Doubly Charмful Baryonic $B$ Decays

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Abstract

There are two apparent puzzles connected with the two-body and three-body doubly charmed baryonic $B$ decays. First, earlier calculations based on QCD sum rules or the diquark model predict $B(B^0 \rightarrow \Xi_c^+\Lambda_c^-) \approx B(B^0 \rightarrow B_cN)$, while experimentally the former has a rate two orders of magnitude larger than the latter. Second, a naive estimate of the branching ratio $O(10^{-9})$ for the color-suppressed three-body decay $B \rightarrow \Lambda_c^+\Lambda_c^-K$, which is highly suppressed by phase space, is too small by five to six orders of magnitude compared to experiment. We show that the great suppression for the $\Lambda_c^+\Lambda_c^-K$ production can be alleviated provided that there exists a narrow hidden charm bound state with a mass near the $\Lambda_c\Lambda_c$ threshold. This new state that couples strongly to the charmed baryon pair can be searched for in $B$ decays and in $p\bar{p}$ collisions by studying the mass spectrum of $D^*(\ast)\overline{D}^{\ast}(\ast)$ or $\Lambda_c\bar{\Lambda}_c$. The doubly charмful decay $B \rightarrow \Xi_c\bar{\Lambda}_c$ has a configuration more favorable than the singly charмful one such as $B \rightarrow \Lambda_c\bar{p}$ since no hard gluon is needed to produce the energetic $\Xi_c\Lambda_c$ pair in the former decay, while two hard gluons are needed for the latter process. Assuming that a soft $q\bar{q}$ quark pair is produced through the $\sigma$ and $\pi$ meson exchanges in the configuration for $B \rightarrow \Xi_c\bar{\Lambda}_c$, it is found that its branching ratio is of order $10^{-3}$, in agreement with experiment.

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I. INTRODUCTION

Recently Belle has observed for the first time two-body and three-body doubly charmed baryonic $B$ decays in which two charmed baryons are produced in the final state $\Xi_c$. The measured branching ratios are

$$
\mathcal{B}(B^- \to \Lambda_c^+ \bar{\Lambda}_c^- K^-) = (6.5^{+1.0}_{-0.9} \pm 0.8 \pm 3.4) \times 10^{-4},
$$

$$
\mathcal{B}(B^0 \to \Lambda_c^+ \bar{\Lambda}_c^- K^0) = (7.9^{+2.9}_{-2.3} \pm 1.2 \pm 4.2) \times 10^{-4},
$$

for three-body decays and

$$
\mathcal{B}(B^- \to \Xi_c^0 \bar{\Lambda}_c^-) \mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+) = (4.8^{+1.0}_{-0.9} \pm 1.1 \pm 1.2) \times 10^{-5},
$$

$$
\mathcal{B}(B^0 \to \Xi_c^+ \bar{\Lambda}_c^-) \mathcal{B}(\Xi_c^+ \to \Xi^- \pi^+ \pi^+) = (9.3^{+3.7}_{-2.8} \pm 1.9 \pm 2.4) \times 10^{-5}
$$

for two-body decays. Taking the theoretical estimates (see e.g. Table III of [3]), $\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+) \approx 1.3\%$ and $\mathcal{B}(\Xi_c^+ \to \Xi^0 \pi^+) \approx 3.9\%$ together with the experimental measurement $\mathcal{B}(\Xi_c^+ \to \Xi^0 \pi^+)/\mathcal{B}(\Xi_c^+ \to \Xi^- \pi^+ \pi^+) = 0.55 \pm 0.16$, it follows that

$$
\mathcal{B}(B^- \to \Xi_c^0 \bar{\Lambda}_c^-) \approx 4.8 \times 10^{-3}, \quad \mathcal{B}(B^0 \to \Xi_c^+ \bar{\Lambda}_c^-) \approx 1.2 \times 10^{-3}.
$$

Therefore, the two-body doubly charmed baryonic $B$ decay $B \to \Lambda_c \bar{\Lambda}_c$ has a branching ratio of order $10^{-3}$, to be compared with $\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+) = (7 \pm 55) \times 10^{-4}$, $\mathcal{B}(B^0 \to \Lambda_c \bar{\Lambda}_c) = (6 \pm 32) \times 10^{-4}$, and $\mathcal{B}(B^0 \to \Lambda_c \bar{\Lambda}_c) = (9 \pm 74) \times 10^{-4}$ for singly charmed baryonic $B$ decays and $\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+) = (4 \pm 4) \times 10^{-4}$, $\mathcal{B}(B^0 \to \Lambda_c \bar{\Lambda}_c) = (2 \pm 5) \times 10^{-4}$ for charmless baryonic $B$ decays. Therefore, we have the pattern

$$
\mathcal{B}(\Xi_c) \sim 10^{-3} \gg \mathcal{B}(\Lambda_c) \sim 10^{-5} \gg \mathcal{B}(\Lambda_c) \sim 10^{-7},
$$

for two-body baryonic $B$ decays.

Using $\mathcal{B}(B^0 \to \Lambda_c^+ \bar{\Lambda}_c^-)$ as a benchmark, one will expect a branching ratio of order $10^{-7}$ for the charmless decay $B \to \Lambda_c \bar{\Lambda}_c$ after replacing the quark mixing angle $V_{cb}$ by $V_{ub}$, provided that the dynamical suppression for the latter is neglected. However, since the doubly charmed baryonic decay mode $\Xi_c \bar{\Lambda}_c$ proceeds via $b \to cs\bar{c}$, while $\Lambda_c \bar{\Lambda}_c$ via a $b \to c\bar{d}u$ quark transition, the CKM mixing angles for them are the same in magnitude but opposite in sign. One may wonder why the $\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+ \pi^+)$ mode has a rate two orders of magnitude larger than $\mathcal{B}(\bar{B} \to \Lambda_c \bar{\Lambda}_c)$. Indeed, earlier calculations based on QCD sum rules [10] or the diquark model [13] all predict that $\mathcal{B}(B \to \Xi_c \bar{\Lambda}_c) \approx \mathcal{B}(\bar{B} \to \Xi_c \bar{\Lambda}_c)$, which is in violent disagreement with experiment. This implies that some important dynamical suppression effect for the $\Xi_c \bar{\Lambda}_c$ production with respect to $\Xi_c \bar{\Lambda}_c$ is missing in previous studies.

