HOW MANY INDEPENDENT BETS ARE THERE?

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We discuss a robust alternative to the ex-post decomposition of a portfolio into the fundamental law coefficients using statistically estimated breadth. The breadth of a market provides a measure of the number of independent bets available to the investor. We use the Keiser-Gutman stopping criterion to select the integer valued effective dimension - eigenvalues greater-than or equal to 1. Such a method for the estimation of breadth relies critically on the appropriate estimation of the covariance matrix of price fluctuations (returns). In a emerging market such as South African it is not surprising to find that the breadth is low because of market concentration, exposure to the global commodity cycle and currency volatility. The implications of refocussing investment objectives on effective dimensionality are further discussed.

Keywords: Effective dimensions; Covariance Estimation; Emerging Markets

1. Introduction

One of the most important issues in asset management is the efficient use of forecast information given constraints on the implementation of portfolios and the nature of the markets in which the information is to be used.

Constraints, such as no short selling, limitations on turn-over, restrictions in implementation due to market impact relating to order-size and market liquidity, and other limitations on the investment style of a manager, all conspire to limit a managers ability to transfer information into portfolio positions. These constraints reduce the efficient use of forecast information.

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The temporal co-movement of asset classes limit the opportunity set which the manager has at her disposal. This in turn reduces the managers ability to transfer asset specific forecast information into useful portfolio views. The limited effective dimension of the opportunity set reduces the usefulness of the forecasts and in turn restrains information transfer.

The relationship that encapsulates the tension between the information content of the forecast models, the ability of the manager to implement those views, and the structure of the market is the fundamental law of active management [3,1]. This 'law' represents the conflict between signal quality, the actual value added by the manager, and the portfolio construction methodology used.

1. The information coefficient (IC) measures the signal quality as the correlation between forecasted residual returns, $\alpha_t$, and the realized returns, $r_t$:
   \[
   IC_t = \rho(r_t, \alpha_t).
   \]
2. The transfer coefficient (TC) measures the efficiency of the portfolio construction is measured as the correlation between active positions, $\Delta \omega_t$, and the forecasted residual returns, $\alpha$:
   \[
   TC_t = \rho(\alpha_t, \Delta \omega_t).
   \]
3. The performance coefficient is a measure of the value added and is the correlation between the active positions, $\Delta \omega_t$, and the realized returns, $r_t$:
   \[
   PC_t = \rho(\Delta \omega_t, r_t).
   \]

The fundamental law of active management relates the effectiveness of the forecasting (IC) in terms of the efficiency of the implementation of the views (TC) and the number of bets actually available to the realized risk adjusted performance of a strategy,

\[
IR = IC \times TC \times \sqrt{N}. \tag{1.1}
\]

2. The Fundamental Law

From Clarke et al. [1] we have that the fundamental law of active management states for Eq. (1.1) that the information ratio (IR) is a function of skill, portfolio construction and the size of the opportunity set. Here the information coefficient (IC) is $IC = \rho(E[R], r)$, the transfer coefficient (TC) is $TC = \rho(\Delta \omega, E[R])$ and the performance coefficient (PC) is $PC = \rho(\Delta \omega, r)$. Here the active weight are $\Delta \omega$ for the controls $\omega$ and are the difference between the benchmark controls and the active positions. The returns realized from bets are $r$, these are the realized residual returns. The returns expected are $E[R]$ where the expectation is evaluated for the next time step, we will use this interchangeably with $\alpha$ in the ex-post case to remind the reader that we are concerned with excess return with respect to some benchmark portfolio. This is not the CAPM $\alpha$ with respect to the broad market portfolio.

The ex-ante performance coefficient is:

\[
PC = TC \times IC. \tag{2.1}
\]

where the realized performance coefficient is simply the cross-correlation between active weights (assumed to be risk-adjusted) and the realized residual returns. The ex-post performance coefficients are found by taking the active weights and the
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realized residual returns of the same underlying variables and finding the correlation between them. The \textit{ex-post} decomposition of the performance coefficient requires the inclusion of a noise term:

\[ \text{PC} = \text{TC} \times \rho(\alpha, r) + \sqrt{1 - \text{TC}^2} \rho(c, r), \quad (2.2) \]

where the noise term is found from: \( c = \Delta \omega - \text{TC}\Delta \omega^* \), the difference between the active weights and the optimal active weights. The noise term \( \rho(c, r) \) can be used interchangeably with \( \rho(b, r) \) where \( b = \Delta \omega - \Delta \omega^* \), the difference between the optimal portfolio and the physical portfolio’s active bets.

