MEASUREMENTS OF THE LIFETIME OF POSITIVE AND NEGATIVE MUONS
AT $\gamma = 30$ IN A CIRCULAR ORBIT

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ABSTRACT

The lifetimes of both positive and negative relativistic ($\gamma = 29.33$) muons have been measured in the CERN Muon Storage Ring with the results

$$\tau^+ = 64.419(58) \mu\text{sec}, \quad \tau^- = 64.368(29) \mu\text{sec}.$$  

The value for positive muons is in accordance with special relativity and the measured lifetime at rest: the Einstein time dilation factor agrees with experiment with a fractional error of $2 \times 10^{-3}$ at 95% confidence. Assuming special relativity, the mean proper lifetime for $\mu^-$ is found to be

$$\tau_0^- = 2.1948(10) \mu\text{sec},$$

the most accurate value reported to date. The agreement of this value with previously measured values of $\tau_0^+$ confirms CPT invariance for the weak interaction in muon decay.

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1. INTRODUCTION

The previous Muon Storage Ring experiment at CERN reported\(^{1}\) a value of the muon lifetime in flight for a relativistic factor \(\gamma = 12\), which agreed within 1\% with the predicted value obtained by applying the Einstein time dilation factor\(^{2}\) to the measured lifetime at rest\(^{3}\). The present letter reports separate measurements for \(\mu^+\) and \(\mu^-\), with a \(\gamma\) factor of 29.33, which are an order of magnitude more precise.

The latest CERN Muon Storage Ring\(^{4,5}\) has a diameter of 14 m with a uniform magnetic field and weak focusing by an electrostatic quadrupole field\(^{6}\). Decay electrons from the stored muons are detected in 20 shower counters inside the ring. The decay times are measured to high precision\(^{7,8}\) for the g-2 experiment so that the muon laboratory lifetime \(\tau\) can be obtained by fitting the observed decay electron time spectrum.

However, this spectrum is distorted by small systematic effects; notably the loss of muons from the trapping region before decay, the time dependence of the background count rate, and the variation of the decay electron detection efficiency as a result of gain changes during the muon storage period. The resulting modification to the normal modulated exponential decay spectrum may be expressed as

\[
N(t) = N_0 \left[ 1 + S \cdot G(t) + F(t) \right] \left[ \exp \left( -t/\tau \right) \left( 1 - A \cos \left( \omega t + \phi \right) \right) + B(t) \right].
\] (1)

The functions \(S \cdot G(t)\) and \(F(t)\) represent the variation of detection efficiency and muon losses, respectively, while \(B(t)\) is the background. For \(\mu^-\) data the background is independent of time and at the negligible level of \(\sim 2 \times 10^{-5}\), but becomes more important in the \(\mu^+\) lifetime data owing to stored proton contamination. The minimization and measurement of these systematic effects are discussed in some detail below.

2. MUON LOSSES

We recall that the muons are injected by the decay in flight of 3 GeV/c pions inflected into the ring\(^{4,5}\). They initially fill the whole available phase space; however particle loss involves preferentially those muon orbits which approach closest to the material limits of the storage region.

To reduce these losses we manipulate the muon orbits in the first few microseconds after injection (the rotation period of the muons is 0.147 \(\mu\)sec) so that the muons which pass within \(\sim 1\) cm of the walls are removed. The remaining muons are left in the central region of the aperture and have a much smaller probability of being lost. This so-called "scraping" is achieved by applying asymmetric voltages to the electrostatic quadrupoles\(^{6}\) used to focus the particles. A vertical
electric field shifts the median plane downwards, and simultaneously a horizontal electric field (outwards on one side of the ring, inwards on the other) shifts the orbit sideways. The muons of large betatron amplitude then strike appropriately located aperture stops. The scraping voltages are turned off with a time constant of 10 µsec, allowing the muon population to return adiabatically to the centre of the storage region.

