FD-Term Hybrid Inflation with Electroweak-Scale Lepton Number Violation

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ABSTRACT

We study $F$-term hybrid inflation in a novel supersymmetric extension of the SM with a subdominant Fayet–Iliopoulos $D$-term. We call this particular form of inflation, in short, $F_D$-term hybrid inflation. The proposed model ties the $\mu$-parameter of the MSSM to an SO(3)-symmetric Majorana mass $m_N$, through the vacuum expectation value of the inflaton field. The late decays of the ultraheavy particles associated with the extra U(1) gauge group, which are abundantly produced during the preheating epoch, could lower the reheat temperature even up to 1 TeV, thereby avoiding the gravitino overproduction problem. The baryon asymmetry in the Universe can be explained by thermal electroweak-scale resonant leptogenesis, in a way independent of any pre-existing lepton- or baryon-number abundance. Further cosmological and particle-physics implications of the $F_D$-term hybrid model are briefly discussed.

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1 Introduction

The inflationary paradigm constitutes an ingenious theoretical framework, in which many of the outstanding problems in standard cosmology can be successfully addressed [1]. The recent WMAP data [2], compiled with other astronomical observations [3, 4], improved upon the precision of about a dozen of cosmological parameters. These include the power spectrum $P_R^{1/2}$ of curvature perturbations, the spectral index $n_s$, the baryon-to-photon ratio of number densities $\eta_B$ and others. The values of these cosmological observables put severe constraints on the model-building of successful models of inflation and their theoretical parameters. For instance, one of the basic requirements for slow-roll inflation is that the so-called inflaton potential be flat. In this respect, supersymmetry (SUSY) emerges as a compelling ingredient in model-building for protecting the flatness of the inflaton potential against quantum corrections.

In addition to the aforementioned element of naturalness, inflationary models would have a greater value if they were predictive and testable as well. One such predictive and perhaps most appealing scenario is the well-celebrated model of hybrid inflation [5]. In this model, the inflaton field $\phi$ can start its slow-roll from values well below the Planck scale $m_{Pl} = 2.4 \times 10^{18}$ GeV. This renders the model very predictive, in the sense that an infinite set of possible higher-dimensional non-renormalizable operators, being suppressed by inverse powers of $1/m_{Pl}$, will not generically contribute significantly to cosmological observables, such as $P_R^{1/2}$ and $n_s$. In the hybrid model, inflation ends through the so-called waterfall mechanism, once the field $\phi$ passes below a critical value $\phi_c$. When this happens, another field $X$ different from $\phi$, with vanishing initial value, develops a tachyonic instability and rolls fast down to its true vacuum expectation value (VEV). Supersymmetric realizations of hybrid inflation from $F$-terms were first analyzed in [6,7], whereas hybrid inflation triggered by a dominant Fayet–Iliopoulos (FI) $D$-term [8] was subsequently considered in [9].

In this paper we study $F$-term hybrid inflation in a novel supersymmetric extension of the Standard Model (SM) that includes a subdominant FI $D$-term. We call this scenario for brevity, the $F_D$-term hybrid model. To account for the low-energy neutrino data, we introduce 3 singlet neutrino superfields $\tilde{N}_{1,2,3}$ that contain 3 right-handed neutrinos $\nu_{1,2,3R}$ and their supersymmetric scalar counterparts $\tilde{N}_{1,2,3}$. Most importantly, the model ties the $\mu$-parameter of the Minimal Supersymmetric Standard Model (MSSM) to an SO(3) symmetric Majorana mass $m_N$, through the VEV of the inflaton field $\phi$ [10,11]. Hence, the $F_D$-term hybrid model naturally predicts lepton-number violation at the TeV or even at the electroweak scale. In this scenario, the non-zero baryon asymmetry in the Universe (BAU), $\eta_B \approx 6.1 \times 10^{-10}$, can be explained by leptogenesis [12,13] and specifically by thermal electroweak-scale resonant leptogenesis [10,14].

In this paper we also study the constraints on the parameters of the $F_D$-term hybrid model that result from a reheat temperature $T_{reh} \lesssim 10^9$ GeV, which is necessary to avoid the well-known gravitino overproduction problem. This consideration puts severe limits on the size of the superpotential couplings of the theory, forcing them all to acquire rather suppressed values, namely to be smaller than about $10^{-5}$ [15]. To overcome this problem of
unnaturalness, the presence of a subdominant FI $D$-term in the $F_D$-term hybrid model is very crucial and provides a new mechanism of relaxing dramatically the above upper limit. More explicitly, the size of the $D$-term controls the decay rates of the ultraheavy fermions and bosons associated with the extra gauge group $U(1)_X$. In the absence of the $D$-term and any other non-renormalizable interaction, these ultraheavy gauge-sector particles are absolutely stable. On the other hand, these particles are abundantly produced during the preheating epoch, thus dominating the energy density of the Universe shortly after the period of the first reheating caused by the perturbative inflaton decays. Therefore, their late decays induced by a non-vanishing $D$-term could give rise to a second reheating phase in the evolution of the early Universe. Depending on the actual size of the FI $D$-term, this second reheat temperature may be as low as 1 TeV, giving rise to an enormous entropy release that can dilute the gravitinos produced during the first reheating to an unobservable level.

The paper is organized as follows: Section 2 presents the model-building aspects of the $F_D$-term hybrid model with electroweak-scale lepton-number violation. Technical details related to the radiatively-induced FI $D$-term are relegated to Appendix A. In Section 3, we estimate the reheat temperature from the perturbative inflaton decays and derive the resulting gravitino constraint on the theoretical parameters. We then discuss the non-perturbative production of the supermassive fields associated with the $U(1)_X$ gauge group during the preheating epoch and how their late decays can help to lower the reheat temperature even up to 1 TeV. Section 4 is devoted to inflation. Here we investigate two regimes: (i) cold hybrid inflation, where dissipative effects can be ignored, and (ii) warm hybrid inflation, where dissipative effects dominate over the expansion rate of the Universe. In Section 5 we illustrate how the $F_D$-term hybrid model can realize thermal electroweak-scale resonant leptogenesis. Finally, Section 6 summarizes our conclusions, including a brief discussion of further possible implications of the $F_D$-term hybrid model for particle physics and cosmology.

