Bounds on masses of bulk fields in string compactifications

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In string compactification on a manifold $X$, in addition to the string scale and the normal scales of low-energy particle physics, there is a Kaluza-Klein scale $\frac{1}{R}$ associated with the size of $X$. We present an argument that generic string models with low-energy supersymmetry have, after moduli stabilization, bulk fields with masses which are parametrically lighter than $\frac{1}{R}$. We discuss the implications of these light states for anomaly mediation and gaugino mediation scenarios.
1. Introduction

It is interesting to ask what constraints string theory can put on models of low-energy physics. In the framework of worldsheet conformal field theory, some powerful results were derived in the 1980s. For instance, one could bound the rank of any non-abelian factors in the 4d gauge group to be $\leq 22$, one could prove that type II theories were incapable of incorporating the field content of the Standard Model [1], and one could make a precise argument substantiating the long-standing belief that quantum gravity does not allow continuous global symmetries [2].

Of course, these results provide a cautionary tale; the advances of the duality revolution extended our understanding of string theory and added ingredients which allow one to circumvent the first two “no-go” theorems just described. Nevertheless, the general program of outlining what is and is not expected to be possible in string-derived effective field theories remains intriguing. It seems quite clear that some 4d effective field theories, coupled to gravity, cannot be UV completed by string theory. The real questions are, what are the precise string theoretic constraints on effective field theories? And, perhaps more importantly, do any of these constraints impinge on effective field theories that have been seriously proposed as models of the world?

In the past few years, there have been many results in this area. Several of these results apply to string cosmology. For instance, there are arguments that, under reasonable assumptions, string theory models of de Sitter space are always metastable with a lifetime short compared to the recurrence time [3,4,5]. In addition, seemingly natural models of large-field inflation based on super-Planckian axion decay constants [7] may not exist in string theory [8]. More elaborate constructions may allow analogues of chaotic inflation in string theory [9,10]; observation of a tensor-to-scalar ratio $r \geq .05$ by the next generation of experiments would make this a central issue [11]. Other popular models of inflation based on brane motion in extra dimensions [12] were similarly constrained by genericity arguments in [13].

There are also interesting results which constrain the geometry of stringy moduli spaces of vacua [14,15,16]. Most recently, there has been a striking claim that theories with abelian gauge fields coupled to quantum gravity, with perturbative gauge coupling

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1 An early discussion of the possible importance of metastability in addressing conceptual challenges of de Sitter quantum gravity appeared in [3].
must manifest a new physical scale \( \sim g M_P \) to avoid the paradoxes associated with remnants \[17\].

In this note, we provide simple arguments that indicate that in string compactification on a manifold \( X \) of radius \( R \) preserving low-energy supersymmetry, there will generically be bulk modes of mass parametrically \(< \frac{1}{R} \). This is relevant in evaluating the prospects for phenomenological scenarios like anomaly mediation \[18,19\] and gaugino mediation \[20\] to be UV completed by string theory. As these scenarios constitute some of the leading ideas for solving the SUSY flavor problem, our result may be relevant in understanding which classes of realistic SUSY models emerge from string theory.

There may be other interesting applications for such a bound. As a simple example, we mention other attempts at sequestering symmetry breaking in extra dimensions, such as (supersymmetric implementations of) “shining” of flavor symmetry breaking \[23\]. In such models, the hierarchy between Standard Model Yukawa couplings is provided by the Yukawa-suppressed couplings between the Standard Model and some distant \( O(1) \) source of flavor violation.

In §2, we review why, in anomaly and gaugino mediation, one wishes to find compactifications where all bulk fields have masses \( \geq \frac{1}{R} \). In §3, we show that known ways of stabilizing the moduli of \( X \) using the ingredients in 10d string theory will generically produce masses which are parametrically \( \ll \frac{1}{R} \) in the regime \( R \gg l_s \) where one can trust classical geometry. It will be clear that the arguments can be extended to constrain other higher-dimensional theories which do not manifest significant non-locality at distances larger than \( R \).

