Core Transitions in the Breakup of Exotic Nuclei

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In order to study nuclei at the limit of stability one needs reliable nuclear reactions models that incorporate the relevant structure degrees of freedom in a consistent manner, in particular the continuum. Theories of nuclear reactions have been repeatedly challenged with the new avenue of experimental work now possible at Radioactive Beam Facilities. Amongst the various reactions, breakup is a prominent role. With breakup reactions, one tries to extract spectroscopic information or capture reaction rates of Astrophysical relevance. In either case, the structure information obtained is model dependent and assumes a single particle description of the projectile as a valence nucleon attached to the ground state of the core. Such a simplification may be the source of lingering disagreements.

When charged particle detectors are coupled with gamma arrays, the states of the core can be disentangled. For the variety of knockout measurements now available, the data are found to contain both elastic breakup (diffraction) and transfer (stripping) contributions, which are typically calculated within an eikonal spectator core model. Only the nuclear reaction cross sections to particular states are computed and other effects need to be added a posteriori, and incoherently (i.e. a priori). One uses the single particle contributions weighted by the composition of the initial state predicted by shell model and neglects interference between the various projectile components and dynamical processes with the target. However, one should expect that, if the energy necessary to excite the core is small, there will be dynamical core excitation/de-excitation during the reaction with the target. When a loosely bound projectile cannot be described as a single particle state, core excited components are mixed in the projectile scattering states as well as the ground state. These can interfere during the reaction and modify the single particle picture.

Exotic systems of the type core + N where core degrees of freedom may play a relevant role include 6Li, 8B, 11Be, 17C, 17F and 19C. In addition, one can expect to find in their spectra states built on excited states of the core. This is the case of a resonance in 11Be which is visible at approximately 3.4 MeV excitation in the spectrum of 2, and such states cannot be understood within the single particle description. This calls for a formulation of breakup where core excitation is consistently included.

As a first example, we look at 9Be(11Be,10Be)X at 60 MeV/u, a reaction already studied in detail. The eikonal model predictions for the total cross sections to particular states are too low: σth = 165 mb to be compared with σexp = 203(31) mb for the ground state of 10Be, and σth = 9 mb to be compared with σexp = 16(4) mb for the 2+ excited state. Estimates of a Coulomb contribution and inelastic core excitation presented in provide a possible explanation for the apparent underprediction of theory, but the problem has been awaiting a consistent formulation. Furthermore, Continuum Discretized Coupled Channel calculations show that the eikonal approximation does not have the desired level of accuracy at this energy. We propose a model where all these corrections and effects are included in a consistent manner.

A recent preliminary study generalizes the spectator core model of to include core excitation. Although the work is performed for nuclear only and within a straight line approximation, it represents an important improvement over previous efforts because a core degree of freedom is introduced consistently in the projectile and the core-target interaction, thus allowing for dynamical core excitation/de-excitation throughout the reaction. The initial results in show an increase of the breakup component of the total cross sections, but no effect on the stripping part. This is an extremely use-

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ful result, that the stripping component seems to be less affected by the various mechanisms discussed above. We will focus here on elastic breakup (diffraction dissociation) only.

As mentioned above, typically breakup models assume a single particle description for the projectile. Only recently are improvements on this approximation being considered. One impressive improvement consists of describing the projectile as a three body system \(^9\)Be adequate for nuclei of Borromean nature such as \(^9\)He. We pursue an alternative improvement, which is to describe the projectile as a multi-component system based on several core states. In this work, we present the eXtended Continuum Discretized Coupled Channel (XCDCC) method, to take into account explicitly core excitation in the breakup reaction of loosely bound systems.

We consider the breakup of a projectile (P), composed of a core (c) plus a valence particle (v), on a target (T). The breakup process is described using a three body Hamiltonian, with core degrees of freedom denoted by \(\xi\): \(H^{3b} = T_R + H_{proj}(r, \xi) + U_{ct}(r, R) + U_{av}(r, R, \xi)\). The coordinates are illustrated in Fig. 1 and, as Jacobi coordinates, can be used for the full three-body wavefunction

\[
\Psi_{J_MR_T}(\mathbf{R}, r; \xi) = \sum_\alpha \Psi^{J_T}_\alpha (\mathbf{R}) \left[ \left( Y_L(\mathbf{R}) \otimes \Phi^{\alpha}_{J_P}(r, \xi) \right)_J \otimes \Phi_{J_T} \right]_{J_MR_T},
\]

where \(L\) is the projectile-target orbital angular momentum, \(J_t\) the total spin of the target, \(J_T\) the total spin of the three-body system, and \(\alpha = \{L, J_P, J, j, i, n\}\).

The projectile states \(\Phi_{J_P}(r, \xi)\) can be either a bound or a scattering state, with several components. They are obtained as coupled channels eigenstates of the projectile Hamiltonian \(H_{proj} = T_r + V_{vc}(r, \xi) + h_{core}(\xi)\):

\[
\Phi^{\alpha}_{J_P}(r, \xi) = \sum_{\alpha} \Psi^{J_T}_\alpha (r)[|Y_l(\mathbf{r}) \otimes \chi_s|_j \otimes \varphi_I(\xi)|_{J_P},
\]

expanded in terms of core eigenstates at energies \(\varepsilon_I\).