When a muon hits an obstacle it loses energy and has a high probability of emerging on the inside of the ring. The muon losses are therefore monitored by placing a detector against the vacuum chamber on the inside of the ring, in a position similar to that occupied by the decay electron detectors. The muon detector consists of a series of four scintillation counters each preceded by two-radiation-length lead converters. Muons are discriminated from electrons on the basis of pulse height. A "muon" is defined as a particle which, for all four counters, gives a pulse height equivalent to that of a minimum ionizing particle, the acceptance ranging from 50% below to 50% above this central value. Measurements in an electron beam show that for the relevant range of decay electron energies, this condition rejects electrons by a factor of \( \sim 700 \). The muon detector is used to optimize the value of the focusing field for minimum muon losses, to test the scraping system, and to measure the loss correction \( F(t) \).

In the optimum running conditions very few muons were lost after 100 µsec. The above rejection ratio and the similarity in acceptance of the muon and electron detectors indicated that \( \geq 80\% \) of the "muon" counts after this time were misidentified electrons. This was confirmed with a small five-gap optical spark chamber placed behind the muon detector. Muons, identified by a straight track with four or five sparks, were found to form only 10% of the late time "muon" counts.

The correction function \( F(t) \) in Eq. (1) is given in terms of the lost muon time spectrum \( M(t) \) (corrected for decay electron background) by the integral

\[
F(t) = \frac{1}{\varepsilon t N_0} \int_{t}^{t_L} M(t) \exp(t/t) \, dt ,
\]

where \( t_L \) is the end of the storage period and where the factor \( \varepsilon \) is the ratio of the detection efficiencies of the muon and electron detectors; its value and hence the absolute calibration of the muon detector is obtained as follows. In data where the losses are relatively high (those taken without scraping in operation), \( F(t) \) can be deduced directly from the decay electron time distribution. Comparison with the values obtained from Eq. (2) provides the calibration of the muon detector and checks that it samples the lost muons in an unbiased way at different times.

This procedure and the effectiveness of the electric scraping are illustrated in Fig. 1. The points (a) represent the fractional excess decay electron counts at early time with respect to a fit to the unscraped data at late time \( (t > 300 \, \mu\sec) \). Taking 40 µsec after injection as \( t = 0 \), gives \( F(0) = 0.03 \) for these data. The
points (b) are those calculated from the lost muon data, and the time dependence of these two data sets are in excellent agreement. The points (c) are the early time excess counts for the scraped data obtained in just the same way as (a). The calibrated muon detector gave a value \( P(0) = 0.001 \) in this case, so the loss level at 40 \( \mu \text{sec} \) after injection is improved by a factor of \( \sim 30 \).

For the late starting times chosen in the analysis, the muon losses contribute at most a shift of 0.02% in the lifetime, as can be seen from the entries in Table 1.

3. PROTON BACKGROUND

For the \( \mu^+ \) data there is another complication in that the time-varying electric field associated with scraping enables protons to be stored in the ring. (The incident \( \pi \) beam is unseparated.) These protons are lost with a characteristic time distribution and so give background in both the decay electron and muon detectors. By assuming that the muon losses are the same for \( \mu^+ \) with identical machine parameters, the time distribution of these lost protons can (by subtraction) be found from the counts recorded in the muon detector. The relative detection efficiency of the muon and electron detectors for these protons is established in a separate experiment where the electric field is switched off \( \sim 1 \) msec after injection. At this time (\( \sim 15.5 \tau \)) only protons are stored, and the ratio of counts seen in the muon and the electron detectors directly gives the efficiency ratio. It turns out that the background level in the decay electron counts due to these "unstable" protons is low (\( \sim 3 \times 10^{-3} \) at 32 \( \mu \text{sec} \) after injection) so the resulting correction to the lifetime is small, as shown in Table 1. The contamination of the \( \bar{\mu} \) data by stored antiprotons was at a negligible level.

4. GAIN EFFECTS

The largest systematic effect which must be considered is the change in the energy acceptance of the decay electron detection system as a function of muon storage time. This effect is due to gain changes following the injection into the ring of one or more \( \sim 10 \) nsec wide bunches containing \( \sim 10^7 \) particles \(^a\); it is minimized by blanking off the dynode chain of the photomultiplier during, and for a few microseconds after, injection.

