QPOs during magnetar flares are not driven by mechanical normal modes of the crust.

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ABSTRACT
Quasi-Periodic Oscillations (QPOs) have been observed during three powerful magnetar flares, from SGR0526-66, SGR1806-20 and SGR1900+14. These QPOs have been commonly interpreted as being driven by the mechanical modes of the magnetar’s solid crust which are excited during the flare. Here we show that this interpretation is in sharp contradiction with the conventional magnetar model. Firstly, we show that a magnetar crustal mode decays on the timescale of at most a second due to the emission of Alfvén waves into the neutron-star interior. A possible modification is then to assume that the QPOs are associated with the magnetars’ global modes. However, we argue that at the frequencies of the observed QPOs, the neutron-star core is likely to support a continuum of Magneto-Hydrodynamic (MHD) normal modes. We demonstrate this on a completely solvable toy model which captures the essential physics of the system. We then show that the frequency of the global mode of the whole star is likely to have a significant imaginary component, and its amplitude is likely to decay on a short timescale. This is not observed. Thus we conclude that either (i) the origin of the QPO is in the magnetar’s magnetosphere, or (ii) the magnetic field has a special configuration: either it is expelled from the magnetar’s core prior to the flares, or it’s poloidal component has very small coherence length.

1 INTRODUCTION
In a prophetic paper, Duncan and Thompson (1992) argued that magnetars—a class of neutron stars with super-strong ($10^{14}$—$10^{15}$G) magnetic fields—must exist. Treated at first with some skepticism, the magnetar paradigm has proved extremely successful in explaining the rich phenomenology of Anomalous X-ray Pulsars (AXPs) and Soft Gamma-ray Repeaters (SGRs) (e.g., Thompson and Duncan 1995, TD). In particular one phenomenon which has an appealing explanation within the magnetar paradigm are Giant Flares from SGRs. They are thought to be powered by an impulsive release of magnetic energy stored in the neutron star. The release may be triggered by a fracture in the magnetically-stressed crust (TD) or by a sudden reconnection in a twisted magnetosphere (Lyutikov 2003). So far 3 Giant Flares have been observed: from SGR0526-66 (Mazets et al. 1979), SGR1900+14 (Hurley et al. 1999, Feroe et al. 1999), and SGR1806-20 (Hurley et al. 2005, Palmer et al. 2005). In each of the flares’ light-curves, there is intriguing evidence for QPOs with frequencies of tens of Hertz.

The QPOs have been widely associated with torsional modes of the neutron-star crust [Barat et al. (1983) made the first suggestion, followed by a more detailed analysis of Duncan (1998), Israel et al. (2005), Strohmayer and Watts (2005) and Piro (2005)]. Indeed, it is attractive to associate a stable QPO with a mechanical mode of a neutron star, and the frequencies of crustal torsional modes are of the right order of magnitude. However, we argue below that this interpretation is not viable within the magnetar paradigm. In the next section, we show that the crustal mode of a magnetar is not stable but decays within seconds by emitting Alfvén waves into the core. This in direct contradiction with the observations. Chris Thompson (private communications) has suggested that one may associate the QPOs with the MHD-

* The 625Hz QPO found by WT is already somewhat problematic for the crustal-mode picture. One can associate it with a crustal shear mode which has 1 radial node ($n=1$; see e.g. Piro 2005). However, a large multitude of $l < 5$, $n = 1$ shear modes exist around that frequency, with spacings of several Hz. There is no physical reason why just one of them should be excited, and not many.
2 DECAY OF MAGNETAR CRUSTAL MODES.

Magnetic fields mechanically connect the elastic crust with the liquid core. Both media can be considered as perfect conductors, and ideal MHD is applicable with magnetic field lines frozen into the media. The Alfven wave speed in the core is given by

\[ v_a = \sqrt{\frac{BB a}{4\pi\rho}} \simeq 3 \times 10^7 \text{ cm/s}. \] (1)

Here \( B = 10^{15} B_{15} \text{ G} \) is the magnetic field inside the core, \( \rho = 10^{14} \rho_{14} \text{ g/cm}^3 \) is the density, \( B_{\text{crit}} \simeq 10^{15} \text{ G} \) is the field inside superconducting fluxtubes, and \( \eta \) is the fraction of the neutron-star core involved in the Alfven-wave motion. If the neutrons are superfluid, then they decouple from the Alfven wave and \( \eta \sim 0.1 \). If protons do not form a superconducting fluid, \( B_{\text{crit}} \) should be substituted by \( B \) in the above equation.

