Threshold Resummation Effects in Direct Top Quark Production at Hadron Colliders

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We investigate the threshold-enhanced QCD corrections to the cross sections for direct top quark productions induced by model-independent flavor changing neutral current couplings at hadron colliders. We use the soft-collinear effective theory to describe the incoming massless partons and use the heavy quark effective theory to treat the top quark. Then we construct the flavor changing operator based on the above effective theories, and resum the large logarithms near threshold arising from soft gluon emission. Our results show that the resummed QCD corrections further enhance the next-to-leading order cross sections significantly. Moreover, the resummation effects vastly reduce the dependence of the cross sections on the renormalization and factorization scales, especially in cases where the next-to-leading order results behave worse than the leading order results. Our results are more sensitive to the new physics effects. If signals of direct top quark production are found in future experiments, it is more appropriate to use our results as the theoretical inputs for extracting the anomalous couplings.

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The production and decay of top quark through FCNC couplings are very sensitive to new physics effects, which have been extensively studied in some new physics models in the literature (see Ref. [1] and references there in). Since we do not know which type of new physics will be responsible for a future deviation from the SM predictions, it is necessary to study the top quark FCNC processes in a model independent way by an effective Lagrangian. For $t$-$q$-$g$ anomalous couplings, the direct top quark production is the most sensitive process. The analysis based on the leading order (LO) cross sections [2] suggests that the anomalous couplings can be detected down to 0.019 TeV and 0.062 TeV for $q = u$ and 0.062 TeV for $q = c$ at the Tevatron Run 2. Studies with a fast detector simulation for ATLAS indicate a similar reach at the LHC [3].

As we know, the LO cross sections for processes at hadron colliders suffer from large uncertainties due to the arbitrary choice of the renormalization scale ($\mu_r$) and factorization scale ($\mu_f$), and are not sufficient for the extraction of the anomalous couplings from experiments. In general, a next-to-leading order (NLO) QCD calculation is capable to reduce the scale dependence significantly. But it is not the case for direct top quark production considered here. In Ref. [4], the cross sections for the direct top production are calculated at NLO in QCD. Their results showed that the NLO QCD corrections reduced the scale dependence of the total cross sections at the Tevatron Run 2. However, the scale dependence was not improved at the LHC, and even became worse in the region $\mu_r = \mu_f < m_t$. It was pointed out [4] that the scale dependence comes mainly from the terms proportional to $\delta(1 - z)$ in the partonic cross sections, and higher order effects are necessary to further reduce such scale dependence in the case of $\mu_r = \mu_f$ at the LHC. In this paper, we report that the threshold resummation effects can remarkably improve the scale dependence of the cross sections.

We consider the process $A + B \rightarrow g(k_1) + q(k_2) \rightarrow t + X$, where $A$, $B$ are the colliding hadrons. At the parton level, the process is induced by the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = -g_s \sum_{q=u,c} \frac{\delta q^a}{\Lambda} T^a \sigma_{\mu\nu} T^a(f_q^q + ih_q^q \gamma_5) q \sigma_{\mu\nu} T^a \bar{t} q + \text{h.c.}, \quad (1)$$

where $\Lambda$ is the new physics scale, $T^a$ are the Gell-Mann matrices satisfying $\text{Tr}(T^a T^b) = \delta^{ab}/2$, $G^a_{\mu\nu}$ are the field strength tensors of gluon fields, $\kappa$ is normalized to be real and positive and $f$, $h$ to be complex numbers satisfying $|f|^2 + |h|^2 = 1$.

The NLO results contain terms like $\frac{m_t^2}{1 - z^2}$, which are singular near the kinematical threshold $z \rightarrow 1$. Here $z = Q^2/s$, $Q^2 = m_t^2$ and $s = (k_1 + k_2)^2$. Physically, these singular terms originate from the emission of soft or collinear gluons. The soft-collinear effective theory (SCET) [5] is a natural framework to deal with the physics of soft and collinear gluons and quarks. In the following, we will use our NLO results to derive a threshold resummation formula for direct top quark production using the SCET.

