Thermal background can solve the cosmological moduli problem

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It is shown that the coherent field oscillation of moduli fields with weak or TeV scale masses can dissipate its energy efficiently if they have a derivative coupling to standard bosonic fields in a thermal state. This mechanism may provide a new solution to the cosmological moduli problem in some special situations [0].

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Modern theories of high energy physics contains a number of scalar fields which have a flat potential and interacts with ordinary particles only with the gravitational strength [1]. In the context of low-energy supersymmetry these moduli fields typically have a flat potential intrinsically and acquire a mass of order of weak or TeV scale when supersymmetry is broken [2], although other mechanisms of moduli stabilization have also been extensively discussed recently [3].

During inflation in the early universe [4], supersymmetry is spontaneously broken in a different manner than it is today due to the large vacuum energy density. Then moduli fields, which we denote by $\phi$, typically acquire a mass of order of the Hubble parameter and they relax to a potential minimum at this stage, which is deviated from today’s value at the current potential minimum by up to the gravitational scale, $\Delta \phi \lesssim M_G = 2 \times 10^{18} \text{ GeV}$. After inflation their mass is turned off to a much smaller value due to the disappearance of vacuum energy density and they do not move until the Hubble parameter decreases to their eventual mass scale which is presumably of order of the weak scale or TeV scale as stated above. The scalar fields then start coherent oscillation with the initial amplitude up to $M_G$, which will dominate the energy density of the universe eventually. According to the conventional estimates, their lifetime is given by

$$\tau_{\phi} \approx \frac{M_G^2}{m_\phi^3} \approx 10^8 \left( \frac{m_\phi}{10^2 \text{GeV}} \right)^{-3} \text{sec},$$

that is, they decay after the primordial nucleosynthesis creating a huge amount of entropy to demolish the successful primordial nucleosynthesis [1].

In this Letter we present a new class of solution to the above cosmological moduli problem by arguing that the previous estimate of the decay rate [1], which has been used in all the other proposed solutions to the problem [2], does not apply in a finite-temperature and finite-density state of the early universe and that it is much more enhanced than in the case of decay in a vacuum. As a result we show that the coherent moduli oscillation may efficiently dissipate its energy density well before the big bang nucleosynthesis in some special situations.

The crucial point is to take moduli decay through derivative coupling correctly into account, such as a coupling with a kinetic term of other fields. Indeed we expect moduli field is coupled with gauge fields through $\phi M_G F_{\mu \nu} F^{\mu \nu}$. It may also be coupled with scalar fields as $\lambda \frac{\phi}{M_G} (\partial \chi)^2$ or $\lambda \frac{\phi}{M_G} \chi \Box \chi$ where $\chi$ is a generic scalar field and $\lambda$ is a dimensionless coupling constant of order unity [6]. Previously, these couplings were expected to give a decay rate no different from [1] at most, because using the equation of motion, $\Box \chi = m_\chi^2 \chi$, it was concluded that the derivative coupling would give a decay rate similar to the coupling $\frac{\lambda}{M_G} m_\chi^2 \chi^2$, which yields [1] for $m_\phi \gtrsim 2 m_\chi$.

Such a naive analysis, however, could only be valid in decay in the vacuum and would not apply in the high-temperature environment in the early universe. If $\chi$ is a standard field in the visible sector, it is strongly coupled with other degrees of freedom in the early universe and rapidly reaches thermal equilibrium. Then they acquire a thermal mass of order of $\sim gT$ in general, where $g$ is a typical gauge coupling and $T$ is the cosmic temperature, so that the right-hand-side of the above equation of motion could be significantly enhanced. Alternatively, one may regard the derivative $\partial$ acting on $\chi$ as not yielding its rest mass energy $m_\chi$ but energy-momentum arising from finite-temperature environment, $\partial \sim T$ modulo some coupling. Then, the strength of interaction $\lambda \frac{\phi}{M_G} (\partial \chi)^2$ is estimated as

$$L_{\text{int}} \approx \frac{\lambda (gT)^2}{M_G} \phi \chi^2.$$

(2)
The decay rate of $\phi$ through the above interaction should read

$$
\Gamma \approx \frac{\lambda^2 g^4 T^4}{8\pi M_G^2 m_\phi} \left[ 1 + 2n_B \left( \frac{m_\phi}{2} \right) \right] C \approx \frac{\lambda^2 g^4 T^5}{2\pi M_G^2 m_\phi^2} C,
$$

where $C$ is a suppression factor due to a large thermal mass of the decay product $\chi$ which has been given in [2] for a specific model. Here $n_B(\omega) = 1/(e^{\omega/T} - 1)$ is the thermal number density of a boson and the factor in the bracket represents the effect of induced emission [8, 9]. Taking $\lambda \sim g \sim 1$, $m_\phi \sim 10^3$ GeV, $T \sim 10^{10}$ GeV, which is a typical radiation temperature at the onset of moduli oscillation $H \sim m_\phi$, we find $\Gamma \sim 3 \times 10^8$ GeV. Thus if $C$ takes an appropriate value, $\phi$ can dissipate its energy right after it starts oscillation.