Our arguments work most simply for models where the extra dimensions are reasonably symmetric (i.e. characterized by a single scale \( R \)). We discuss asymmetric models in §4 (focusing on heterotic M-theory for concreteness). In such models, say with two characteristic radii \( R >> R_X \), one can imagine two different conjectures. A strong conjecture would posit that there are always masses parametrically \(< 1/R \). A weak conjecture would instead be that there are always masses parametrically \(< 1/R_X \). In some circumstances, the success of the extra-dimensional mediation mechanisms may only require violations of the strong conjecture, or may even be consistent with the strong conjecture if the light fields have special constraints on their couplings. We find that both conjectures hold in

Other discussions of possible difficulties in implementing anomaly mediation in string theory appear in \[21,22\].
commonly studied models. For the strong conjecture, one can find regimes where the question becomes numerical instead of parametric for a subset of the moduli fields (while others remain parametrically lighter than $1/R$). If the numerical factors conspire properly for the moduli whose masses are $\sim 1/R$, and if special considerations allow one to argue that the remaining parametrically lighter moduli have constrained (flavor-blind) couplings to the Standard Model (as sometimes happens [24]), one could potentially make a working model of anomaly mediation. We close with a brief discussion in \S 5.

2. Solving the flavor problem with bulk locality

The MSSM and its extended counterparts typically introduce many new opportunities for flavor violation into low-energy physics. For instance, if the soft-breaking terms include a generic squark mass matrix, the resulting flavor violation is unacceptable. For one discussion of this issue with further references, see [25].

Several natural mechanisms have been proposed to surmount this problem. The basic idea is to ensure that the messenger of SUSY breaking from some hidden sector to the Standard Model, couples to squarks in a flavor-blind way. The gauge interactions of the Standard Model itself clearly do this. So one promising mechanism which can surmount the flavor problem is gauge mediation, where SUSY breaking is first transmitted to the Standard Model gauge multiplets, whose gauge couplings to matter particles then lead to squark and slepton masses. It seems plausible that pseudo-realistic models of gauge mediation can be naturally embedded into string theory [26].

Two other very interesting proposals for solving the flavor problem, anomaly mediation and gaugino mediation, use the physics of extra dimensions. In anomaly mediation [18,19], the fact that the “conformal compensator” multiplet of supergravity couples universally to mass dimension, can be used to impart universal masses to sparticles if the dominant F-term arises in the conformal supermultiplet. The couplings of this field to squarks and sleptons are Planck suppressed, and for the flavor-blind contribution of anomaly mediation to dominate over gravity mediation, one needs to sequester away the SUSY breaking sector. That is, if $Z$ is the chiral superfield with the dominant F-term, one needs to forbid non-universal, radiatively induced terms of the form

$$\int d^4\theta \ c_i \frac{1}{M^2} Z^i Z Q_i^\dagger Q_i,$$  

(2.1)
where $M$ is the scale of high-energy physics that has been integrated out in writing down the effective field theory (typically it could be the Planck scale or the string scale), $Q_i$ are Standard Model particles, and $c_i$ are $\mathcal{O}(1)$ coefficients. Terms with the structure (2.1) are generically induced by integrating out high-scale physics.

In order to forbid such terms, it was suggested in [18] that one should find models where the SUSY-breaking hidden sector and the Standard Model arise on branes which are well-separated in the compactification manifold $X$ of radius $R \gg \frac{1}{M}$. Suppose they are separated by a distance $d \leq R$. Assume furthermore that all bulk moduli fields $\phi_a$ have masses

$$ m_a > \frac{1}{d}. \quad (2.2) $$

The couplings between the field $Z$, localized on the hidden brane, and the fields $Q_i$, localized on “our” brane, which are generated by exchange of the bulk field $\phi_a$ will then be suppressed by the factor

$$ c_i \sim e^{-m_a d} \ll 1 \quad (2.3) $$

if (2.2) is satisfied. The dangerous flavor-violating effects can therefore be suppressed.\(^3\)

Anomaly mediation, by itself, leads to tachyonic sleptons. With some cleverness, reasonable extensions of the minimal scenario can be constructed that avoid this problem. Specific extensions are discussed in, for instance, [27].