As for the three-body decay $B \to \Lambda_c \bar{\Lambda}_c K$, its branching ratio is estimated to be of order $10^{-9}$, which is extremely small due to the tiny phase space available for this decay and the color-suppression effect. The puzzle is that why the measured rate is much larger than the naive expectation?
A crucial ingredient for understanding the baryonic $B$ decays is the threshold or low mass enhancement behavior of the baryon-pair invariant mass in the spectrum for $B \rightarrow B_1 B_2 M$: It sharply peaks at very low values. That is, the $B$ meson is preferred to decay into a baryon-antibaryon pair with low invariant mass accompanied by a fast recoiled meson. Therefore, some three-body final states have rates larger than their two-body counterparts, e.g. $p\bar{p}K^\pm \gg p\bar{p}$, $\Lambda\bar{p}\pi^\pm \gg \Lambda\bar{p}$, $\Sigma_c\bar{p}\pi^\pm \gg \Sigma_c\bar{p}$. This phenomenon can be understood in terms of the threshold effect, namely, the invariant mass of the dibaryon is preferred to be close to the threshold. The configuration of the two-body decay $B \rightarrow B_1 B_2$ is not favorable since its invariant mass is $m_B$. In $B \rightarrow B_1 B_2 M$ decays, the effective mass of the baryon pair is reduced as the emitted meson can carry away a large amount of energies. The two-body decay pattern also follows from the low-mass enhancement effect: The energy release is least for the $B$ decay into two charmed baryons and becomes very large when the final-state baryons are charmless.

Although the gross feature of the baryonic $B$ decays can be qualitatively comprehended in terms of the near threshold effect, how to quantitatively evaluate their absolute decay rates and how to realize the low mass enhancement effect require detailed dynamical studies. In the present work we will focus on the doubly charmful baryonic $B$ decays, namely, $B \rightarrow \Xi_c\bar{\Lambda}_c$ and $B \rightarrow \Lambda_c\bar{\Lambda}_cK$ in Secs. II and III, respectively, aiming to resolve the aforementioned two puzzles connected with them. Sec. IV gives the conclusion. The evaluation of the delta functions occurring in the phase space integral is discussed in the Appendix.

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1 The three-body decay is usually referred to the nonresonant one. The relation $\Lambda_c\bar{p}\pi^- \gg \Lambda_c\bar{p}$ is trivial as the former arises mostly from resonant contributions.
II. THREE-BODY DECAYS

We consider the decay $B^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^- K^-$, which proceeds through the internal $W$-emission diagram Fig. 1(a). It turns out this diagram is factorizable. In the weak Hamiltonian approach, the factorizable amplitude reads

$$A(B^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^- K^-) = \frac{G_F}{\sqrt{2}} V_{cs} V_{cb}^* a_2 \langle \Lambda_c^+ \bar{\Lambda}_c^- |(\bar{c}c)|0\rangle \langle K^-|(s\bar{b})|B^-, \rangle,$$

where $(\bar{q}_1 q_2) \equiv \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$, and the effective Wilson coefficient $a_2$ indicates that this decay is color suppressed. The matrix elements can be parametrized as

$$\langle \Lambda_c(p_1)|\bar{\Lambda}_c(p_2)|(\bar{c}c)|0\rangle = \bar{u}_{\Lambda_c}(p_1) \left[ f_1(q^2) \gamma_\mu + i \frac{f_2(q^2)}{2m_{\Lambda_c}} \sigma_{\mu\nu} q^\nu - \left( g_1(q^2) \gamma_\mu + \frac{g_3(q^2)}{2m_{\Lambda_c}} q_\mu \right) \gamma_5 \right] v_{\Lambda_c}(p_2),$$

$$\langle K^-(p_K)|(s\bar{b})|B^-(p_B)\rangle = F_1^{BK}(q^2)(p_B + p_K)_\mu + \left( F_0^{BK}(q^2) - F_1^{BK}(q^2) \right) \frac{m_B^2 - m_K^2}{q^2} q_\mu,$$

with $q = p_B - p_K = p_1 + p_2$. In terms of the form factors, the decay amplitude has the expression

$$A(B^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^- K^-) = \frac{G_F}{\sqrt{2}} V_{cs} V_{cb}^* a_2 \bar{u}_{\Lambda_c} |a \bar{q} K + b - (c \bar{q} K + d) \gamma_5| v_{\Lambda_c},$$

with

$$a = 2 F_1^{BK}(q^2) \left[ f_1(q^2) + f_2(q^2) \right],$$

$$b = 2 F_1^{BK}(q^2) f_2(q^2)(p_2 - p_1) \cdot p_K/(2m_{\Lambda_c}),$$

$$c = 2 F_1^{BK}(q^2) g_1(q^2),$$

$$d = 2m_{\Lambda_c} g_1(q^2) \left[ F_1^{BK}(q^2) + (F_0^{BK}(q^2) - F_1^{BK}(q^2)) \frac{m_B^2 - m_K^2}{q^2} \right] + g_3(q^2) F_0^{BK}(q^2)(m_B^2 - m_K^2)/(2m_{\Lambda_c}).$$

There are numerous estimates of the $B \rightarrow K$ transition form factors. We will follow [11] where the form factors are evaluated using the relativistic covariant light-front quark model.

Due to the heavy mass of $\Lambda_c$, the phase space of the $\Lambda_c^+ \bar{\Lambda}_c^- K^-$ decay is about one hundred times smaller than, say, that of the $\Lambda \bar{p} \pi^+$ [12]. The $\Lambda_c^+ \bar{\Lambda}_c^-$ form factors, if any, can therefore be taken as constants whose values are determined at the threshold over the phase space. To achieve a rate that is at least comparable with that of the $\Lambda \bar{p} \pi^+$ whose branching ratio is of order $3 \times 10^{-6}$ [12], one would need the $\Lambda_c^+ \bar{\Lambda}_c^-$ form factors to be more than one hundred times larger than those of the $\Lambda \bar{p} \pi^+$ near the $\Lambda \bar{p}$ threshold, which is quite unlikely since the $\Lambda \bar{p}$ form factors have their maximum values already about $O(1)$. Besides, $O(10^2)$ form factors would just give a rate of $\Lambda_c^+ \bar{\Lambda}_c^- K^-$ comparable to $\sim O(10^{-6})$, not to mention the remaining factor of $O(10^2)$ difference between the rates of $\Lambda \bar{p} \pi^+$ and of $\Lambda_c^+ \bar{\Lambda}_c^- K^-$. Therefore we conclude that the suppression from the $\Lambda_c^+ \bar{\Lambda}_c^- K^-$ phase space is so strong that $\Lambda_c^+ \bar{\Lambda}_c^-$ pair is unlikely to be produced dominantly through the direct three-body decay processes. The great suppression, however, seems to hint strongly that a $c\bar{c}$-content resonance with the width comparable to the nearby resonances like $\psi(4415)$ could be located around the threshold of $\Lambda_c^+ \bar{\Lambda}_c^- (\sim 4.6$ GeV), and the whole process takes place dominantly via the charmonium-like resonance as shown in Fig. 1(b).
Let us assume the resonance, \(X_{c\bar{c}}\), exists with a mass \(m_{X_{c\bar{c}}} \gtrsim 2m_{c}\) and a width \(\Gamma_{X_{c\bar{c}}}\). Let us further assume that this resonance is a spin-1 particle with \(J^P = 1^-\) or \(1^+\), as inspired from the observation that all the charmonia near the \(\Lambda_c^+ \bar{\Lambda}_c^-\) threshold are spin-1 particles. The decay amplitude then reads