We introduce the parameter $\lambda^2 \sim 1 - z \gg \Lambda_{\text{QCD}}/Q$ in SCET$_1$ and match the full theory operator $\lambda^2$ onto the operator in the effective theory at leading order in $\lambda$. This step is similar as the ones for the deep inelastic scattering (DIS) process in Ref. [6] and the Drell-Yan process in Ref. [7]. However, there is a non-trivial point in our problem concerning the top quark field. In general, the top quark field can not be described in the SCET.
However, in the threshold region \( z \lesssim 1 \), the top quark is nearly at rest, which implies that its momentum can be written as \( q = m_t v + k \), where the residue momentum \( k \sim Q \lambda^2 \) comes from its interactions with soft gluons. This situation is much similar as the bottom quark in \( B \) decays. Therefore, we can treat the top quark field using the heavy quark effective theory (HQET) by a systematic expansion in \( 1/m_t \). Thus the effective operator can be written as

\[
\mathcal{L}_{gq} = g_s \frac{\kappa}{\Lambda} \bar{t}_\mu \Gamma^\mu B_{\mu\nu} C_{gq}(Q^2, \mu) W_{\nu}^\dagger \xi_n
\]

\[
\equiv g_s \frac{\kappa}{\Lambda} C_{gq}(Q^2, \mu) T_{gq},
\]

(2)

where \( \Gamma^\mu = \frac{1}{2} (\not\! t + i g_s \not\! g_5 n_\nu \sigma^{\mu\nu} ) \) is the heavy quark field in HQET describing the top quark, \( \xi_n \) is the collinear quark field, \( B_{\alpha\beta} \) is related to the field strength tensor of the collinear gluon field in SCET and \( W_{\nu} \) denotes a Wilson line which are required to ensure gauge invariance of the operator. \( n \) and \( \tilde{n} \) are two light-cone vectors satisfying \( n^2 = \tilde{n}^2 = 0 \) and \( n \cdot \tilde{n} = 2 \). \( C_{gq} \) is the matching coefficient which are obviously equal to unity at tree level. We can determine the \( \mathcal{O}(\alpha_s) \) matching conditions by evaluating the on-shell matrix elements of operators in both full theory and SCET. In dimensional regularization (DR), the facts that the IR structure of the full theory and the effective theory is identical and the on-shell integrals are scaleless and vanish in SCET and HQET imply that the IR divergence of the full theory is just the negative of the UV divergence of SCET. Thus, from the \( \mathcal{O}(\alpha_s) \) virtual corrections, we can obtain the NLO matching coefficient and the anomalous dimension of the effective operator \( T_{gq} \):

\[
C_{gq}(Q^2, \mu) = 1 + \frac{\alpha_s}{12\pi} \left[ -12 \ln \frac{\mu^2}{Q^2} - \frac{13}{2} \ln^2 \frac{\mu^2}{Q^2} - 23 + \frac{55\pi^2}{12} \right],
\]

(3)

\[
\gamma_1(\mu) = \frac{\alpha_s}{6\pi} \left[ 13 \ln \frac{\mu^2}{Q^2} + (6\beta_0 + 10) \right],
\]

(4)

where \( \beta_0 = 11/4 - \gamma_E/6 \).

Next, we perform the usual field redefinition to decouple the collinear and soft fields and match the operator to \( \text{SCET}_{1\text{F}} \). This matching is trivial: the matching coefficient is unity and the anomalous dimension is the same as Eq. (4) and we will still denote the new operator as \( T_{gq} \). Now we calculate the cross section in SCET with \( T_{gq}(\mu) \) at the scale \( \mu \sim Q \lambda^2 \), and then match the result onto the product of two PDFs, which means

\[
\sigma_{\text{SCET}} = \mathcal{M}(z, \mu) \otimes [f_g(z, \mu) \otimes f_q(z, \mu)]
\]

\[
\equiv \mathcal{M}(z, \mu) \otimes \mathcal{F}(z, \mu),
\]

(5)

where \( \mathcal{M}(z, \mu) \) is the matching coefficient. Since the virtual corrections in SCET vanish at NLO in DR, \( \sigma_{\text{SCET}} \) can be obtained by calculating the real gluon emission diagrams in SCET and taking into account the renormalization of the operator. The result is

\[
\frac{\delta_{\text{SCET}}}{\delta_0} = \frac{\alpha_s}{6\pi} \left[ \frac{26}{(1-z)_+} - (6\beta_0 + 6)\delta(1-z) \right]
\]

\[
+ \frac{8 - 13\pi^2}{4} + 4 \ln \frac{\mu^2}{Q^2} + \frac{13}{2} \ln^2 \frac{\mu^2}{Q^2} \delta(1-z)
\]

\[
- \frac{26}{(1-z)_+} \ln \frac{\mu^2}{Q^2} - \frac{8}{1-z}_+ + 52 \ln(1-z) + \frac{27}{2} \ln(1-z) - \frac{28 \ln z}{1-z} + 7 \right],
\]