Precisely the same idea of sequestering enables models of gaugino mediation [20] to solve the flavor problem. In such models, SUSY is again broken on a distant hidden sector brane. However, in these scenarios, the Standard Model gauge fields and gauginos live in (at least part of) the bulk of $X$. The gauge multiplet couples directly to the hidden brane, which yields a gaugino mass. This then generates flavor-blind Standard Model splittings as in gauge mediation. One finds that the sparticle masses are suppressed by powers of $1/M$ in a way that depends on the number of extra dimensions where the gaugino lives. Again, for the effects of gaugino mediation to dominate bulk exchange, the regime (2.2) for masses of bulk moduli is favored.

In the following sections, we argue that achieving the bound (2.2) for bulk moduli masses in string theory is difficult. More precisely, the scaling of all known effects which can generate a potential for bulk moduli, leave some masses which are parametrically smaller than (2.2) in the well-studied classes of supersymmetric string models. We will see that this result follows from the simple fact that string models give rise to local effective theories at distances large compared to the string scale.

\(^3\) The reader may wonder about the possible effects of Kaluza-Klein modes. It is argued in [18] and in more detail in [24] that integrating these out does not generate the dangerous operators.
3. The scaling of energy densities in string theory

Let us assume that the bulk geometry is well described by a 6d compact manifold $X$ which preserves supersymmetry at the KK scale $\frac{1}{R}$. In our discussion we will assume $X$ is a Calabi-Yau space but relaxing this assumption would not modify our conclusions.

The 10d metric takes the form

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + R^2 \tilde{g}_{mn}(y)dy^m dy^n$$  \hspace{1cm} (3.1)

where $y^m$ are coordinates on $X$ and $R$ is the overall radial dilaton. Here $A(y)$ is a possible warp factor. The scenarios of interest do not rely on a heavily warped geometry and we will henceforth set $e^A \sim 1$.

The field content of the theory depends on our precise choice of string theory. For definiteness we will mostly consider the wide class of IIB Calabi-Yau orientifolds whose effective field theories were derived in \[28,29\]. It will be clear that our arguments could be generalized to other known classes of geometric compactifications with minimal modifications. We make some comments about another specific class of models in §4.

In IIB string theory on a Calabi-Yau space $X$, the relevant fields include a dilaton-axion, the Kähler and complex structure moduli of $X$, and two three-form field strengths $H_{NS}$ and $F_{RR}$. There is also a five-form field strength $F_5$ which is self-dual; since $X$ has no nontrivial five-cycles it will play no role in our discussion.

Our question will be, how does the potential for the bulk moduli (the axio-dilaton, the Kähler moduli, and the complex structure moduli) scale with $R$? The following discussion is very similar to the one in \[3\], where the focus was on the potential for the radial dilaton. A nice pedagogical description of similar results appears in \[30\].

The basic point is the following. Below the scale $1/R$ there is a 4d supersymmetric effective field theory. The scalar potential for bulk moduli

$$V = e^{\frac{K}{M_P^2}}\left( \sum_{i,j} g^{ij} D_i W D_j \overline{W} - 3 \frac{|W|^2}{M_P^2} \right)$$  \hspace{1cm} (3.2)

can be computed from a superpotential $W$ and a Kähler potential $K$. The finite list of ingredients in 10d supergravity, together with the known possible stringy corrections to the superpotential and the known expansion parameters which control corrections to $K$, allow us to categorize the possible scalings of $V$ with $1/R$. We will see that the masses of bulk moduli end up parametrically below the scale $1/R$. Of course this was necessary to have
a 4d effective theory incorporating the moduli at all (since one could not self-consistently keep fields with \( m \sim 1/R \) without keeping the KK tower as well). We will set \( g_s \sim 1 \) for this discussion, but let us note that in models with weak string coupling, there could be further suppression of the bulk moduli masses by powers of \( g_s \).

### 3.1. Potentials from bulk fluxes

Turning on magnetic fluxes in the Calabi-Yau space \( X \) generates a potential for many of the moduli fields. Such models have been extensively studied, some representative references include [31, 28].

The scaling of the potential arising from the 3-form fluxes can be found as follows. Start with the 10d Lagrangian

\[
\int d^{10}x \sqrt{-g_{10}} \left( F_{ijk} g_{10}^{il} g_{10}^{jm} g_{10}^{kn} F_{lmn} + \cdots \right) \tag{3.3}
\]

The reduction to 4d using the ansatz (3.1) is straightforward. The three factors of \( g^{-} \) give a \( \frac{1}{R^6} \). The volume of \( X \) gives a compensating factor of \( R^6 \). This naively gives an overall \( R^0 \) scaling.