The optimal active weights are those determined by some optimal selection criteria, one such \textit{ex-ante} definition of \( \Delta \omega \) is given as \( \Delta \omega^* \propto \Sigma^{-1} \alpha \) following the unconstraint mean-variance criteria. For our \textit{ex-post} purposes we define: \( \Delta \omega^* = r / \sum r \) for all \( r > 0 \).

3. Enhancement to the traditional application

Using historical constituent data on an active fund and the corresponding weightings in an underlying nominated benchmark, as well as historical price time-series data, we can derive all of: IR, \textit{ex-post} PC, \( \Delta \omega \) and \( \Delta \omega^* \). We have two unknowns: the transfer coefficients and the information coefficients. These need to be estimated from the complete solutions of Eq. 1.1.

Few fund managers know there transfer coefficients and the only available information, if any at all, are the rankings of underlying return expectations rather than the return expectations themselves. We denote the \textit{ranking} of each underlying as \( \Omega \). The term-structure of the rankings of each underlying is \( \Omega(t) \) a vector valued function of term.

The transfer coefficient and the information coefficients are simply scalars on \((-1,1)\), the product along with the breadth explains the information ratio. As a proxy to ‘expected risk-adjusted’ returns we simply as \( \Omega \). The transfer coefficient is estimated as \( \rho(\Omega, \Delta \omega) \) and the information coefficient as \( \rho(\Omega, r) \) with three key departures from convention:

1. Correlation Estimation: Both TC and IC are drawn from the domain \((-1,1)\). Is there a more rigorous way for the estimation of the correlation function \( \rho \)? Typically a Pearson product-moment correlation is used to estimate TC and IC. Pearson’s correlation make the restrictive assumption about the normally distributed nature of the underlying variables. We are dealing with rankings in the situation where the distributional properties can be argued as being non-gaussian; we utilize a non-parametric rank correlation. In this regard it becomes convenient to use the Spearman rank correlation.

2. Independence of TC and IC estimation: It is convention that one coefficient is derived and the other solved from Eq. 1.1. If both are estimated then inconsistencies will emerge in the fundamental law Eq. 1.1. As we do not know \textit{a priori} which is better to derive first, we can choose to weight both derivations by their
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statistical significance. The co-efficient with the greater significance will then dominate the other.

3. Breadth estimation: The key variable is the measure of the number of independent bets in the investors opportunity set. Traditionally it has been convention to use $\sqrt{N}$ as a measure of breath where N is the number of underlyings in the opportunity set, not the number of independent bets. We believe this to be flawed, particularly when implemented in markets with concentration or limited liquidity.

These three departures encapsulate separate debates.

First, the real problem with the estimation of the TC and IC coefficients is the repeatability of the statistics - the assumed property of scale invariance that renders interpretation to these co-efficients over time. One can debate the philosophical implications of handling TC and IC in this manner but it should be acknowledged that we are dealing with several crude estimation assumptions anyway. What we propose is part refinement (a non-parametric derivation of coefficients using rankings) and part necessity (weighting coefficients by their t-statistics). The key concerns about the repeatability of the statistics is significantly more problematic when one uses parametric methods regardless of their statistical significance.

Second, we believe that there are better ways to represent independence than the original construction of Grinold [3]. For our purposes, as well as for ease of replication, we make use of the principle of "effective dimensionality": given a return matrix $X$, we us the singular-value decomposition to factorize $X$ as $X = U\Sigma V^T$ for eigenvectors $\Sigma = \text{diag}(\Sigma)$ and eigenvalues $U$. The unitary matrix $U$ spans the subspace where the variations in the data are the largest. Each eigenvalue has an associated eigenvector. We then utilize the Keiser-Gutman stopping criterion to select those eigenvalue greater than or equal to one, the number of such eigenvalues corresponds to an estimation of the effective dimensions of the subspace - the N in the fundamental equation of active management Eq. 1.1.

Third, although there are alternative means of defining the effective number of stocks in a portfolio using entropy measures [7]. These all ultimately revolve around the degree of localization of the portfolio controls under optimization, and hence pivot on the appropriate definition of entropy. Although we do not use this approach here it is worthwhile understanding the idea - for a portfolio with N independent bets the portfolio weights would provide a localization proportional to $\omega^T \omega$ where for an equally weight portfolio $\omega = 1/N$ this then results in a localization of $1/N$ and for all bets in a single asset, a localization of 1. Localization is a reasonable proxy for entropy.