2 The $F_D$-Term Hybrid Model

The $F_D$-term hybrid model may be defined by the superpotential

$$W = \kappa \hat{S} (\hat{X}_1 \hat{X}_2 - M^2) + \lambda \hat{S} \hat{H}_u \hat{H}_d + \frac{\rho}{2} \hat{S} \hat{N}_i \hat{N}_i + h^u_{ij} \hat{L}_i \hat{H}_u \hat{N}_j + W_{\text{MSSM}}^{(\mu=0)},$$

(2.1)

where $W_{\text{MSSM}}^{(\mu=0)}$ is the MSSM superpotential without the $\mu$ term:

$$W_{\text{MSSM}}^{(\mu=0)} = h^u_{ij} \hat{Q}_i \hat{H}_u \hat{U}_j + h^d_{ij} \hat{H}_d \hat{Q}_i \hat{D}_j + h_l \hat{H}_l \hat{L}_l \hat{E}_l.$$  

(2.2)

In (2.1), $\hat{S}$ is the SM-singlet inflaton superfield, and $\hat{X}_1$ and $\hat{X}_2$ is a chiral multiplet pair of the so-called waterfall fields which have opposite charges under the additional $U(1)_X$ gauge group. The superpotential (2.1) of the model is uniquely determined by the $R$
transformation: \( \hat{S} \rightarrow e^{i\alpha} \hat{S}, \hat{X}_{1,2} \rightarrow e^{i\beta} \hat{X}_{1,2}, \hat{L} \rightarrow e^{i\alpha} \hat{L}, \hat{Q} \rightarrow e^{i\alpha} \hat{Q}, \) with \( W \rightarrow e^{i\alpha} W, \) whereas all other fields remain invariant under an \( R \) transformation. As a consequence of the \( R \) symmetry, higher-dimensional operators of the form \( \hat{X}_1 \hat{X}_2 \tilde{N}_i / m_{\text{Pl}}, \) for example, are not allowed.

In addition, the model contains a subdominant FI \( D \)-term, \(-\frac{1}{2} g \hat{m}_{\text{FI}} D, \) giving rise to the \( D \)-term potential

\[
V_D = \frac{g^2}{8} \left( |X_1|^2 - |X_2|^2 - m_{\text{FI}}^2 \right)^2.
\]

The FI \( D \)-term plays no role in the inflationary dynamics, as long as \( g m_{\text{FI}} \ll \kappa M. \) In Appendix A, we show how a subdominant \( D \)-term can be generated radiatively after Planck-scale heavy degrees of freedom have been integrated out. The presence of the \( D \)-term is important to break an accidental discrete charge symmetry that survives after the spontaneous symmetry breaking (SSB) of the \( U(1)_X. \) Such a breaking is crucial to render all \( U(1)_X \) gauge-sector particles unstable. As we will see in Section 3, an upper limit on the size of the FI term is obtained by requiring a sufficiently low reheat temperature, e.g. \( T_{\text{reh}} \lesssim 10^9 \) GeV, in order to suppress the gravitino abundance to an unobservable level.

From (2.1) it is straightforward to read the Lagrangian of the inflationary soft SUSY-breaking sector,

\[
-\mathcal{L}_{\text{soft}} = M_S^2 S^* S + \left( \kappa A_\kappa S X_1 X_2 + \lambda A_\lambda S H_u H_d + \frac{\rho}{2} A_\rho S \tilde{N}_i \tilde{N}_1 - \kappa a_S M^2 S + \text{H.c.} \right),
\]

where \( M_S, A_{\kappa, \lambda, \rho} \) and \( a_S \) are soft SUSY-breaking mass parameters of order \( M_{\text{SUSY}} \sim 1 \) TeV. In the regime \( |S| \gg M \) relevant to inflation, the dominant tree-level and one-loop contributions to the renormalized effective potential may be described by

\[
V_{\text{inflation}} \approx \kappa^2 M^4 \left[ 1 + \frac{1}{64 \pi^2} \left( 4 \kappa^2 + 8 \lambda^2 + 6 \rho^2 \right) \ln \left( \frac{|S|^2}{M^2} \right) \right] - \left( \kappa a_S M^2 S + \text{H.c.} \right) + V_{\text{SUGRA}},
\]

where \( V_{\text{SUGRA}} \) denotes the supergravity (SUGRA) correction that results from the Kähler potential. Assuming a minimal Kähler potential, the SUGRA correction of interest to us takes on the simple form \([6, 16, 17]\)

\[
V_{\text{SUGRA}} = \kappa^2 M^4 \frac{|S|^4}{2 m_{\text{Pl}}^2},
\]

where \( m_{\text{Pl}} = 2.4 \times 10^{18} \) GeV is the reduced Planck mass. Possible one-loop contributions to \( V_{\text{inflation}} \) from \( A_{\kappa, \lambda, \rho} \)-terms become significant only for relatively low values of \( M, \) e.g. \( M \lesssim 10^9 \) GeV, for \( \kappa, \lambda, \rho \sim 1, \) and may therefore be neglected. At the tree level, however, only the tadpole term \( \kappa a_S M^2 S \) may become relevant for values of \( \kappa \lesssim 10^{-4}, \) whereas the other soft SUSY-breaking terms are negligible during inflation \([15]\).

We now investigate the stability of the inflationary trajectory in the presence of the Higgs doublets \( H_{u,d} \) and the right-handed scalar neutrinos \( \tilde{N}_{1,2,3}. \) Specifically, the initial condition for inflation is

\[
\text{Re} S^{\text{in}} = |S^{\text{in}}| \gg M, \quad X_{1,2}^{\text{in}} = 0, \quad H_{u,d}^{\text{in}} = 0, \quad \tilde{N}_{1,2,3}^{\text{in}} = 0.
\]
At the end of inflation, one should ensure that the waterfall fields acquire a high VEV, i.e. \( X_{1,2}^{\text{end}} = M \), while all other fields have small VEVs, possibly of the electroweak order. To achieve this, we have to require that the Higgs-doublet and the sneutrino mass matrices stay positive definite throughout the inflationary trajectory up to the critical value \( |S_c| \approx M \), whereas the corresponding mass matrix of \( X_{1,2} \) will be the first to develop a negative eigenvalue and tachyonic instability close to \( |S_c| \). In this way, it will be the fields \( X_{1,2} \) which will first start moving away from 0 and set in to the ‘good’ vacuum \( X_{1,2}^{\text{end}} = M \), instead of having the other fields, e.g. \( H_{1,2} \) and \( \tilde{N}_1, \tilde{N}_2, \tilde{N}_3 \), go to a ‘bad’ vacuum where \( X_{1,2}^{\text{end}} = 0 \), \( H_{1,2}^{\text{end}} = \frac{\kappa}{\lambda} M \) and \( \tilde{N}_{1,2,3}^{\text{end}} = \frac{\kappa}{\rho} M \). To see this, let us write down the mass matrix in the background Higgs-doublet field space \((H_d^\dagger, H_u)\) as

\[
M^2_{\text{Higgs}} = \begin{pmatrix}
|\lambda|^2 S^2 & -\kappa \lambda (M^2 - X_1 X_2) + \lambda A \lambda S \\
-\kappa^* \lambda^* (M^2 - X_1^* X_2^*) & |\lambda|^2 S^2
\end{pmatrix} .
\]

(2.8)

The requirement of positive definiteness may be translated into the simple condition:

\[
|\lambda| |S|^2 \geq |\kappa (M^2 - X_1 X_2) - A \lambda S| .
\]

(2.9)

From this last inequality, we may see that the condition \( \lambda \gtrsim \kappa \) is sufficient for ending hybrid inflation to the ‘good’ vacuum. Likewise, one obtains a condition analogous to (2.9) from the sneutrino mass matrix, which amounts to having \( \rho \gtrsim \kappa \). The above two restrictions on the superpotential couplings \( \lambda \) and \( \rho \) will be imposed throughout our analysis.