However, it is important to remember that one should measure energies in 4d Einstein frame. The naive reduction gives an Einstein term with an \( R^6 \) in front. Doing the Weyl rescaling of the four dimensional metric by \( 1/R^6 \) to reach the Einstein frame and re-writing the potential \( \sqrt{-g_4} V \) in terms of the Einstein metric, yields a factor of \( R^{-12} \).

Hence, the overall energy from the three-form fluxes scales like \( \frac{1}{R^{12}} \). To find the scale of the moduli masses \( m \) which result from this potential, one equates

\[
M_P^2 m^2 \sim M_P^4 \left( \frac{\alpha'}{R^2} \right)^6 . \tag{3.4}
\]

Recall that

\[
M_P^2 \sim \frac{R^6}{(\alpha')^4} . \tag{3.5}
\]

We conclude that the moduli which receive masses from three-form fluxes in string theory have

\[
m \sim \frac{\alpha'}{R^3} . \tag{3.6}
\]

This is parametrically \( \ll \frac{1}{R} \) at large \( R \). For a more detailed analysis which reaches the same conclusion, see e.g. §5.4 of [29].
A different way to derive the moduli mass (3.6) that does not use the Weyl rescaling is to find the Lagrangian for the moduli and read off the mass from it. The bulk moduli in question, the complex and Kähler structure moduli, are fluctuations of the metric. Hence their kinetic term comes from expanding the GR action \( \int_{X \times R^4} \sqrt{g_{10}} R \) in small fluctuations. Reducing this to four dimensions, the kinetic term gets a factor of \( M_P^2 \) in front

\[
L_{\text{kinetic}}(\phi) \sim M_P^2 (\partial \phi)^2, \tag{3.7}
\]

just like the 4d GR action, which is essentially the kinetic term for fluctuations of the 4d metric. Let us assume that the moduli potential from the fluxes has scale \( V_0 \). Its form near a local minimum, assuming a generic shape of the potential, is

\[
V(\phi) \sim V_0 \phi^2, \tag{3.8}
\]

since \( V \sim V_0 \) for \( \phi \sim 1 \). Normalizing \( \phi \rightarrow \phi/M_P \) to get canonical kinetic terms yields a general formula for the moduli mass

\[
m_\phi \sim \frac{\sqrt{V_0}}{M_P}. \tag{3.9}
\]

The complex structure moduli considered above have \( V_0 \sim (\alpha')^{-2} \) and \( M_P^2 \sim R^6 / (\alpha')^4 \), so that \( m_\phi \sim \alpha'/R^3 \), in agreement with (3.4).

3.2. Potentials from brane worldvolume fluxes

Many pseudo-realistic string models incorporate D-branes wrapping cycles in \( X \). D-branes come equipped with worldvolume gauge fields. So, for instance, in type IIB string theory, in addition to the bulk three-form field strengths one can in general also consider two-form field strengths of the brane worldvolume gauge fields. In the class of type IIB compactifications described in [28], the unique possibility consistent with Lorentz invariance is nontrivial gauge field strengths supported on D7 brane worldvolumes. The effects of these fluxes have been discussed extensively in recent literature, see for instance [32,33].

The 4d description of the potential arising from D7 brane gauge fields is somewhat complicated, but if one is only interested in scalings as we are, the result is rather simple. The energy stored in a worldvolume gauge field \( F \), on a D7 brane wrapping the 4-cycle \( C \), is

\[
E \sim \int d^4x \int_C \sqrt{-g} g^{mn} g^{mq} g^{nr} F_{qr}. \tag{3.10}
\]
After the reduction to four dimensions and Weyl rescaling to Einstein frame, this contribution to the 4d potential scales as $1/R^{12}$, similarly the contribution from the bulk fluxes described in §3.1. Hence, the mass of the moduli generated by D7-brane worldvolume fluxes is

$$m \sim \frac{\alpha'}{R^3}. \quad (3.11)$$

### 3.3. Quantum corrections to $W$

The flux superpotential in IIB Calabi-Yau models does not depend on the Kähler moduli of $X$. However, there are model-dependent nonperturbative corrections to $W$ which do depend on Kähler moduli. It is proved in [34] that these come entirely from a class of Euclidean D3 branes wrapping certain 4-cycles $C_\alpha$ in $X$. The authors of [35] discuss the extension of this nonrenormalization theorem to vacua with nonzero fluxes.

