DESCRIBING THE BARYON SPECTRUM WITH $1/N_c$ QCD

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This talk outlines recent advances using QCD in the $1/N_c$ limit aimed at understanding baryon scattering processes and their embedded short-lived baryon resonances. In this presentation we emphasize developing qualitative physical insight over presenting results of detailed calculations.

1. Introduction
When addressing an audience of baryon resonance experts, it is hardly necessary to emphasize the elusive nature of the $N^*$s as both experimental and theoretical objects: Owing to their extremely short $O(10^{-23}\text{ s})$ lifetimes, they are often barely discernable, lurking in baryon scattering amplitudes like strangers in a fog. My previous talk write-ups on this material have been geared exclusively towards theory audiences, but an $N^*$ conference is attended by a large number of experimentalists as well, who view theory talks with an eye toward picking up new notions of physical understanding for the phenomena that they study, rather than focusing on calculational detail. I therefore wish to focus here on the qualitative description of the motivation behind and the results of my recent work with Tom Cohen on excited baryons. The reader who craves more detail is welcomed to peruse Refs. or the original works.

2. Two Physical Pictures for $N^*$s
The most frequently invoked picture for baryons is that suggested by the constituent quark model, in which the light (masses $\sim 5\text{ MeV}$) fundamental quarks of the QCD Lagrangian somehow agglomerate with the multitude of gluons and virtual quark-antiquark pairs to form constituent ($\sim 300\text{ MeV}$) quarks. In order to be discernable as distinct entities, such pseudoparticles
must nevertheless remain weakly bound to one another. If this physical picture is valid, then the baryon, originally a complicated many-body object only describable using full quantum field theory, reduces to a simple three-particle quantum-mechanical system interacting through a potential, not unlike a miniature atomic nucleus. In this case the baryon excited states consist of orbital and radial excitations of the three constituents. Inasmuch as the constituent quark masses are larger than the energies that bind them, the baryons fill well-defined multiplets based upon approximate invariances of the state under quark spin flips, quark flavor substitutions, and spatial exchanges, the SU(6) × O(3) symmetry.

Constituent quark models therefore predict numerous excited hadron multiplets, the lowest of which have indeed been observed. For example, the ground states, consisting of the nucleons, the Δ resonances (related to the nucleons by a spin flip), and their strange partners, fill a spin-flavor-space symmetric \((56, 0^+)\) of SU(6) × O(3), while the lightest excitations appear to fill the orbitally-excited mixed-symmetry multiplet \((70, 1^-)\) or a radially-excited \((56, 0^+)\). However, higher in the spectrum the picture becomes much murkier, with numerous partly-filled multiplets as well as predicted multiplets whose members remain unobserved.

Alternately, the chiral soliton picture for baryons, starting directly from a hadronic perspective, recognizes that hadrons rather than quarks are the states observed in nature. Solitons are semiclassical finite-energy solutions to a field theory, which is to say that they are non-dissipating “lumps” of energy (such as a lump in a rug placed in a room too small: It can be moved from place to place, but not eliminated). Chiral Lagrangians, which have been so successful in delimiting light meson dynamics, admit solitonic solutions that couple to mesons according to chiral symmetry constraints. Their semiclassical nature is guaranteed if they are heavy compared to the mesons, just as is physically true for the baryons. In the best-studied variant, the Skyrme model, the solitons are shown to carry fermionic statistics.

The basic soliton configuration, called a hedgehog, turns out not to possess a single well-defined isospin or spin quantum number, but rather a quantum number that is the magnitude of their vector sum \(K \equiv I + J\), sometimes called the grand spin. Physical baryon states with particular spin and isospin eigenvalues are then recovered by forming a judicious linear combination of hedgehog states of different \(K\); these “judicious” couplings are none other than Clebsch-Gordan coefficients (CGC). The couplings of mesons to the underlying hedgehog, as arise in scattering processes, also induce spin and isospin CGC.
Excited baryons in chiral soliton models appear as rotational or vibrational excitations of the basic hedgehog configuration. Much of the particular spectrum generated by such excitations depends strongly upon the details of the dynamical “profile” functions multiplying the hedgehog, making predictions of baryon resonance multiplets in chiral soliton models less than robust.