As mentioned above, after the end of inflation, one has \( X_{1,2}^{\text{end}} = M \), giving rise to a high mass for the inflaton field, i.e. \( 2|\kappa|^2 M^2 |S|^2 \). Combining this fact with the soft SUSY-breaking tadpole \( -\kappa a S M^2 S \) and the trilinear coupling \( \kappa A \lambda S X_1^{\text{end}} X_2^{\text{end}} \), one gets a VEV for the inflaton [18]:

\[
v_S \equiv \langle S^{\text{end}} \rangle = \frac{1}{2|\kappa|} \left| A \lambda - a S \right| ,
\]

(2.10)

where we have neglected the VEVs of the Higgs doublets \( H_{u,d} \). The VEV of \( S \) induces an effective \( \mu \)-term and an SO(3) symmetric lepton-number-violating Majorana mass \( m_N \) of the electroweak order [10]:

\[
\mu = \lambda v_S , \quad m_N = \rho v_S .
\]

(2.11)

If \( \rho \) and \( \lambda \) are comparable in magnitude, then these two mass parameters are tied together and can naturally be of the TeV or even of the electroweak scale.

In Sections 3 and 4, we will derive the constraints on the key theoretical parameters \( \kappa, \lambda, \rho \) and \( M \) from the requirement of a low reheat temperature, \( T_{\text{reh}} \lesssim 10^9 \text{ GeV} \), and successful inflation.

### 3 Preheating and Second Reheating

In the SUGRA framework, the reheat temperature is constrained by the fact that an overabundant amount of gravitinos may destroy, through their possible late decays, the
successful predictions of Big Bang nucleosynthesis [19]. This possibility is avoided, if the gravitino abundance $Y_3/2$ is small enough, i.e. $Y_3/2 < 10^{-12}$. The latter may be translated to an upper limit on the reheat temperature, i.e. $T_{\text{reh}} \lesssim 10^9$ GeV. If the gravitinos are stable, the above limit may be relaxed by one order of magnitude to $\sim 10^{10}$ GeV. This depends on whether the so-called next-to-lightest supersymmetric particle (NLSP) has a small branching fraction to hadronic decay modes [20]. In addition to the above upper limit, the reheat temperature $T_{\text{reh}}$ is also constrained from below, depending on the mechanism of baryogenesis. Thus, for successful electroweak-scale resonant leptogenesis, a lower bound of order TeV on $T_{\text{reh}}$ should be considered.

In the following we will study the post-inflationary dynamics. To this end, let us define the fields:

$$X_\pm = \frac{1}{\sqrt{2}} (X_1 \pm X_2) = \langle X_\pm \rangle + \delta X_\pm ,$$

$$\delta X_\pm = \frac{1}{\sqrt{2}} (R_\pm + iI_\pm) . \quad (3.1)$$

As mentioned in the introduction, inflation ends, once the inflaton field, $\phi = \sqrt{2} \text{Re} S$, rolls below a critical value $\phi_c \approx \sqrt{2} M$. Then, the waterfall regime begins, where the waterfall fields $S$ and $R_+$ evolve rapidly (we use the gauge freedom to ensure that all VEVs point to real directions). Ignoring small corrections due to a non-vanishing FI $D$-term, $m_{F_1}$, the VEVs of $S$ and $R_+$ oscillate around zero, whereas $X_+$ attains its $U(1)_X$-breaking VEV, $\langle X_+ \rangle = \sqrt{2} M$.

The masses of the waterfall- or $\kappa$-sector fields $\phi$ and $R_+$ are equal to $m_\kappa = \sqrt{2} \kappa M$. The inflaton $\phi$ decays predominantly into pairs of charged and neutral higgsinos, $\tilde{h}^\pm_{u,d}$, $\tilde{h}_0^{u,d}$, and into pairs of right-handed Majorana neutrinos $\nu_{1,2,3R}$. The decay width of the inflaton is given by

$$\Gamma_\phi = \frac{1}{32\pi} \left( 4\lambda^2 + 3\rho^2 \right) m_\kappa . \quad (3.2)$$

It turns out that the field $R_+$ decays into the scalar SUSY partners of the aforementioned fields at the same rate. Hence, we find

$$\Gamma_\phi = \Gamma_{R_+} \equiv \Gamma_\kappa . \quad (3.3)$$

The reheat temperature resulting from the perturbative decays of the $\kappa$-sector fields may usually be estimated by

$$T_{\text{reh}}^{\kappa} = \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \sqrt{\Gamma_\kappa m_{Pl}} , \quad (3.4)$$

where $g_* = 228.75$ is the number of the relativistic degrees of freedom in the supersymmetric model under consideration. The gravitino bound then implies that

$$\kappa \left( \lambda^2 + \frac{3}{4} \rho^2 \right) \lesssim 3 \cdot 10^{-15} \times \left( \frac{T_{\text{reh}}^{\kappa}}{10^9 \text{ GeV}} \right)^2 \left( \frac{10^{16} \text{ GeV}}{M} \right) . \quad (3.5)$$
\[
\begin{array}{|c|c|c|c|}
\hline
\text{Sector} & \text{Boson} & \text{Fermion} & \text{Mass} \\
\hline
\text{Waterfall} & S, R_+, I_+ & \psi_\kappa = \left( \frac{1}{\sqrt{2}} \left( 1 - \frac{v}{2M} \right) \psi_{X_1} + \left( 1 + \frac{v}{2M} \right) \psi_{X_2} \right) & \sqrt{2}\kappa M \\
(\kappa\text{-sector}) & & & \\
\hline
\text{U}(1)_X \text{ Gauge} & V_\mu, R_- & \psi_g = \left( \frac{1}{\sqrt{2}} \left( 1 + \frac{v}{2M} \right) \psi_{X_1} - \left( 1 - \frac{v}{2M} \right) \psi_{X_2} \right) & gM \\
(g\text{-sector}) & & & \\
\hline
\end{array}
\]

Table 1: Particle spectrum of the waterfall and \(U(1)_X\) gauge sectors after inflation, where \(V_\mu\) denotes the \(U(1)_X\) gauge boson and \(\lambda\) its associate gaugino.

If \(\kappa \approx \lambda \approx \rho\), this amounts to being each individual coupling smaller than about \(10^{-5}\), for \(M = 10^{16}\) GeV and \(T_{\text{reh}}^\text{\(g\)}} \leq 10^9\) GeV.