The D3-brane instantons generate a correction to the superpotential

$$\Delta W = \sum_\alpha f_\alpha(z_\alpha) \text{Exp}(-\text{vol}(C_\alpha)) + \cdots \quad (3.12)$$

where $z_\alpha$ are complex structure and brane position moduli, $f$ is a one-loop determinant, and $\cdots$ denotes additional terms which are exponentially smaller at large volume than the terms we kept. The leading terms clearly scale like $e^{-R^4/(\alpha')^2}$ and vanish exponentially quickly at large volume. Any masses arising predominantly from (3.12) will scale as

$$m \sim e^{-R^4/(\alpha')^2} \quad (3.13)$$

up to power law prefactors at large $R$. Of course these masses are parametrically $\ll \frac{1}{R}$.

### 3.4. Quantum corrections to $K$

In models arising from supergravity or string theory, the additional terms in the 4d potential (3.2) arising from Kähler covariantization of the partial derivatives

$$\partial_i W \rightarrow D_i W = \partial_i W + \frac{K}{M_P} W \quad (3.14)$$

Strong IR dynamics in gauge theories arising from D7 branes wrapping 4-cycles $C$ in $X$ can also contribute to the superpotential for Kähler moduli. Such contributions are also exponentially suppressed at large $\text{vol}(C)$. 

(as well as from the $e^K$ prefactor in (3.2)) can play an important role in the effective potential. Fields which do not receive a mass from the leading order potentials generated by e.g. fluxes, are quite plausibly stabilized in many models by these supergravity corrections to global quantum field theory. It is then interesting to ask for the form of such corrections.

For IIB orientifolds, the leading such corrections were computed in [36]. A nice summary of the resulting potentials appears in [37]. For further work on these corrections and their physical applications see [38,39,40]. The basic structure is the following. Defining the complex scalar

$$\rho = \left( \int C_4 \right) + i \frac{R^4}{(\alpha')^2}$$

(3.15)

(where the integral defining Re($\rho$) is taken over an appropriate 4-cycle and gives rise to an axion field), one finds

$$K = -3 \log (\rho - \bar{\rho} + f_1(z_a)) + \frac{f_2(z_a)}{\rho - \bar{\rho}} + \frac{f_3(z_a)}{(\rho - \bar{\rho})^{3/2}} + \frac{f_4(z_a)}{(\rho - \bar{\rho})^2} + \cdots \quad (3.16)$$

Again $z_a$ runs over the complex moduli, brane positions, and the axio-dilaton.

For any fields which have not received a potential from the bulk fluxes of §3.1, including the correction to $K$ in the expansion of the 4d potential (3.2) can yield a mass. Integrating out the fields which receive a mass from three-form fluxes, [37] finds a potential which takes the schematic form

$$V = c_1 \frac{R^{18}}{R^{18}} + c_2 \frac{R^{20}}{R^{20}} + \cdots \quad (3.17)$$

where $c_{1,2}$ depend on combinations of the remaining light fields.

Because of the $R^{-18}$ fall-off of the leading term in (3.17), any fields which receive their leading mass from such corrections are parametrically lighter than those of §3.1, and have masses $\ll \frac{1}{R}$.

4. Asymmetric models: Heterotic M-theory

So far, we have assumed that $X$ is fairly isotropic so that it can be characterized by a single Kaluza-Klein scale $1/R$. In some cases, it is natural to assume that the compactification manifold has some dimensions much larger than others. A frequently discussed example is the compactification of M-theory on a Calabi-Yau manifold $X$ of radius

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5 In some models where even non-perturbatively $W$ is independent of $\rho$, these corrections may suffice to fix $\rho$. This requires competition between different orders of perturbation theory. To find reliable vacua of this sort one requires $|c_2| \gg |c_1|, c_2 > 0, c_1 < 0$, which can plausibly happen in some fraction of models.
$R_X \sim \ell_{11}$ times an interval of length $R \gg R_X$ separating the two $E_8$ walls \[11\]. This higher dimensional setup could possibly lead to a model of anomaly mediation in M-theory \[18\].

