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MINIMAL INTERACTIONS AT VERY HIGH TRANSFERS

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CM-P00056982

A B S T R A C T

It is shown that both large transfer electron-proton scattering and large angle proton-proton scattering may be consistent with, or close to, the lower limits obtained from dispersion theory.

For form factors the fastest allowed decrease is $\exp -\sqrt{|q^2|}$ where q is the four-momentum transfer of the photon. For scattering amplitudes one gets $\exp -C(\theta) \sqrt{s}$ or $\exp -C(\theta) \sqrt{s} \log s$ or $\exp -C(\theta) \sqrt{s} (\log s)^2$ according to various detailed assumptions, where s is the square of the c.m. energy and θ the c.m. scattering angle.

The conditions to impose on the growth of the form factors and the scattering amplitude outside the physical region to obtain these results are considerably weaker than the usual polynomial boundedness condition, and are likely to be satisfied by the actual form factors or scattering amplitudes.

I. INTRODUCTION

Experiments on high energy proton-proton scattering at large angles have shown that the fixed angle, differential, cross-section decreases very fast with energy ¹⁾. In particular, a very good fit of all existing data has been obtained by Orear ²⁾ in the form

$$\frac{d\sigma}{d\Omega} \sim \exp - c k_T \quad (1)$$

where $k_T = k \sin \theta$, k c.m. momentum and θ c.m. scattering angle. It has been noticed by Kinoshita ³⁾ that this is not very far from the lower limit obtained from the Mandelstam representation by Cerulus and the author ⁴⁾ which, at least for θ between 0 and 60° , behaves like

$$\exp - [c k_T \log s/s_0] \quad (2)$$

This, of course, is an asymptotic statement with all the defects attached to it. In particular s_0 is unknown. However, if one adjusts s_0 to be a sufficiently small value, one can manage to fit the data slightly better with Eq. (2) as with Eq. (1). We shall see later that the correct lower limit is in fact certainly higher than (2).

On the other hand, the behaviour (1) has been explained by Hagedorn and Fast ⁵⁾ on the basis of a statistical model.

Now, recently, Yang and Wu ⁶⁾ have speculated, again on the basis of statistical considerations, that the nuclear electromagnetic form factor should decrease as fast as $\exp - \sqrt{|q^2|}$ where q^2 is the square of the four momentum transfer and have shown that indeed this gives a very reasonable fit of the data, in particular the recent high q^2 measurements. So, according to their interpretation large angle scattering and high transfer form factors are linked together.

Here we want to present an alternative interpretation that both large angle scattering and high q^2 form factors are illustrations of "minimal interaction" according to the denomination of Kinoshita³⁾. The treatment of form factors is in fact considerably simpler than that of the scattering amplitude and so we shall start with form factors.

II. FASTEST ALLOWED DECREASE OF FORM FACTORS

Any nucleon form factor $f(t)$ where t is the square of the momentum transfer (t negative in the physical region) is generally assumed to be analytic in a cut plane with a cut from $t = 4\mu^2$ (μ = pion mass) to $t = +\infty$ with a finite number of subtractions, that is to say that outside the physical region $f(t)$ is bounded by $C|t|^N$. This latter assumption is always unpleasant but, fortunately, we shall see that it can be considerably weakened. More precisely, we shall only require this :

1) in any direction of the t plane $f(t)$ does not increase faster than $\exp C|t|^{\frac{1}{2}}$;

2) on the cut

$$\int_{4\mu^2}^{\infty} \frac{\log^+ |f(t)| dt}{(1+t)^{3/2}}$$

converges, where

$$\log^+ |f(t)| = \log |f(t)| \quad \text{for } |f(t)| \geq 1$$

$$= 0$$

$$\text{for } |f(t)| \leq 1$$

This means, for instance, that if $f(t)$ is less than

$$\exp \left[\frac{C|t|^{1/2}}{(\log t)^{1+\varepsilon}} \right]$$

condition 2) is satisfied.