So far, we have only considered the post-inflationary dynamics of the \(\kappa\)-sector fields, \(S, R_+\) and \(I_+\), to which all the energy of the inflationary potential is stored at the onset of the waterfall regime. We now turn our attention to the \(g\)-sector, namely to the particles associated with the extra \(U(1)_X\) gauge group. This distinction of the different fields involved after inflation is made clear in Table 1. Thus, the \(g\)- or \(U(1)_X\) gauge-sector contains the \(U(1)_X\) gauge boson \(V_\mu\), the Dirac fermion \(\psi_g\), which consists of the gaugino \(\lambda\) and the fermionic superpartner of \(X_-\), and the scalars \(R_-\) and \(I_-\); the field \(I_-\) is a massless would-be Goldstone boson which becomes the longitudinal component of \(V_\mu\). Each of the \(g\)-sector particles has a mass \(m_g = 2^{-1/2}g\langle X_+ \rangle\). In fact, during the waterfall transition, their masses evolve rapidly from 0 to \(gM\). As we will see below, this rapid non-adiabatic mass variation triggers the so-called preheating mechanism, through which the \(g\)-sector particles can be produced in sizeable amounts. Their decays can only be induced by the presence of a non-vanishing \(D\)-term, which breaks explicitly a discrete charge symmetry in the \(F\)- and the \(D\)-term sectors which would remain otherwise intact even after the SSB of the \(U(1)_X\).

To make this last point explicit, let us express the relevant \(F\)- and \(D\)-term potential in terms of the fields \(X_\pm\) defined in (3.1):

\[
V_{FD} = \frac{\kappa^2}{4} \left| X_+^2 - X_-^2 - 2M^2 \right|^2 + \frac{g^2}{8} \left( X_+^2X_- + X_-X_+ - m_{FI}^2 \right)^2. \tag{3.6}
\]

It is obvious that the potential \(V_{FD}\) possesses an additional discrete charge symmetry under the transformation, \(X_\pm \rightarrow \pm X_\pm\), if the FI mass term vanishes, \(m_{FI}^2 = 0\). In the absence of a FI term, this symmetry will still survive even after the SSB of the \(U(1)_X\).
along the flat direction $\langle X_1 \rangle = \langle X_2 \rangle = M$, or equivalently when $\langle X_+ \rangle = \sqrt{2} M$ and $\langle X_- \rangle = 0$. As a consequence, the $U(1)_X$ gauge boson $V_\mu$, the scalar field $R_- = \sqrt{2} \text{Re}(X_-)$ and their fermionic superpartner $\psi_g$ are all stable with a mass $g M$. This feature is highly unsatisfactory for the hybrid model without a FI term, since these particles can be produced in large numbers during the preheating process, and since they are very massive, they could dominate and so overclose the Universe at later times.

The presence of the FI term $m_{\text{FI}}$ breaks explicitly the above discrete charge symmetry and so provides a new decay mechanism for making these particles unstable. To leading order in the expansion parameter $m_{\text{FI}}/M$, the potential $V_{FD}$ given in (3.6) can be minimized using the linear field decompositions

$$X_+ = \sqrt{2} M + \delta X_+ , \quad X_- = \frac{v}{\sqrt{2}} + \delta X_- ,$$

where $v = m^2_{\text{FI}}/(2M)$. Table 1 exhibits the particle spectrum of the waterfall and $U(1)_X$ gauge sectors to leading order in $m_{\text{FI}}/M$. Unlike the case of a vanishing FI $D$-term, the scalar field $R_-$ of mass $g M$ will now decay into pairs of two lighter scalars, $R_+$ and $I_+$, of mass $\sqrt{2} \kappa M$, assuming that $g \gg \kappa$. The strength of this coupling is given by the Lagrangian

$$\mathcal{L}_{\text{int}} = g^2 m^2_{\text{FI}} \frac{R_- (R_+^2 + I_+^2)}{8 M} .$$

The $D$-term induced decay width of the $R_-$ particle can readily be found to be

$$\Gamma_{R_-} = \frac{g^3}{128 \pi} \frac{m^4_{\text{FI}}}{M^3} ,$$

and the same rate also holds true for the decay of $I_-$, or equivalently for the longitudinal polarization of $V_\mu$. Correspondingly, the decays of the $g$-sector fermions $\psi_g$ are induced by the Lagrangian

$$\mathcal{L}_{\text{int}} = - \frac{g}{8} \left( \frac{m_{\text{FI}}}{M} \right)^2 (R_+ - i I_+) \bar{\psi}_g \frac{1 - \gamma_5}{2} \psi_\kappa + \text{H.c.} .$$

Neglecting soft SUSY-breaking, we find that $\Gamma_{\psi_g} = \Gamma_{R_-} \equiv \Gamma_g$.

If the decay rate $\Gamma_g$ of the $g$-sector particles is sufficiently low, they may dominate the energy density of the Universe at later times, eventually leading to a second reheating phase due to their out of equilibrium decays. To offer an initial estimate, consider that, after the first reheating, the energy density $\rho_\kappa$ of the waterfall-sector fields gets distributed among their decay products and so diluted as relativistic radiation $\propto a^{-4}$, where $a$ is the usual cosmological scale factor describing the expansion of the Universe. Meanwhile, the energy density $\rho_g$ of the ultraheavy $g$-sector particles produced via preheating scales as $\propto a^{-3}$, such that $\rho_g/\rho_\kappa \propto a$. Moreover, during a radiation-dominated epoch, the dependence of

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1Observe that an analogous discrete charge symmetry also survives after SSB in the so-called $D$-term inflationary model [9], where $M = 0$ and $m_{\text{FI}} \neq 0$. In this case, the waterfall fields $X_{1,2}$ transform as $X_{1,2} \to \pm X_{1,2}$, while their VEVs after inflation are $\langle X_1 \rangle = m_{\text{FI}}$ and $\langle X_2 \rangle = 0$. 

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the Hubble expansion rate $H$ on $a$ is $H \propto a^{-2}$. Let us therefore denote with $H_{\text{reh}}$ the Hubble rate at the first reheating of the Universe and $H_{\text{eq}}$ the Hubble rate at the time, when $\varrho_g = \varrho_\kappa$. Then, the U(1)$_X$ gauge-sector particles will dominate the energy density of the Universe, when

$$H_{\text{eq}} = H_{\text{reh}} \left( \frac{\varrho_g^0}{\varrho_\kappa^0} \right)^2 \gg \Gamma_g,$$

where the superscript 0 stands for the energy density right after preheating. Note that $\varrho_g/\varrho_\kappa$ is conserved until the time of the first reheating, since both $\varrho_g$ and $\varrho_\kappa$ scale as $a^{-3}$ during this period.

The $g$-sector particle production via preheating can be computed numerically [21], by first solving for the mode functions and then using these to calculate the Hamiltonian energy density. For the evolution of the VEVs $\langle X_1 \rangle \approx \langle X_2 \rangle$, we assume that they initially undergo strong damping due to tachyonic preheating [22]. This phenomenon can be mimicked by setting

$$\langle X_1 \rangle = \langle X_2 \rangle = \begin{cases} 0, & \text{for } t \leq -\pi/(4\sqrt{2}\kappa M), \\ \frac{1}{2} M \left[ 1 + \sin(\sqrt{2}\kappa M t) \right], & \text{for } -\pi/(4\sqrt{2}\kappa M) < t < \pi/(4\sqrt{2}\kappa M), \\ M, & \text{for } t \geq \pi/(4\sqrt{2}\kappa M), \end{cases}$$

More precise forms of field evolutions may be obtained using numerical simulations [22]. For an initial estimate, however, only the velocity of the transition is important. In Fig. 1 we display the energy densities $\varrho_F$ and $\varrho_B$ of the $g$-sector fermions $\psi_g$ and bosons $R$ and
$V_\mu$ (produced via preheating), normalized to the energy density $\varrho_{WF} \equiv \varrho_\kappa$ carried by the waterfall-sector particles, as functions of the $U(1)_X$ gauge coupling $g$, for $\kappa = 10^{-3}$. For the given profile (3.12) of field evolutions, these normalized energy densities depend only very weakly on $\kappa$.