To find the masses of complex structure moduli of $X$ which receive a mass from $G_4$ flux, one can do an analysis very similar to that in §3.1. The allowed modes of $G_4$ have a single leg on the interval. Therefore, the scaling of the potential arising from G-flux will be

$$V \sim \frac{1}{R^2} \int_{X \times S^1/Z_2} \sqrt{-g} G_{ijkl}G_{im}g_{jn}g_{kp}g_{lq}G_{mnpq} \sim \frac{1}{R^3} \quad (4.1)$$

where the prefactor to the integral comes from the Weyl rescaling to 4d Einstein frame. Since $M_P^2 \sim R$, the moduli masses arising from fluxes turn out to scale as

$$m \sim \frac{1}{R} \quad (4.2)$$

It then becomes a detailed numerical question, whether or not the suppression (2.3) kicks in for a given complex structure mode.

The Kähler moduli of $X$, however, do not receive a mass from G-flux. In perturbative string theory they could, in some models, develop a superpotential at the level of worldsheet instantons \[42\]. In the strong coupling regime, the worldsheet instantons become membrane instantons extended across the interval. Hence the moduli masses will be exponentially small at large $R$

$$m \sim \exp(-R/\ell_{11}) \quad (4.3)$$

Kähler potential corrections could potentially give these moduli masses larger than (4.3) but parametrically smaller than (4.2). The discussion of these corrections would be analogous to the one in §3.4.

The models commonly discussed in the literature incorporate gauge bundles $E \to X$, breaking the $E_8$ gauge symmetries, which have $c_1(E) = 0$. In models involving a gauge bundle with $c_1(E) \neq 0$\[3\], there is an additional contribution to some Kähler moduli masses from the $E_8$ gauge fluxes. The potential due to gauge fluxes (with field strength $F$) scales as

$$V \sim \frac{1}{R^2 R_X^{12}} \int_X \sqrt{g_{10}} F_{ij}g^{ik}g^{jl}F_{kl} \sim \frac{1}{R_X^{10} R^2} \sim \frac{1}{R^2} \quad (4.4)$$

\[6\] See e.g. \[43\] for a detailed discussion.
where the factor $\frac{1}{R^2 R_X}$ comes from Weyl rescaling into 4d Einstein frame and, in the last equality, we set $R_X \sim \ell_{11}$. Hence, in this class of models, some Kähler moduli get masses

$$m \sim \ell_{11}^{-1} \sqrt{\frac{\ell_{11}}{R}}, \quad (4.5)$$

which lifts them above the bound (2.2).

This is promising. However, the potential induced by nontrivial $c_1(E)$ just imposes the geometric condition (required for supersymmetry)

$$c_1(E) \wedge J \wedge J = 0 \quad (4.6)$$

where $J$ is the Kähler form of $X$. This means that the leading order effect of gauge fluxes does not induce a potential for the overall volume modulus of $X$, since any solution of (4.6) can be scaled up $J \rightarrow \lambda J$ while preserving the condition (4.6). So the overall volume modulus of $X$ should receive its leading mass from some other source or from a quantum correction to the potential generated by gauge flux, and will generically have a mass which is parametrically lighter than $1/R$.

There is also a modulus controlling $R$, the length of the interval. It enjoys various special properties, as described in [24]. For instance, this field has flavor-blind couplings to the observable $E_8$, and in fact does not have dangerous dimension-six couplings at all, at leading order. It is therefore not important for this field to satisfy (2.2). More generally, one could conjecture that generic fields which control separations of the Standard Model brane from other branes, but do not control the geometry or field theory couplings of the Standard Model brane itself, would enjoy universal couplings to the different generations and may safely violate (2.2). In contrast, any bulk moduli controlling the geometry or couplings of the Standard Model brane should be required to satisfy (2.2), since they “know about” the origin of flavor and a priori can introduce flavor violations into the SUSY-breaking soft terms.\footnote{We thank R. Sundrum for helpful discussions of this distinction.} We conclude that, if in a given example the subset of Kähler modes of $X$ which violate (2.2) also enjoy the special properties described in [24], and if the numerical factors in (4.2) are favorable, one could potentially find a working model in this asymmetric regime.

We now make some remarks about the extension of this logic to type II models. We will discuss IIB, the mirror statements apply in IIA. In IIB models, the Standard Model is
typically realized on some stack of D-branes. To leading order, the superpotential couplings are controlled by complex structure moduli while the gauge couplings and D-terms depend on Kähler moduli [15]. For example in the particular D3-brane model of [46], the $dP_8$ blow-up modes control the pattern of gauge symmetry breaking due to FI terms, while the 8 deformations of complex structure control Yukawa couplings. These modes should be required to satisfy (2.2). For other bulk moduli, the question would be more model dependent. Some subset could enjoy the special properties discussed in [24].

5. Discussion

The results of §3-4 basically arise because string theory, at distances large compared to $l_s$, behaves very much like a local field theory. For instance, the known corrections to the 4d effective superpotential which do not have a local 10d description (e.g. which involve Euclidean brane instantons) are exponentially small at $R > l_s$. But local contributions to the energy density in a compactification on $X$ grow at most as fast as $\text{vol}(X)$. The Weyl rescaling to go to 4d Einstein frame, which rescales the 4d effective potential by $(\text{vol}(X))^{-2}$, more than overcomes this fastest possible growth, yielding potentials which vanish quickly as $R$ increases. Since the bulk masses come from the dependence of these potentials on the bulk moduli $\phi_a$, it becomes very challenging to find models where the bulk moduli have masses $\geq \frac{1}{R}$. It may be possible to prove more rigorous bounds in some spacetimes by using the AdS/CFT correspondence [47].

Our approach here was to apply simple scaling arguments to various ingredients which can be invoked in different well-known classes of string compactifications. If indeed all low-energy supersymmetric models have some bulk field with mass which is parametrically smaller than the bound (2.2), it would be nice to find a conceptual proof of this result. The locality argument above eliminates many but not all possibilities. For instance, the potential energy from zero-form fluxes enjoys sufficiently slow fall-off $\sim 1/R^6$ to potentially give high-scale bulk masses. However, we are not aware of any models which incorporate these in a way that actually leads to moduli masses satisfying (2.2).

There are a few obvious caveats to the arguments presented here. Firstly, we have only discussed parametric scalings. It may be that numerical factors, in a regime where $R$ is greater than $l_s$ but not very large, can sufficiently enhance moduli masses to allow the Yukawa suppression (2.3) to kick in.
Secondly, one can imagine scenarios where the number of moduli is extremely small and couplings of the moduli to the branes are highly constrained by symmetry arguments. For example, in [24] it was argued that in five dimensional models compactified on an interval, the overall radion controlling the separation of the “end of the world” branes has highly constrained couplings to the branes, which are flavor-blind at leading order. Such modes will only need to satisfy a weaker bound than (2.2), and hence supersymmetric string models with very few moduli in the leading approximation (e.g. a hypothetical Calabi-Yau space with $h^{1,1} = 1$ and $h^{2,1} = 0$) could provide interesting exceptions to our reasoning.

Thirdly, one could always consider highly anisotropic compactifications. The models of §4 fall into this class, but other anisotropic models may be equally or more interesting in this regard. In the case of gaugino mediation, assuming one wishes to preserve the successful high-scale grand unification resulting from logarithmic running of the MSSM couplings [18], there is not much room for anisotropy. The large extra dimensions should not be larger than the GUT scale, however the GUT scale is quite close to the string scale, so one does not want the extra dimensions in which gauge fields propagate to be substantially larger than $l_s$. In the case of anomaly mediation, the situation is less constrained. It is conceivable that consideration of highly anisotropic models, combined with special arguments about the couplings (to the Standard Model fields) of the moduli controlling the sizes of the large dimensions, could yield exceptions to our arguments.

Perhaps more generally, one should keep in mind that the history of “no-go” arguments in string theory is somewhat tortured, suggesting a no-go theorem for no-go theorems. In addition to the examples provided in the introduction, there are the famous examples involving chirality in compactifications of 11d supergravity [19], warped compactifications [60], and many others. In this spirit, one can view the arguments here as a provocation to find new classes of string models where large bulk masses can be achieved and the clever constructions of [18, 19, 20] can be realized.

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One could loosen this constraint by either giving up unification, or by adding extra ingredients to the MSSM to yield low-scale unification. Neither option is particularly attractive.
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