\[
A_{X_{c\bar{c}}}(B^- \to \Lambda_c^+ \bar{\Lambda}_c^- K^-) = \frac{G_F}{\sqrt{2}} V_{cs} V_{cb}^* a_2 (K^- | \bar{s} \gamma_\mu (1 - \gamma_5) b | B^- )
\]

\[
\times m_{X_{c\bar{c}}} f_{X_{c\bar{c}}}(q^2 - m_{X_{c\bar{c}}}^2 + im_{X_{c\bar{c}}} \Gamma_{X_{c\bar{c}}}) i \bar{u}_{\Lambda_c}(p_1) M_\nu v_{\Lambda_c}(p_2), \tag{11}
\]

where

\[
M_\nu = h_{1,2}^{\Lambda_c \bar{\Lambda}_c V} \gamma_\nu + \frac{i h_{1,2}^{\Lambda_c \bar{\Lambda}_c A}}{2m_{c}} \sigma_{\mu\rho} q^\rho,
\]

when \(X_{c\bar{c}}\) is a vector (\(X_{c\bar{c}} = V\)), and

\[
M_\nu = \left( h_{1}^{\Lambda_c \bar{\Lambda}_c A} \gamma_\nu + \frac{i h_{2}^{\Lambda_c \bar{\Lambda}_c A}}{2m_{c}} q_\nu \right) \gamma_5,
\]

when \(X_{c\bar{c}}\) is an axial-vector (\(X_{c\bar{c}} = A\)) particle. \(f_{X_{c\bar{c}}}\) is the decay constant for \(X_{c\bar{c}}\), and \(h_{1,2}^{\Lambda_c \bar{\Lambda}_c V}\) and \(h_{1,2}^{\Lambda_c \bar{\Lambda}_c A}\) represent the dimensionless \(\Lambda_c \bar{\Lambda}_c X_{c\bar{c}}\) strong couplings. Since the allowed phase space is very small, the strong couplings can effectively be treated as constants within this region. The decay constant \(f_{X_{c\bar{c}}}\) comes from the factorization of the amplitude of \(B^- \to X_{c\bar{c}} K^-\), followed by the strong decay \(X_{c\bar{c}} \to \Lambda_c^+ \bar{\Lambda}_c^-\). Since the chirality-flipping baryon vector form factor \(f_2(q^2)\) is in general suppressed by two more powers of the dibaryon invariant mass \(q^2\) than \(f_1(q^2)\) and since \(q^2 \sim 4m_{\Lambda_c}^2\) is large in \(B \to \Lambda_c \bar{\Lambda}_c K\) decays, we expect the contributions from \(X_{c\bar{c}}\) coupled to \(\Lambda_c \bar{\Lambda}_c\) through \(h_{2}^{\Lambda_c \bar{\Lambda}_c V}\) to be small and hence can be neglected in our calculation. The \(h_{2}^{\Lambda_c \bar{\Lambda}_c A}\) term in the axial-vector decay amplitude can also be dropped since \(q_\nu (-g^{\mu\nu} + q^\rho q^\nu / m_{\Lambda_c}^2) \sim 0\) due to the fact that \(m_{\Lambda_c}^2 \sim q^2\) within the phase space. The decay amplitude \(A_{X_{c\bar{c}}}\) then becomes

\[
A_{V(A)}(B^- \to \Lambda_c^+ \bar{\Lambda}_c^- K^-) = \frac{G_F}{\sqrt{2}} V_{cs} V_{cb}^* a_2 m_{V(A)} f_{V(A)}(\gamma_5 + \Gamma_{V} m_{V(A)}^0) v_{\Lambda_c}, \tag{14}
\]

with

\[
\mathcal{M}_V = 2F_1^{BK}(q^2) \left( \frac{-h_{1}^{\Lambda_c \bar{\Lambda}_c V}}{q^2 - m_{V}^2 + im_{V} \Gamma_{V}} \right),
\]

\[
\mathcal{M}_S = 0,
\]

\[
\mathcal{M}_A = 2F_1^{BK}(q^2) \left( \frac{-h_{1}^{\Lambda_c \bar{\Lambda}_c A}}{q^2 - m_{A}^2 + im_{A} \Gamma_{A}} \right),
\]

\[
\mathcal{M}_P = \left[ F_1^{BK}(q^2) \frac{q^2 - (m_B^2 - m_K^2)}{q^2} + F_0^{BK}(q^2)(m_B^2 - m_K^2) \left( \frac{1}{q^2} - \frac{1}{m_{A}^2} \right) \right]
\]

\[
\times 2m_{A} \left( \frac{-h_{1}^{\Lambda_c \bar{\Lambda}_c A}}{q^2 - m_{A}^2 + im_{A} \Gamma_{A}} \right). \tag{15}
\]

The minus signs in front of the strong couplings \(h_{1}^{\Lambda_c \bar{\Lambda}_c V,A}\) come from the minus sign of the \(g^{\mu\nu}\) part of the \(X_{c\bar{c}}\) propagator in Eq. (11).
FIG. 2: Branching fractions of $B_{X_{cc}}(B^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^- K^-)$ as a function of the mass and the decay width of the intermediate resonance $X_{cc}$ when it is (a) a vector ($X_{cc} = V$) and (b) an axial-vector ($X_{cc} = A$) particle with $f_V \cdot h_1^{\Lambda_c \bar{\Lambda}_c V} = f_A \cdot h_1^{\Lambda_c \bar{\Lambda}_c A} = 4$ GeV.