A continuum projectile state is characterised by \(\{J_P, i, n\}\), where \(i\) refers to the asymptotic energy, and \(n\) denotes the channel with a plane wave component. It is composed of projectile radial wavefunctions \(f_{a;n}(r; k_{an})\) for each \(a = \{lsj, I\}\) that are solutions of coupled equations which, since the state of the core can change, are

\[
E_k - \varepsilon_a - \varepsilon_n + \frac{\hbar^2}{2\mu_{vc}} \left( \frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} \right) f_{a;n}(r; k_{an}) = \sum_{\alpha'} V_{a;\alpha'}(r) f_{\alpha';n}(r; k_{\alpha'n}) ,
\]

where the coupling potentials are matrix elements of \(V_{rc}(r, \xi)\). From these equations we obtain the S-matrix \(S_{a;\alpha'n}\).

Given the importance of continuum-continuum couplings in the CDCC formulation of breakup for halo nuclei \(^{12}\)C, we need to transform the projectile scattering states into square integrable functions, otherwise continuum-continuum couplings would diverge. We define a coupled channel bin as

\[
u^i_{a;n}(r) = \sqrt{\frac{2}{\pi(k_{i-1}-k_i)}} \int_{k_{i-1}}^{k_i} dk' e^{-i\delta_n(k)} f_{a;n}(r; k_{an})
\]

where \(k\) is the core-valence relative momentum, \(k_{an}^2 = k^2 - \frac{\hbar^2}{2\mu_{vc}} (\varepsilon_a - \varepsilon_n)\). From \(S_{a;\alpha'n}\) we obtain \(\delta_n(k)\), the diagonal valence-core phase shift of channel \(a = n\). Coupled channel bins defined in this way are complex.

Substituting the three body wavefunction Eq. 1 into the three body Schrödinger equation \(H^{3b}\Psi_{J_P} = E\Psi_{J_P}\), one obtains an equation set similar to the standard CDCC equations of \(^{14}\), with coupling potentials \(U_{\alpha;\alpha'}(R) = \langle \alpha|U_{ct}(r, R, \xi) + U_{av}(r, R)|\alpha'\rangle\) containing both Coulomb and nuclear interactions between the projectile fragments and the target. The techniques for solving this equation are the same as in \(^{14}\). However there are essentially two differences in these eXtended CDCC equations: first they span a larger number of projectile coupled components, corresponding to core excitation, and second the interaction core-target depends on core degrees of freedom \(\xi\). In our calculations, the couplings \(U_{\alpha;\alpha'}(R)\) are further expanded in multipoles and non-trivial algebra is necessary to simplify the problem, but we leave the details on the evaluations of these matrix elements for a longer publication \(^{15}\). A new generation of the code FRESCO \(^{16}\) was developed to incorporate these aspects, namely coupled channel bins and core excitation CDCC couplings. The code was further optimized and parallelized so that realistic calculations could become feasible.

The first application of XCDCC is to the breakup components of \(^9\)Be(\(^{11}\)Be,\(^{10}\)Be)X at 60 MeV/u \(^{11}\). As the spin \(s\) of the neutron has an insidious effect, we set it to \(s = 0\). We include only the first excited state of the core as in \(^{15}\). For our case the ground state of \(^{11}\)Be contains two components: an s-wave neutron coupled to \(^{10}\)Be(0\(^+\)) and a d-wave neutron coupled to \(^{10}\)Be(2\(^+\)). The model for n-\(^{10}\)Be is based on \(^{15}\), but the depths of the Woods-Saxon interactions need to be adjusted to give a positive parity bound state energy at \(E_+ = -0.5\) MeV and a negative parity level at \(E_- = -0.18\) MeV, in the \(s = 0\)
The results are shown in table I. Using a single particle model for $^{11}$Be, and introducing ground state occupation probabilities consistent with the ones produced by the CC model, namely 88.3\% s-wave and 11.7\% d-wave, the predicted cross sections are 109 mb to the g.s. of the core, and 1 mb to the first excited state (model SPIS). In comparison, when core excitation is included consistently, the XCDCC calculations predict exactly the same cross section to the g.s. of the core, but a large increase (a factor of 8) in the cross section to the $^{10}$Be 2$^+$ state. In both SPIS and XCDCC breakup, the probabilities for seeing the 2$^+$ state, $P_2 = \sigma_2/\sigma_{tot}$, are much lower than in the ground state. This reflects the fact that the partial cross section decreases rapidly with increasing single-particle Q-value for breakup.