The above results strongly suggest that the $U(1)_X$ gauge-sector particles, $\psi_g$, $R_-$ and $V_\mu$, if sufficiently long-lived, will dominate the energy density of the early Universe. We may estimate the second reheat temperature $T_{reh}^g$ caused by their late decays, by employing a formula very analogous to (3.4). Solving this last relation for the ratio $m_{FI}/M$ yields

$$\frac{m_{FI}}{M} \approx 8.4 \times 10^{-4} \times \left(\frac{0.5}{g}\right)^{3/4} \left(\frac{T_{reh}^g}{10^9 \text{ GeV}}\right)^{1/2} \left(\frac{10^{16} \text{ GeV}}{M}\right)^{1/4}.$$  

(3.13)

For second reheat temperatures of cosmological interest, i.e. $1 \text{ TeV} \leq T_{reh}^g \leq 10^9 \text{ GeV}$, we obtain the combined constraint for $M = 10^{16} \text{ GeV}$:

$$10^{-6} \lesssim \frac{m_{FI}}{M} \lesssim 10^{-3}.$$  

(3.14)

From our discussion in this section, it is evident that the late decays of the ultraheavy $U(1)_X$ gauge-sector fields, which are copiously produced during the preheating epoch, will give rise to a second reheating phase in the evolution of the early Universe at a temperature $T_{reh}^\kappa \ll T_{reh}^g$. This makes the $F_D$-term hybrid model an interesting cosmological scenario that could even lead to a complete relaxation of the strict bound (3.5) on the couplings $\kappa$, $\lambda$, $\rho$. The reason is that gravitinos, which are produced very efficiently at high reheat temperatures $T_{reh}^\kappa > 10^9 \text{ GeV}$, will now be diluted by the large entropy release from the late decays of the $g$-sector particles. In this way, the so-called gravitino overproduction problem can be completely avoided. A detailed study of this topic will be given elsewhere [23].

4 Inflation

In this section we will discuss the additional constraints on the theoretical parameters of the $F_D$-term hybrid model from the power spectrum $P_{\mathcal{R}}^{1/2}$ and the spectral index $n_s$. We distinguish two possible regimes of inflation: (i) the cold hybrid inflation (CHI), where dissipative effects on inflation are negligible, e.g. for $\kappa$, $\lambda$, $\rho \lesssim 10^{-2}$ and (ii) the warm hybrid inflation (WHI), where dissipation might dominate over the expansion rate of the Universe [24].

4.1 Cold Hybrid Inflation

In models of hybrid inflation, the spectral index $n_s$ may well be approximated as follows [1]:

$$n_s - 1 = \frac{d \ln P_{\mathcal{R}}^{1/2}}{d \ln k} \approx 2\eta ,$$  

(4.1)
where \( k \) is the comoving wavenumber at the horizon exit and

\[
\eta = m_{\text{Pl}}^2 \frac{V_{\phi \phi}}{V} \tag{4.2}
\]
is the so-called \( \eta \)-parameter. In (4.2), \( V \) denotes the inflationary potential, and \( V_{\phi} = dV/d\phi \), \( V_{\phi \phi} = d^2V/d\phi^2 \) etc. The current WMAP data [2] show a strong preference for a red-tilted spectrum, with \( n_s - 1 \leq 0 \), implying that \( V_{\phi \phi} \leq 0 \). The actual value is \( n_s = 0.98 \pm 0.02 \) [4].

The \( T_{\text{reh}} \) constraint (3.5) on the theoretical parameters imply that \( \kappa, \lambda, \rho \ll \sim 10^{-5} \). In this case, the radiative correction to the potential becomes subdominant and may be ignored to a good approximation. The potential driving inflation simplifies considerably to

\[
V_{\text{inflation}} = \kappa^2 M^4 - \sqrt{2} \kappa aS M^2 \phi + \frac{1}{2} M_S^2 \phi^2 + \frac{\kappa^2 M^4}{8 m_{\text{Pl}}^4} \phi^4 , \tag{4.3}
\]

where \( \phi = \sqrt{2} \text{Re} S \) is the inflaton field canonically normalized. For \( M_S < 1 \text{ TeV}, \kappa \gg 10^{-6} \) and \( M \gg 10^{15} \text{ GeV} \), the soft SUSY-breaking term \( M_S \) can be omitted. The inflationary potential \( V_{\text{inflation}} \) of (4.3) generically leads to a blue-tilted spectrum, i.e. \( n_s - 1 = \frac{1}{2} \eta > 0 \), which is slightly disfavoured by the recent WMAP data.

In the following, we will concentrate on the regime where the loop correction dominates the slope of the potential, such that a negative value for \( n_s - 1 \) becomes possible. This possibility arises for \( 10^{-4} \leq \kappa, \lambda, \rho \leq 10^{-2} \). Naively, such large values of the parameters lead to a too high reheat temperature \( T_{\text{reh}} \), i.e. \( T_{\text{reh}} \gg 10^{10} \text{ GeV} \). However, as we have discussed in Section 3, the presence of a subdominant \( D \)-term renders the stable \( U(1)_X \) gauge-sector fields unstable, and so a large amount of entropy can be released from their late decays, leading to a \( T_{\text{reh}} \) which may even be as low as 1 TeV.

Our results simplify considerably if one assumes that the slope of the inflationary potential given in (2.5) is dominated by the \( \lambda \)-dependent term. To be specific, the number of \( e \)-folds \( N_e \) is given by

\[
N_e = \frac{1}{m_{\text{Pl}}^2} \int_{\phi_{\text{end}}}^{\phi_N} d\phi \frac{V}{V_{\phi}} \approx \frac{2\pi^2}{\lambda^2} \frac{\phi_N^2}{m_{\text{Pl}}^2} . \tag{4.4}
\]

Notice that at the horizon exit, it is \( \phi_N = \sqrt{N_e/2} (\lambda/\pi) m_{\text{Pl}} \) and \( \phi_N \ll 10^{-1} m_{\text{Pl}} \), for \( \lambda \ll 0.1 \) and \( N_e = 60 \). Hence inflation starts at values of \( \phi_N \) well below \( m_{\text{Pl}} \). In terms of the number of \( e \)-folds \( N_e \), the power spectrum \( P^{1/2}_\mathcal{R} \) of the curvature perturbations may now be given by

\[
P^{1/2}_\mathcal{R} = \frac{1}{2\sqrt{3} \pi m_{\text{Pl}}^3} \left| \frac{V}{V_{\phi}} \right|^{3/2} \approx \left( \frac{2N_e^3}{3} \kappa \left( \frac{M}{m_{\text{Pl}}} \right)^2 \right) = 5 \times 10^{-5} . \tag{4.5}
\]