In order to examine if the above naive estimate is correct, we employ nonequilibrium field theory to calculate the dissipation rate of a modulus $\phi$ in the presence of a derivative interaction. For simplicity we consider the following model consisting of two scalar fields, $\phi$ and $\chi$.

$$
\mathcal{L} = \frac{1}{2} \left( \partial_\mu \phi \right)^2 - \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} \left( \partial_\mu \chi \right)^2 - \frac{1}{2} m_\chi^2 \chi^2 + \frac{\lambda}{M_G} \phi \partial_\mu \left( \partial^\mu \chi \right)^2 - \mathcal{L}_{\text{int}},
$$

where $\mathcal{L}_{\text{int}}$ is interaction of $\chi$ which thermalizes it.

We calculate the dissipation rate of $\phi$ under the following setup appropriate to the specific problem we are working on. First we neglect cosmic expansion since we are interested only in the case dissipation time is shorter than the cosmic expansion time. Second we assume $\chi$ is in a thermal state with a specific temperature $T = \beta^{-1}$ owing to the rapid thermalizing interaction due to the self-coupling. Finally, we consider the situation the parametric resonance [10] is ineffective which is the case for the modulus mass range of interest [11].

We calculate an effective action for $\phi$ and derive an equation of motion for its expectation value using the closed time-path formalism [12, 13]. Although this method has been applied to various cosmological problems by a number of authors [14, 13], to our knowledge, derivative coupling at finite temperature has not been investigated in this context yet. The one-loop effective action relevant to dissipation due to the derivative coupling is given by

$$
\Gamma[\phi_c, \phi_\Delta] = -\int d^4x \phi_\Delta(x)(\square + m_\phi^2)\phi_c(x)
- \int d^4x d^4x' C(x - x') \theta(t - t') \phi_\Delta(x) \phi_c(x') + \frac{i}{2} \int d^4x d^4x' D(x - x') \phi_\Delta(x) \phi_\Delta(x') + \cdots .
$$

Here $\phi_\Delta$ and $\phi_\Delta'$ are mean and difference of the field variable in the forward time branch $(t = -\infty$ to $+\infty$, $\phi_+$, and that in the backward time branch $(t = +\infty$ to $-\infty$, $\phi_-$, namely, $\phi_\Delta \equiv (\phi_+ + \phi_-)/2$ and $\phi_\Delta' \equiv \phi_+ - \phi_-$, respectively. $\phi_+$ and $\phi_-$ should be identified with each other in the end. The kernels in [13] are defined by

$$
C(x - x') = \frac{\lambda^2}{M_G^2} \eta^\mu_1 \eta^\nu_2 \eta^\rho_3 \eta^\sigma_4 \text{Im} \left[ G^F_{\mu_1 \nu_2}(x - x') G^F_{\rho_3 \sigma_4}(x - x') \right], \quad \text{for } t - t' > 0,
$$

$$
D(x - x') = \frac{\lambda^2}{2M_G^2} \eta^\mu_1 \eta^\nu_2 \eta^\rho_3 \eta^\sigma_4 \text{Re} \left[ G^F_{\mu_1 \nu_2}(x - x') G^F_{\rho_3 \sigma_4}(x - x') \right].
$$

Here $G^F_{\mu_\nu}(x, x')$ is a finite-temperature Feynman propagator of field derivatives defined by

$$
G^F_{\mu_\nu}(x, x') \equiv \langle \beta | T \partial_\mu \chi(x) \partial_\nu \chi(x') | \beta \rangle = \partial_\mu \partial_\nu G^F_\chi(x - x') + i\delta_{\mu_0} \delta_{\nu_0} \delta^4(x - x'),
$$

with $G^F_\chi(x) \equiv \langle \beta | T \chi(x) \chi(x') | \beta \rangle$.

The effective action [13] is complex-valued as a manifestation of the dissipative nature of the system. We cannot obtain any sensible equation of motion by simply differentiating with respect to a field variable because we are dealing with a real scalar field and its equation of motion should be real-valued. As shown in [14], one can obtain a real-valued effective action by introducing a random Gaussian variable $\xi(x)$, which represents fluctuation related to dissipation, with the dispersion $\langle \xi(x) \xi(x') \rangle = D(x - x')$. As a result the equation of motion for the expectation value $\phi_c(x)$ is given by

$$
(\square + m_\phi^2) \phi_c(x) + \int_{-\infty}^t dt' \int d^4x' C(x - x') \phi_c(x') = \xi(x).
$$
Hereafter we omit the suffix $c$.

As described in \cite{7, 9}, this equation can easily be solved using Fourier transform. As a result, we find the dissipation rate of zero-mode modulus oscillation is related to the imaginary part of $\phi'$s self energy and given by

$$
\Gamma_\phi = i \frac{\hat{C}_0(m_\phi)}{2m_\phi},
$$

(10)

with $\hat{C}_0(m_\phi)$ being the $k = 0$ mode of the Fourier transform, $\hat{C}_k(\omega)$, of the memory kernel $C(x)$.