We take $f_V \cdot h_1^{\Lambda_c \bar{\Lambda}_c V} = f_A \cdot h_1^{\Lambda_c \bar{\Lambda}_c A} = 4$ GeV and show in Fig. 2 the plots of branching ratios for each kind of resonance as functions of both the mass and the width of the resonance. The branching fractions depend on $(f_{X_{cc}} \cdot h_1^{\Lambda_c \bar{\Lambda}_c V})^2$ and $(f_{X_{cc}} \cdot h_1^{\Lambda_c \bar{\Lambda}_c A})^2$ proportionally. We notice that the width of the axial-vector resonance $\Gamma_A$ is more than an order of magnitude smaller than $\Gamma_V$ when both $B_V$ and $B_A$ are around the experimental value $B \sim 7 \times 10^{-4}$. This is due mainly to the smallness of $B_A$ which suffers from the destructive interference between comparable $M_A$ and $M_P$ contributions in the decay rate. Fig. 3 shows the decay rates from $M_A$, $M_P$ and from $\text{Re} M_A M_P^*$ alone without taking into account the resonance effect. One can see that the $M_A$ and $M_P$ contributions are comparable while the interference term $\text{Re} M_A M_P^*$ gives almost twice the negative contribution of either $M_A$ or $M_P$, resulting in a large cancellation with $M_A$ and $M_P$. Therefore, in order to counteract this cancellation, one needs a smaller width of the resonance such that $|1/(q^2 - m_A^2 + i m_A \Gamma_A)|^{-2}$ becomes more singular in the allowed range of $q^2$. There is, however, no such interference found in $B_V$ as $M_V$ stands alone in the vector-induced decay amplitude after taking $h_2^{\Lambda_c \bar{\Lambda}_c V} = 0$ in Eq. (12).

Therefore, the above analysis seems to imply the existence of a narrow hidden charm bound state with a mass of order $4.6 \sim 4.7$ GeV that couples strongly with the charmed baryon pair. Recall that many new charmonium-like resonances with masses around 4 GeV starting with $X(3872)$ and so far ending with $Y(4260)$ have been recently observed by BaBar and Belle. These charmonium-like states are above the $D\bar{D}$ threshold but below the two-charmed-baryon threshold. The new state we have put forward is just marginally above the $\Lambda_c \bar{\Lambda}_c$ threshold. In principle, this new state can be searched for in $B$ decays and in $p\bar{p}$ collisions by studying the mass spectrum of $D^{(*)} \bar{D}^{(*)}$ or $\Lambda_c \bar{\Lambda}_c$. 

6
4.6
[0x0]4.65
[0x0]4.7
[0x0]4.75
m_{\Lambda_c \bar{\Lambda}_c} (GeV)
\begin{align*}
\frac{4.6}{4.65/\text{CapLambda}} c/\text{Space/CapLambda/OverBar/OverBar/OverBar} c/\text{LParen1 GeV}/\text{RParen1} - 2 - 1 0 1 2
\end{align*}

rates (arbitrary unit)

FIG. 3: Decay rates contributed from $M_V$ (short-dash), $M_A$ (solid), $M_P$ (long-dash), sum of the previous two (dotted) and from $2\text{Re}M_A M_P^*$ (dot-dashed) alone without taking into account the resonance effect, i.e. $(q^2 - m_A^2 + i m_A \Gamma_A)^{-1}$ is replaced by any constant in each term.

III. TWO-BODY DECAYS

The two-body doubly charmed baryonic $B$ decays $B^- \to \Xi^0 \bar{\Lambda}_c^-$ and $B^0 \to \Xi^+ \bar{\Lambda}_c^-$ receive contributions from the internal $W$-emission (see Fig. 4) and weak annihilation. The latter contribution can be safely neglected as it is not only quark-mixing but also helicity suppressed. It should be stressed that, in contrast to the internal $W$-emission in mesonic $B$ decays, internal $W$-emission in baryonic $B$ decay is not necessarily color suppressed. This is because the baryon wave function is totally antisymmetric in color indices. One can see from Fig. 4 that there is no color suppression for the meson production. In the effective Hamiltonian approach, the relevant weak Hamiltonian is

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* (c_1 O_1 + c_2 O_2) \to \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* (c_1 - c_2) O_1,$$

where $O_1 = (\bar{c}b)(\bar{s}c)$ and $O_2 = (\bar{c}c)(\bar{s}b)$. In the above equation, we have used the fact that the operator $O_1 - O_2$ is antisymmetric in color indices (more precisely, it is a color anti-triplet). Therefore, the Wilson coefficient for the tree-dominated internal $W$-emission is $c_1 - c_2$ rather than $a_2 = c_2 + c_1/3$. This is indeed the case found in the pole model calculation in [15].

Since the internal $W$-emission in Fig. 4 is not factorizable, it is difficult to evaluate its amplitude directly. Pole model has been applied in [15] to compute $B \to B_1 B_2$. However, the strong coupling involved in this model is unknown and hence it has to be fixed from other processes, e.g. the 3-body baryonic $B$ decays. Since the CKM angles for $B^0 \to \Xi^0 \bar{\Lambda}_c^-$ and $B^0 \to \Lambda^+ \bar{p}$ are the same in magnitude (but opposite in sign), the pole model does not explicitly explain why the former has a rate much larger than the latter. In particular, the dynamical suppression of $\Lambda^+_c \bar{p}$ relative to $\Xi^+_c \bar{\Lambda}_c^-$ is not clearly manifested in the pole model calculation. In order to understand why $\Xi^+_c \bar{\Lambda}_c^- \gg \Lambda^+_c \bar{p}$, let us re-examine Fig. 4.

There are several possibilities for the quark-antiquark pair creation in Fig. 4. In one case, $q\bar{q}$ is picked up from the vacuum via the soft nonperturbative interactions so that it carries the vacuum quantum numbers $^3P_0$. It is also possible that the quark pair is created perturbatively via one
FIG. 4: (a) $\bar{B}^0 \rightarrow \Xi_c^+ \bar{\Lambda}^-_c$ and (b) $B^- \rightarrow \Xi_c^0 \bar{\Lambda}^-_c$ as proceeding through internal $W$-emission diagrams.

gluon exchange with one-gluon quantum numbers $^3S_1$. It is not clear which mechanism, the $^3P_0$ or $^3S_1$ model, dominates the 2-body baryonic $B$ decays, though in practice the $^3P_0$ model is simpler. Since the energy release is relatively small in charmed baryonic $B$ decay, the $^3P_0$ model seems to be more relevant. In the present work, we also consider the possibility that the $q\bar{q}$ pair is produced via a light meson exchange. The $q\bar{q}$ pair created from soft nonperturbative interactions tends to be soft. For an energetic proton produced in 2-body $B$ decays, the momentum fraction carried by its quark is large, $\sim \mathcal{O}(1)$, while for an energetic charmed baryon, its momentum is carried mostly by the charmed quark. As a consequence, the doubly charmed baryon state such as $\Xi_c \bar{\Lambda}_c$ has a configuration more favorable than $\Lambda_c \bar{p}$.