The timescale for an Alfven wave to cross the neutron-star core is \( \sim 0.05 \text{s} \). Therefore, even if the oscillation was initially localized in the crust, the rest of the core will get involved in less than a second. It is instructive to estimate the power \( W \) radiated in the Alfven waves from the crust:

\[ W \sim \rho \omega^2 \zeta^2 v_a A, \] (2)

where \( \omega \) is the angular frequency of the oscillation, \( \zeta \) is the amplitude of the crustal oscillation at the crust-core interface, and \( A \) is the inner surface area of the crust. The timescale on which the energy is drained from the crustal mode is given by

\[ \tau \sim \frac{M}{\rho v_a A} \sim 0.01 \text{s}. \] (3)

Here \( M \) is the mass of the crust.

In a completely rigorous treatment of the problem various corrections of order 1 would occur. Partial reflection of the Alfven waves is possible, but it will not be a major correction since the wavelength and the relevant scale-heights (density, magnetic pressure) are comparable to the neutron-star radius, and thus there is no impedance mismatch which is needed for significant reflection. We note that the situation here is qualitatively different from that considered by Blaes et al. (1989), who considered small-wavelength shear waves propagating from the depth of the crust into the neutron-star magnetosphere, and found large reflection coefficients. Here we deal with global modes of the crust, which feature almost radius-independent horizontal displacements. The motion of the inner surface of the crust, together with frozen-in magnetic field-lines, should be considered as boundary condition for launching the Alfven waves into the core.

The geometry of the magnetic field is another obvious factor, but for a generic configuration we do not expect a qualitative change in radiated Alfven-wave power. If however magnetic field was confined to the crust or had a very small coherence length in its poloidal component, then the radiated Alfven-wave power could decrease significantly. We do not know any compelling argument for either. Thus we conclude that even if the crustal mode was originally excited, magnetic stresses would significantly reduce the mode amplitude in a fraction of a second and redistribute the energy within a liquid core. Therefore, barring special magnetic-field geometry, a pure crustal mode cannot be associated with a stable QPO during the flare. One may hope to associate the global MHD-elastic mode with the QPO. This proposal (due to Chris Thompson) runs into potential difficulties which we discuss in the next section.

3 GLOBAL MODES.

Computing global MHD modes of a star is clearly a difficult task. In recent tour-de-force study Reese, Rincon, and Rieutord (2004, RRR) have found the eigenmodes of a shell of a conductive incompressible fluid with a dipole magnetic field. [see also Rincon and Rieutord 2003, and Levin and D’Angelo (2004)]. However, we are not just limited by purely computational difficulties. In a magnetar core the magnetic pressure is sub-dominant, and there is virtually no conversion of the Alfven waves into magnetosonic waves. The core responds to the wave as an incompressible fluid, and the wave propagates along the field lines. Since different field lines have different lengths in the core, and different average Alfven velocities along them, one might suspect that the core supports a continuum of the Alfven modes. This is hard to prove generally, but below we, in the spirit of Appendix B in RRR, present a toy-model example where the Alfven-mode spectrum can be computed explicitly.

Consider a perfectly conducting incompressible fluid sandwiched in a box, with top and bottom plates also being perfect conductors; see Figure 1. The magnetic field \( \vec{B}(y) \) is vertically directed and is a function of \( y \) only, threading both top and bottom plates. Gravity is zero, and magnetic

\[ \vec{B}(y) \]

\[ \text{top} \]

\[ \text{bottom} \]
pressure gradient is trivially compensated by the fluid pressure gradient. Consider now a z-independent Lagrangian displacement $\zeta(x, y)$ in the $z$ direction. The equation of motion is

$$\frac{\partial^2 \zeta}{\partial t^2} = c^2 \left( \frac{\partial^2 \zeta}{\partial x^2} - \gamma \frac{\partial \zeta}{\partial t} \right)$$  \hspace{1cm} (4)

where $c(y) = \sqrt{T(y)/\rho(y)}$ and $\gamma$ is a small damping constant added for generality; here $T = B^2/(4\pi)$ is the magnetic tension (in a superconducting fluid $T = B B_{\text{crit}}/(4\pi)$) and we have allowed $y$-dependence of the fluid density $\rho$. Clearly, Eq. (4) separates in $x$ and $y$, with eigenfunctions

$$\zeta_n_{\text{xy}}(x, y) = \sin(n\pi x/L_x)\delta(y - y_0) \exp(\iomega_{n_{xy}} t),$$  \hspace{1cm} (5)

with eigenfrequencies

$$\omega_{n_{xy}} = \pm \sqrt{(n^2\pi^2/L_x^2)T(y_0)/\rho(y_0) - \gamma^2/4 + i\gamma/2}.$$  \hspace{1cm} (6)

Here $L_x$ is the box height, $n$ is the integer number, and $y_0$ is the $y$-coordinate where the eigenfunction is localized. The spectrum is continuous!

It is notable that for a spherical shell numerical results of RRR indicate that the spectrum of toroidal modes is also continuous [and the eigenmodes are singular, as in Eq. (5) above]. The continuity of part of the spectrum is a topological property, which should remain when external parameters (the shape of the box, density profile, etc) change continuously. Since the basic MHD physics in our example, in the fluid shell of RRR, and in the magnetar core are similar, we believe that the magnetar core supports a continuum of MHD modes.