In order to take thermal effects of $\chi$ correctly into account, we should use the full dressed propagator, $G_{\mu
u}^{(d)}(x, x')$, to calculate $\hat{C}_k(\omega = m_\phi)$ \cite{7}. It is obtained by calculating

$$
G_{\mu
u}^{(d)}(x, x') = \langle \beta | T \partial_\mu \chi(x) \partial_\nu \chi(x') \rangle \exp \left( -i \int \mathcal{L}_{\text{int}} d^4 x \right) |\beta\rangle,
$$

(11)

using the Matsubara representation \cite{10} and resummation. As a result, the spatial Fourier mode of the finite-temperature dressed propagator is given in terms of the spectral representation as

$$
G_{\mu
u}^{(d)}(p, t) = i \int \frac{d\omega}{2\pi} \left\{ (1 + n_B(\omega)) \theta(t) + n_B(\omega) \theta(-t) \right\} \left[ \frac{1}{(\omega + i\Gamma_p)^2 - \omega_p^2} - \frac{1}{(\omega - i\Gamma_p)^2 - \omega_p^2} \right] p_{\mu} p_{\nu} + i\delta_{\mu\nu} \delta_{00} \right) e^{-i\omega t},
$$

(12)

with $p_{\mu} = (\omega, p), \omega_p^2 = p^2 + m_\chi^2 + \Sigma_R(p) + \Gamma_p^2$ and $\Gamma_p = -\Sigma_L(p)/(2\omega)$, where $\Sigma_R(p)$ and $\Sigma_L(p)$ are real and imaginary parts of $\chi$'s self energy. The real part typically scales as $\Sigma_R(p) \sim g^2 T^2$.

Inserting (12) into (10), the dissipation rate of the coherent field oscillation is given by

$$
\Gamma_\phi = \frac{\lambda^2}{2m_\phi M_G^2} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{4\omega_p^2} \left\{ (2p^2 + m_\phi^2)^2 [2n_B(\omega_p') + 1] \left[ \frac{2\Gamma_p}{(m_\phi - 2\omega_p')^2 + (2\Gamma_p)^2} - \frac{2\Gamma_p}{(m_\phi + 2\omega_p')^2 + (2\Gamma_p)^2} \right] + \left[ -2(2p^2 + m_\phi^2)^2 n_B(\omega_p') e^{\beta\omega_p' - m_\phi + 4(2p^2 + m_\phi^2)\omega_p' \Gamma_p} \right] \right\},
$$

(13)

to the first order in $\Gamma_p$. Here $m_\chi^2 = m_\chi^2 + \Sigma_R$ is the finite-temperature mass. We find that the first two terms are at most of order of $T^3/M_G^2$ for $\Gamma_p \lesssim T$. The last term is also of the same order of magnitude if $\Gamma_p$ is much larger than $m_\phi$ \cite{17}, but it may have a different dependence and could be larger for a sufficiently small $\Gamma_p$. So we concentrate on it hereafter.

Assuming that there are $N$ degrees of decay modes with the same strength, the last term of (13) reads

$$
\Gamma_\phi = \frac{N\lambda^2}{2M_G^2 m_\phi} \int_{m_T}^{\infty} \frac{d\omega}{2\pi^2 \omega} \sqrt{\omega^2 - m_T^2 m_\phi^4 n_B(\omega) e^{\beta\omega}} \frac{2m_\phi \Gamma_p}{m_\phi^2 + (2\Gamma_p)^2},
$$

(14)

The last factor is maximal and equal to $1/2$ when $m_\phi = 2\Gamma_p$. In this situation (14) is given by

$$
\Gamma_\phi \approx \frac{N\lambda^2 m_\phi^3 T}{16\pi m_\phi M_G^2} \sim \frac{T^4}{M_G^2 m_\phi}.
$$

(15)

Hence it could be enhanced by a factor of $T/m_\phi$ compared with other terms. Indeed it can be larger than the Hubble parameter, $H \approx T^2/M_G$, if

$$
\frac{N\lambda^2 g^4}{16\pi} > \frac{m_\phi}{H},
$$

(16)

where we have replaced $m_T$ by $gT$. Since the right-hand-side is larger than unity in the oscillation regime, from the above condition we need $N \gg 16\pi/(\lambda^2 g^3)$. That is, assuming the coupling constants are of order unity, $N$ should be larger than 50 or so.

Thus in such a rather special situation, thermal effects may dissipate the moduli oscillation efficiently. For this to be the case the imaginary part of the thermal correction to the mass of the decay product $\Gamma_p$ must be sufficiently small to satisfy $\Gamma_p \sim m_\phi$. 

In summary, we have calculated thermal effects on the dissipation of modulus oscillation and have shown that they can dissipate its energy efficiently in some special situations.

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