The system gain is measured as a function of time by means of light-emitting diodes (LEDs). The injected beam and all other conditions are the same as in normal data-taking. The signals from the LEDs are distinguished from decay electron counts by timing; a series of pulses are input to LEDs on the photomultipliers of the electron detectors at a number of fixed times throughout the muon storage interval. The output signal from each electron detector is split and input to two discriminators with a threshold separation of 3 dB. The LED pulse heights are adjusted so that all pulses fire the lower threshold discriminators and \( \sim 50\%

\(^a\) For more details of the effect of high particle flux on the gain of photomultipliers, see Ref. 9.
of the pulses fire the higher one, i.e. the threshold of the latter is at the
maximum of the LED pulse-height spectrum where the sensitivity to gain changes
is greatest. A subsidiary calibration measurement of the pulse-height distribu-
tion for each LED-photomultiplier combination is made to establish the quantita-
tive relation between the ratio of the counting rates output from the two dis-
criminators and the gain change $\Delta G$.

The gain curve is measured typically at 11 points between 7 and 580 $\mu$sec for
each counter, with a precision per point of $\sim 0.1\%$. From the gain curves thus
obtained, a counter selection is made to remove from the analysis counters with a
steep gain variation. These counters are the ones which are struck preferentially
by the injected beam. The mean gain curve of the remaining counters is typically
flat to within $\sim 0.1\%$ over the whole muon storage interval. The mean gain curve
$G(t)$ is multiplied by the factor $S = \Delta N/(N\Delta G)$, the fractional change of counting
rate of the decay electrons per unit gain change, to give the correction function
in Eq. (1). For the electron energy threshold chosen for the lifetime analysis,
$S = 1.00 \pm 0.02$.

The error in the gain correction was estimated by repeated measurement under
identical running conditions. From the consistency of the slopes of mean gain
curves measured at intervals of both a few days and several hours apart, this
error is conservatively estimated to be $\sim 0.03\%$ in $\tau$.

5. RESULTS

Values of the muon lifetime in flight $\tau$ were found by fitting the experimental
decay electron time distribution to the six-parameter function of Eq. (1) using the
maximum likelihood method and varying the six parameters $N_0$, $\tau$, $A$, $\omega_0$, $\phi$, $B_0$, where
$B_0$ is the time-independent part of $B(t)$.

The results obtained for the lifetime for the two $\mu^+$ and four $\mu^-$ runs which
were separately analysed are presented in Table 1. The uncorrected lifetime, and
the corrections with estimated systematic errors due to muon loss and gain effects,
are shown for each run. In addition, for the $\mu^+$ data, the proton background cor-
rection and error are shown. The starting times quoted for the fits vary from run
to run as, according to the beam intensity used, the muon detector took varying
times to regain full efficiency after injection. In all cases the upper limit of the
time range of the fit is 650 $\mu$sec.

The weighted average values of the lifetimes in flight for $\mu^+$ together with
the combined average are given in the first line of Table 2.

In the second line of Table 2 are given the corresponding lifetimes at rest
$\tau_0$ calculated via the Einstein\textsuperscript{2}) relation:

$$\tau_0 = \tau/\gamma, \quad \gamma = \left[1 - (v/c)^2\right]^{-\frac{1}{2}}.$$
The average value of $\gamma$ for the circulating muons is found by analysis of the bunch structure of the stored muons (see Fig. 2) over a period of $\sim 40$ $\mu$sec after injection. The mean rotation frequency $\bar{f}_{\text{rot}}$ is related to the average $\gamma$-value $\bar{\gamma}$ by the expression:

$$\bar{\gamma} = \frac{2\lambda f_p}{g f_{\text{rot}}} ,$$

where $f_p$ is the proton magnetic resonance frequency [corrected to vacuum], $g$ is the g-factor of the muon, and $\lambda = \mu_p / \mu$ is the ratio of the muon and proton magnetic moments. The result for $\bar{\gamma}$ is found to be:

$$\bar{\gamma} = 29.327(4) ,$$

where the quoted error corresponds to an uncertainty of $\pm 1$ mm in the mean radius of the distribution of circulating muons.

The last line in Table 2 gives the most accurate published values for the $\mu^+$ and $\mu^-$ lifetimes measured at rest. Comparing the high precision value of the $\mu^+$ lifetime at rest, $\tau_0^+$, with the value found in this experiment we obtain:

$$\frac{\tau_0^+ - \tau^+ / \bar{\gamma}}{\tau_0^+} = (2 \pm 9) \times 10^{-4} .$$

At 95% confidence the fractional difference between $\tau_0^+$ and $\tau^+ / \bar{\gamma}$ is in the range (-1.6 to 2.0) $\times 10^{-3}$. To date, this is the most accurate test of relativistic time dilation using elementary particles.

