Full $fp$-shell Study of Even-Even $^{48-56}$Ti Isotopes

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The purpose of this letter is to study the reduced transition probabilities and level schemes of even-even $^{48-56}$Ti isotopes using the new version of OXBASH for $^{54,56}$Ti calculated in this work compared with the most recently available experimental data and with the previous theoretical work in Ref. 9 using GXPF1A, GXPF1 and KB3G interactions.

I. INTRODUCTION

The structure of neutron-rich nuclei has recently become the focus of much theoretical and experimental effort. Central to the on-going investigation is the expectation that substantial modifications can occur to the intrinsic shell structure of nuclei with a sizable neutron excess 1.

Interactions between protons and neutrons have been also invoked to account for the presence of a sub-shell gap at $N=32$ in neutron-rich nuclei located in the vicinity of the doubly-magic nucleus $^{48}$Ca.

Full $pf$-shell model study of $^{48}$Ca were performed by Carrier and Zuker 8 by modifying Kuo-Brown (KB) 4 to KB1 and KB3. The isobaric chains $^{49,50,51}$Ca and $^{52}$Ti studied by Poves et al. 9 using KB3 and FPD6 6 and their new released version KB3G.

Reduced transition probabilities to the first $2^+$ state in $^{52,54,56}$Ti and the development of shell closure at $N=32$, 34 were studied by Dinca et al. 7 both experimentally and theoretically using the most recently modified interaction labeled GXPF1A done by Honma et al. 9. They confirm the presence of a sub-shell closure at neutron number $N=32$ in neutron-rich Ti nuclei above $^{48}$Ca and this observation are in agreement with the shell model calculations using the most recent effective interaction, also they conclude that the data do not provide any direct indication of the presence of additional $N=34$ sub-shell gap in the Ti isotopes and that the measured $B(E2; 0^+_g \rightarrow 2^+_1)$ probabilities highlight the limitations of the present large-scale calculations as they are unable to reproduce in detail the magnitude of the transition rates in semi-magic nuclei and their strong variation across the neutron-rich Ti isotopes.

The purpose of this letter is to study the reduced transition probabilities and level schemes of even-even $^{48-56}$Ti isotopes using the new version of OXBASH for windows 10. The level schemes of selected states of $^{54,56}$Ti calculated in this work compared with the most recently available experimental data and with the previous theoretical work in Ref. 9 using GXPF1A, GXPF1 and KB3G interactions.

II. SHELL MODEL CALCULATIONS

The calculations were carried out in the D3F7 model space with the FPD6 Hamiltonian 6 using the code OXBASH 11 for $^{48}$Ti, while F7P3 model space employed with effective interaction FPD6 for $^{50}$Ti.

For $^{48}$Ti the core is considered as $^{32}$S with 16 nucleons outside core, while for $^{50}$Ti the core was taken as $^{40}$Ca and 10 nucleons outside the core.

The core was taken as $^{48}$Ca for the three nuclei $^{52}$Ti, $^{54}$Ti and $^{56}$Ti and the model space is (HO) with FPD6 effective interaction. The effective interaction GXPF1 14 were used also to calculate the level spectra for $^{54,56}$Ti for the purpose of comparison with Ref. 9.

III. RESULTS AND DISCUSSION

The test of success of large-scale shell model calculations is the prediction of the first $2^+$ level and the transition rates $B(E2; 0^+_g \rightarrow 2^+_1)$ using the optimized effective interactions for the description of $fp$-shell nuclei.

Figure 1 presents the comparison of the calculated $E_x(2^+_1)$ energies with FPD6 from the present work with the experiment, the work of Dinca et al. 7 and with the most recent calculations using the new effective interaction labeled GXPF1A 14. The comparison shows that FPD6 effective interaction is better than GXPF1 except for $^{54}$Ti at $N=32$ shell closure, GXPF1 is better in reproducing the $E_x(2^+_1)$ level. The modified effective interaction GXPF1A is more successful in description of all the mass region $A=48-56$, but only at $N=32$ shell
gap GXPF1 is more successful in reproducing $E_x(2^+_1)$ for $^{54}_{Ti}$.

The new effective interaction GXPF1A which is the improved type of GXPF1 are the most convenient one for the whole chain of Ti isotopes for the mass region $A=48-56$, but still can not reproduce the shell gap at $N=32$ like GXPF1. Our work is also fail to reproduce the shell gap at $N=32$.

Figure 2 shows the large-scale shell model calculations of the reduced transition strengths $B(E2; 0^+_{x.s.} \rightarrow 2^+_1)$ that have been performed by adopting the effective charges for proton is $e_p=1.15\text{e}$ and for neutron $e_n=0.8\text{e}$ as suggested in Ref.[13] and also these values were used in the calculations of the previous work using GXPF1 and GXPF1A in Ref.[14].