(6)

where \( \delta_0 = (8\pi^2/3)\alpha_s(\kappa/\Lambda)^2 z \). Matching the cross section onto the product of two PDFs, all the poles are cancelled by the renormalization of the PDFs,

\[
Z_F(z) = \delta(1-z) + \frac{\alpha_s}{6\pi} \frac{26}{(1-z)_+} + (6\beta_0 + 6)\delta(1-z).
\]

The resulting matching coefficient is given by

\[
\mathcal{M}(z, \mu) = \frac{\alpha_s}{6\pi} \left[ \frac{8 - 13\pi^2}{4} + 4 \ln \frac{\mu^2}{Q^2} + \frac{13}{2} \ln^2 \frac{\mu^2}{Q^2} \right]
\]

\[
\times \delta(1-z) - \frac{26}{(1-z)_+} \ln \frac{\mu^2}{Q^2} - \frac{8}{1-z}_+ + 52 \ln(1-z) + \frac{27}{2} \ln(1-z) - \frac{28 \ln z}{1-z} + 7 \right].
\]

(7)

To factorize the convolution in Eq. (5), we make Mellin transformation into moment space and neglect terms that vanish in the limit \( N \to \infty \). The moment of \( \mathcal{M} \) is

\[
\mathcal{M}_N(\mu) = \frac{\alpha_s}{6\pi} \left[ 8 + 13\pi^2 \frac{12}{12} + 26 \ln^2 \frac{\tilde{N}_\mu}{Q} + 8 \ln \frac{\tilde{N}_\mu}{Q} \right],
\]

(8)

where \( \tilde{N} = N e^{\gamma_E} \) and \( \gamma_E \) is the Euler constant. To avoid large logs we choose the matching scale \( \mu \sim Q/\tilde{N} \). The running of the moments of \( \mathcal{F} \) is governed by it anomalous dimension

\[
\gamma_2(\mu) = \frac{\alpha_s}{3\pi} \left[ -26 \ln \tilde{N} + 6\beta_0 + 6 \right].
\]

(9)

Combining the above results, we can write down the resummed cross section in moment space

\[
\sigma_N^{\text{SCET}} = \sigma_0(\mu) \left| C_{gq}(Q^2, \mu) \right|^2 \left[ 1 + \mathcal{M}_N(\mu) \right] \mathcal{F}(\mu)
\]

\[
= \sigma_0(\mu_r) \left| C_{gq}(Q^2, \mu_r) \right|^2 e^{I_1}
\]

\[
\times \left[ 1 + \mathcal{M}_N(Q/\tilde{N}) \right] e^{I_2} \mathcal{F}(\mu_f),
\]

(10)

with

\[
I_1 = \int_{Q/\tilde{N}}^{\mu_r} \frac{d\mu}{\mu} 2\gamma_1(\mu), \quad I_2 = \int_{\mu_f}^{Q/\tilde{N}} \frac{d\mu}{\mu} \gamma_2(\mu),
\]

(11)

where \( \mu_r \) and \( \mu_f \) correspond to the renormalization and factorization scales in the full theory, respectively, which are expected to satisfy \( \mu_r \sim \mu_f \sim Q \gg Q/\tilde{N} \gg \Lambda_{\text{QCD}} \).
To reach the accuracy of next-to-leading logarithms (NLL) in the exponents, we need the coefficient of \( \ln(\mu^2/Q^2) \) in \( \gamma_1(\mu) \) and that of \( \ln N \) in \( \gamma_2(\mu) \) up to two-loop. It can be shown that \( \gamma_1 \) and \( \gamma_2 \) have the form

\[
\gamma_1(\mu) = A_1(\alpha_s) \ln \frac{\mu^2}{Q^2} + A_0(\alpha_s),
\]

\[
\gamma_2(\mu) = B_1(\alpha_s) \ln N + B_0(\alpha_s),
\]