Then according to a theorem by Boas ⁷⁾ such a function cannot decrease faster than $\exp -C|t|^{\frac{1}{2}}$ for $t \rightarrow -\infty$. For those who are not familiar with these theorems it is enough to try functions like $\exp -(4\mu^2 - t)^\alpha$. Then for $t > 4\mu^2$ the modulus of the function is given by $\exp [-\cos(\alpha\pi) (t - 4\mu^2)^\alpha]$ for $\alpha \leq \frac{1}{2}$ the modulus is bounded on the cut, for $\alpha > \frac{1}{2}$ the modulus of $f(t)$ increases faster than $\exp C(t - 4\mu^2)^{\frac{1}{2}}$, $C > 0$ which is rejected.

Now, the minimal interaction assumption consists in saying that $f(t)$ has the fastest decrease allowed in the physical region, i.e. :

$$f(t) \sim \exp -C|t|^{\frac{1}{2}} \quad (3)$$

which is precisely the behaviour proposed by Yang and Wu. Then, if such is the case, the discontinuity across the cut must have a rather complicated behaviour. First, it must oscillate infinitely many times, at least in the case where the function is bounded by a polynomial because otherwise, as has been shown by Y.S. Jin and the author ⁸⁾, $f(t)$ cannot decrease faster than t^{-N-1} where N is the number of oscillations. Whether this shows a connection with the statistical approach or not will be discussed in the last Section. Here we want only to emphasize that if the decrease at large t is "minimal" it is hopeless to simulate the left-hand cut by a few poles except perhaps for small t , where only the nearest oscillations will be felt, the further away oscillations averaging to zero.

Second, we notice that the discontinuity on the cut, i.e., the various intermediate states contributing to the form factors, determines completely the amplitude. Indeed, let us consider two form factors $f_1(t)$ and $f_2(t)$ having the same left-hand cut. Then $f_1(t) - f_2(t)$ is an entire function which decreases like $\exp -|t|^{\frac{1}{2}}$ for $t \rightarrow -\infty$ and increases at most like $\exp |t|^{\frac{1}{2}}$ in any complex direction. Such a function is identically zero ⁹⁾. So there is no arbitrary subtraction constant (if the number of subtractions is finite) and more generally no arbitrariness. The discontinuity on the cut fixes, for instance, the electric charge or the magnetic moment according to the form factor one considers. This, of

course, does not mean that in low q^2 fits the electric charge or the magnetic moment should not be forced to have the correct value by means of a subtraction because then only the nearest contribution of the cut is taken into account.

Finally, let us notice that this very fast decrease is allowed only because the sign of the spectral function of the form factor (in contrast with the case of the propagator) is not necessarily positive. As was noticed for instance by Salam ¹⁰⁾, the fast decrease of form factors is the only hope to make converge theories which are superficially divergent.

Let us now discuss shortly what happens in the limiting case : on the cut $f(t)$ is bounded only by $\exp C t^{\frac{1}{2}}$ so that condition 2) is violated, and outside the cut $f(t)$ is bounded by $\exp C |t|^{\frac{1}{2}}$. Then the result is slightly weakened and $f(t)$ is allowed to decrease as fast as

$$\exp - \left[|t|^{\frac{1}{2}} \log |t| \right] \quad (4)$$

for $t \rightarrow -\infty$.

Proof :

Suppose that $f(t)$ behaves like $\exp -|t|^{\frac{1}{2}} \log |t| C(|t|)$ where $C(|t|)$ is a monotonous increasing function of $|t|$ and goes to infinity for $t \rightarrow -\infty$. Then

$$\phi(t) = f(t) \exp C' \sqrt{4\mu^2 - t} \log(4\mu^2 - t)$$

goes to zero as $t \rightarrow -\infty$ and, on the right-hand cut is bounded by

$$\exp C t^{\frac{1}{2}} \exp - C' \pi t^{\frac{1}{2}}$$

By taking $\eta C' > C$ it also goes to zero as $t \rightarrow +\infty$. Now, in the upper half t plane it is bounded by $\exp |t|^{\frac{1}{2}} \log |t|^{\frac{1}{2}}$ and therefore the Phragmén-Lindelöf condition is fulfilled ⁹⁾ and $\phi(t) \rightarrow 0$ in

all directions of the upper and lower half plane. Since $\phi(t)$ is analytic and bounded in all the cut plane and decreases like $\exp -(C'-C)|t|^{\frac{1}{2}}$ on both sides of the cut ⁹⁾ it is identically zero. This result will be useful later.