This result is compared in Table 3 with other limits on the validity of special relativity calculated from lifetime measurements given in the existing literature for $\pi^\pm$, $K^\pm$, $\nu_S^{\pm}$.

The entries in the table are the values of, and 95% confidence limits for $[\tau_0(\gamma_1) - \tau_0(\gamma_2)] / \tau_0(\gamma_1)$, where $\tau_0(\gamma_1, 2)$ are the lifetimes at rest as calculated via the Einstein relation using measurements on particles with mean $\gamma$-values $\gamma_1, 2$. The present experiment improves on the previously existing upper limits on violations of special relativity by about an order of magnitude.

The $\mu^+$ measurement differs in one important respect from the other results quoted in Table 3. The $\pi^\pm$, $K^\pm$, $\nu_S^0$ measurements in flight were performed in beams for which the decaying particle, regarded as a clock, was in an inertial (unaccelerated) frame. In the muon experiment, however, the particles are subjected to a constant transverse acceleration of $10^{21}$ cm sec$^{-2}$. The muons perform a round trip and so when compared with a muon decaying at rest in the laboratory, simulate closely the
so-called twin paradox which was already discussed in Einstein's first paper\textsuperscript{2}).

Other experiments\textsuperscript{21,22}) have confirmed the correctness of relativistic time dilation for clocks in circular motion to \(\sim 10\%\) at low velocities\textsuperscript{*}). The present experiment is unique in its use of an ultra-relativistic clock (\(\gamma \gg 1\)) and its much greater precision \(\sim 0.1\%\).

The possibility also exists that very large accelerations may modify in some way the internal constitution of particles\textsuperscript{25-28}). No such effects, in so far as they affect the particle lifetime, are seen in this experiment where the transverse acceleration is \(\sim 10^{18}\text{g}\).

Some authors\textsuperscript{29,30}) have suggested that just as special relativity breaks down at very large distances owing to gravitational effects, there may also be a breakdown below some fundamental distance \(\alpha\). Rédei\textsuperscript{31}) has calculated the effect of such a breakdown on the muon lifetime in flight, and finds a \(\gamma^2\) correction to the Einstein formula:

\[
\tau = \gamma \tau_0 (1 + 2.5 \times 10^{24} \gamma^2 \alpha^2),
\]

where \(\alpha\) is in centimetres. From our value of \(\tau^+\) a limit can be set on \(\alpha\):

\[
\alpha \leq 9.6 \times 10^{-16} \text{ cm (95\% confidence level).}
\]

Assuming the correctness of special relativity, the \(\mu^-\) lifetime at rest \(\tau^-\) given in Table 2 is the most precise value so far reported. Comparing this value with the high-precision measurement of \(\tau_0^+\) it is found that

\[
\frac{\tau_0^- - \tau_0^+}{\tau_0^+} = -0.00105(46).
\]

Or, at 95\% confidence level, the fractional difference of \(\tau_0^-\) and \(\tau_0^+\) is in the range

\[-0.002\text{ to }-0.00013.\]

The limit set on this quantity, which should vanish\textsuperscript{32}) by CPT invariance of the weak interaction, is comparable to that set by the best direct measurement\textsuperscript{3}) of the ratio \(\tau_0^-/\tau_0^+\).

\textsuperscript{*}) A similar conclusion follows from observations of the second-order (transverse) Doppler effect in the temperature dependence of the Mössbauer effect\textsuperscript{23}) and of the hydrogen maser frequency\textsuperscript{24}).
Acknowledgements

We would like to express our warm appreciation to those who have contributed to this measurement of the muon lifetime. In particular, we wish to thank F. Wickens for early calculations on the electric field configuration; E.M. McMillan for studies of various mechanisms of muon losses; R.W. Williams for analysis of the muon losses from the decay electron data; J. Lindsay for the preparation of an excellent blanking scheme for photomultipliers and S. Wojcicki for his early contributions to the gain measurements.
REFERENCES


2) A. Einstein, Ann. Phys. (Germany) 17, 891 (1905).


