Moduli-Induced Gravitino Problem

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We investigate the cosmological moduli problem by studying a modulus decay in detail and find that the branching ratio of the gravitino production is generically of $O(0.01 - 1)$, which causes another cosmological disaster. Consequently, the cosmological moduli problem cannot be solved simply by making the modulus mass heavier than 100 TeV. We also illustrate our results by explicitly calculating the branching ratio into the gravitinos in the mixed modulus–anomaly/KKLT- and racetrack-type models.

The cosmological moduli problem is one of the most challenging puzzles in particle physics and cosmology. In this letter, we show that the problem is even more difficult than usually thought.

In supergravity/superstring theories, generically there exist moduli fields which have flat potentials and obtain masses from supersymmetry (SUSY) breaking and non-perturbative effects. During an inflationary period, a modulus field $X$ is likely to develop a large expectation value. After the end of the inflation, it starts a coherent oscillation and soon dominates the energy density of the universe. Due to the interaction suppressed by the Planck scale $M_P = 2.4 \times 10^{18}$ GeV, the decay rate of the modulus $X$ is extremely small:

$$\Gamma_X = \frac{c}{4\pi} \frac{m_X^3}{M_P^2},$$  \hfill (1)

which leads to an onset of a radiation-dominated universe with a very low temperature:

$$T_X = \left(\frac{\pi^2 g_*/90}\right)^{-1/4} \sqrt{M_P \Gamma_X} \approx 5.5 \times 10^{-3} \text{ MeV} \cdot e^{\frac{\pi}{2} \left(\frac{m_X}{1 \text{ TeV}}\right)^2}.$$  \hfill (2)

Here, $c$ is an order one coefficient and $g_*$ is the effective number of massless degrees of freedom. This is cosmologically unacceptable because a successful big–bang nucleosynthesis (BBN) requires that the (last) radiation–dominated universe starts with temperature higher than $\sim 5$ MeV.

As is clear from Eq. (2), a simple solution would be to assume that the modulus $X$ is ultra heavy $a$:

$$m_X \lesssim 100 \text{ TeV} \rightarrow T_X \gtrsim O(\text{MeV}).$$  \hfill (3)

Actually, there have been proposed scenarios with such a large modulus mass (cf. [3, 5, 7, 9]).

However, there exists yet another serious cosmological obstacle even for heavy moduli scenarios. The new problem is caused by the gravitino which is produced by the modulus decay. Indeed, as we will show, the branching ratio of the modulus decay into the gravitino is generically quite large

$$B_{3/2} \equiv \text{Br}(X \rightarrow \text{gravitino}) = O(0.01 - 1),$$  \hfill (4)

which causes serious problems after the modulus decay. We call this problem the moduli–induced gravitino problem.

The gravitino production via modulus decay and its cosmological implications have been previously discussed in Refs. [10, 11], taking $\text{Br}(X \rightarrow \text{gravitino}) \ll 1$. The main purpose of this letter is to show that Eq. (4) holds in a generic setup, and to emphasize how disastrous its consequences are. We also exemplify explicit results in the mixed modulus–anomaly/KKL T mediation [6, 7] and in the racetrack setups.

Let us first estimate the branching ratio of a modulus decay into gravitino(s). We consider a heavy modulus scenario, $m_X \gtrsim 100 \text{ TeV}$ [cf. Eq. (3)]. On the other hand, the gravitino is likely to be (much) lighter than 100 TeV, because too large gravitino mass requires a fine-tuning in the Higgs sector due to the anomaly-mediated effects. Thus, we assume $m_X \gg m_{3/2}$ hereafter. After choosing the unitary gauge in the Einstein frame, where the goldstino component is completely absorbed into the gravitino under the super–Higgs mechanism [12], the relevant gravitino-modulus couplings are [10, 11, 13]

