GROUP THEORETICAL METHODS

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I. DEFINITIONS

a) General requirements

Let me get rid quickly of generalities. Groups that have been investigated in the last years are practically all Compact Lie Groups, which means among other things, that they are continuous and described by unitary matrices $U^\dagger U = 1$. We have then no problem as to the invariance of the free field theory. This, moreover, is a nice restriction in practice for it turns out that we are left with a very limited choice among groups or families of groups so that the problem of which one, if any, applies to physics can, in principle, be solved by successive elimination.

A relatively minor point is unimodularity: should we require $\det U = 1$ or not? I have no time to discuss this but in practice it makes little difference, so you can do as you like. The golden rule here is to minimize the mathematics involved. If one does require $\det U = 1$ (and this is usually, a simplification) then it turns out that the baryon number $B$ cannot be incorporated in the group and that its conservation must be treated as a separate postulate.

b) Definitions

Let us take isospin group as an example (which corresponds to $O_3$, i.e., to 3 dimensional rotations). You have there a series of representations...
As for the group itself it has a certain number of parameters which is also, of course, the number of its infinitesimal generators \( X_K \) (\( X_K = I_K \) for isospin). This number is 3 in the \( O_3 \) case, 6 in the \( O_4 \) case, 8 in \( SU_3 \) case, etc. All this is familiar.

Less familiar but extremely important for us are the following concepts:

i) the rank of the group: it is the number of independent additive quantum numbers whose conservation is implied by the invariance under the transformations of the group. For instance, isospin is a group of rank 1 because there is only one such number, namely \( I_3 \) (\( I \) is not additive in our sense: it is the length of a vector).

ii) the regular representation of the group.

Let \( \psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix} \) be any isospin multiplet \( (N, \vec{n}, \text{etc.}) \). Consider the three entities

\[ (\psi, I_K \psi). \]

These objects together form a triplet, i.e., constitute a basis for the representation 3 of the group. The representation that can be constructed by this procedure is called the regular (or "adjoint") representation. Its dimension is, of course, equal to the number of parameters in the group.
In the case of a larger group than isospin and of which isospin is a subgroup, the representations are associated with supermultiplets. Each supermultiplet is made, of course, by the association of isospin multiplets.

Table I gives some information pertaining to useful groups. $SU_3$ is the group of three by three unitary

<table>
<thead>
<tr>
<th></th>
<th>number of parameters</th>
<th>rank</th>
<th>dimensions of representations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SU_2 \cong O_3$</td>
<td>3</td>
<td>1</td>
<td>$1, 2, \frac{3}{2}, 4, 5, \ldots$</td>
</tr>
<tr>
<td>$SU_3$</td>
<td>8</td>
<td>2</td>
<td>$1, 3, \frac{8}{2}, 10, \ldots$</td>
</tr>
<tr>
<td>$O_5$</td>
<td>10</td>
<td>2</td>
<td>$1, 4, 5, \frac{10}{2}, \ldots$</td>
</tr>
<tr>
<td>$G_2$</td>
<td>14</td>
<td>2</td>
<td>$1, 7, \frac{14}{2}, \ldots$</td>
</tr>
</tbody>
</table>

Table I

unimodular matrices; $G_2$ is the first of Cartan’s exceptional groups, $O_n$ is the group of rotations in an $n$-dimensional space.

c) Partially conserved currents and vector mesons

Let $X_1 \ldots X_K \ldots$ be the infinitesimal generators of a group. If the Lagrange function $L$ is invariant under that group, it is easily shown that the currents

$$J^{(k)}_{\alpha} = \sum_{i} \left\{ \bar{\psi}_i X_\alpha \gamma_\mu \psi_i + i\left[ \bar{\psi}_i X_\alpha \gamma_\mu \psi_i - (\partial_\mu \bar{\psi}_i X_\alpha \psi_i) \right] \right\}$$

are conserved, i.e., satisfy

$$\frac{\partial J^{(k)}_{\alpha}}{\partial X_\alpha} = 0$$

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(here the summation $\sum_i$ extends over all supemultiplet fields $\psi_i, \bar{\psi}_i$ that appear in $L$). In the case of isospin these $3$ $J$'s are the familiar conserved vector currents. In the case of a larger group it is immediately seen that the number of such $J$'s is just the number of parameters of the group and that indeed by construction they are in the regular representation of the group.