and \( 4A_1(\alpha_s) = -B_1(\alpha_s) \). Thus we can extract \( A_1 \) from the information of \( B_1 \) at the two-loop level. \( B_1 \) is related to the coefficients of \( 1/(1-z) \) in the DGLAP splitting kernels. Using the two-loop splitting functions together with Eq. (4) and (9), we can get the coefficients

\[
A_1^{(1)} = -\frac{1}{4} B_1^{(1)} = \frac{13}{6},
\]

\[
A_1^{(2)} = -\frac{1}{4} B_1^{(2)} = \frac{A_1^{(1)}}{2} \left[ C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} n_f \right],
\]

\[
A_0^{(1)} = \beta_0 + \frac{5}{3}, \quad B_0^{(1)} = 2\beta_0 + 2,
\]

where \( A_1 = \sum (\alpha_s/\pi)^n A^{(n)} \) and similar for \( B_1, A_0 \) and \( B_0 \). Note that the coefficients \( A_1 \) and \( B_1 \) only depend on the initial states and can be related to the quark-quark coefficients and the gluon-gluon coefficients in a straightforward way. For example, \( A_1^{(1)} = (A_{1g}^{(1)} + A_{1gg}^{(1)})/2 \equiv (C_F + C_A)/2 \).

Now we can evaluate the integrals in Eq. (11) using the two-loop evolution of \( \alpha_s \) in the \( \overline{\text{MS}} \) scheme. Keeping only terms up to NLL in the exponents in Eq. (11), we obtain

\[
I_1 + I_2 = \ln N g_0^{(1)}(\beta_0 \alpha_s(\mu_R) \ln N/\pi)
+g_0^{(2)}(\beta_0 \alpha_s(\mu_R) \ln N/\pi) + \mathcal{O}(\alpha_s(\alpha_s \ln N)^k),
\]

\[
g_0^{(1)}(\lambda) = \frac{A_1^{(1)}}{\beta_0 \lambda} [2\lambda + (1 - 2\lambda) \ln(1 - 2\lambda)],
\]

\[
g_0^{(2)}(\lambda) = -\frac{2A_1^{(1)}}{\beta_0} \ln(1 - 2\lambda) + \frac{A_1^{(1)}}{\beta_0^2} [2\lambda + \ln(1 - 2\lambda)]
+ \frac{1}{2} [2\lambda(1 - 2\lambda) - A_1^{(1)} \ln(1 - 2\lambda)]
- \frac{1}{2} \ln^2(1 - 2\lambda)
- \frac{A_1^{(1)}}{\beta_0} \ln(1 - 2\lambda) \ln \frac{\mu^2}{Q^2}
- \frac{A_1^{(1)}}{\beta_0} 2\lambda \ln \frac{\mu^2}{\mu_f^2}
+ \frac{B_0^{(1)} - 2A_1^{(1)}}{2\beta_0} \ln(1 - 2\lambda),
\]

which are the same as the results obtained within full QCD except the coefficients A and B. The NLL cross section in moment space is then given by

\[
\sigma_N^{\text{NLL}} = \sigma_0 C(\alpha_s, Q^2, \mu_R, \mu_f) \exp \left( g_0^{(1)} \ln N + g_0^{(2)} \right) \mathcal{M}_N(\mu_f),
\]

where \( C \) represents the contributions from \( C_{gg}, \mathcal{M}_N, \) as well as the terms neglected in the exponents. To obtain the physical cross section, we perform the inverse Mellin transformation back to the x-space

\[
\sigma^{\text{NLL}}(\tau) = \frac{1}{2\pi i} \int_C dN \tau^{-N} \sigma_N^{\text{NLL}}.
\]

We use the tricks introduced in Ref. [11] to evaluate the N-integral numerically. Finally, The resummed cross section at NLL accuracy is defined to be the NLL cross section plus the remaining terms in the NLO result which are not resummed, i.e.,

\[
\sigma^{\text{Resum}} = \sigma^{\text{NLL}} + \sigma^{\text{NLO}} - \sigma^{\text{NLL}}\bigg|_{\alpha_s=0} - \alpha_s \frac{\partial \sigma^{\text{NLL}}}{\partial \alpha_s} \bigg|_{\alpha_s=0}.
\]

We now present the numerical results of the NLL threshold resummation for the direct top quark production at the Tevatron Run 2 and the LHC, respectively. In our numerical calculations, we take the top quark mass \( m_t = 174.3 \text{ GeV} \) [12], which is different from the value 178.0 GeV used in Ref. [11]. We use both the CTEQ6 PDFs [13] and the MRST2004 PDFs [14] to evaluate the total cross sections.

In Table I, we list the LO, NLO and resummed cross sections for the renormalization and factorization scales \( \mu_R = \mu_F = m_t \) using the two PDF sets. It can be seen that the threshold resummation effects further enhance the NLO cross sections by about 30%. When the resummed results are compared with the LO ones, the enhancement can even reach 100% in most of cases. We also note that the discrepancies between the different PDF sets are somewhat larger in the case of resummed than the case of LO or NLO, especially at the LHC. This may be due to the different fitting methods used in the PDF sets. We believe that when the LHC data be used in the global fitting of the PDFs, these discrepancies will be improved.

Despite the uncertainties from the PDFs, we can estimate the theoretical uncertainties by investigating the sensitivity of the results to the renormalization and factorization scales. We define \( R \) as the ratio of the cross sections (LO, NLO, resummed) to their values at central scale, \( \mu_R = \mu_F = m_t \), always assuming \( \mu_T = \mu_f = \mu \) for simplicity. Figure 4 and Figure 5 show the ratio \( R \) as functions of the renormalization and factorization scales for subprocesses \( gu \to t \) and \( gc \to t \) at the Tevatron Run 2 using the two PDF sets, respectively. We can find that, for both subprocesses, the threshold resummation effects further decrease the scale dependence of the NLO cross sections remarkably. For example, the variations of the cross sections in the region \( m_t/2 \leq \mu \leq 2m_t \), for subprocess \( gu \to t \), are about 28% for NLO cross section and only 9% for resummed one. In the case of \( gc \to t \), they are about 11% and 2%, respectively.

In Figure 4 and Figure 5 the ratio \( R \) is shown for the cases at the LHC. First, we note again that the NLO corrections do not reduce the scale dependence of the LO cross sections. In fact, the variations of the LO and NLO cross sections between \( m_t/2 \) and \( 2m_t \) are nearly the
The table below shows the LO, NLO and resummed cross sections for direct top quark production at the LHC and Tevatron Run 2. The cross sections are given in pb for both colliders and for the subprocesses $gu \rightarrow t$ and $gc \rightarrow t$. The cross sections are calculated using CTEQ and MRST PDFs.

**Table I:** The LO, NLO and resummed cross sections for direct top quark production at the LHC and Tevatron Run 2. Here $\mu_r = \mu_f = m_t$.

<table>
<thead>
<tr>
<th>subprocess</th>
<th>PDF</th>
<th>LHC $\frac{n/\Lambda}{0.01 \text{TeV}^{-2}}$</th>
<th>Tevatron $\frac{n/\Lambda}{0.01 \text{TeV}^{-2}}$</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>LO</td>
<td>NLO</td>
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<tr>
<td>$gu \rightarrow t$</td>
<td>CTEQ</td>
<td>12.9</td>
<td>17.0</td>
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<tr>
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</tr>
<tr>
<td>$gc \rightarrow t$</td>
<td>CTEQ</td>
<td>1.71</td>
<td>2.53</td>
</tr>
<tr>
<td></td>
<td>MRST</td>
<td>1.68</td>
<td>2.38</td>
</tr>
</tbody>
</table>

In summary, we have calculated the NLL threshold resummation effects in the direct top quark productions induced by model-independent FCNC couplings at hadron colliders in the framework of SCET and HQET. Our results show that the resummation effects increase the total cross sections by about 30% in general compared to the NLO results. Moreover, the resummation effects significantly reduce the dependence of the cross sections on the renormalization and factorization scales. Especially, the resummation effects have good control over the scale dependences in the region $\mu < m_t$ at the LHC, where the NLO results behave worse than the LO ones.

If future experiments at hadron colliders find signals of the direct top quark production, the anomalous couplings can be extracted by measuring the cross sections and comparing with the theoretical predictions. Thus, the precision of the extracted anomalous couplings directly depends on the accuracy of the theoretical predictions. Since the resummation effects increase the cross sections and reduce the theoretical uncertainties, our results are more sensitive to the new physics effects and it is more appropriate to use our results as the theoretical inputs.
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