$$e^{-1} \mathcal{L} = -\frac{1}{8} \epsilon_{\mu\nu\rho\sigma} \left(G_X \partial_\mu X - G_{\bar{X}} \partial_\mu \bar{X}\right) \bar{\psi}_i \gamma_\mu \nu \psi_\sigma - \frac{1}{8} e^{G/2} \left(G_X X + G_{\bar{X}} \bar{X}\right) \bar{\psi}_i \left[\gamma^\mu, \gamma^\nu\right] \psi_\nu,$$  \hfill (5)

where $\psi_\mu$ is the gravitino field. Here and hereafter, we set $M_P = 1$ unless explicitly written. The function $G = K + \ln |W|^2$ is the total Kähler potential, and $G_i$ denotes a derivative of $G$ with respect to the field $i$. The real and imaginary components of $X = (X_R + iX_I)/\sqrt{2}$ have the decay rate $b$,

$$\Gamma(X_{R,I} \rightarrow 2\psi_{3/2}) \simeq \frac{1}{288\pi} \frac{|G_X|^2}{g_{XX}} \frac{m_X^3}{m_{3/2}^2},$$  \hfill (6)

$^a$ See Refs. [3, 5] for other solutions.

$^b$ The branching ratio of the single gravitino production is suppressed due to the phase space: $\text{Br}(X \rightarrow \psi_{3/2}) \lesssim (m_{3/2}/m_X)^2$. 

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in the limit of $m_X \gg m_{3/2}$ after the canonical normalization, $\hat{X} = \sqrt{g_{XX}} X$, where $g_{ij} = K_{ij}$ is the Kähler metric. We emphasize that the gravitino mass in the denominator arises from the longitudinal component of the gravitino, which corresponds to the goldstino, $\psi_\mu \sim (p_\mu/m_{3/2}) \chi_{GS}$, in the massless limit. This factor magnifies the decay rate by a factor of $\sim (m_X/m_{3/2})^4$, compared to the previous results \[11, 11, 14\].

The auxiliary field of the modulus, $G_X$, represents its fractional contribution to the total amount of the SUSY breaking. It may be small enough to suppress the gravitino-production rate. However, we can see that in the framework of the 4D $N = 1$ supergravity, $G_X$ is generically bounded below, $G_X \gtrsim m_{3/2}/m_X \equiv R^{-1}$. The scalar potential is given by $V = e^{G'(G'G_i - 3)}$ and the (almost) vanishing vacuum energy demands that at least one of the auxiliary fields, $G_i$, should take $G_i^2 \sim O(1)$, where $G_i = \sqrt{g_{ij}} G_j$. If the modulus field plays the role, it dominantly decays into the gravitino [cf. Eqs. (1) and (6)] and hence clearly $B_{3/2} \approx 1$. Instead, by introducing a hidden sector field $Z$ with $G_{Z} \sim O(1)$, the modulus-auxiliary field $G_X$ can be small $^c$. In the following discussion, we assume that the modulus field does not decay into the hidden-sector field for simplicity. In order to see how small $G_X$ can be, we investigate the condition to minimize the potential, $V_X = 0$, leading to

$$G_X \nabla_X G_X + G^Z \nabla_X G_Z + G_X = 0,$$  
(7)

where $\nabla_i G_j = g_{ij} - \Gamma^k_{ij} G_k$ with the connection, $\Gamma^k_{ij} = g^{kk'} g_{i,j}$. We assume that the Kähler potential includes no enhancement factor, and especially $g_{XX}, g_{ZZ} \sim 1$ and $K_X \sim 1$. The first term in Eq. (7) then becomes $G^X R$, because $m_{XX}/m_{3/2} \sim |\nabla_X G_X| + O(G_X)$. Barring cancellation, the second term is given by $\sqrt{|H_{Z}/H_{W} W_{XZ} V_{WZ}^X 3^2 X Z|}$, which is $O(1)$ unless the hidden sector takes a special form. Thus, we arrive at $|G_X| \sim R^{-1}$ for $|g^{XX}| < R^{-1}$ and $|G_X| \gtrsim R^{-1}$ for $|g^{XX}| \gtrsim R^{-1}$. As a result, Eq. (8) becomes

$$\Gamma(X_{R, I} \rightarrow \text{radiation}) \approx \frac{3}{8\pi} \frac{1}{g_{XX} |X_{R}|^2} m_X^3.$$  
(8)

It is important to note that the production rate of the gravitino channel is one of the dominant processes in the modulus decay. Actually, it becomes an order of $0.01 - 1$.