The occupation probability (or for that matter the spectroscopic factor) is not an observable, therefore it is not good practice to compare theory and experiment at this level. Rather one should include an appropriate reaction model and compare cross sections. If we take the eikonal prediction for nuclear breakup directly from \[1\] and include occupation probabilities consistent with our $^{11}$Be coupled channel model, we obtain cross sections lower than the XCDCC predictions (see first row of table II). The differences between the theoretical cross sections predicted in the Eikonal model and the data \[1\] were attributed to the Coulomb breakup and inelastic excitation of the core. In XCDCC one can turn off the various couplings to understand their relative importance. We find that even when core excitation couplings are not included in the core-target interaction, the cross section to the $2^+$ is increased over the SPIS prediction, due to constructive interference between the projectile’s components in the continuum. There are also nuclear-Coulomb interference effects. Consequently, inelastic and Coulomb contributions should not be added incoherently. These are automatically contained in the XCDCC predictions.

For a meaningful comparison with the data one needs to construct the stripping component, since the neutron was not detected in the measurement \[1\]. However XCDCC produces the elastic breakup component only.

<table>
<thead>
<tr>
<th>model</th>
<th>interaction</th>
<th>$V(\pi)$</th>
<th>$V(-\pi)$</th>
<th>$R$</th>
<th>$a$</th>
<th>$\beta_2$</th>
</tr>
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<tr>
<td>CC</td>
<td>$^{10}$Be${0^+,2^+}+n$</td>
<td>55.25</td>
<td>47.00</td>
<td>2.483</td>
<td>0.65</td>
<td>0.67</td>
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<tr>
<td>SP</td>
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<td>30.48</td>
<td>2.736</td>
<td>0.67</td>
<td>0.0</td>
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<tr>
<td>SP</td>
<td>$^{10}$Be${2^+}+n$</td>
<td>75.07</td>
<td>39.95</td>
<td>2.736</td>
<td>0.67</td>
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</tr>
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</table>

TABLE I: Potential parameters for $^{10}$Be+n with and without deformation of the core. Also given are the parameters for the core-target optical potential.

- The neutron-target optical potential parameters are taken from \[19\]. As the target $^9$Be is a spectator, its spin $J_z$ is neglected, as in Refs. \[1, 6, 8, 10, 11\]. For the $^{10}$Be-$^9$Be we take the potentials from \[20\] that reproduce unpublished data for the elastic scattering of $^{10}$Be on $^{12}$C at 59 MeV/u. This potential is directly used in the SP calculations. For the XCDCC calculations, we deformed both the nuclear and the Coulomb parts using the deformation lengths consistent with our structure model for $^{11}$Be \[21\] and refit the potential to reproduce the same elastic distribution. The resulting potential is given in the last row of table I.

- We briefly describe the model space for solving the XCDCC equations. We take partial waves for $J_P \leq 4$ organized in 178 bins as schematically shown in Fig. 2. The evaluation of the couplings $U_{a\nu} (R)$ involve an integration in $r$ which is performed up to $r_{bin} = 70$ fm, and a multipole expansion which we truncate at $\Lambda_{max} = 4$. The projectile-target relative angular momentum is taken up to $L_{max} = 3000$ and the corresponding distorted waves are matched to Coulomb functions at $R_{asym} = 500$ fm. The XCDCC calculations were performed on a SGI Altix 3700.

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In conclusion, we find that the amount of core excitation is modified in the breakup reaction. This process occurs both through constructive interference of various components in the projectile and through the interaction of the core with the target. The eXtended Continuum Discretized Coupled Channel method was developed specifically to handle the problem of core excitation in the breakup of loosely bound projectiles. Effects of core excitation in the projectile bound and scattering states are explicitly taken into account. Through the interaction with the target, the core can excite or de-excite during the reaction. We have applied XCDCC to the breakup of $^{11}$Be on $^9$Be at 60 MeV/u, where the final $^{10}$Be state is identified. Theoretical predictions within a truncated model space agree with the data for the cross sections to the first two individual $^{10}$Be states.

As compared to the preliminary calculations of Batham et al. $^{11}$, XCDCC provides more detailed cross sections, namely partial cross sections to each core state. As a consequence we now understand that the increase of the total cross section when including core excitation comes mainly from an increase of the core excited component. Thus one can think of it as production of core excitation. This process was not understood before.

Despite the computational challenge, XCDCC consists on a significant improvement to previous theories. Given the possibility of producing core excitation, previous spectroscopic analyses and extractions of astrophysical S-factors need to be revisited. Other reactions that can be usefully studied with XCDCC include $^{11}$Be(p,p')$^{11}$Be*, $^{12}$C($^{11}$Be,$^{10}$Be+n), the various modes of $^9$B breakup, and work along these lines is underway. So far, the method is limited to the inclusion of states of the core that can be modelled as collective excitations, but it could easily be adapted to including a better description of the core, as long as a complete set of core + N (bound and scattering states) could be obtained.

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\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
core state & $\sigma_{\text{in}}$ & $\sigma_{\text{at}}$ & $\sigma_{\text{th}}$ & $\sigma_{\text{exp}}$ \\
\hline
0$^+$ & 109 mb & 91 mb & 200 & 203(31) \\
2$^-$ & 8 mb & 6 mb & 14 & 16(14) \\
\hline
\end{tabular}
\caption{Comparison of calculated cross sections, sum of XCDCC breakup and Eikonal stripping, with the data $^1$ for $^9$Be($^{11}$Be,$^{10}$Be)$X$ at 60 MeV/u.}
\end{table}