Evidently, for \( N_e = 60 \) and \( M = 10^{16} \text{ GeV} \), the parameter \( \lambda \) cannot be by more than one order of magnitude larger than \( \kappa \), i.e. \( \lambda \ll 10 \kappa \). Finally, the spectral index \( n_s \) in terms of \( N_e \) may be expressed as follows:

\[
n_s - 1 = - \frac{1}{N_e} \approx -0.02 \tag{4.6},
\]

for \( N_e = 50-60 \). In this CHI regime, the model predicts a red-tilted spectrum, as currently favoured by the WMAP data.
4.2 Warm Hybrid Inflation

It has been extensively argued [24] that dissipative effects due to radiation production of massless particles during inflation may dominate over the expansion rate $H$ of the Universe. This form of inflation is known as warm inflation. Although a firm first principles derivation for the existence of a strong dissipative regime of inflation is still missing, it might be worth presenting tentative results for such a possible situation, using the semi-empiric formalism on warm inflation developed in [24].

In the framework of WHI, dissipation occurs from the radiation produced by the decays of the excited $H_u$ doublet of mass $\lambda S$. Specifically, the interactions relevant to WHI are

$$\mathcal{L}_{\text{int}}^{\text{WHI}} = |S|^2 \left[ |\lambda|^2 |H_u|^2 + |\rho|^2 \left( \sum_{i=1}^{3} |\tilde{N}_i|^2 \right) \right] + \left( h_t H_u \tilde{Q}_t t_R + h_{ij} \tilde{L}_i \tilde{h}_u \tilde{N}_j + \text{H.c.} \right). \quad (4.7)$$

The dominant decay mode will be $H_u \rightarrow Q_t t_R$ [25]; the other possible decay channel $\tilde{N}_j \rightarrow L_i \tilde{h}_u$ is Yukawa-coupling suppressed and kinematically allowed only when $\rho > \lambda$. Adapting the results of [24, 25] to our model, the dominant friction term for $|S| \gg M$ is given by

$$Y_S \approx \frac{\sqrt{\pi} \alpha_{\lambda}^{3/2}}{20 \sqrt{2}} \frac{\alpha_t}{\lambda} \phi, \quad (4.8)$$

where $\alpha_{\lambda} = \lambda^2/(4\pi)$ and $\alpha_t = h_t^2/(4\pi)$. The dynamics of warm inflation is governed by the following two equations:

$$\ddot{\phi} + 3H (1 + r) \dot{\phi} + V_\phi = 0, \quad (4.9)$$

$$\dot{\rho}_{\text{rad}} + 4H \rho_{\text{rad}} = Y_S \dot{\phi}^2, \quad (4.10)$$

where $r = Y_S/(3H)$, with $H^2 \approx \kappa^2 M^4/(3m_{Pl}^2)$. In the strong dissipative regime where $r \gg 1$, inflation usually ends when $\rho_{\text{rad}} > \rho_{\text{vac}} \approx \kappa^2 M^4$.

Assuming conditions of slow roll during WHI, i.e. $\eta/r^2 \ll 1$, we may determine the number of $e$-folds by

$$N_e = \frac{1}{m_{Pl}^2} \int_{\phi_{\text{end}}}^{\phi_N} d\phi \frac{(1 + r) V}{V_\phi} = \frac{\pi \alpha_{\lambda}^{1/2} \alpha_t}{60 \kappa} \frac{\phi_N^3}{m_{Pl} M^2}. \quad (4.11)$$

In the limit $r \gg 1$, the power spectrum $P_{R}^{1/2}$ due to WHI is approximately given by

$$(P_{R}^{1/2})_{\text{WHI}} \approx \left( \frac{3\pi}{4} \right)^{1/4} \sqrt{\frac{T_{\text{rad}}}{H}} r^{5/4} \left( P_{R}^{1/2} \right)_{\text{CHI}}. \quad (4.12)$$

The temperature $T_{\text{rad}}$ associated with radiation production can be calculated from (4.10), by solving the approximate equation

$$\rho_{\text{rad}} = \frac{\pi^2}{30} g_\ast T_{\text{rad}}^4 \approx \frac{3r}{4} \dot{\phi}^2, \quad (4.13)$$

$^2$A detailed calculation based on a two-particle irreducible effective action in an expanding deSitter background metric would be highly preferable.
where $\dot{\phi} \approx -V_{\phi}/(3rH)$ is evaluated at the horizon exit. Putting everything together, we find
\[
(P_{1/2})_{\text{WHI}} \approx g_s^{-\frac{1}{8}} \mathcal{N}_e^{\frac{5}{8}} (2\kappa)^{1/4} \alpha_X^{5/8} \alpha_t^{1/2} \left( \frac{M}{m_{\text{Pl}}} \right)^{1/2} = 5 \times 10^{-5} . \tag{4.14}
\]
It is interesting to observe that WHI leads to a viable inflationary scenario even for strong couplings, e.g. for $\kappa, \alpha_X, \alpha_t \sim 1$. In this case, the U(1)$_X$-breaking scale $M$ will be as low as $10^{10}$ GeV, in agreement with the earlier discussion in [25]. Obviously, it would be difficult to associate such a low scale for $M$ with gauge coupling unification. Finally, the spectral index $n_s$ in WHI is calculated in terms of $\mathcal{N}_e$ to be: $n_s - 1 \approx -5/(4\mathcal{N}_e) \approx -0.025$.

5 Baryon Asymmetry in the Universe

As discussed in Section 3, the late decays of the U(1)$_X$ gauge-sector particles may lead to a second reheating phase in the evolution of the early Universe, giving rise to a very low final reheat temperature $T_{\text{reh}}$. Depending on the size of the $D$-term, $T_{\text{reh}}$ may even be as low as 1 TeV. In such a case, the BAU may be explained by thermal electroweak-scale resonant leptogenesis [10, 14]. The $F_D$-term hybrid model under study can realize such a scenario even within a minimal SUGRA framework, where all soft SUSY-breaking parameters are constrained at the gauge-coupling unification point $M_X$, which can be chosen to be $M = M_X \approx 10^{16}$ GeV. Instead, electroweak baryogenesis is no longer viable in minimal SUGRA, since it requires an unconventionally large hierarchy between the left-handed and right-handed top squarks [26].

An advantageous feature of resonant leptogenesis is that the predictions for the BAU are almost independent of any pre-existing lepton- or baryon-number abundance. This kind of fixed-line attractor behaviour is a consequence of the quasi-in-thermal equilibrium dynamics governing the heavy Majorana neutrino sector. It results from the fact that the heavy neutrino decay widths can be several orders of magnitude larger than the expansion rate $H$ of the Universe. A detailed analysis of this dynamics was presented in [10], where single lepton-flavour and freeze-out sphaleron effects were systematically considered for the first time. In particular, it was shown that single lepton-flavour effects resulting from the Yukawa-neutrino couplings $h_{\nu ij}$ can have a dramatic impact on the predictions for the BAU, enhancing its value by many orders of magnitude. From the model-building point of view, phenomenologically rich scenarios are now possible with testable implications for high-energy colliders [27] and low-energy observables, such as $\mu \rightarrow e \gamma$, $\mu \rightarrow eee$ and $\mu \rightarrow e$ conversion in nuclei [28].

