13}$ and its upper value is $|U_{e3}(\theta)|^2 = \sin^2 \theta_{13} \lesssim 5 \times 10^{-2}$ (99.73% C.L.). The neutrino mass is then bounded from above by $m_{\nu} \lesssim 1.2 \times 10^{-2}\text{eV}$, where the best fit for the neutrino oscillations parameters give

$$\Delta m_{\text{atm}}^2 \sim 2.5 \times 10^{-3}\text{eV}^2, \quad \Delta m_{\text{sol}}^2 \sim 5 \times 10^{-5}\text{eV}^2, \quad \tan^2 \theta_{\text{sol}} \sim 3.4 \times 10^{-1}.$$  

One can easily see that the limit can be relaxed to $m_{\nu} \lesssim$ a few $10^{-3}\text{eV}$ if $|U_{e3}(\theta)| \sim 3 \times 10^{-2}$.

For the inverted mass hierarchy $m_2 \sim m_3 \simeq \sqrt{\Delta m_{\text{atm}}^2}$ and $m_1 \ll m_{2,3}$ ($\Delta m_{32}^2 \simeq \Delta m_{\text{sol}}^2$ and $\Delta m_{21}^2 \simeq \Delta m_{\text{atm}}^2$), the neutrino mass is expressed in the form [42]

$$m_{\nu}^2 = m_1^2 + (1 - |U_{e1}|^2) (\sin^2 \theta_{\text{sol}} \Delta m_{\text{sol}}^2 + \Delta m_{\text{atm}}^2),$$

(IV.3)

where $|U_{e1}| \lesssim 5 \times 10^{-2}$. The neutrino mass is then expected to be $m_{\nu} \sim 5 \times 10^{-2}\text{eV}$.

The $m_{\nu}$ bounds that we have determined are at least one order of magnitude smaller than the sensitivity of present experiments and can not yet be measured in the laboratory. They are however consistent with what has so far been ascertained. We point out that the results we have found follow in a natural way from Caianiello’s model, and that they are completely consistent with the notion that there is a MA linked to the mass of particles. It should be interesting to investigate the joined effects of MA and neutrino oscillations within the core of a supernova, along the line of [43]. This goes beyond the scope of the present paper.

With the caveats discussed in Section III, the model may be applied to photons with effective mass. One obtains a lower bound on the electron density in intergalactic space that is compatible with WMAP measurements.
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APPENDIX

We now calculate the term $a^2$ in (II.3) for photons. For convenience, we perform the calculations using the Schwarzschild metric. Then, in the plane $\theta = \pi/2$, we obtain

$$a^2 = g_{00} \dot{t}^2 + g_{11} \dot{r}^2 + g_{33} \dot{\varphi}^2,$$  \hspace{1cm} (A.1)

where

$$\dot{\varphi} \equiv \frac{d\varphi}{ds} = \frac{B}{r^2}, \quad \dot{t} \equiv \frac{dt}{ds} = \frac{A}{1 - 2MG/r},$$  \hspace{1cm} (A.2)

$$\dot{r} \equiv \frac{dr}{ds} = \dot{t} \frac{dr}{dt} = \dot{t} \left(1 - \frac{2GM}{r}\right) \left[1 - \frac{b^2}{r^2} \left(1 - \frac{2GM}{r}\right) \left(1 - \frac{2GM}{b}\right)^{-1}\right]^{1/2}.$$  

$b$ is the impact parameter and $A$ and $B$ are infinite constants [11] when $ds = 0$. It is also useful to remember that

$$r^2 \frac{d\varphi}{dt} = r^2 \frac{\dot{\varphi}}{\dot{t}} = \frac{B}{A} \left(1 - \frac{2GM}{r}\right),$$  \hspace{1cm} (A.3)

remains finite as $ds \to 0$ and that $B/A$ is a finite constant whose value $b/\sqrt{1 - 2GM/b}$ can be determined by requiring that $dr/dt = 0$ at $r = b$. We show below that the ratio $a^2/A_\gamma^2$, where $A_\gamma^2$ is the photon’s MA, remains finite. By using the fact that the components of the four-velocity in (A.2) depend on $r$ only and that therefore $\dot{t} = \dot{r} \dot{d}/dr$, $\dot{r} = \dot{r} \dot{d}/dr$, and $\dot{\varphi} = \dot{r} d\varphi/dr$, we can write (A.1) in the form
\[ a^2 \simeq \frac{A^4}{r^4} \left[ (2GM)^2 - 4b^2 \left( 1 + \frac{2GM}{b} \right) \right] \] (A.4)

to leading order in \( 2GM/r \) and \( 2GM/b \). For purely radial motion, we find the result of Carmeli [44]

\[ a^2 \simeq A^4 \left( \frac{2GM}{r^2} \right)^2, \] (A.5)

from which it also follows that \( A^2_\gamma = A^4/(2GM)^2 \) and that the ratio

\[ \frac{a^2}{A^2_\gamma} \leq \left( \frac{2GM}{r} \right)^4 \]

remains finite. We finally obtain from (II.3)

\[ \tilde{h}_{\mu\nu}k^\mu k^\nu = \frac{a^2}{A^2_\gamma} \eta_{\mu\nu}k^\mu k^\nu \leq k^2 \left( \frac{2GM}{r} \right)^4 = 0 \] (A.6)

for on-shell photons.
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TABLE I. Observation times and neutrino energies in the Kamioka experiment [1,4].

<table>
<thead>
<tr>
<th>Event No.</th>
<th>$t_{obs}$ (s)</th>
<th>Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>21.3 ± 2.9</td>
</tr>
<tr>
<td>2</td>
<td>0.107</td>
<td>14.8 ± 3.2</td>
</tr>
<tr>
<td>3</td>
<td>0.303</td>
<td>8.9 ± 2.0</td>
</tr>
<tr>
<td>4</td>
<td>0.324</td>
<td>10.6 ± 2.7</td>
</tr>
<tr>
<td>5</td>
<td>0.507</td>
<td>14.4 ± 2.9</td>
</tr>
<tr>
<td>6</td>
<td>0.686</td>
<td>7.6 ± 1.7</td>
</tr>
<tr>
<td>7</td>
<td>1.541</td>
<td>36.9 ± 8.0</td>
</tr>
<tr>
<td>8</td>
<td>1.728</td>
<td>22.4 ± 4.2</td>
</tr>
<tr>
<td>9</td>
<td>1.915</td>
<td>21.2 ± 3.2</td>
</tr>
<tr>
<td>10</td>
<td>9.219</td>
<td>10.0 ± 2.7</td>
</tr>
<tr>
<td>11</td>
<td>10.433</td>
<td>14.4 ± 2.6</td>
</tr>
<tr>
<td>12</td>
<td>12.439</td>
<td>10.3 ± 1.9</td>
</tr>
</tbody>
</table>
FIG. 1. Behaviour of $F(x, n)$ for $x \epsilon [1.5, 3.5]$ and $n \epsilon [3.5, 10]$. 
FIG. 2. $t_{em}(s)$ vs. $(m_\nu/10^{-3}\text{eV})^{-4}$ for the eight events analyzed: 1.solid black line 2.dashed black line 3.dot-dashed black line 4.dotted black line 5.solid grey line 7.dashed grey line 8.dot dashed grey line 9.dotted grey line.
FIG. 3. Minimum time interval over which neutrinos of the Kamioka experiment could have been emitted as a function of $(m_\nu/\sqrt{\Delta})^{-4}$ in unit of $10^{-3}$ eV.