The other decay processes are suppressed, except for a possible decay into the Higgs(-ino) $^1$H. However, this decay channel is model-dependent, and does not change the above result $c = O(1)$.

Let us show how such large branching ratio into the gravitinos jeopardizes the success of the standard big bang cosmology. We consider the constraints from (i) the speed-up effect, (ii) BBN, and (iii) the lightest SUSY particle (LSP) abundance. First let us consider the so-called speed-up effect which modifies mostly the $^4$He abundance. This sets a bound on the abundance of the gravitinos with $m_{3/2} \lesssim 20$ TeV (cf. \[13\]), since they decay after the neutron-proton transformation decouples. The observational data put an upper bound on the ratio of the energy densities of the gravitinos to the standard model particles at the BBN epoch \[14\] as

$$f_{3/2} \equiv \frac{\rho_{3/2}}{\rho_{SM}|_{BBN}} \lesssim 0.2, \quad (95\%\text{C.L.}).$$  
(12)

In our heavy moduli scenario, however, the ratio is bounded below: $f_{3/2} \geq B_{3/2}/(1 - B_{3/2})$, where the equality holds if the gravitinos are still relativistic at the BBN epoch and most of the superpartner of the SM particles directly produced from the modulus decay soon annihilate into the SM particles. Thus we obtain

$$B_{3/2} < 0.2,$$  
(13)

\[11, 11, 14\]

\[12\]

\[13\]
irrespective of whether the gravitino is stable or unstable. For $B_{3/2} > 0.2$, the gravitinos from the modulus decay always upset the standard BBN, as long as $m_{3/2} \lesssim 20$ TeV.

Now we discuss the cases of the stable and unstable gravitinos separately. First we take up the unstable gravitinos, which is the case if the gravitino mass is heavier than the LSP mass, $m_{3/2} > m_{\text{LSP}}$. The gravitino-to-entropy ratio is given by

$$Y_{3/2} \equiv \frac{n_{3/2}}{s} \simeq 2B_{3/2} \frac{3T_X}{4m_X},$$

$$\simeq 2.6 \times 10^{-7} \epsilon B_{3/2} \left( \frac{m_X}{10^3 \text{ TeV}} \right)^{3/2}. \quad (14)$$

The BBN severely constrains $Y_{3/2}$ [13, 17]. Even if we adopt the recent analysis on $^4$He abundance which has taken account of possible large systematic error [18], $Y_{3/2}$ must be smaller than $2 \times 10^{-12}$ at 95% C.L. for $m_{3/2} \simeq 30$ TeV [17], and the bound becomes much severer for smaller $m_{3/2}$. Therefore the branching ratio into the gravitinos must be extremely small:

$$B_{3/2} < 10^{-5} \epsilon c^{-1/2} \left( \frac{10^3 \text{ TeV}}{m_{\text{LSP}}} \right)^{3/2}, \quad (15)$$

for $m_{\text{LSP}} < m_{3/2} \leq 30$ TeV. Here $\epsilon \leq 1$ parameterizes the BBN bound: $\epsilon = 1$ for $m_{3/2} \simeq 30$ TeV, and $10^{-5} \lesssim \epsilon \ll 1$ for $m_{3/2} < 30$ TeV. In addition, the abundance of the LSPs from the gravitino decay is (cf. [13])