3. Large $N_c$ QCD and the $1/N_c$ Expansion

While both the constituent quark and chiral soliton models warrant attention for incorporating observable features of baryons, they remain just that—models. In both cases, an expansive literature demonstrates that one may refine the models by including subleading effects, but it is not a priori obvious which corrections are essential for understanding baryon dynamics. Instead, we prefer to obtain a method directly from QCD that combines the best features of both pictures. Ab initio lattice calculations applied to excited baryons hold great promise for the future, but even when completed will provide numerical results rather than definitive dynamical statements.

Large $N_c$ QCD, obtained by supposing that QCD had not but some larger number $N_c$ of color charges, is not a model but rather an extension of the field theory representing strong interactions. It is physically useful if i) physical observables have well-defined limits as $N_c \to \infty$ [i.e., with small $O(1/N_c)$ corrections], and ii) the values of these observables do not change excessively as $N_c$ is allowed to decrease from a large value down to 3. The key question then becomes whether one can recognize in observables unambiguous signatures of this expansion in powers of $1/N_c$, and in fact the $(56, 0^+)$ baryons provide ample evidence in their spectra and couplings.

We first require a few fundamental baryon results. For $N_c$ colors, the baryons contain at least $N_c$ quarks, the number required to form a colorless state. Baryons have $O(N_c^0)$ masses, and meson couplings that are $O(N_c^{1/2})$ (trilinear) and $O(N_c^0)$ (quartic). The latter fact implies that ordinary baryon resonances, since they appear in baryon-meson scattering amplitudes, have masses above the ground states and widths each of $O(N_c^0)$. The baryons themselves, despite having large masses at large $N_c$, maintain an essentially constant $[O(N_c^0)]$ size, which follows from the suppression of multiple-quark interactions by powers of $N_c$. Lastly, order-by-order unitarity in $N_c$ powers in baryon-meson scattering processes (called consistency conditions) require the ground-state multiplet to have not only spin-$\frac{1}{2}$ but spin-$\frac{3}{2}$ members as well, the large $N_c$ analogue to the $56$ [for $N_c > 3$ the
completely symmetric SU(6) multiplet also contains up to spin $\frac{N}{2}$ states].

Both the quark model and the chiral soliton model have straightforward extensions to arbitrary $N_c$. Of course, $N_c$ must be odd for baryons to remain fermions. In the quark model case, one may define quantum fields with all the properties of constituent quarks by noting that ground-state baryons carry precisely the quantum numbers of $N_c$ quarks (which remains true for $N_c = 3$; this of course was the original motivation of the quark model), and dividing the baryon into $N_c$ non-overlapping “interpolating fields” that exhaust its wave function. Using this definition for the quarks, the suppression of multiquark operators by powers of $1/N_c$ allows one to conclude that effects carrying the spin-flavor quantum numbers of such operators are also suppressed. If the states are stable against strong decays (as is the case for the ground-state multiplet), one may construct a Hamiltonian for which these baryons are the asymptotic states, and matrix elements are computed by means of the Wigner-Eckart theorem. For example, the nucleon and $\Delta$ masses are split only at $O(1/N_c)$ because this is the order of the lowest-order (hyperfine) Hamiltonian operator distinguishing their masses; the exact coefficient remains incalculable unless the strong interactions can be solved from first principles, but if the $1/N_c$ expansion is valid, then it should be a typical hadronic scale (a few hundred MeV) times an $O(1)$ number. Indeed, the observed $N$-$\Delta$ splitting follows this pattern.\textsuperscript{17}

One may attempt an extension of this approach to the excited baryons. A large body of literature\textsuperscript{18} treats (for example) the lightest negative-parity resonances as filling the analogue to the $(70, 1^-)$, a symmetrized core of $N_c - 1$ quarks and one excited quark. While this approach has yielded many interesting phenomenological insights, its strict application seems sensible only when i) the excited baryons are also asymptotically stable states of a Hamiltonian, and ii) can be represented uniquely as 1-quark excitations of a ground state (\textit{i.e., configuration mixing} with states having 2 or more excited quarks but the same overall quantum numbers are ignored).

Chiral soliton models also combine efficiently with the $1/N_c$ expansion. Indeed, much of the interest in such models during the early 1980s centered on the fact that the semiclassical nature of the solitons was consistent with the heaviness of large $N_c$ baryons, in that many of their predictions turned out to be independent of the particular choice of profile function.\textsuperscript{19} Subsequent work\textsuperscript{20} showed that quark and soliton models for ground-state baryons share common group-theoretical features in the large $N_c$ limit. But
these results apply only to the ground-state multiplet, whose members are related by various rotations of the basic hedgehog state.

4. Resonances in the $1/N_c$ Expansion

Since soliton models can be used to study baryon scattering amplitudes, it begs the question whether one can use these models to reach beyond the ground states and obtain definite statements about resonances with a degree of model independence inherited from large $N_c$. A successful picture for resonances ought not put them in by hand; they are intrinsically excitations in baryon scattering amplitudes and should be generated as complex-valued poles ($z_R = M_R + i\Gamma_R$) within them. Work along these lines in the mid-1980s began with Ref. 21 and rapidly progressed to focus upon model-independent group-theoretical features. In particular, from this approach one finds a number of linear relations between distinct partial-wave amplitudes.

The central feature driving these works is the underlying conservation of $K$-spin. As we have seen, not only the composition of baryon states from the hedgehog, but also the couplings of baryon-meson scattering processes, introduce group-theoretical factors. Carefully combining them yields the full set of baryon partial wave amplitudes written as linear combinations of a smaller set of underlying reduced amplitudes labeled by $K$, while composing the CGC leads to coefficients that are purely group-theoretical $6j$ and $9j$ factors. As a trivial example, for $\pi N$ scattering one obtains $S_{11} = S_{31}$.

Based upon interesting regularities noted for scattering processes viewed in the $t$-exchange channel, $K$-spin conservation (expressed in terms of the usual $s$-channel quantum numbers) was shown to be equivalent to the $t$-channel rule $I_t = J_t$. It was not until several years later, however, that the $I_t = J_t$ rule was shown to follow directly from large $N_c$ consistency conditions, completing the ingredients of the proof that underlying $K$-spin conservation is a direct result of the large $N_c$ limit.

To say that full baryon partial waves are linearly related for large $N_c$ means that a resonant pole occurring in any one of them must appear in at least one of the others, or more fundamentally, in one of the reduced amplitudes. However, since a given reduced amplitude contributes to multiple partial waves, the same resonant pole appears in each one: Large $N_c$ baryon resonances appear in multiplets degenerate in both mass and width.

Large $N_c$ baryon resonances are not the exclusive provenance of soliton models; if one considers the large $N_c$ generalization of the ($70$, 1) using the Hamiltonian approach described above, one finds that only 5 distinct
mass eigenvalues occur up to $O(N_c^0)$ inclusive, the level at which distinct resonances of the ground states split in mass. When one examines all partial waves in which states carrying these quantum numbers can occur, one finds that all of the states in the multiplet are induced by one pole in each of the reduced amplitudes with $K = 0, \frac{1}{2}, 1, \frac{3}{2},$ and $2$ (and only $K = 0, 1, 2$ occur for the nonstrange states). From the point of view of large $N_c$, the irreducible multiplet $(70,1^-)$ of SU(6)×O(3) is therefore actually a reducible collection of 5 distinct irreducible multiplets, which are labeled by $K = 0, \frac{1}{2}, 1, \frac{3}{2},$ and $2$; let us label the masses as $m_K$. When SU(3) flavor symmetry is invoked, $K$ may also be defined for strange states, where it is simply defined as the magnitude of $I + J$ for the nonstrange member of the SU(3) multiplet. A similar pattern, which we call compatibility, occurs for every SU(6)×O(3) multiplet, each of which decomposes at large $N_c$ into a collection of irreducible multiplets labeled by $K$: Each quark-model multiplet forms a collection of distinct resonance multiplets. This result generalizes the one discussed above, that the ground-state multiplet in large $N_c$ forms a complete $(56,0^+)$ (in this case, only $K = 0$ appears).

5. Phenomenological Consequences

The quark and chiral soliton approaches thus find common ground for large $N_c$ by having compatible resonance multiplets. But this is a formal result; to find phenomenological successes, one needs to go no further than examining which reduced amplitudes appear in a given partial wave amplitude.

To illustrate this point, let us consider the lightest $I = \frac{1}{2}, J = \frac{3}{2}$ (N$_{1/2}$) negative-parity states. It turns out for any $N_c \geq 3$ that $(70,1^-)$ contains precisely 2 N$_{1/2}$ states; for $N_c = 3$ these are $N(1535)$ and $N(1650)$. Using only the group theory imposed by the $N_c \to \infty$ limit, $\eta N$ states at large $N_c$ allow only $K = 0$ amplitudes, while the process $\pi N \to \pi N$ allows only $K = 1$. Thus, only the resonance of mass $m_0$ appears in $\eta N$ amplitudes, and only $m_1$ appears in $\pi N \to \pi N$. As is well known to this audience, $N(1535)$ lies just barely above the $\eta N$ threshold and yet decays to it as frequently as to the heavily phase-space favored $\pi N$ channel. Alternately, the $N(1650)$ has a $\pi N$ branching ratio many times larger than for $\eta N$ despite a much more comparable phase space in these channels. The $N(1535)$ $\pi N$ and $N(1650)$ $\eta N$ couplings thus arise only through subleading corrections of the size expected from the $1/N_c$ expansion.

Results of this sort also appear among the strange resonances. In particular, the $N(1535)$ appears to be just the nonstrange member of an entire
As evidence, note that the Λ(1670) lies only 5 MeV above ηΛ(1116) threshold, and yet this channel has a 10–25% branching ratio.

Even stranger selection rules occur when full SU(3) group theory is taken into account. For example, one can prove for N\textsubscript{c} arbitrary that resonances in SU(3) multiplets whose highest hypercharge states are nonstrange (8 and 10) decay preferentially (by a factor N\textsuperscript{1/3}) with a π or η, while those whose top states are strange (1) prefer K decays by O(N\textsuperscript{2/3}). Evidence for this peculiar prediction is borne out by the Λ(1520): Its branching ratios for KN and Σπ are roughly equal, but when the near-threshold p\textsuperscript{2L+1} behavior for this d wave is taken into account, one finds the effective coupling constant ratio g(Λ(1520) → KN)/g(Λ(1520) → Σπ) ∼ 4–5 = O(N\textsubscript{c}), as advertised.

1/N\textsubscript{c} corrections may be incorporated by noting the demonstration that the I\textsubscript{t} = J\textsubscript{t} rule is equivalent to the large N\textsubscript{c} limit, also shows amplitudes with |I\textsubscript{t} − J\textsubscript{t}| = n to be suppressed by at least 1/N\textsuperscript{n}. To incorporate all possible O(1/N\textsubscript{c}) effects one simply appends to all possible amplitudes with I\textsubscript{t} = J\textsubscript{t} those with I\textsubscript{t} − J\textsubscript{t} = ±1. The number of reduced amplitudes then increases while the number of observable partial waves of course remains the same, making linear relations tougher to obtain; for example, no such 1/N\textsubscript{c}-corrected relations occur among πN → πN, but πN → πΔ relations do occur, and definitely improve by about a factor of 3 when the 1/N\textsubscript{c} corrections are taken into account.

We have noted that configuration mixing between different states with the same overall quantum numbers can be a nuisance within specific models by requiring additional assumptions. A true advantage of treating excited baryons as resonances in partial wave amplitudes is that configuration mixing can occur naturally. As an example of this philosophy, if one model predicts an especially narrow excited baryon [say, a width of O(1/N\textsubscript{c})], and if there exist broad resonances [O(N\textsuperscript{0})] in the same mass region with the same overall quantum numbers, then generically the states mix and produce two broad resonances. In the quark picture, for example, this mixing occurs any time one can find a Hamiltonian operator with transition matrix elements of O(N\textsuperscript{0}) between the two states.