The solid line in Fig.2 is the present calculations using the effective interaction FPD6 compared with the most recently measured experimental data and with the previous work using GXPF1 and the new modified interaction GXPF1A. Our calculations produced staggering in the calculation of $B(E2)$ and it is in better agreement with experimental data as compared with the previous theoretical work [7] even when they choose the modified interaction GXPF1A, but our work compared with the recent theoretical work of Poves et al. [8] their calculations using KB3G effective interaction are in better agreement with the experiment for the nuclei $^{48,50,54,56}_{Ti}$, but not $^{52}_{Ti}$ at $N=30$ our results are in better agreement with experiment. Although that GXPF1A effective interaction is in better agreement in reproducing the first $2^+$ level in all even-even Ti isotopes for the mass region $A=48-56$ but still not able in reproducing the experimental data for the $B(E2; 0^+_{x.s.} \rightarrow 2^+_1)$ transition strengths. The difference between our calculations and the previous theoretical work from Ref.[14] is mainly attributed to the difference of the location of the single-particle energies $f_{7/2}, p_{3/2}, f_{5/2}$ and $p_{1/2}$ for the effective interactions FPD6, GXPF1 and the modified one GXPF1A which effect significantly the predication of level excitations and transition strengths $B(E2)$.

The calculated FPD6 and GXPF1 energy levels are compared with the experimental data and the previous work using GXPF1A, GXPF1 and KB3G as shown in Fig. 3. The agreement is excellent for $J^\pi=0^+, 2^+$, $4^+$ and $6^+$ sequence with FPD6 effective interaction. In order to improve the description of $E_x(2^+_1)$ for $^{56}_{Ti}$, one possible choice is to lower the single particle energy of the $f_{5/2}$ orbit by 0.8 MeV, as suggested in Ref.[18].

The reduction of $f_{5/2}$ orbit by 0.8 MeV improve the prediction of $E_x(2^+_1)$ as shown in Fig. 4 for $^{56}_{Ti}$ and it remedies this discrepancy by about 0.2 MeV. However, such a modification improve the prediction of $E_x(2^+_1)$ in $^{54}_{Ti}$ also, but it is fail completely in description of high spin states.

It can be seen in Fig.4 that GXPF1 predicts $E_x(2^+_1)$ better than FPD6 and almost its prediction as compared with previous work using GXPF1A is excellent, but it is not good in description of high spin states of $^{56}_{Ti}$ and still FPD6 is in better agreement in describing the high spin states. Besides FPD6 predicts the level sequence $J^\pi=8^+, 7^+, 9^+$, while GXPF1 predicts $J^\pi=9^+, 8^+, 7^+$. 

![FIG. 1: Systematics of $E_x(2^+_1)$ for eve-even Ti isotopes. Experimental data (closed circles) are compared with present work (solid line), the previous work using GXPF1 (dashed-dot-line) and GXPF1A (dashed line). Experimental data are taken from Refs.[12, 13].](image1)

![FIG. 2: Comparison of the large-scale shell model calculations using FPD6 (squares) with the experimental $B(E2; 0^+_{x.s.} \rightarrow 2^+_1)$ transition strengths (closed circles) for the chain of even-even Ti isotopes and with the previous work using the effective interactions GXPF1 (diamonds) and GXPF1A (stars) and with the work from Ref.[8] using KB3G effective interaction. Experimental data are taken from Refs.[14, 15].](image2)
FIG. 3: Comparisons between shell-model calculations with FPD6 and GXPF1 effective interactions (present work) with the experimental energy levels for the positive parity states of $^{54}$Ti and with the theoretical work using GXPF1, KB3G and GXPF1A (previous work) Ref.[9]. Experimental data are taken from Ref.[17].

FIG. 4: Calculated energy levels of $^{56}$Ti with two effective interactions FPD6 and GXPF1 compared with the experimental data and with the previous theoretical work using GXPF1, KB3G and GXPF1A Ref.[9]. Experimental data are taken from Ref.[18].

IV. SUMMARY

Large-scale shell model calculations by adopting FPD6 and GXPF1 effective interactions were used to calculate the level excitation and transition strengths $B(E2; 0^+_{g.s.} \rightarrow 2^+_1)$ for the mass region $A=48-56$ for the even-even Ti isotopes. The comparison of the calculated $B(E2; 0^+_{g.s.} \rightarrow 2^+_1)$ with the measured experimental data even with the small staggering prove the conclusions made by Refs.[7,8] that there is limitations of the present large-scale calculations to reproduce in detail the magnitude of the transition rates in the semi-magic nuclei and their strong variation across the neutron-rich Ti isotopes.

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