$$Y_{\text{LSP}} |_{\psi_{3/2}} \simeq \min \left[ Y_{3/2}, \sqrt{\frac{45}{8\pi^2 g_*} \frac{1}{M_P T_{3/2}} (\langle \sigma_{\text{ann}} v \rangle)} \right], \quad (16)$$

where $\langle \sigma_{\text{ann}} v \rangle$ is the thermally averaged annihilation cross section of the LSP, and $T_{3/2}$ is the decay temperature of the gravitino. Since the LSP abundance must be smaller than the dark matter abundance, we have another constraint on $B_{3/2}$:

$$B_{3/2} < 1.8 \times 10^{-5} c^{-1/2} \left( \frac{10^3 \text{ GeV}}{m_{\text{LSP}}} \right) \left( \frac{\Omega_{\text{DM}} h^2}{0.13} \right) \left( \frac{10^3 \text{ TeV}}{m_X} \right)^{3/2 \beta}, \quad (17)$$

for $m_{3/2} > m_{\text{LSP}}$. Here $\Omega_{\text{DM}}$ is the density parameter of the dark matter, $h$ is the present Hubble parameter in units of 100km/sec/Mpc, and we have assumed $\langle \sigma_{\text{ann}} v \rangle < 10^{-6}$ GeV$^{-2}$ and $m_{3/2} < 100$ TeV. We can see that [13, 15] and [17] rule out the unstable gravitinos, unless $B_{3/2}$ is extraordinarily small in spite of our result $B_{3/2} = O(0.01 - 1)$.

Next we consider the stable gravitinos, which is the case if the gravitino is the LSP. A constraint then comes from the requirement that the gravitino abundance should not exceed the dark matter abundance, and we only have to replace $m_{\text{LSP}}$ with $m_{3/2}$ in [17]:

$$B_{3/2} < 1.8 \times 10^{-2} c^{-1/2} \left( \frac{10^3 \text{ TeV}}{m_{3/2}} \right) \left( \frac{\Omega_{\text{DM}} h^2}{0.13} \right) \left( \frac{10^3 \text{ TeV}}{m_X} \right)^{3/2 \beta}, \quad (18)$$

for the stable gravitino. Furthermore, the bound on the gravitino abundance is severer by an order of magnitude for $m_{3/2} \lesssim 100$ MeV ($m_X/10^3 \text{ TeV}$)$^{-1/2}$, due to the present large free-streaming velocity [20]. See Fig. 1.

From [13] and [17], we conclude that large $B_{3/2}$ encounters trouble even for the stable gravitinos.

Lastly we comment on the lightest superpartner of the SM particles, denoted by $\chi$, produced from the modulus decay through the gauginos. From the discussion above, the number of $\chi$ generated from the decay of unit quantum of the modulus field is order unity. If $\chi$ is the LSP and stable, it must be electrically neutral. To satisfy $\Omega_{\chi} h^2 < 0.13$, the pair annihilation cross section must be large [14, 15], which in turn constrains the mass spectrum of the SUSY particles. In the case of the gravitino LSP, there is a strict BBN bound on the abundance and lifetime of the next-to-LSP (NLSP) $\chi$. For a stable NLSP $\tau$, we find upper bounds on the gravitino mass, $m_{3/2} \lesssim (0.3 - 1)$ GeV for $m_X = 10^3$ TeV and 100 GeV $\lesssim m_{\tau} \lesssim 1$ TeV $d$. For a neutralino NLSP, the bound becomes severer.

We summarize the bounds considered above in Fig. 1 from which we can see how serious the cosmological moduli problem becomes as a result of our finding that $B_{3/2}$ should be $O(0.01 - 1)$. Note that the bounds on $B_{3/2}$ [cf. 13, 17] and [18] become severer for heavier modulus mass and larger $c$, although the dependence is weak.