The existence of well-defined multiplets of resonances at large N\textsubscript{c} is also an aid to searching for exotic states. For example, let us suppose that the pentaquark candidate Θ\textsuperscript{+} (1540) were confirmed with hypercharge +2, I = 1/2, J = 1/2, and either parity. Then large N\textsubscript{c}, independently of any model, mandates that it must have I = 1, J = 1/2, 3/2 and I = 2, J = 3/2 partners with the same mass [up to O(1/N\textsubscript{c}) corrections, less than about 200 MeV]
and the same width [which of course can magnify or shrink in response to nearby thresholds, again indicating $O(1/N_c)$ differences].

Studies of baryon scattering amplitudes are not limited only to couplings with mesons. As long as the quantum numbers and the $1/N_c$ couplings of the field to the baryons is known, precisely the same methods apply. Processes such as photoproduction, electroproduction, real or virtual Compton scattering are then open to scrutiny. In the case of pion photoproduction, the photon carries both isovector and isoscalar quantum numbers, with the former dominating by a factor $N_c$. Including the leading and first subleading isovector and the leading isoscalar amplitudes then gives linear relations among multipole amplitudes with relative $O(1/N_c^2)$ corrections. Some of the relations obtained this way (e.g., the prediction that isovector amplitude combinations dominate isoscalar ones) agree quite impressively with data. Some, however, do not appear to the eye to fare as well. In those cases, the threshold behaviors still agree quite well, followed by seemingly disparate behavior in the respective resonant regions. Does this mean that the $1/N_c$ expansion is failing? Not so: The disagreements come from resonances in the different partial waves whose masses are split at $O(1/N_c)$, giving critical behavior occurring in different places in distinct partial waves. When this effect is taken into account by extracting couplings on resonance (as presented by the Particle Data Group), the linear relations good to $O(1/N_c^2)$ do indeed produce results that agree to within 10–15%.

6. Looking Ahead

A very brief summary tells us where this program is at the current time: We now have at our disposal the correct large $N_c$ method of studying baryon resonances of finite widths model-independently, i.e., in the context of a full quantum field theory. Multiplets of resonances degenerate in masses and widths naturally arise in this approach, and are similar but not identical to old quark-model multiplets. The first phenomenological results have been very encouraging, demonstrating that the $1/N_c$ expansion continues to bear a rich harvest for the excited states. Not only the resonances themselves, but the partial wave amplitudes in which they appear, can be studied using the same methods.

The most important issue yet unsolved in this program is how to treat spurious states, i.e., those that occur only for $N_c > 3$. Indeed, we were loose in our notation when we spoke of, for example, the SU(6) or the SU(3), which contain (due to quark combinatorics) many more than the
given number of states when \( N_c > 3 \). As commented above, we obtain interesting results for specific states occurring with the same multiplicities for all \( N_c \geq 3 \), such as negative-parity \( N_{1/2} \)'s. However, many more high-spin and high-isospin states occur for \( N_c > 3 \). Which ones survive at \( N_c = 3 \) and which ones do not? Since this is a difference between \( N_c \to \infty \) and \( N_c = 3 \), it represents a special kind of \( 1/N_c \) correction yet to be mastered.

All results thus far obtain from 2-to-2-particle scattering processes. In fact, multiparticle processes such at \( \pi N \to \pi \pi N \) are not substantially more difficult in many cases of interest. For example, if the \( \pi \pi \) pair is identified by reconstruction as originating from a \( \rho \), then the suppressed width [\( O(1/N_c) \)] of \( \rho \) allows the process to be studied in factorized form.

The reader should note that physical input within this method has been virtually nil: Only the imposition of an organizing principle, around suppressions in powers of \( 1/N_c \), has occurred. In this sense, the \( 1/N_c \) methods employed thus far have the flavor of chiral Lagrangians, which obtain results using only symmetries and a low-momentum expansion. Indeed, one thrust of future work will be the folding of chiral symmetry (e.g., low-energy theorems) into the \( 1/N_c \) expansion; our preliminary examination suggests this to be a promising direction.

The essential tools thus appear to be in place to disentangle the fundamental features of the \( N^* \) spectrum using a systematic approach, much as chiral Lagrangians have done for the light mesons. Given sufficient time and resources, it is a program well within the reach of the \( N^* \) community.

Acknowledgments

This work was supported in part by the National Science Foundation under Grant No. PHY-0456520.

References

11. See C. Morningstar, these proceedings.