We will not reiterate all these results here, but only underline some of the key model-building aspects related to the neutrino sector of the $F_D$-term hybrid model. The $F_D$-term hybrid model contains a $3 \times 3$ Majorana mass matrix $M_S$, which is SO(3) symmetric at the gauge-coupling unification point $M_X = M \approx 10^{16}$ GeV, i.e. $M_S = m_N 1_3$. The parameter $m_N = \rho v_S$ is a universal Majorana mass whose natural value is of the order of the soft SUSY-breaking or the electroweak scale, i.e. $m_N \sim M_{\text{SUSY}}$ or $m_t$. The SO(3) symmetry of the heavy neutrino sector is broken explicitly by the Yukawa neutrino couplings $h_{\nu ij}$. In
order to explain the low-energy light neutrino data, the breaking of the SO(3) symmetry should proceed via an intermediate step, namely SO(3) should first break into its subgroup SO(2) ≃ U(1)l. This can be achieved by coupling all lepton doublets \( L_{e,\mu,\tau} \) to the linear combination:

\[
\frac{1}{\sqrt{2}} (\nu_{2R} + i\nu_{3R}).
\]

These Yukawa couplings could be as large as the \( \tau \)-Yukawa coupling \( h_\tau \), i.e. \( h_{12}^\nu = i h_{13}^\nu \sim 10^{-2} \). As a consequence of the U(1)l symmetry, the resulting light neutrino mass matrix \( m_\nu \) vanishes identically to all orders in perturbation theory. The remaining U(1)l symmetry can be broken by smaller Yukawa couplings of the order of the electron Yukawa coupling \( h_e \), i.e. \( h_{11}^\nu = \varepsilon_i \sim 10^{-6} - 10^{-7} \), which arise when one couples \( L_{e,\mu,\tau} \) to \( \nu_{1R} \) [29].

Further breaking of the U(1)l symmetry is induced in the heavy-neutrino sector by renormalization-group and threshold effects while running \( M_S \) from \( M \) to \( m_t \) [30]. Thus, \( M_S \) will generically modify to:

\[
M_S = m_N 1_3 + \Delta M_S,
\]

where one typically has \( (\Delta M_S)_{ij}/m_N \sim 10^{-5} - 10^{-7} \). Taking the effect of U(1)l-breaking parameters \( (\Delta M_S)_{ij} \) and \( \varepsilon_i \) into account, one obtains a light neutrino mass matrix which can comfortably accommodate the low-energy light neutrino data, e.g. with an inverted hierarchical light neutrino spectrum [10]. On the other hand, the heavy neutrino sector of the \( F_D \)-term hybrid model consists of 3 nearly degenerate heavy Majorana neutrinos \( N_{1,2,3} \) of mass \( m_{N_{1,2,3}} \approx m_N \), which can give rise to successful baryogenesis through thermal electroweak-scale resonant leptogenesis [29].

6 Conclusions

We have studied \( F \)-term hybrid inflation in a novel supersymmetric extension of the SM, to which a subdominant FI \( D \)-term was added. We called this particular form of inflation \( F_D \)-term hybrid inflation. The \( F_D \)-term hybrid model we have been analyzing in this paper ties the \( \mu \)-parameter of the MSSM to an SO(3) symmetric Majorana mass \( m_N \), through the VEV of the inflaton field. As a consequence, the model predicts naturally lepton-number violation at the electroweak scale.

In order to obtain predictions for the observables \( P_{R}^{1/2} \), \( n_s \) and \( \eta_B \) compatible with global cosmological analyses [4], as well as interesting particle-physics phenomenology that could be tested in laboratory experiments, one needs to make certain assumptions for the model of \( F_D \)-term hybrid inflation:

(i) Successful hybrid inflation relies on the assumption that the inflaton field is displaced from its minimum in the beginning of inflation, whereas all other non-inflaton fields have zero VEVs, according to (2.7).

(ii) The present \( F_D \)-term hybrid scenario utilizes a minimal Kähler potential, where terms of order \( H^2|S|^2 \) in the potential are set to zero or assumed to be negligible. This consideration introduces some tuning in general SUGRA models with non-minimal Kähler potentials.

(iii) In order to get a red-tilted spectrum with negative \( n_s - 1 \), one has to assume that the radiative corrections dominate the slope of the inflationary potential. This possibility
arises for superpotential couplings: $10^{-4} \lesssim \kappa, \lambda, \rho \lesssim 10^{-2}$.

(iv) Even though a bare $D$-tadpole may be present as a bare parameter in the tree-level Lagrangian, we have considered here, however, the possibility that such a term is generated radiatively after heavy degrees of freedom have been integrated out. These heavy degrees of freedom are assumed to be Planck-mass chiral superfields which are oppositely charged under the $U(1)_X$ and which break explicitly the discrete charge symmetry discussed after (3.6) and in Appendix A.

(v) We have assumed that the coupling $\rho$ of the inflaton to neutrino superfields is $SO(3)$ symmetric or very close to it. After the inflaton receives a VEV, one ends up with 3 nearly degenerate heavy Majorana neutrinos with masses at the electroweak scale. This enables one to successfully address the BAU within the thermal electroweak-scale resonant leptogenesis framework (see our discussion in Section 5). As has also been discussed in Section 5, if one assumes that the neutrino-Yukawa couplings $h^\nu_{ij}$ have a certain hierarchical structure controlled by the approximate breaking of global flavour symmetries, the model can have further testable implications for $e^+e^-$ colliders and low-energy experiments of lepton flavour and/or number violation.

The requirement for a sufficiently low reheat temperature $T_{\text{reh}} \lesssim 10^9$ GeV, which does not lead to overproduction of gravitinos, provides an important constraint on the basic theoretical parameters $\kappa, \lambda$ and $\rho$. The naive limits on these couplings derived from reheating due to perturbative inflaton decay are very strict, i.e. $\kappa, \lambda, \rho \lesssim 10^{-5}$. These limits may be completely avoided by considering the late decays of the $U(1)_X$ gauge-sector particles which are induced by a non-vanishing FI $D$-term $m^2_{\text{FI}}$. Their decay rates depend crucially on $m^2_{\text{FI}}$. As mentioned above in point (iv) and in Appendix A, the generation of a FI $D$-tadpole and its size may be engineered by adding Planck-scale heavy degrees of freedom to the theory and by subjecting these into extended $R$ symmetries. In this way, a phase of second reheating takes place in the evolution of the early Universe, which can lead to a significant lowering of the reheat temperature even up to 1 TeV.

The $F_D$-term hybrid model with electroweak-scale lepton number violation can easily be embedded within a minimal SUGRA theory, where all soft SUSY-breaking parameters are constrained at the gauge coupling unification point $M_X$ which can be chosen to be $M \approx 10^{16}$ GeV. Instead, electroweak baryogenesis is not viable in a minimal SUGRA scenario of the MSSM. Moreover, the CP-odd soft SUSY-breaking phases required for successful electroweak baryogenesis face severe constraints from the non-observation of the electron and neutron electric dipole moments, even though the latter arise diagrammatically at the 2-loop level [31].

The $F_D$-term hybrid model under discussion conserves $R$-parity. The reason is that all superpotential couplings either conserve the $B-L$ number or break it by even number of units. Specifically, the operator $\hat{S}\hat{N}_i\bar{N}_i$ breaks explicitly $L$, as well as $B-L$, by 2 units. Consequently, the lightest supersymmetric particle (LSP) of the spectrum is expected to be stable, thus providing a viable candidate to address the so-called Cold Dark Matter (CDM) problem. The new aspect of our model is that right-handed sneutrinos could be the LSPs, opening up new possibilities in the phenomenology of CDM and its detection.
From the particle-physics point of view and in the low-energy limit where the waterfall sector has decoupled and the $\rho$-coupling neglected for simplicity, the $F_D$-term hybrid model becomes identical to the so-called Minimal Nonminimal Supersymmetric Standard Model (MNSSM) in the decoupling limit of a large tadpole [32]. In particular, in the framework of WHI discussed in Section 4, the coupling $\lambda$ can be sizeable, i.e. $\lambda \sim 0.6$. In this case, the Higgs phenomenology of the MSSM will modify drastically, despite the decoupling of the singlet Higgs states. One striking possibility in the MNSSM is that the charged Higgs boson $H^+$ could be lighter than the SM-like Higgs boson [33], thus pointing to particular collider phenomenologies [34]. However, even within the traditional scenario of CHI, where $\kappa, \lambda \lesssim 10^{-2}$, the $F_D$-term hybrid model will favour particular benchmark scenarios of the MSSM. For example, if $\lambda \gg \kappa$, the $F_D$-term hybrid model may account for a possible large value of the $\mu$-parameter. Specifically, if $\lambda = 4\kappa$, one gets from (2.10) the hierarchy $\mu \approx 4M_{\text{SUSY}}$, which is the so-called CPX benchmark scenario [35] describing maximal CP violation in the MSSM Higgs sector at low and moderate values of $\tan \beta$.

A possible natural solution to the famous cosmological constant problem is expected to provide further constraints on the model building of cosmologically viable models in future. Nevertheless, the $F_D$-term hybrid model presented in this paper constitutes a first attempt towards the formulation of a minimal Particle-Physics and Cosmology Standard Model, whose validity could, in principle, be tested in laboratory experiments and further vindicated by astronomical observations.

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\section*{A \hspace{1em} D–Term Engineering}

The generation and the size of a D-term may be engineered by adding Planck-scale heavy degrees of freedom to the theory and by subjecting these into extended $R$ symmetries.

To elucidate our point, let us first consider a model augmented by a pair of oppositely charged superfields $\hat{X}_{1,2}$, with $U(1)_X$ charges: $Q(\hat{X}_2) = -Q(\hat{X}_1) = Q(\hat{X}_1) = -Q(\hat{X}_2) = 1$. The extended superpotential $W$ of our interest is

\[ W = \kappa \hat{S} \left( \hat{X}_1 \hat{X}_2 - M^2 \right) + \xi m_{\text{Pl}} \hat{X}_1 \hat{X}_2 + \xi_1 (\hat{X}_1 \hat{X}_1)^2 + \xi_1' (\hat{X}_2 \hat{X}_2)^2. \quad (A.1) \]

This form of the superpotential may be enforced by the $R$ symmetry: $\hat{S} \to e^{i\alpha} \hat{S}$, $\hat{X}_{1,2} \to e^{\pm i\beta} \hat{X}_{1,2}$, $\hat{L} \to e^{i\alpha} \hat{L}$, $\hat{Q} \to e^{i\alpha} \hat{Q}$, with $W \to e^{i\alpha} W$. As before, all remaining fields are considered to be neutral under the $R$ symmetry. Notice that the same $R$-symmetry allows for the operator $\kappa' S (\hat{X}_1 \hat{X}_2)^2 / m_{\text{Pl}}^2$. The presence of this superpotential term can trigger shifted hybrid inflation, where the gauge symmetry $U(1)_X$ is broken along the inflationary trajectory, thereby inflating away unwanted topological defects \cite{36}.

A D-term will now be generated after integrating out the Planck-scale superfields $\hat{X}_{1,2}$. The loop-induced D-tadpole $m_{\text{FI}}^2$ is found to be

\[ m_{\text{FI}}^2 \approx \frac{\xi_1^2 - \xi_1'^2}{8\pi^2} \frac{M^4}{m_{\text{Pl}}^2} \ln \left( \frac{m_{\text{Pl}}}{M} \right). \quad (A.2) \]

For $M = 10^{16}$ GeV, we find that $m_{\text{FI}}/M \lesssim 10^{-3}$ for $\xi_1, \xi_1' \lesssim 0.3$. Observe that if $\xi_1 = \xi_1'$, the discrete charge symmetry discussed after (3.6) gets restored again and $m_{\text{FI}}$ vanishes identically.

The size of the D-term may be suppressed further, if the Planck-mass chiral superfields $\hat{X}_{1,2}$ possess higher $U(1)_X$ charges. In general, one may assume that the $U(1)_X$ charges of $\hat{X}_{1,2}$ are: $Q(\hat{X}_2) = -Q(\hat{X}_1) = n$, where $n \geq 1$. In addition, we require for $\hat{X}_{1,2}$ to transform under $U(1)_R$ as follows:

\[ \hat{X}_{1,2} \to e^{\frac{i}{2} [a \mp (n+1)\beta]} \hat{X}_{1,2}, \quad (A.3) \]

while $\hat{S}$, $\hat{X}_{1,2}$ and all other fields transform as before. With this symmetry restriction, the superpotential reads:

\[ W = \kappa \hat{S} \left( \hat{X}_1 \hat{X}_2 - M^2 \right) + \xi m_{\text{Pl}} \hat{X}_1 \hat{X}_2 + \xi_n (\hat{X}_1)^2 (\hat{X}_1)^{n+1} + \xi_n' (\hat{X}_2)^2 (\hat{X}_2)^{n+1}. \quad (A.4) \]

In this case, the loop-induced D-term is given by

\[ m_{\text{FI}}^2 \approx \frac{\xi_n^2 - \xi_n'^2}{8\pi^2} \frac{M^{2(n+1)}}{m_{\text{Pl}}^2} \ln \left( \frac{m_{\text{Pl}}}{M} \right). \quad (A.5) \]

To obtain a small ratio $m_{\text{FI}}/M \sim 10^{-6}$, with $\xi_n, \xi_n' \sim 1$, one would need $n = 5, 6$. Finally, it is important to remark that the loop-induced D-term does not lead to spontaneous breakdown of global supersymmetry.

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