It is then an easy matter to show $^1$ that by commuting these currents with each other at equal time, one reconstructs the commutation relations of the $X_k$'s, i.e., the algebra of the group.

Going further in the same direction one may associate vector mesons with the partially conserved currents. This is achieved, in the same way as in the usual electromagnetic gauge theory, by making the parameters of the group depend on the position (the so-called Yang-Mills trick $^2, ^3$, generalized to groups larger than isospin $^1, ^4, ^9$). These vector mesons are then, of course, in the regular representation also. We do not know, unfortunately, the true significance of the Yang-Mills trick, and its relation to quantized fields and to massive vector bosons has been questioned for some time — and also, with fresh arguments, by Ogievetskij and Polubarinov and by J.C. Taylor at this conference.
II. FRONTDOOR AND BACKDOOR

I claim that there are, in fact, not just one but two doors, at least, by which group theory can hope to enter the domain of elementary particle physics. The frontdoor, say, is strong interactions and the backdoor is weak interaction currents but you may, of course, exchange these appellations. I shall try to described what one sees when one enters through each of these doors and ask the important question: are they doors to the same house or to different houses? But let us first compare them as to their principles.

a) The approach through strong interactions

The idea here is that for some reason or other the strong interactions might be very roughly invariant under some group larger than the isospin group. Of course, many of the more quantitative consequences of this invariance, such as equalities between masses, cross-sections, etc., may be spoiled by the fact that it is so rough and approximate. But if we want the model to be useful we must set a kind of minimum requirement. For instance, we may say that whenever the group leads to a qualitative prediction, whenever it forbids a particular reaction, then this reaction should, in the actual world, be less frequent than an allowed reaction with the same kinematics.

If even that were not true, nothing would remain.

Now let us assume that each physical particle is, at least to a good approximation, a member of some definite representation. Each quantum number introduced by the group invariance has then immediate physical significance. Now it is well known that the strong interactions show no sign of conserving any additive quantum number other than $I_3$ and hypercharge $U$ (apart from $B$ already discussed). For this reason we get an imperative requirement:

the group should be of rank 2
There are only four such groups

\[ 0_4 \quad \text{SU}_3 \quad 0_5 \quad 0_2 \]

so that we have a limited choice and life should be easy.

Let me open a parenthesis here: because of the fact that the group symmetry is badly violated, it might happen that some of the physical particles ride over two representations. Then the group would not need to be of rank two and, as pointed out by Lee and Yang \(^{5}\), this would be a way to save global symmetry. I cannot discuss these schemes today and so I want to make the explicit assumption that this phenomenon does not occur.

So really we have the situation mentioned above. I now close the parenthesis.

b) The approach through the weak interaction currents

It is due to Gell-Mann and its principle is entirely different. There we take the weak vector currents, which in principle are measurable quantities, and commute them with each other at equal times. This gives us new currents which, in turn, we commute with the old ones, and we pursue these operations until they give us no new currents. If nature is kind to us this will happen after a finite number of operations. What we have constructed is then a Lie algebra corresponding to a group with a finite number of parameters. So this is a way to introduce a group which is practically independent of the strong interactions, and which, therefore, these interactions may violate as much as they like. The group thus introduced has no compelling reason to be of rank 2.

On the other hand it is easily shown that, by construction, the group admits the weak vector currents in its regular representation. Now, this turns out to be a very stringent condition: this is because the weak vector currents begin to be well known. Recent experiments by the Wisconsin and Padua groups \(^{6}\) and by others \(^{7}\) have established the existence of both \( |\Delta I| = 3/2 \) and

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\[ |\Delta I| = 1/2 \text{ vector currents.} \]  
(The existence of \[ |\Delta I| = 1/2 \text{ axial currents} \]  
is well-known from \( K^+ \) decay, but this is of course a completely disconnected 
 piece of information.)

c) Same house or different houses ?

If, by chance, these two approaches led us to the same group they would reinforce each other and it would be quite gratifying. So, do they ?
Well, we have to inquire which ones, if any, of the four groups listed in a) have both \[ |\Delta I| = 3/2 \]  and \[ |\Delta I| = 1/2 \]  currents in their regular representations ? The answer to this question is that unfortunately none of them has.

Well, let us not be taken aback ! The principles of these two approaches are very different after all, so that nothing tells us that they should both stand or fall together, nor that, if they both stand, they should necessarily lead to the same groups. They may be doors to different houses after all. Anyhow we are entitled to examine them separately.
III. THE "WEAK INTERACTION CURRENTS" APPROACH

Table II classifies groups according to the isotopic spin content of their regular representations. In this table we meet many old friends.

| $|\Delta I|$ | 2 | 3 | 4 | ... |
|----------|---|---|---|-----|
| 3/2      | $G_2$ | $O_7$ | $O_8$ | $SU_3 \times G_2$ |
| 1/2 and 3/2 | none | | | $O_5 \times G_2$ |

Table II

The first column has been included for completeness, and also because, in my opinion, the existence of the $|\Delta I|=1/2$ vector currents is experimentally established on less firm grounds than the existence of $|\Delta I|=3/2$ currents. If, however, we take the experimental data at their face value we may forget this column. The next bet is then $O_7$. This group has been considered a long time ago by Tjonno $^8$, Behrends $^9$, Feaslee $^{10}$ and many others. Of course, $O_7$ would not work for the strong interactions but one might speculate that perhaps the strong interactions know only a subgroup of $O_7$. Then one is led to $G_2$ again. The importance of $G_2$ in connection with weak and strong interactions was first stressed by Behrends and Sirlin $^{11}$. $SU_3$, unfortunately, is a subgroup of $O_7$ but not with the correct isotopic spin assignments.

Gürsey $^{12}$ has suggested some time ago a model based on $O_8$. This model is very attractive because it gives not only the vector currents but also the axial vector currents, with definite predictions as to their isotopic content. Here also a strong interaction subgroup would be $G_2$.  

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All these models have this point in common that, along with the \( |\Delta I| = 3/2 \) vector currents, they require \( |\Delta S| = 2, \Delta I = 0 \) vector currents. The existence of

\[
\Xi \rightarrow N + e + \nu
\]  

(1)
is thus made extremely likely. The Gürsey model makes the additional prediction that there are neither \( |\Delta I| = 3/2 \) nor \( |\Delta S| = 2 \) axial currents. The angular correlations in (1) or the branching ratios in \( K \ell_4 \) could help testing this\(^{13}\).

In principle we should, of course, go on exploring higher rank groups as to the isotopic spin content of their higher representations. This, however, has not yet been done.
IV. THE "STRONG INTERACTION" APPROACH

a) Possible relation with $S$ matrix theory

Lagrangians having fallen into discredit, we badly need a way to
describe our approximate symmetries in some modern language. The paper presented
by Cutkosky, Kalckar and Tarjanne at this conference may perhaps be considered as
a hint in that direction.

For their investigation they use the group $SU_n$, with $n$ arbitrary.
They show by means of a typical "bootstrap" calculation that it is self-consistent
to assume that the baryons are in the so-called "fundamental" representation –
of dimension $n$ – and the vector and pseudoscalar mesons in the regular
representation.

For that purpose they first assume that such mesons exist. They then
investigate what force is created between a baryon and an antibaryon, or between
two bosons, by the exchange of these mesons. They find it is attractive and the
strongest if, precisely, the system is in a state corresponding to the regular
representation. They thus create the mesons and in particular the vector mesons
whose existence has been assumed for the proof, and find self-consistency as
regard to the assignment of particles to representations.

The feasibility of such calculations is not, of course, limited to
$SU_n$ and to these assumptions and, in fact, the authors also get interesting
results for the octet model in $SU_3$ and for other groups.

Miyamoto has done something rather similar, which, however, is more in
the original spirit of the Sakata model in that he shows some preference for
treating the mesons as composite and the baryons as elementary. The force which
binds baryons and antibaryons is also attributed principally to one of the vector
mesons which is to emerge from the model, so that again we have a self-consistent
theory and Miyamoto has calculated what should be the coupling constants in order
that the distribution of masses should be that which we observe.
b) Weak interactions

The group being defined through the strong interactions, we have here a complete freedom to put the weak interaction currents in any representation we like. Now, Radicatti, Ruegg and Speiser, in a contribution to this conference, suggested that we should put some limits to this freedom by using the following idea: if a weak current coupled to leptons is made a member of a given representation, then all the other members of this representation (at least those having the same $\Delta Q$) should participate also in the leptonic weak interactions. This is a very mild requirement (which, in fact, we have been using already) and it is interesting that it leads them all the same to one nice prediction, that again if $\Delta S = -\Delta Q$ currents exist, then $|\Delta S| = 2$ currents should also show up. In other words group theory, whichever way you look at it, always seems to indicate that reaction (1) is very likely to take place with appreciable probability.

c) Possible symmetry schemes

They are given, together with the group they belong to, by Table III, which also lists the names of the principal contributors:

<table>
<thead>
<tr>
<th>$\Sigma_2$</th>
<th>$\Sigma_3$</th>
<th>$\Sigma_5$</th>
<th>$\Sigma_G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega$</td>
<td>(N)</td>
<td>(N)</td>
<td>(N)</td>
</tr>
<tr>
<td>$N - \Xi$</td>
<td>(Sakata)</td>
<td>Octet</td>
<td>(E)</td>
</tr>
<tr>
<td>Symmetry</td>
<td></td>
<td></td>
<td>(E)</td>
</tr>
<tr>
<td>Salam and</td>
<td></td>
<td></td>
<td>(E)</td>
</tr>
<tr>
<td>Polkinghorne</td>
<td></td>
<td></td>
<td>(E)</td>
</tr>
<tr>
<td>14)</td>
<td></td>
<td></td>
<td>(E)</td>
</tr>
<tr>
<td></td>
<td>Ohmuki et al. 15)</td>
<td>Ne'eman 20)</td>
<td>Behrends</td>
</tr>
<tr>
<td></td>
<td>Wess 16)</td>
<td>Gell-Mann 21)</td>
<td>Drittelein</td>
</tr>
<tr>
<td></td>
<td>Yamaguchi 17)</td>
<td></td>
<td>Fronsdal</td>
</tr>
<tr>
<td></td>
<td>Okun 18)</td>
<td></td>
<td>and</td>
</tr>
<tr>
<td></td>
<td>Salam and Ward 19)</td>
<td></td>
<td>B.W. Lee 22)</td>
</tr>
<tr>
<td></td>
<td>Gell-Mann 1)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table III
I shall not discuss the first of these schemes. It is a good scheme and nothing serious can be said against it, except that it is just too old. It has faded out of actuality.

d) $SU_3$ : general

This is, as you all know, the pet group of theoreticians.

The Sakata model puts the three fundamental baryons, $p$, $n$, $\Lambda$, in the "$3$" representation and the mesons are then in the "$\bar{3}$". The Octet Scheme puts everything in the "$\bar{3}$". Both schemes, therefore, predict the same things for mesons, which is essentially that mesons of the same spin and parity should appear in octets, as in Table IV.

<table>
<thead>
<tr>
<th>I</th>
<th>U</th>
<th>$^0$</th>
<th>$^-$</th>
<th>$^-$</th>
<th>$^+$ ???</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$\eta$</td>
<td>$\omega$</td>
<td>($K^0 \bar{K}^0$)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>$\pi$</td>
<td>$\rho$</td>
<td>$\zeta$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>$K$</td>
<td>$K^*$</td>
<td>$K^*$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>-1</td>
<td>$\bar{K}$</td>
<td>$\bar{K}^*$</td>
<td>$K^*$</td>
<td></td>
</tr>
</tbody>
</table>

Table IV

This fits nicely with experiment. What is more, a remarkable relation seems to hold between the masses of the particles in a given octet. This famous "mass relation" \(^1\),\(^23\) is the special beauty of $SU_3$. It was derived in a first order perturbation calculation, however, so that nobody understands why it is so good.

Finally, notice that the vector mesons are in the regular representation. This is what should be if we believe in the generalized Yang-Mills trick.
e) Sakata model

Up to here, then, the Sakata and Octet models are on the same footing. The Sakata model, however, makes more predictions. This is very nice in principle but it is also dangerous. It makes it more vulnerable.

And here I have to bring you sad news: that the Sakata model is presently in a bad shape. Let me just present you the facts.

The experimental facts are due to the Ecole Polytechnique and CERN groups working here on $\bar{p}p$ annihilation at rest. The data recently communicated to me by Morrisson are as follows:

\[
\begin{array}{c}
\pi^+ \pi^- & K^+ K^- & K^0\bar{K}^0 \\
6 & 2 & 1
\end{array}
\]

Here we are especially interested in the last two modes whose kinematics are practically identical. And now I disclose the awful truth which is that Levinson, Lipkin, Meshkov, Salam and Munir\(^{24}\) have shown that in the Sakata model the last reaction is forbidden.

Now this looks very serious. Why have we been rejecting (in this strong interaction approach) groups of higher ranks? Just because they lead to selection rules that nature does not seem to follow (remember for instance the famous Pais remark about doublet symmetry.

\[
\begin{array}{c}
Z^+ \pi^- & \Sigma^- \pi^+ \\
\exp : & 1 & 1
\end{array}
\]
Well, here we have, precisely, a forbiddenness of that sort and, as you see, nature does not follow it either so that it seems consistent to reject the scheme. Still, I do not want to be too drastic. Perhaps we are not at high enough energies. I just want to stress that in order to make up our mind completely, we ought to know the spin of the $\Xi$. This is because in the Sakata model the only nice place to put the $\Xi$ is together with the pion nucleon 3-3 resonance, and this implies, of course, that it should have spin 3/2, whereas in the other models it is associated with the nucleon and has spin $\frac{1}{2}$.

f) Scheme connected with $O_5$

I shall return to the octet in a moment. Let me first get rid of the schemes connected with $O_5$.

The first one ($N \Lambda \Xi \Sigma$) is very easy to get rid of. It resembles the Sakata model in that it may also be viewed as a composite model, but with $N$ and $\Xi$ playing a more symmetrical role. It turns out that it should follow the same fate: one can easily prove that this scheme also forbids $\bar{p}p \rightarrow K^0 K^0$.

The second scheme associates $N$ and $\Xi$, and also $K$ and $\bar{K}$, inside the 4 dimensional representation. The situation here is somewhat more tricky. First, let me tell you that this is a good scheme. Some of its virtues are: a) that it does not require the pions and kaons to have the same mass and, b) that it very naturally forbids $X_1^* \rightarrow \Sigma + \Pi^0$. It may, however, be found at fault because it forbids

$$\bar{p} + p \rightarrow K^0 + K^0 + x \Pi^0 \; ; \; x \; \text{arbitrary}.$$

Now this is very easy to check and at first sight the experimental situation looks rather bad. The CERN and Ecole Polytechnique data, kindly communicated to me by Armenteros, give about a hundred of such events. Before drawing a conclusion one should, however, a) compare with the corresponding
data on charged $K$, b) try to keep only really high energy events. I would not, therefore, conclude that this scheme is disproved but I would like to conclude that we have the means to disprove it and that we definitely should do so.

g) $G_2$ and octet

Let us, for simplicity, assume the result to be negative. Then, at the end of this unfortunately most incomplete survey, we are left with two models

\[ \text{octet and } G_2 \]

between which we have to choose at the most one.

To guide us in this choice the mesons are of no great help unless we believe in the Yang-Mills trick. If we do, then $G_2$ is bad because the vector mesons should then sit in the regular representation 14, which, as we have already seen, contain no $I = \frac{1}{2}$. But if we do not, then all the eight mesons families can, of course, be split into singlets and triplets: for this, one has just to replace the dotted line by an unbroken line in Table IV.

We may then think of looking at the baryon resonances. Now I believe that in the future, when we know the full story about them, they can help us in checking these groups. Up to now, however, it turns out that you can fit them in several ways and in several schemes. For that reason, as well as for lack of time, I shall not discuss them in the talk.

Finally, we may think of looking at high energy, high momentum transfer limits in cross-sections, form factors and the like. Here, in the absence of any reliable theory, let me take the naïve approach that when energies and so on
are orders of magnitude higher than the mass differences, the group symmetries might show up better. If this is the case, then Table V gives some simple predictions common to the octet and $G_2$ and some simple predictions from $G_2$ alone (I do not know of any simple prediction from the octet that would not also follow from $G_2$).

<table>
<thead>
<tr>
<th>Some high energy predictions from</th>
<th>$G_2$ alone</th>
</tr>
</thead>
<tbody>
<tr>
<td>both Octet and $G_2$</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{K^+} = \sigma_{\pi^-}$ (tot. cross-sections, Ref. 25)</td>
<td></td>
</tr>
<tr>
<td>$\frac{\sigma(K^- \rightarrow \Xi^- K^0)}{\sigma(K^- \rightarrow \Sigma^- n^+)} = 1$ (Ref. 25)</td>
<td></td>
</tr>
<tr>
<td>$\frac{\sigma(pp \rightarrow \Xi^- \Xi^0)}{\sigma(pp \rightarrow \Sigma^- \Sigma^-)} = 1$</td>
<td></td>
</tr>
<tr>
<td>$\frac{\sigma(\Sigma^+_p \rightarrow \Sigma^+_n \pi^+)}{\sigma(\Sigma^+_p \rightarrow p \Xi^0 K^+)} = 1$ (Ref. 26)</td>
<td></td>
</tr>
<tr>
<td>$\frac{\sigma(K^- \rightarrow \Xi^- K^0 + x \Lambda^0)}{\sigma(K^- \rightarrow \Xi^- n^+ + x \Lambda^0)} = 1$</td>
<td></td>
</tr>
<tr>
<td>$\sigma(K^- \rightarrow nK^0 + x \Lambda^0)$</td>
<td></td>
</tr>
<tr>
<td>$\sigma(K^- \rightarrow \Sigma^0 K^- + x \Lambda^0)$</td>
<td></td>
</tr>
<tr>
<td>$\frac{\sigma(pp \rightarrow \Xi^- \Xi^0 \Xi^- + x \Xi^0)}{\sigma(pp \rightarrow \Sigma^- \Sigma^- + x \Xi^0)} = 1$</td>
<td></td>
</tr>
<tr>
<td>$\frac{\sigma(pp \rightarrow K^0 K^- + x \Lambda^0)}{\sigma(pp \rightarrow \Xi^- + x \Lambda^0)} = 1$</td>
<td></td>
</tr>
<tr>
<td>etc...</td>
<td></td>
</tr>
</tbody>
</table>

Table V

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V. CONCLUSIONS

As you see, the $G_2$ group is nice in that it comes out in the open so to speak, with definite predictions of its own, that we may test. Under the assumption, therefore, that this general approach is right we may hope to find, eventually, what is the group.

The octet model on the other hand is a shy person who does not want to commit himself \(^{27}\). We can force him to speak out his mind, however, by looking at these difficult things that appear in Table V, 1st column, or by looking at the baryon resonances when we know more about them. I have no time to enter into this subject, but I would just like to say that things look rather nice there. In particular, this new $\Xi^\ast$ seems to fit nicely in. (See Gell-Mann's remark after Snow's report.) Anyhow, let me just tell the experiment- alists that, whichever way we look at our groups, we always expect several $\Xi^\ast$. So we should thank them for having produced one and ask them to, please, produce some more.
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