In order to evaluate Fig. 4 for the decay $\bar{B} \rightarrow \Xi_c \bar{\Lambda}_c$, we need to know the distribution amplitudes of the charmed baryon $\mathcal{B}_c$ and the $B$ meson. For the wave functions of $\mathcal{B}_c = \Xi_c^0, \Lambda_c$, they have the forms [11, 17] 

\begin{align*}
\langle \Xi_c^0(p)|c_\alpha^a(z_1)s_\beta^b(z_2)d_\gamma^c(z_3)|0\rangle &= \frac{e^{abc}}{6} \frac{f_{\Xi_c}}{4} \left[ \bar{u}_{\Xi_c}(p) \right]_\alpha C^{-1}\gamma_5(\not{p} + m_{\Xi_c}) \gamma_\beta \Psi_{\Xi_c}(z_1, z_2, z_3), \\
\langle \Lambda_c(p')|c_\alpha^a(z_1')\bar{s}_\beta^b(z_2')d_\gamma^c(z_3')|0\rangle &= \frac{e^{abc}}{6} \frac{f_{\Lambda_c}}{4} \left[ \bar{v}_{\Lambda_c}(p') \right]_\alpha [(p' - m_{\Lambda_c})\gamma_5 C]_{\beta\gamma} \Psi_{\Lambda_c}(z_1', z_2', z_3'),
\end{align*}

(17) 

where $c, q$ and $d$ are the quark fields, $a, b$, and $c$ the color indices, $\alpha, \beta$ and $\gamma$ the spinor indices, $C$ the charge conjugation matrix, and $f_{\mathcal{B}_c}$ the decay constant. Following [18] we can write

\begin{align*}
\Psi_{\Xi_c}(z_1, z_2, z_3) &= \int [dx][d^2 k_\perp] e^{i\Sigma_k \cdot z_i} \Psi_{\Xi_c}(x_1, x_2, x_3, k_{1\perp}, k_{2\perp}, k_{3\perp}), \\
\Psi_{\Lambda_c}(z'_1, z'_2, z'_3) &= \int [dx'][d^2 k'_{\perp}] e^{i\Sigma'_{k'} \cdot z'_i} \Psi_{\Lambda_c}(x'_1, x'_2, x'_3, k'_{1\perp}, k'_{2\perp}, k'_{3\perp}),
\end{align*}

(18) 

where $p^{(t)} = (p^{(t)+}, p^{(t)-}, 0_\perp)$ is the momentum of $\Xi_c^0 (\bar{\Lambda}_c)$ and $k_1^{(t)}, k_2^{(t)}, k_3^{(t)}$ the momenta of the constituent quarks of the baryons, which are taken to be [17, 22]

\begin{align*}
k_1 &= (x_1 p^+, p^-, k_{1\perp}) , \quad k_1' = (p^+, x'_1 p'^-, k'_{1\perp}), \\
k_{2(3)} &= (x_{2(3)} p^+, 0, k_{2(3)\perp}) , \quad k_{2(3)}' = (0, x_{2(3)} p'^-, k'_{2(3)\perp}),
\end{align*}

(19) 

\begin{align*}
[dx^{(t)}] &= dx_1^{(t)} dx_2^{(t)} dx_3^{(t)} \delta \left( 1 - \sum_{i=1}^3 x_i^{(t)} \right), \\
[d^2 k_{\perp}] &= d^2 k_{1\perp} d^2 k'_{2\perp} d^2 k'_{3\perp} \delta^2 \left( k_{1\perp} + k'_{2\perp} + k'_{3\perp} \right),
\end{align*}

(20) 

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In a meson wave function is expressed as $\sigma$, the Eq. (23) is the color factor. For the with $\Gamma\eta\eta$ respectively, where Eq. (17) has been used. Due to the symmetry property of the $\Lambda\pi\pi\pi\pi\pi\pi$ given in Eq. (28), the factor of $(\pi\pi)$ between the soft $F$ contribution is the same as the $\bar{q}qF$ soft quark pair and the spectator as shown in Fig. 5.

\[ A(B^+ \to \Xi^0\bar{\Lambda}^-) = A_\sigma + A_{\pi^0} + A_{\pi^-}, \]

where, for instance, in the case that the decay proceeds through the exchange of $\sigma$ or $\pi^0$

\[ A_{\sigma(\pi^0)} = \frac{G_F}{\sqrt{2}} v_{us} V_{us}^* (c_1 - c_2) \int dz \ dz' \ e^{i(k_1 + k_3)\cdot z} e^{i(k_2 - p_\ell)\cdot z'} g_{\sigma(\pi)qq}^2 \frac{f_\pi}{2} (z - z') \]

\[ \times \langle \Xi^0|c^b(0)\bar{s}^b(0)\bar{c}^b(z)|0\rangle \langle \Lambda_c|c^b(0)u^{d}_{\bar{f}}(z')d^{\bar{f}}_{f}(z)|0\rangle \langle 0|u^{d}_{\bar{f}}(z')b^b(0)|B^-\rangle \]

\[ \times [\gamma_\mu(1 - \gamma_5)]_{\alpha\beta} [\gamma_\nu(1 - \gamma_5)]_{\beta\delta} \Gamma_{\gamma'\gamma'} \Gamma_{\eta'\eta'} \]

\[ = \frac{G_F}{\sqrt{2}} v_{us} \frac{f_B f_{\Xi^-}\bar{F}_{\Lambda_c}}{64 \ \frac{c_1 - c_2}{18}} \int d\xi \ \int [dx][dx'][d^2k_{\perp}][d^2k'_{\perp}](2\pi)^4 \]

\[ \times \delta^4(p_\ell - k_2' - k_3' - k_3) \Psi_{\Xi^-}(x, k_{\perp}) \Psi_{\Lambda_c}(x', k'_{\perp}) \Phi_B(\xi) \]

\[ \frac{g_{\sigma(\pi)qq}^2}{p_{\sigma(\pi)} - m_{\sigma(\pi)} + i m_{\sigma(\pi)} \Gamma_{\sigma(\pi)}} \]

\[ \times \bar{u}_{\Xi^-} \left[ a_{\sigma(\pi)} + b_{\sigma(\pi)} \gamma_5 \right] v_{\Lambda_c} \]

with $\Gamma_{\eta'\eta'} = \Gamma_{\gamma'\gamma'} = 1$ for $\sigma$ and $\Gamma_{\eta'\eta'} = -\Gamma_{\gamma'\gamma'} = i \gamma_5$ for $\pi^0$ in Eq. (22).