15) K. Borer and F. Lange, Absolute calibration of the NMR magnetometers and of the magnetic field measuring system used in the CERN Muon Storage Ring, to be published in Nuclear Instrum. Methods.


Table 1

<table>
<thead>
<tr>
<th>Run</th>
<th>Charge of ( \mu )</th>
<th>Starting time of fit</th>
<th>Uncorrected lifetime (statistical error)</th>
<th>( \mu )-loss correction</th>
<th>p-background correction</th>
<th>Gain correction</th>
<th>Fully corrected lifetime (total error)</th>
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<td></td>
<td>+</td>
<td>+</td>
<td>64.406(66)</td>
<td>64.295(66)</td>
<td>64.444(55)</td>
<td>64.366(84)</td>
<td>64.278(60)</td>
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<td>50</td>
<td>64.336(36)</td>
<td>64.009(8)</td>
<td>0.000(8)</td>
<td>64.306(64)</td>
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<td></td>
<td></td>
<td>90</td>
<td>64.312(8)</td>
<td>0.012(8)</td>
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Results for the muon lifetime in flight. All entries in usec.
### Table 2
Lifetime results ($\mu$sec)

<table>
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<tr>
<th></th>
<th>$\mu^+$</th>
<th>$\mu^-$</th>
<th>Weighted average $\mu^+ + \mu^-$</th>
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<tbody>
<tr>
<td>Lifetime in flight</td>
<td>64.419(58)</td>
<td>64.368(29)</td>
<td>64.378(26)</td>
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<td>(this experiment)</td>
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<td></td>
<td></td>
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<tr>
<td>Lifetime at rest</td>
<td>2.1966(20)</td>
<td>2.1948(10)</td>
<td>2.1952(9)</td>
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<tr>
<td>(this experiment)</td>
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<td></td>
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<tr>
<td>Lifetime at rest</td>
<td>2.1971(8)</td>
<td>2.198(2)</td>
<td>-</td>
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<td>(previous best measure-</td>
<td></td>
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<td>ments, Refs. 13 and 3)</td>
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Table 3
Tests of relativistic time dilation from particle lifetime measurements. Values of $\tau_0 = \tau/\gamma$ are compared at different $\gamma$ values

<table>
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<th>Particle</th>
<th>References</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$[\tau_0(\gamma_1) - \tau_0(\gamma_2)]/\tau_0(\gamma_1)$ (per cent)</th>
<th>95% confidence limits (per cent)</th>
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<td>$\pi^\pm$</td>
<td>14, 15</td>
<td>1.0</td>
<td>2.44</td>
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<td>-4.3 to -0.7</td>
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<td>$K^-$</td>
<td>16, 17</td>
<td>1.0</td>
<td>3.38, 4.17</td>
<td>0.9 ± 0.3</td>
<td>0.3 to 1.5</td>
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<td>$K_S^0$</td>
<td>18, 20</td>
<td>1.63</td>
<td>15.2</td>
<td>0.5 ± 0.6</td>
<td>-0.7 to 1.7</td>
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<td>$K_L^0$</td>
<td>18, 19</td>
<td>1.63</td>
<td>20.2</td>
<td>0.2 ± 0.7</td>
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<td>$\mu^+$</td>
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<td>29.3</td>
<td>0.02 ± 0.09</td>
<td>-0.16 to 0.20</td>
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Figure captions

Fig. 1 : Ratio of the decay electron time distribution (obs) to an exponential fit (fit) in the time range 300-650 μsec. The g-2 modulation is included in the fit. The excess counts at early times over the extrapolated fit function show the effect of muon losses. (a) obs/fit no scraping. (b) muon loss function as calculated from the muon detector time spectrum. (c) obs/fit with scraping.

Fig. 2 : Bunch structure in the muon storage ring from 6-10 μsec after injection. Each bunch corresponds to one turn in the ring. Such distributions are analysed to give the mean rotation frequency \( \tilde{f}_{\text{rot}} \) of the stored muons, and hence the mean \( \gamma \)-factor \( \bar{\gamma} \).
Fig. 1
Fig. 2