For a general portfolio of correlated bets the inverse of the localization gives a measure of the effective number of asset: $N_{eff} = (\omega^T \omega)^{-1}$, required to build a smaller portfolio with risk-return properties not dissimilar from that of the full optimal portfolio constituted using the full N assets. This does not necessarily reflect the effective number of bets available, but reflects the effective number of bets
required to replicate the optimal portfolio based on a estimated covariance matrix and vector of views of future asset class performance. One would suspect that \( \omega^* = \frac{1}{\lambda} \Sigma^{-1} \mu \) for risk aversion \( \lambda \) should provide an effective number of assets comparable to our definition based on counting eigenvalues greater than one. However our approach is not dependent on a required set of asset views.

Ultimately all questions about the effective number of bets are: *What is the smallest \( M \) resulting from uniquely arranging \( N \) objects (assets) into \( M \) clusters based on the uncertain co-movement of the objects.* This in turn implies clarification of the notion of measures of co-movement:

1. Which measure of correlation is appropriate to specify object co-movement and,
2. which likelihood function based on this measures is appropriate to specify the cluster structure.

These questions have been at the heart of a study on South African data by Wilcox et al [6].

For the sake of brevity, we demonstrate and discuss the refined representation of breadth below in an empirical example. We would like to open up the debate on our ideas of more rigorous correlation estimation. Concerns regarding the independence of TC and IC are already present in the literature [2] - our proposal suggests one way in which the independence between the two is reinforced.

### 3.1. Empirical example of the enhanced application

We utilize return data from the Johannesburg Stock Exchange (JSE) and the Bond Exchange of South Africa (BESA) for the purposes of demonstrating both our breadth computations as well as the evidenced effect of limited breadth within the South African marketplace. We use daily data for a period of 3.6 years (Jan 2002 - present) for 37 of the most liquid equity stocks on the JSE. The most liquid equity index is termed the Top-40 index, but several of the constituents were listed post-2002. The period of 3.6 years is arbitrarily chosen as a cut off point where most of the equity counters currently trading are subsumed in the analysis.

We commence by computing the effective dimensionality of this sample of 37 single-stocks from the estimated correlation matrix. A projection of the single-stocks (variables) onto the 2-D eigenvector space is represented in Figure 1. The projection shows that two gold stocks, ANG and HAR, represent a different cluster amongst the group.

A scree-plot is used to map the decay of the eigenvalues by the dimensionality of the data set in Figure 2.

Using our Keiser-Gutman criterion, we compute the effective dimensionality of the dataset as no more than 9 dimensions. This translated into an effective breadth of 3. The conventional use of the fundamental law of active management would estimate breadth here at \( \sqrt{37} = 6 \), twice that evidenced here.

Next, we repeat the selfsame exercise as above, but now consider jointly the
selfsame period of nine total return series of the seven dominant government-issued bonds along with our 37 single-stocks. A projection of the underlyings onto the 2D eigenvector space is noted in Figure 3.

In Figure 4 a scree-plot is once again used to map the decay of the eigenvalues by the dimensionality of the data set.

The effective dimensionality is estimated as no more than 10 dimensions, translating into an effective breadth of 3.162. Conventional analysis here would infer a breadth of 6.6. Note how the analysis suggests both that South African bonds do not present much of a diversification enhancement to an equity portfolio and that replication of the self-same communalities exist. The reasons for these anomalies are easily explained by the dominant role that the local exchange rate plays on both equity and bond valuation and the impact of interest rates and commodity pressures on both.

4. Conclusion

Breadth in our approach takes on a meaning that is quite different from that usually used, but is closer to the spirit of the idea, as it measures the benefit of real diversification. The SVD approach to the problem handles independence of the basis of

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Fig. 1. Eigenvector projection of Equities: The projection of the single-stocks (variables) onto the 2-D eigenvector space is provided as a biplot. Note the clustering of the bulk of the counters in two of the four quadrants. Note also that two stocks, notably AngloGold (ANG) and Harmony Gold (HAR), represent a different cluster amongst the group of 37.
spanning eigenvectors correctly whereas the notion typically assumed, that gambles are independent by their very nature, is simply incorrect.