Note that the factor of $1/18$ in Eq. (23) is the color factor. For the $\pi^\pm$ exchange, the terms $g_{\pi\eta\eta}$, $\Gamma_{\gamma\gamma'}$ and $\langle \Lambda_c|c^b(0)u^{d}_{\bar{f}}(z')d^{\bar{f}}_{f}(z)|0\rangle$ in $A_{\pi^0}$ are replaced by $\sqrt{2} g_{\pi\eta\eta}$, $-\Gamma_{\gamma\gamma'}$ and $\langle \Lambda_c|c^b(0)u^{d}_{\bar{f}}(z')d^{\bar{f}}_{f}(z)|0\rangle = -\langle \bar{c}_c \rho(0)u^{d}_{\bar{f}}(z')d^{\bar{f}}_{f}(z')|0\rangle$, respectively, where Eq. (17) has been used. Due to the symmetry property of the $\Lambda_c$ wave function given in Eq. (23), the $\pi^\pm$ contribution is the same as the $\pi^0$ one except for an enhancement factor of $(\sqrt{2})^2$ arising from isospin. Due to the tiny mass difference between $\pi^0$ and $\pi^-$ and the
extremely narrow widths of these two particles, we have \( A_{\pi^-}/A_{\pi^0} = 2 \) to a very good precision. The momentum labels of the quarks in \( \Xi_c \) and \( \Lambda_c \) are depicted in Fig. 5. \( p_b \) and \( p_{\ell} \) are the momenta of the \( b \) quark and the light spectator quark of the \( B \) meson defined after Eq. (21), respectively, \( g_{\sigma(\pi)q\bar{q}} \) is the coupling of the \( \sigma (\pi) \) meson with the \( q\bar{q} \) pair, and

\[
a_\sigma = -4m_{\Xi_c} \left[ (m_B + m_{\Lambda_c})^2 - m_{\Xi_c}^2 \right], \quad b_\sigma = -4m_{\Xi_c} \left[ (m_{\Lambda_c} + m_{\Xi_c})^2 - m_B^2 \right], \quad (24)
\]

for \( \sigma \) and

\[
a_\pi = 4m_{\Xi_c} \left[ m_B^2 - (m_{\Xi_c} - m_{\Lambda_c})^2 \right], \quad b_\pi = 4m_{\Xi_c} \left[ m_{\Xi_c}^2 - (m_{\Lambda_c} - m_{\Xi_c})^2 \right], \quad (25)
\]

for both \( \pi^0 \) and \( \pi^- \).

Our results are consistent with heavy quark effective theory. It has been shown that in the heavy quark limit, the decay amplitude can be expressed as \( \bar{u} [A + B \gamma_5] v \) with [19]

\[
A = 2\sqrt{m_B} [\alpha (r_1 - r_2) - \beta], \quad B = -2\sqrt{m_B} [\alpha (r_1 + r_2) - \beta], \quad (26)
\]

where \( r_1 = m_{\Xi_c}/m_B, \quad r_2 = m_{\Lambda_c}/m_B \) and \( \alpha, \beta \) are two unknown parameters. It is easily seen that, Eqs. (24), (25) and (26) agree with each other after setting \( \alpha = \alpha_\sigma + \alpha_\pi, \quad \beta = \beta_\sigma + \beta_\pi \), with \( \alpha_\sigma = \beta_\sigma \propto 1 + r_1 + r_2 \) for \( \sigma \) and \( \alpha_\pi = \beta_\pi \propto 1 + r_1 - r_2 \) for both \( \pi^0 \) and \( \pi^- \), where the overall coefficients can be easily determined from Eqs. (23), (24) and (25).

To proceed with the numerical calculations, we need first deal with the delta functions that impose constraints on the integration limits as well as relations between integral variables. We show in the Appendix the decay amplitude and the integral variables as a result of the delta function integrations. The wave functions are adopted to be

\[
\Psi_{B_c}(x_1, x_2, x_3) = \int [d^2k_{\perp}] \Psi_{B_c}(x_i, k_{\perp}) = N_{B_c} x_1 x_2 x_3 \exp \left[ -\frac{\hat{m}_c^2}{2\beta^2 x_1} - \frac{\hat{m}_2^2}{2\beta^2 x_2} - \frac{\hat{m}_3^2}{2\beta^2 x_3} \right], \quad (27)
\]

with

\[
\Psi_{B_c}(x_i, k_{\perp}) = \frac{N_{B_c}}{(2\pi\beta^2)^2} \exp \left[ -\frac{k_{\perp}^2 + \hat{m}_c^2}{2\beta^2 x_1} - \frac{k_{\perp}^2 + \hat{m}_2^2}{2\beta^2 x_2} - \frac{k_{\perp}^2 + \hat{m}_3^2}{2\beta^2 x_3} \right], \quad (28)
\]

for the charmed baryon [20] and

\[
\Phi_B(x) = N_B x^2 (1-x)^2 \exp \left[ -\frac{1}{2} \frac{x^2 m_B^2}{\omega_B^2} \right] \quad (29)
\]

for the \( B \) meson [21]. The wave functions obey the normalization

\[
\int [dx] \Psi(x_1, x_2, x_3) = 1, \quad \int_0^1 dx \Phi_B(x) = 1. \quad (30)
\]

The other input parameters are specified as follows. The decay constants for the charmed baryons can be related to that of the \( \Lambda_b \) baryon via the relation [22]

\[
f_{B_c} m_{B_c} = f_{\Lambda_b} m_{\Lambda_b}, \quad (31)
\]

valid in the heavy quark limit. Using \( f_{\Lambda_b} = 2.71 \times 10^{-3} \text{ GeV}^2 \) obtained from a fit of the PQCD calculation for \( \Lambda_b \rightarrow \Lambda_c \) decays to \( B(\Lambda_b \rightarrow \Lambda_c l\bar{\nu}) \) [22], it is found that \( f_{\Lambda_c} = 6.7 \times 10^{-3} \text{ GeV}^2 \) and \( f_{\Xi_c} = 6.2 \times 10^{-3} \text{ GeV}^2 \), which are roughly \( \sim 1.3 - 2.3 \) times that of the results from QCD sum
TABLE I: Predictions on the branching ratios of $B^- \to \Xi_c^0 \bar{\Lambda}^-_c$ and $\bar{B}^0 \to \Xi_c^+ \bar{\Lambda}^-_c$ decays. The first and second errors come from the theoretical uncertainties in the parameters $\beta$ and $\omega_b$, respectively, which are taken to be $\beta = 1.20 \pm 0.05$ GeV and $\omega_b = 0.40 \pm 0.05$ GeV, and the third error from the baryon decay constants. Results shown in second and third rows are from $\pi$ or $\sigma$ exchange alone, respectively.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Theory (10^{-3})</th>
<th>Expt (10^{-3})</th>
<th>Mode</th>
<th>Theory (10^{-3})</th>
<th>Expt (10^{-3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B(B^{-} \to \Xi_c^0 \bar{\Lambda}^-_c)$</td>
<td>$2.2^{+0.6+1.1+1.6}_{-0.6-1.9-1.9}$</td>
<td>$\approx 4.8$</td>
<td>$B(\bar{B}^0 \to \Xi_c^+ \bar{\Lambda}^-_c)$</td>
<td>$2.0^{+0.5+4.7+5.6}_{-0.6-1.7-1.7}$</td>
<td>$\approx 1.2$</td>
</tr>
<tr>
<td>$B(B^{-} \to \Xi_c^0 \bar{\Lambda}^-<em>c)</em>\pi$</td>
<td>$1.8^{+0.5+2.2+5.0}_{-0.5-1.6-1.6}$</td>
<td></td>
<td>$B(\bar{B}^0 \to \Xi_c^+ \bar{\Lambda}^-<em>c)</em>\pi$</td>
<td>$1.7^{+0.5+3.9+4.7}_{-1.4-1.4-1.5}$</td>
<td></td>
</tr>
<tr>
<td>$B(B^{-} \to \Xi_c^0 \bar{\Lambda}^-<em>c)</em>\sigma$</td>
<td>$0.2^{+0.0+0.4+0.6}_{-0.1-0.1-0.2}$</td>
<td></td>
<td>$B(\bar{B}^0 \to \Xi_c^+ \bar{\Lambda}^-<em>c)</em>\sigma$</td>
<td>$0.2^{+0.0+0.4+0.6}_{-0.0-0.1-0.2}$</td>
<td></td>
</tr>
</tbody>
</table>

rules \cite{23}. In our calculations we shall employ the value of the $\Lambda_b$ baryon decay constant which is in the middle of the range that has the PQCD value as the lower bound and the highest value from QCD sum rules as the upper bound. The deviations of the lower and the upper bounds from this central value are then taken as one of the theoretical errors in our model.

For the $B$ meson, we use $f_\pi = 0.2$ GeV. For the coupling $g_{\sigma qq}$, the linear sigma model leads to $g_{\sigma NN} = \sqrt{2} m_N/f_\pi$ with $f_\pi = 132$ MeV, and $g_{\pi NN} = \sqrt{2} m_N g_A/f_\pi$ with $g_A \simeq 1.25$ from the Goldberger-Treiman relation. Hence, it is reasonable to take $g_{\sigma(\pi) qq} = g_{\sigma(\pi) NN}/3$ in the constituent quark model. For the $\sigma$ meson, we use $\Gamma_\sigma \approx m_\sigma = 600$ MeV \cite{11}. The constituent quark masses appearing in the $B_c$ wave function are taken to be $\hat{m}_u = \hat{m}_d = 0.33$ GeV and $\hat{m}_s = 0.55$ GeV \cite{20}, while $\hat{m}_c = m_{B_c}$ is employed.

The decay rate is given by

$$\Gamma(B \to B_1 \bar{B}_2) = \frac{p_c}{4\pi} \left\{ |A|^2 \frac{(m_B + m_1 + m_2)^2 p_c^2}{(E_1 + m_1)(E_2 + m_2)m_B^2} + |B|^2 \frac{(E_1 + m_1)(E_2 + m_2 + p_c^2)^2}{(E_1 + m_1)(E_2 + m_2)m_B^2} \right\}, (32)$$

where $p_c$ is the c.m. momentum, $E_i$ and $m_i$ are the energy and mass of the baryon $B_i$, respectively. The results of calculations are summarized in Table I. The theoretical errors come from the uncertainties in the parameters $\beta$, $\omega_b$, which are taken to be $\beta = 1.20 \pm 0.05$ GeV \cite{23}, $\omega_b = 0.40 \pm 0.05$ GeV, and the baryon decay constants. It is clear that the pion exchange gives the dominant contribution owing to its narrow width. The prediction is in agreement with experiment for $\Xi_c^0 \bar{\Lambda}^-_c$, but a bit small for $\Xi_c^0 \bar{\Lambda}^-_c$.

The above calculation is not applicable to the two-body decay $\bar{B}^0 \to \Lambda_c^+ \bar{p}$ with one charmed baryon in the final state. This is because two hard gluons are needed to produce an energetic antiproton: one hard gluon for kicking the spectator quark of the $B$ meson to make it energetic and

\begin{itemize}
  \item[2] The $\Lambda_b$ decay constant is found to be in the range $(2.0 - 3.5) \times 10^{-2}$ GeV\r superscript{3} in QCD sum rules \cite{23}. It differs from the $\Lambda_b$ decay constant in this work by a factor of $\Lambda_b$ mass. After normalizing it to having the same dimension as the decay constants in this work, the above range turns out to be about $1.3 - 2.3$ times that of the decay constant from PQCD.
  \item[3] As shown in \cite{20}, the parameter $\beta$ is of order 1 GeV for light baryons. Just as the meson case \cite{11}, $\beta$ should become larger for the heavy baryons.
\end{itemize}
the other for producing the hard $q\bar{q}$ pair. The pQCD calculation for this decay will be much more involved (see e.g. [1] for pQCD calculations of $\Lambda_b \rightarrow \Lambda J/\psi$) and is beyond the scope of the present work. Nevertheless, it is expected that $\Gamma(\bar{B}_c \rightarrow \Xi_c \bar{\Lambda}_c) \ll \Gamma(\bar{B}_c \rightarrow \Lambda_c \bar{\Lambda}_c)$ as the former is suppressed by order of $\alpha_s^4$. This dynamical suppression effect for the $\Lambda_c \bar{p}$ production relative to $\Xi_c \bar{\Lambda}_c$ has been neglected in the previous studies based on QCD sum rules [9] and on the diquark model [10].

IV. CONCLUSIONS

In this work we have studied the two-body and three-body doubly charmed baryonic $B$ decays, namely, $\bar{B} \rightarrow \Xi_c \bar{\Lambda}_c$ and $\bar{B} \rightarrow \Lambda_c \bar{\Lambda}_c K$, aiming to resolve the puzzles associated with them. We point out that the suppression from the $\Lambda_c^+ \bar{\Lambda}_c^- K^-$ phase space is so strong that $\Lambda_c \bar{p}$ is unlikely to be produced dominantly through the direct three-body decay processes. Nevertheless, the great suppression for the $\Lambda_c^+ \bar{\Lambda}_c^- K$ production can be alleviated provided that there exists a narrow hidden charm bound state with a mass near the $\Lambda_c \bar{\Lambda}_c$ threshold, of order $4.6 \sim 4.7$ GeV. This new state that couples strongly to the charmed baryon pair can be searched for in $B$ decays and in $p\bar{p}$ collisions by studying the mass spectrum of $D^{(*)} \bar{D}^{(*)}$ or $\Lambda_c \bar{\Lambda}_c$.