We started this section by asking whether a global magnetar MHD-elastic mode can be responsible for an observed QPO. We can now see that such a mode of the whole magnetar will be coupled to a continuum of the MHD modes in the core. Experience from Quantum Mechanics tells us that a mode coupled to a continuum of other modes decays if its frequency lies within the continuum (cf. Fermi’s Golden Rule). This is likely the situation here: the QPO frequencies are tens of Hertz, above the base Alfvén-mode frequency (but obviously beyond that of infinite number of higher-order Alfvén modes). Let us illustrate explicitly the decay of a global mode by returning to the simple example in Figure 1. We now model the presence of a crust by assuming that the top plate is allowed to move in $z$-direction but is also a harmonic oscillator (with restoring force provided by some external spring). The equation of motion of the top plate is given by

$$\frac{d^2 Z}{dt^2} = -\omega_0^2 Z - F/M,$$  \hspace{1cm} (7)

where $Z$ is the top plate’s displacement, $\omega_0$ is the proper frequency of the oscillator, $F$ is the force due to magnetic stresses from the fluid below, and $M$ is the plate’s mass. When the top plate participates in a global mode of frequency $\omega$, then $Z = Z_0 \exp(\i\omega t)$. It is straightforward, by using Equation (4), to solve for the fluid motion:

$$\zeta(x, y) = \frac{\sin[k(y)L_y]}{\sin[k(y)L_x]} Z,$$  \hspace{1cm} (8)

where

$$k(y) = \sqrt{\omega^2 - \i\gamma/c(y)} \approx \frac{\omega}{c(y)} \left(1 - \frac{i\gamma}{2\omega}\right).$$  \hspace{1cm} (9)

The back-reaction force $F$ can now be computed:

$$F = -\int T(y) \left( \frac{\partial \zeta}{\partial x} \right)_{x=L_x} dy dz$$

$$= -\i L_z \int T(y)k(y) \cot[k(y)L_x] dy$$  \hspace{1cm} (10)

where $L_z$ is the $z$-dimension of the top plate. Let $\gamma = 0$. Then if $\omega$ real and in resonance with at least one of the continuum modes then $\text{cot} k L_x$ will have a pole singularity at some $y = y_0$ in the range of integration. The singularity is lifted if $\gamma$ is non-zero; in the limit of $\gamma \ll \omega$ the integration will produce an imaginary component of the amplitude of $F$:

$$\Im[F \exp(-\i\omega t)] = -\i L_z \zeta_0 T(y_0)k(y_0) \pi$$  \hspace{1cm} (11)

Comparing the above equation with the Eq. (7), we see that our assumption that $\omega$ real was not self-consistent. Within an order-of-magnitude the imaginary part of $\omega$ is

$$\Im(\omega) \approx \frac{T L_z L_y}{M \omega}.$$  \hspace{1cm} (12)

Substituting the parameters relevant for a magnetar ($B = 10^{15}$, $L_z = L_y = L_x = 10^6$cm, $M = 3 \times 10^{33}$g, $\omega \sim 300$rad/s), we get

$$\Im(\omega) \approx 10^8.$$  \hspace{1cm} (13)

Even allowing for several corrections of order 1, we see that a global mode is expected to decay within a second. This is much shorter than the timescale on which the QPO is observed, $\sim 100$s.

One can ask the following question: is it possible that a global mode is not coupled to the MHD continuum in the core? After all, along with the MHD continuum the core can probably support discrete MHD modes as well (such modes are seen in RRR’s numerical work). However, while some global mode may be decoupled from the continuum, the mode of our interest must involve the motion of the crust, in order to produce the observed QPO. Unless magnetic field in the core has high degree of symmetry (which seems unlikely, especially if one believes that it was generated by turbulent dynamo), this crustal motion is bound to couple the QPO-generating global mode to the continuum of the core’s MHD modes via the field-lines threading the crust. Thus the decay of a global mode seems generic.

4 MAGNETAR: A TUNING FORK OR WET SPAGHETTI?

Observations argue for the former: QPO frequencies are stable over tens of seconds, something strongly indicative of a mechanical mode. However, we have shown that the theory argues for the latter. Strong MHD crust-core coupling destroys the stability of a purely crustal mode, and the presence of a continuum of MHD modes in the core makes a global crust-core mode decay within a second. So what is the origin of the magnetar QPOs? It can still be mechanical, but only if the magnetic field geometry is very different from what has been assumed—for example, if the magnetic field is largely confined to the crust or has an extremely incoherent poloidal component. Alternatively, the QPOs may
have a magnetospheric origin; this was mentioned as a possibility in Barat et al. (1983), but has remained unexplored.

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References