With breadth assuming smaller numbers than most worked examples, TC and IC are likely to be elevated to explain the same information ratio. This could be interpreted in terms of the prevalence of active concentrated strategies as opposed to diversified active strategies. When the opportunity set is limited one needs to put in place concentrated bets to achieve the same results. Concentrated bets are known to push up against investment constraints, this in turn can be used to motivate the low transfer co-efficients manifested in emerging markets.

It may be that it become more difficult to pick bets that the market will like, rather than bets that individual agents in the market like simply because fewer bets are available.

One common criticism with the approach proposed in this paper is that the coefficients that are derived here cannot be easily compared with previous studies. This is because treating breadth as the number of independent bets, rather than the number of bets will generate coefficient’s with different absolute values that cannot be meaningfully contrasted with other studies. Which approach should the diligent practitioner choose? The debate concerns using a methodology that is more correct in terms of the interpretation of breadth, or using a methodology that is popular

Fig. 2. Scree plot of Equities: The scree-plot mapping the decay of the eigenvalues by the dimensionality of the data set. The effective dimensionality of the dataset is found to be no more than 9 dimensions when using the Keiser-Gutman criterion. This is an effective breadth of 3.
and treats independence colloquially and incorrectly.

One of the key results arising from this investigation into the fundamental law in the context of an emerging market such as South Africa is the limitation of breadth in the opportunity set. This breadth limitation is due to a concentration of capital in a handful of local stocks within the South African market. Some 33% of the market capitalization is contained in the top-5 underlyings. Interestingly, most of these stocks are dual-listed overseas, hence further limiting the breadth expansion when international assets are included in a resident South African portfolio. This breadth limitation has implications for the variety and nature of the risk associated with investment strategies.

A common misperception prevalent in the literature regarding the benefits of diversification is that skill is scaleable over breadth [5]. Hence, diversification is a free lunch offered to a 'diversified' portfolio in the sense that a larger number of bets effected with the same skill will produce higher IRs. Hence, a fund manager with a skill level of getting 60% of his bets correct will have an IR of 0.90 with a portfolio of 20 instruments, an IR of 0.45 with five bets and an IR of 0.20 with one bet. This scenario is easily proved. However, the error is made in assuming that one’s IC remains constant as breadth increases. Clearly, it cannot. For every

Fig. 3. Eigenvector projection of Equities and Bonds: A projection of the underlyings onto the 2-D eigenvector space is provided again, but now we include 7 dominate government-issued bonds along with the 37 single-stocks.
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added dimension of independence, one requires a novel skillset. In the context of asset management, the implications of this error in the context of a limited breadth debate is twofold.

First, real diversification into breadth to achieve an optimal risk-adjusted return (IR) requires skill (IC). If IC is compromised by breadth increasing, as we expect it to be, it can be argued that 'diversification' is actually a recipe for mediocrity amongst professional fund managers in the most general case. A prefatory analysis of several South African fund managers show different levels and persistence of IC for different sectoral bets. It would be of specific interest to quantify where the value resides within such institutions, and how to best extricate this value-add in the context of a balanced (mutual fund) mandate.

Second, less breadth will exists within any one asset class than the fundamental law implies. For a unit of capital, shifting allocation within an asset class will increase the breadth less (if at all) than shifting allocation across asset classes (the idea of tactical asset allocation). In the context of the South African marketplace, limited diversification exists within a highly concentrated equity market such that a shift from one security to another represents more of a bet about the relative spreads between their expected returns than it does anything about diversification. In fact,

Fig. 4. Scree plot of Equities and Bonds: The scree-plot showing the decay of the eigen-values in the equity-bond universe. The effective dimensionality is estimated as no more than 10 dimensions, translating into an effective breadth of 3.162.
a pair-trading strategy (in its own right) creates a dimension of independence that is uncorrelated with either of the two original underlyings, but may be correlated with other positions.

Lastly, it should be noted that tactical asset allocation can facilitate a rapid breadth expansion by translating 'possible' breadth into 'realized' breadth. The pros and cons of asset allocation need to be considered in the selfsame context of the skill that managers have in timing the movements in various asset classes versus the diversification benefits of so doing. In this sense, our proposed modification to the fundamental law of active management provides a generalizable framework in which both static, dynamic and tactical asset allocation can be thoroughly and correctly investigated. In this context, IC(t), the information coefficient as a function of term, is the basis on which any analysis needs to be focussed. The prospects of utilizing the fundamental law in this manner are particularly piquant and we hope that this research will stimulate some further work in this area.

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