The doubly charmed decay such as $\bar{B} \rightarrow \Xi_c \bar{\Lambda}_c$ has a configuration more favorable than the singly charmed one such as $\bar{B}^{0} \rightarrow \Lambda_c \bar{p}$ even though they have the same CKM angles in magnitude. This is because no hard gluon is needed to produce the energetic $\Xi_c \bar{\Lambda}_c$ pair in the former decay, while two hard gluons are needed for the latter process. Therefore, $\Lambda_c \bar{p}$ is suppressed relative to $\Xi_c \bar{\Lambda}_c$ due to a dynamical suppression from $O(\alpha_s^4)$. Assuming that a soft $q\bar{q}$ quark pair is produced through the $\sigma$ and $\pi$ meson exchanges in the configuration for $\bar{B} \rightarrow \Xi_c \bar{\Lambda}_c$, it is found that $B(\bar{B} \rightarrow \Xi_c \bar{\Lambda}_c) \sim 10^{-3}$.

Acknowledgments

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Note added: After this paper was submitted for publication, Belle published the updated version of [1], in which the spectrum of the $\Lambda_c \bar{\Lambda}_c$ pair in the $\bar{B} \rightarrow \Lambda_c \bar{\Lambda}_c K$ decays is shown for the first time. According to Belle’s observation no new resonance with a mass near the $\Lambda_c \bar{\Lambda}_c$ threshold was found (see Fig. 3 in version 2 of [1]). This implies the failure of naive factorization for this decay mode and may hint at the importance of nonfactorizable contributions such as final-state effects. For example, the weak decay $B \rightarrow D^{(*)} \bar{D}^{(*)}$ followed by the rescattering $D^{(*)} \bar{D}^{(*)} \rightarrow \Lambda_c \bar{\Lambda}_c K$ [24] or the decay $B \rightarrow \Xi_c \bar{\Lambda}_c$ followed by $\Xi_c \bar{\Lambda}_c \rightarrow \Lambda_c \bar{\Lambda}_c K$ may explain the large rate observed for $B \rightarrow \Lambda_c \bar{\Lambda}_c K$. 

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APPENDIX A:

After integrating out the delta functions in Eq. (23), we obtain, taking decay via $\sigma$ as an example,

$$A_{\sigma}(B^- \to \Xi_c^0 \Lambda^-) = \frac{G_F}{\sqrt{2}} V_{td} V_{us} \frac{f_B f_{\Xi_c} f_{\Lambda}}{1152} (c_1 - c_2)(2\pi)^4 2 \int_0^{v^-/p^-} \frac{d\xi \int_0^{1-\xi v_B^+/p^+} dx_1}{x_1} \int_0^{\xi v_B^+/p^-} dx_2 \int_0^{p^+} \frac{d\xi}{p^-} \int_0^{\xi v_B^+/p^-} dx_3 \int_0^{p^+} \frac{d\xi}{p^-} \int_0^{p^+} \frac{d\xi}{p^-}$$

$$\times (2\pi) \left( \frac{1}{2} \right)^3 \int_0^{\infty} dk_{3\perp}^2 \int_0^{\infty} dk_{2\perp}^2 \int_0^{2\pi} d\theta_{23} \int_0^{\infty} dk_{3\perp}^2 \int_0^{2\pi} d\theta_{33'}$$

$$\times \Psi_{\Xi_c}(x, k_{\perp}) \Psi_{\Lambda_c}(x', k'_{\perp}) \Phi_B(\xi) \frac{(g_{qq})^2}{p_{\sigma}^2 - m_{\sigma}^2 + i m_{\sigma} \Gamma_{\sigma}} v_{\Xi_c}(a_{\sigma} + b_{\sigma} \gamma_5) v_{\Lambda_c}, \quad (A1)$$

where $p^{(i)} = (p^{(i)+}, p^{(i)-}, 0)_{\perp}$ is the momentum of $\Xi_c^0$ ($\Lambda^-_c$) and $k^{(i)}_1, k^{(i)}_2, k^{(i)}_3$ the momenta of the constituent quarks of the baryons, which are taken to be

$$k_1 = (x_1 p^+, p^-, k_{1\perp}), \quad k'_1 = (p'^+, x'_1 p'^-, k'_{1\perp}),$$

$$k_{2(3)} = (x_{2(3)} p^+, 0, k_{2(3)\perp}), \quad k'_{2(3)} = (0, x'_{2(3)} p'^-, k'_{2(3)\perp}). \quad (A2)$$

The integrations of the delta functions yield the following relations

$$x_1^{(i)} = 1 - x_2^{(i)} - x_3^{(i)}, \quad x_3 = \frac{\xi v_B^+}{p^+}, \quad x'_3 = \frac{\xi v_B^-}{p'^-} - x'_2,$$

$$k_{1\perp}^{(i)} = - (k_{2\perp}^{(i)} + k_{3\perp}^{(i)}), \quad k_{2\perp}^{(i)} = - (k_{3\perp}^{(i)} + k_{3\perp}^{(i)}) \Rightarrow k_{1\perp}^{(i)} = k_{3\perp}^{(i)}, \quad (A3)$$

and the limits of integrations as shown in Eq. (A1). Thus,

$$P_{\sigma}^2 = (k_3 + k_3')^2 = \left( \frac{\xi v_B^+}{p^+} \right) p^+ p^- \left( \frac{\xi v_B^-}{p'^-} - x'_2 \right) p'^+ p'^- - \left( k_{2\perp}^2 + k_{3\perp}^2 + 2 |k_{3\perp}| |k'_{3\perp}| \cos \theta_{33'} \right),$$

$$k_{2\perp}^2 = k_{2\perp}^2 + k_{3\perp}^2 + 2 |k_{3\perp}| |k'_{3\perp}| \cos \theta_{23},$$

$$k_{2\perp}^2 = k_{3\perp}^2 + k_{3\perp}^2 + 2 |k_{3\perp}| |k'_{3\perp}| \cos \theta_{33'}, \quad (A4)$$

where $\theta_{23}$ and $\theta_{33'}$ are the angles of $k_{2\perp}$ and of $k'_{3\perp}$ as measured against $k_{3\perp}$, respectively. Note that the integration ranges of $x_2$ and $x'_2$ are constrained by the delta function in Eq. (23), while the integration range of $\xi$ is restricted by \((1 - \xi v_B^+/p^+) \geq 0\) and $\xi v_B^+/p^- \leq 1$. 

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