\[\text{FIG. 1: The cosmological bounds on } m_{3/2} \text{ and } B_{3/2}. \text{ Shaded regions are excluded by cosmological arguments. See the text for details. The horizontal dashed line denotes the BBN bound from NLSP decay into gravitinos for } m_{\text{NLSP}} = 100 \text{ GeV. We have chosen } m_X = 10^3 \text{ TeV and } c = 1 \text{ as representative values. The bounds become severer for larger } m_X.\]
Let us explicitly calculate the branching fraction of the gravitino in some models. The relevant terms in the Lagrangian of the mixed modulus–anomaly/KKL T mediation model are given by

\begin{align}
    f &= -3(X + \bar{X})^{n/3} + C_Z(X + \bar{X})^4|Z|^2, \quad (19) \\
    W &= W_0 + ae^{-bX} + W_{\text{Hidden}}(Z), \quad (20)
\end{align}

where the Kähler potential is defined as \( f = -3e^{-K/3} \). Here coefficients \( b \) and \( C_Z \) are real with \( b X_R \gg 1 \), while \( a \) and \( W_0 \) are complex. Then the modulus mass is

\[ m_X^2 \approx 2(b X_R)^2 m_{3/2}^2. \quad (21) \]

The gravitino mass \( m_{3/2} \) is much larger than the soft mass scale \( m_0 \). On the other hand, \( G_X \) is

\[ G_X \approx -\frac{2n + 3l}{2X_R} \frac{1}{b X_R}, \quad (22) \]

where we assumed \( \langle Z \rangle \ll 1 \) and the vanishing cosmological constant. Thus the branching ratio becomes

\[ \text{Br}(X_I \to 2\psi_{3/2}) \approx \frac{(2n + 3l)^2/216}{1 + (2n + 3l)^2/216}. \quad (23) \]

In the original KKL T model with the lift-up potential due to \( D3 \) brane, which is realized by choosing \( n = 3 \) and \( l = 0 \), the branching ratio is \( B_{3/2} \approx 0.14 \).

In the racetrack-type models, the Kähler potential is the same as Eq. (19) but the superpotential is

\[ W = a_1 e^{-b_1X} + a_2 e^{-b_2X} + W_{\text{Hidden}}(Z), \quad (24) \]

where \( a_i \)'s are complex and \( b_i \)'s are real numbers. Here \( b_i X_R \gg 1 \). The modulus mass and \( G_X \) are

\[ m_X^2 \approx \frac{4}{n^2} (b_1 X_R)^2 (b_2 X_R)^2 m_{3/2}^2, \quad (25) \]

\[ G_X \approx -\frac{n(2n + 3l)}{2\sqrt{2} X_R} \frac{1}{(b_1 X_R)(b_2 X_R)}. \quad (26) \]

Therefore the branching ratio becomes the same as Eq. (23). For \( n = 1 \) and \( l = 0 \), one obtains \( B_{3/2} \approx 0.018 \).

In summary, we have shown that the branching ratio of the modulus decay into gravitinos is \( O(0.01 - 1) \), and hence heavy modulus scenario is plagued with the moduli-induced gravitino problem. Here let us comment on possible ways out. Unless either \( B_{3/2} \) or the gravitino mass is extremely small, there is no way to circumvent the problem other than introducing something that dilutes the modulus field and the subsequently produced gravitinos and (N)LSPs. One of the candidates is the thermal inflation \( \phi \). Another is Q-balls in the Affleck-Dine mechanism \( \psi_3 \). In the latter case, the Q-balls can not only dilute the unwanted gravitinos but also generate the baryon asymmetry successfully. Detailed discussion is beyond the scope of this letter and will be presented elsewhere \( \psi_3 \).

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Note Added: Shortly after our letter, Ref. \( \psi_3 \) appeared, pointing out the same problem. They also discussed the gravitino much heavier than 100 TeV as a possible solution to the moduli-induced gravitino problem. More recently, Ref. \( \psi_3 \) has pointed out \( G_X \) can be much smaller than \( R^{-1} \) in some situations. However, for general non-renormalizable couplings in \( K \), our estimate on the gravitino production still holds. See \( \psi_3 \) for details.

References:


