Experimental tests on the lifetime Asymmetry

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Abstract: The experimental test problem of the left-right polarization-dependent lifetime asymmetry is discussed. It is shown that the existing experiments cannot prove whether the lifetime asymmetry is correct after analyzing the measurements of the neutron, the muon and the tau lifetime, as well as the $g-2$ experiment. Possible experiments are proposed which might test the lifetime asymmetry—a parity-violation phenomenon that might be overlooked for nearly 50 years.

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1 Introduction

The left-right polarization-dependent lifetime asymmetry has been proposed based on theoretical analyses and concrete calculations for decays of muons within the framework of the electroweak standard model (SM) \cite{1,2,3,4}. It is pointed out that the right-handed (RH) polarized fermion lifetime $\tau_{Rh}$ is different from the left-handed (LH) polarized fermion lifetime $\tau_{Lh}$, i.e.

\begin{equation}
\tau_{Rh} = \frac{\tau}{1-\beta} \quad \text{and} \quad \tau_{Lh} = \frac{\tau}{1+\beta},
\end{equation}

where $\tau$ is the average lifetime, $\tau = \gamma \tau_o = \tau_o/\sqrt{1-\beta^2}$, $\beta$ is the velocity of the fermions measured in a natural system of units in which $c=1$ and $\tau_o$ is
the lifetime in the fermion rest frame. It is shown that the lifetime of RH polarized fermions is always greater than that of LH polarized fermions in flight with the same speed in any inertial frame. As shown in Fig. 1 when \( \beta = 0 \), we find \( \tau_{Rh} = \tau_{Lh} = \tau_0 \); when \( \beta \to 1 \), we find \( \tau_{Rh} \to \infty, \tau_{Lh} \to \infty \). In particular, when \( \beta = \frac{1}{2} \), the lifetime of the LH polarized fermions has a minimum value \( \tau_{Lh} = \tau_{min} = 0.77\tau_0 \) and when \( \beta \simeq 0.84 \), equals \( \tau_0 \) again.

Figure 1: The lifetime as a function of fermion velocity \( \beta \). (a) The lifetime \( \tau_{Rh} \) of right-handed polarized fermions. (b) The lifetime \( \tau_{Lh} \) of left-handed polarized fermions. (c) The lifetime \( \tau \) of unpolarized fermions.

Parity violation causes \( \tau_{Lh} \) and \( \tau_{Rh} \) to be different. The lifetime asymmetry is defined as

\[
A \equiv \frac{\tau_{Rh} - \tau_{Lh}}{\tau_{Rh} + \tau_{Lh}} = \beta. \tag{2}
\]

Obviously, the lifetime asymmetry, \( A \), is proportional to velocity \( \beta \) of fermion.

Similarly, for antifermions in flight, the left-right lifetimes asymmetry are given by

\[
\tau_{Rh} = \frac{\tau}{1 + \beta}, \quad \text{and} \quad \tau_{Lh} = \frac{\tau}{1 - \beta}. \tag{3}
\]
As is well known, parity nonconservation implies that fermions exhibit asymmetrical behavior with respect to the right and the left in the weak interactions. The existing experiments do indicate that fermions from decay processes are polarized in their direction of motion and their angular distributions with respect to the momentum of decaying particles are forward-backward asymmetrical. However, the measurement on the lifetime of polarized fermions in flight has not yet been found in literature. So-called left-right lifetime symmetry is so far only a theoretical prediction or hypothesis unverified by experimental evidence. We actually lack any direct experimental evidence either to validate or to invalidate the lifetime asymmetry.

In this paper, many experiments, the measurement of the lifetime of the neutrons, the $\mu$ leptons, the $\tau$ leptons and the $g$-2 experiment, will be analyzed in detail. The measurement of the left-right cross section asymmetry in $Z$ boson production by $e^+e^-$ collisions will also be discussed. The results show that these existing experiments cannot test the lifetime asymmetry. Finally, some possible experiments will be suggested which might test the lifetime asymmetry. If the lifetime asymmetry is indeed found, it could have important consequences, both theoretical and experimental. Because the lifetime asymmetry is a natural result of the standard model, its measurement will demonstrate the utility of a new and powerful technique for testing the SM.

2 The measurement of the neutron and the $\mu$ lifetime

In order to test the lifetime asymmetry the fermions must be polarized and the experiments must be performed in their motion, i.e., the fermions must be moving with respect to the detector. As seen from formula (2), the lifetime asymmetry $\beta$ increases with the increase of velocity $v$, and is negligible at low energy. Published neutron lifetimes were measured by using thermal or cold neutron beams [5, 6]. The energy of these beams is below 1 eV and too law for the lifetime asymmetry effect to be observed.

The velocity of high energy muons can exceed $\beta = 0.9$, so the polarized muons are an ideal object to test the lifetime asymmetry. Unfortunately, all the experiments for measuring the lifetime of muons were performed in the muon rest frame. In these experiments, the muons, formed by forward decay
in flight of pions inside cyclotron, were stopped in a nuclear emulsion, sulphur, Aluminum, calcium or polyethylene target etc. To research the polarization effects of muon decays, muons were stopped in a carbon target in which there is no depolarization of the muons. Therefore, this experiment was also performed in the muon rest frame.

In CHARM experiment [8, 9], the positive muons were produced by scattering of antineutrinos on nucleons and found to have positive helicity. The decay process \( \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \) of the muons was investigated by a measurement of the time distribution of the positrons and from which a muon lifetime has been derived. It is noteworthy that the polarized muons were stopped in target material Marble (CaCO3) and “the decay positron had to be recorded within the time window \( 0.94 < t < 4.89 \mu\text{sec} \) after the muon came to rest.” Therefore, the measurement was performed in the muon rest frame and independent of lifetime asymmetry.

3 The muon \( g - 2 \) experiment

By the \( g - 2 \) experiment [10, 11, 12, 13], the anomalous magnetic moment of the muons was measured and the muon lifetime in flight in a circular orbit provided an important check on the time dilation effect predicted by the special theory of relativity. Could it also provide an excellent check on the lifetime asymmetry?

The muon beam was formed from decays \( \pi^+ \rightarrow \mu^+ + \nu_\mu \) of a secondary pion beam. Since the pion has spin zero, the neutrino and muon must have antiparallel spin vectors. The positive pions decayed in flight and those decay muons projected in the forward direction were selected, thus the muon and neutrino have parallel momentum vectors in the laboratory frame. The neutrino has negative helicity, so that the positive muons \( \mu^+ \) must have positive helicity, i.e. RH polarized, and might be partially polarized. According to the lifetime asymmetry, Eq. (3), the decay rate of RH polarized positive muons, which are antiparticles, is given by

\[
\lambda = \frac{1 + P \beta}{\gamma \tau_0},
\]

where \( P \) is the degree of muon polarization and \( 0 \leq P \leq 1 \).

However, a muon in the muon storage ring will describe a circular path and meanwhile, owing to its magnetic moment, the muon spin vector will
precess about the field direction. Because the momentum and the spin vector
do not keep “in step”, the angle, $\omega_\alpha t$, between them is a function of time
and the helicity of muon does not remain constant. Here $\omega_\alpha$ is the angular
frequency difference between the spin precession frequency and the cyclotron
frequency. Therefore, the decay rate should be substituted by

$$\lambda = \frac{1 + P \beta \cos \omega_\alpha t}{\gamma \tau_0},$$

which is a function of time too.

The decay formula is given by

$$\frac{dN}{N} = -\lambda dt = -\frac{1 + P \beta \cos \omega_\alpha t}{\gamma \tau_0} dt.$$  \hspace{1cm} (6)

After integrating we obtain

$$N = N_0 e^{-\frac{P \beta}{\gamma \tau_0 \omega_\alpha} \sin \omega_\alpha t} - e^{-\frac{t}{\gamma \tau_0}}.$$  \hspace{1cm} (7)

In the g-2 experiments, $P = 0.97$, $\beta = 0.9994$, $\gamma \tau_0 = 64.4 \mu s$, $\omega_\alpha = 1.4378/\mu s$
(see ref. [12]), therefore we have

$$\frac{P \beta}{\gamma \tau_0 \omega_\alpha} = 0.01047.$$  \hspace{1cm} (8)

The exponential term

$$N_1 = e^{-\frac{P \beta}{\gamma \tau_0 \omega_\alpha} \sin \omega_\alpha t}$$

is related to the lifetime asymmetry. It changes with time in the range
$0.98959 - 1.01053$ and its average value is $1.00006$. The muon lifetime is a
fitted parameter in the $g-2$ experiment. We can obtain the muon lifetime
and its error only if we carry out the fit to the data sets by the function
in which $N_1$ is included. However, $N_1$ is not sensitive to the fitted lifetime
because it equals $1$ approximately and its change with time is expected to
be extremely small. Therefore, the lifetime asymmetry cannot yet be found
out by the $g-2$ experiment under the condition described above.

In Eqs. (8) and (9), we find $\beta/\gamma = \beta \sqrt{1 - \beta^2} = 0$ when $\beta = 0$ and $\beta = 1$.
However, when $\beta = \frac{1}{\sqrt{2}} \approx 0.707$, it has a maximum value $\beta/\gamma = 0.5$. Under
this condition it might be possible to use the $g-2$ experiment as a test of
the lifetime asymmetry.
4 The measurement of the $\tau$ lifetime

The lifetime of $\tau$-leptons in flight has been measured using a sample from process $e^+e^- \rightarrow Z^0 \rightarrow \tau^+\tau^-$. \[14, 15, 16\] In these measurements, three different techniques, decay length, impact parameter and impact parameter difference, were used. Directly or indirectly, all three methods involve the measurement of an average decay length. As viewed from the measurement of the lifetime asymmetry, however, many problems need to be discussed.

First, the polarization of $\tau$ lepton originating from $Z^0$ decay should be concretely analyzed. The $\tau$ polarization depends on the initial state polarization via angular momentum conservation. Since the initial particle, $Z^0$ boson, has spin 1, its helicity, $h$, can have ±1 and 0. Along or near the incidence direction of electron beam in the collider, $\tau^-$ and $\tau^+$ must be both RH polarized when $h = +1$, LH polarized when $h = -1$, and one is RH polarized while another is LH polarized when $h = 0$, according to the conservation law of angular momentum. In first and second situations, the effects of the lifetime asymmetry of positive and negative $\tau$ leptons are counteracted one another. Only in third situation, might the effect of the lifetime asymmetry be observed. In which if $\tau^-$ is LH polarized and $\tau^+$ is RH polarized the lifetime of $\tau$ leptons in the laboratory frame will be reduced by a factor of $(1 + \beta)$; by contrast if $\tau^-$ is RH polarized and $\tau^+$ is LH polarized the lifetime will be lengthened by a factor of $(1 - \beta)$. [see Eqs.(1) and (3)]

Second, the effect of the lifetime asymmetry might be quite small. Some experiments have measured the polarization of $\tau$ leptons produced in $Z$ decays and found that $\tau^-$ is partially LH polarized and $\tau^+$ is partially RH polarized. However, their polarization data are quite scattered and small as well. For example, the average polarizations were found to be only $P = 0.152$ \[17\], 0.01 \[18\], 0.24 \[19\] and 0.132 \[20\], respectively. In other words that the average decay length or the lifetime of the $\tau$ leptons would be reduced by a factor of $(1 + P\beta)$. Therefore, the polarization must be measured precisely, and because it might be quite small the samples should be identified and treated very carefully in the fit. Unfortunately, we unable to find the information about polarization measurements in the papers of measuring the $\tau$ lifetime.

Third, there were some uncertainties in the fitting procedure. In order to obtain the average decay length, a maximum likelihood fit was applied to the observed decay length distribution. In these experiments, however, the authenticity of the fitted results is suspicious because the measurement errors
are quite big and the samples are selected again and again. For example, in the L3 experiment only observed decay length values with an error less than 5 mm were accepted for the fit, but the average decay lengths of the fit were 2.245 mm and 2.265 mm, respectively \cite{14}. In MAC experiment, the fit included only the 532 events having assigned the observed decay length error less than 3 mm, but the average decay length was 0.777 mm \cite{15}. Obviously these error values of samples are much greater than the average values of the samples themselves. On the other hand, after each iteration, some samples were removed again \cite{16}. In the fitting process, for example, about 10% of the samples were removed \cite{15}. It is well known that an input decay length is sensitive to the fitted result and plays an important role in selecting a sample being either accepted or rejected. If considering the lifetime asymmetry in these experiments, the assigned input decay length might not be reasonable and the removed samples might be important to determine the correct value of an average decay length.

To conclusion, these experiments cannot demonstrate the lifetime asymmetry to be either correct or wrong.

It is worth note that the SLD Collaboration has discussed in detail the reactions \cite{21, 22, 23}

\begin{equation}
\nonumber\epsilon^-_{L,R} + e^+ \rightarrow Z^0 \rightarrow \mu^+ \mu^- \text{ and } \tau^+ \tau^-,
\end{equation}

where \( L(R) \) refers to the left(right)-handed electron beam polarization. Parity violation in the production of \( Z^0 \) bosons from electron-positron collisions gives rise to asymmetrical polarization-dependent cross sections. The left-right cross section asymmetry is defined as

\begin{equation}
A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R},
\end{equation}

where \( \sigma_L \) and \( \sigma_R \) are the \( e^+e^- \) production cross section for \( Z^0 \) bosons with LH and RH polarized electron beams, respectively. The SLD measured \( A_{LR} \) to be \( 0.15056 \pm 0.00239 \) \cite{24}. The experiments showed that \( \sigma_L > \sigma_R \). It is due to the weak coupling strength of LH chirality states to the \( Z^0 \) boson being stronger than that of RH chirality states in the neutral weak currents. This characteristic is much more evident in the charged weak currents, in which there exist only LH chirality states whereas the weak coupling strength of RH chirality states to the \( W^\pm \) boson is equal to zero. The decay processes are caused by the charged weak currents and it can be further deduced from this
result that the decay cross section of LH polarized fermions is also greater than that of RH polarized fermions. Therefore the SLD$^2$ experiments have indirectly proved that the lifetime asymmetry should be correct.

5 The measurement of the lifetime asymmetry

To decide unequivocally whether the lifetime asymmetry is correct, one must perform some experiments to determine whether lifetime differentiate the right from the left. A relatively simple possibility is to measure the angular distribution of the electrons or positrons coming from the decays of polarized muons.

In the SM, muon decay is described by the $V - A$ interaction and the muon decay rate is given by

$$\Gamma = \frac{4 \, G^2}{(2\pi)^5} \frac{1}{E_p} \int \frac{d^3q}{E_q} \frac{d^3k}{E_k} \frac{d^3k'}{E_{k'}^\prime} \delta^4(p - q - k - k') \mathcal{F}. \quad (12)$$

where $p$, $q$, $k$ and $k'$ are 4-momenta, while $s$, $s'$, $r$ and $r'$ are spin indices for $\mu$, $e$, $\nu_\mu$ and $\bar{\nu}_e$, respectively. Because the polarization of muons in flight must be described by the helicity state $\mathcal{F}$, the decay amplitude of LH and RH polarized muons are

$$\mathcal{F}_{Lh} = (1 + \beta)[(p \cdot k')(q \cdot k) + m_\mu (e \cdot k')(q \cdot k)],$$

$$\mathcal{F}_{Rh} = (1 - \beta)[(p \cdot k')(q \cdot k) - m_\mu (e \cdot k')(q \cdot k)], \quad (13)$$

respectively, where $m_\mu$ is muon mass. If the muons are partially polarized, the formulas above should be replaced by

$$\mathcal{F}_{Lh} = (1 + P \beta)[(p \cdot k')(q \cdot k) + m_\mu (e \cdot k')(q \cdot k)],$$

$$\mathcal{F}_{Rh} = (1 - P \beta)[(p \cdot k')(q \cdot k) - m_\mu (e \cdot k')(q \cdot k)], \quad (14)$$

where $P$ is the degree of muon polarization and $0 \leq P \leq 1$.

Neglecting electron mass and taking the simplest case of $p : p = p_z$, after integrating calculation the differential decay rates of polarized muons or the angular distributions of electrons emitted by the muons $\mu^-$ during decay are
given by

\[ \sigma_L^\mu^-(\theta) = \frac{d\Gamma_Lh(\mu^-)}{d\cos\theta} = \frac{\Gamma_0(1 + P\beta)}{2\gamma^3(1 - \beta \cos\theta)^2} \left(1 + \frac{1}{3}P\frac{\cos\theta - \beta}{1 - \beta \cos\theta}\right), \]

\[ \sigma_R^\mu^-(\theta) = \frac{d\Gamma_Rh(\mu^-)}{d\cos\theta} = \frac{\Gamma_0(1 - P\beta)}{2\gamma^3(1 - \beta \cos\theta)^2} \left(1 - \frac{1}{3}P\frac{\cos\theta - \beta}{1 - \beta \cos\theta}\right), \]  

where \( \theta \) is the angle between the electron momentum and the muon momentum, the relativistic factor \( \gamma = \frac{1}{\sqrt{1 - \beta^2}} \) and \( \Gamma_0 \) is the decay rate in the muon rest frame, which is given by

\[ \Gamma_0 = \frac{G^2m_\mu^5}{192 \pi^3}. \]  

Similarly, for polarized muons \( \mu^+ \), the angular distributions of positrons emitted by the muons \( \mu^+ \) during decay are given by

\[ \sigma_L^\mu^+(\theta) = \frac{d\Gamma_Lh(\mu^+)}{d\cos\theta} = \frac{\Gamma_0(1 - P\beta)}{2\gamma^3(1 - \beta \cos\theta)^2} \left(1 - \frac{1}{3}P\frac{\cos\theta - \beta}{1 - \beta \cos\theta}\right), \]

\[ \sigma_R^\mu^+(\theta) = \frac{d\Gamma_Rh(\mu^+)}{d\cos\theta} = \frac{\Gamma_0(1 + P\beta)}{2\gamma^3(1 - \beta \cos\theta)^2} \left(1 + \frac{1}{3}P\frac{\cos\theta - \beta}{1 - \beta \cos\theta}\right), \]  

It can be see from Eqs. (15) and (17) that the angular distributions of electrons and positrons are of the similar form and they always tend to be emitted along the direction which is antiparallel to the muon spin-vector, like that in nuclear \( \beta \) decay. The decay rate asymmetry can be expressed as

\[ A_L = \frac{\sigma_L^\mu^-(\theta) - \sigma_L^\mu^+(\theta)}{\sigma_L^\mu^- (\theta) + \sigma_L^\mu^+(\theta)} = P\beta, \quad \text{and} \quad A_R = \frac{\sigma_R^\mu^+(\theta) - \sigma_R^\mu^-(\theta)}{\sigma_R^\mu^+ (\theta) + \sigma_R^\mu^-(\theta)} = P\beta. \]  

When \( \cos\theta = \beta \), instead of Eqs. (15) and (17) we obtain

\[ \sigma_L^\mu^- (\cos\theta = \beta) = \frac{1}{2}(1 + P\beta)\gamma\Gamma_0, \]

\[ \sigma_R^\mu^- (\cos\theta = \beta) = \frac{1}{2}(1 - P\beta)\gamma\Gamma_0, \]

\[ \sigma_L^\mu^+ (\cos\theta = \beta) = \frac{1}{2}(1 - P\beta)\gamma\Gamma_0, \]

\[ \sigma_R^\mu^+ (\cos\theta = \beta) = \frac{1}{2}(1 + P\beta)\gamma\Gamma_0. \]  

Obviously, the condition, \( \cos\theta = \beta \), would reduce the asymmetries due to angular distribution and then the decay rate asymmetry can also be expressed
as

$$A_{LR}(\cos \theta = \beta) = \frac{\sigma_{LR}^{-}(\cos \theta = \beta) - \sigma_{LR}^{+}(\cos \theta = \beta)}{\sigma_{LR}^{+}(\cos \theta = \beta) + \sigma_{LR}^{-}(\cos \theta = \beta)} = P \beta,$$

$$A_{RL}(\cos \theta = \beta) = \frac{\sigma_{RL}^{+}(\cos \theta = \beta) - \sigma_{RL}^{-}(\cos \theta = \beta)}{\sigma_{RL}^{+}(\cos \theta = \beta) + \sigma_{RL}^{-}(\cos \theta = \beta)} = P \beta.$$  \hspace{1cm} (20)

The quantities $A_L$, $A_R$, $A_{LR}(\cos \theta = \beta)$ and $A_{RL}(\cos \theta = \beta)$ are all proportional to the product $P \beta$. They can be obtained by counting electrons or positrons from decays for each of the two longitudinally polarized muon beam. If these quantities are not equal to zero, one would then have a definite proof of lifetime asymmetry. The experiments for measuring these quantities might be simple and easy because the measurements only require one detector to count the numbers of the electrons or positrons. Furthermore in these measurements nearly no exact knowledge of detector efficiency and background is needed. Of course, the luminosity, polarization and momentum of the muon beam should be precisely measured.

### 6 Summary and discussion

All the existing experiments cannot be employed for testing the lifetime asymmetry. Only when $\beta \approx 0.707$, can the influence of the lifetime asymmetry on the $g - 2$ experiment be observed. The measurement of the left-right cross section asymmetry produced by $e^+ e^-$ collisions has already indirectly demonstrated the lifetime asymmetry. In the measurements of the tau lifetime there exist many uncertain factors. Moreover the tau decay is a multiplicity process, not like muon decay, and includes many modes, the electro decay, the muon decay, the hadron decay and so on. Each decay mode makes different contribution to the decay lengths and tau decays into hadrons are sensitive to the strong coupling constant [14]. The purely leptonic processes obey the pure $V - A$ theory and are the simplest weak interaction processes. The muon decay, as a typical sample, is always used for researching the theory of weak interactions and also the best selection for testing the lifetime asymmetry. It can be seen that the muons must be highly polarized and moving with velocities approaching $c$ and along a straight line in order to measure the lifetime asymmetry. A direct demonstration of the lifetime asymmetry could come true by measuring the differential decay rate asymmetry, $A_L$, $A_R$, $A_{LR}(\cos \theta = \beta)$ and $A_{RL}(\cos \theta = \beta)$, of the polarized muon beam. The ex-
experiments proposed might prove to be quite feasible and provide a sensitive
test of the standard model in particle physics.

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