III. THE LARGE ANGLE SCATTERING AMPLITUDE : A SIMPLIFIED TREATMENT

Here we shall closely follow the form factor case by making the drastic assumption that the scattering amplitude takes the simple form $sf(t)$ where s is the square of the c.m. energy and $t = -2k^2(1 - \cos\theta)$, θ being the scattering angle and k the c.m. momentum transfer. This assumption can be justified empirically for finite, physical t , by noticing that there is no evidence for the shrinking of the $p\pi$ diffraction peak ¹¹⁾, and that the shrinking of the pp diffraction peaks seems to become less and less marked as the energy increases ¹¹⁾. It has been argued by Gribov ¹²⁾ that this assumption is not tenable outside the physical region if one insists on the validity of the Mandelstam representation. However, his argument is valid only if the form $sf(t)$ still holds on the cut in the t plane of the true scattering amplitude $f(s, t)$. So we shall disregard Gribov's objection and stick to $sf(t)$. What is still more daring is that we shall study the behaviour of $sf(t)$ for fixed angle, i.e., for $t \sim \text{const} \times s$, and accept that this gives indications on the behaviour of the true scattering amplitude $f(s, t)$ at fixed angle. A more rigorous treatment will be given in the next Section anyway.

Then, if one accepts this, the argument is extremely simple because we just need to copy what has been said in the previous Section : $f(t)$ is analytic in a cut plane, with a cut from $t = 4\mu^2$ to $t = +\infty$. So

- i) if $f(t)$ is polynomial bounded or, more generally, if $f(t) < \exp(|t|^{\frac{1}{2}})$ and

$$\int_{4\mu^2}^{\infty} \frac{\log^+ |f(t)| dt}{(1+t)^{3/2}}$$

converges, then $f(t)$ cannot decrease faster than $\exp -C|t|^{\frac{1}{2}}$ for $t \rightarrow -\infty$. Therefore the fixed angle scattering amplitude cannot decrease faster than

$$s \exp[-C \sqrt{2(1-\cos\theta)} \sqrt{s}]$$

for fixed angle, and, for not too large angles [anyway the $sf(t)$ behaviour certainly does not hold beyond $\theta = 90^\circ$]

$$\sqrt{2(1-\cos\theta)} \simeq \sin\theta$$

so that we get essentially

$$s \exp - C K_T \quad (5)$$

which is the Orear fit.

For angles beyond 90° and for a symmetric (or antisymmetric) scattering amplitude, we should take $sf(u)$ where $u = -2k^2(1+\cos\theta)$ so that we shall essentially get the same result. It is certainly possible to fit the data more generally by

$$s [f(t) + f(u)]$$

- ii) if $f(t)$ increases like $\exp|t|^{\frac{1}{2}}$ on the cut the result is slightly weakened and we get a large angle behaviour

$$s \exp(-C K_T \log K_T) \quad (6)$$

If one does not like the assumption $s f(t)$ and if one wants to take for instance a single dominant Regge pole :

$$f(s, t) \simeq s^{\alpha(t)} f(t)$$

the results are essentially unchanged since $\alpha(t)$ is supposed to be larger than -1 and we get, apart from a power of s which is irrelevant, essentially the same large angle behaviour.

In these considerations we see that the large angle scattering amplitude appears as the unavoidable tail of the diffraction peak. I.e., the diffraction peak cannot die off too fast; otherwise, either the fixed transfer scattering amplitude goes to zero at high energies [and we know that at least for $t=0$ it cannot go to zero faster than $1/s^2$, otherwise it is identically zero⁸⁾], or $f(t)$ has a nasty behaviour outside the physical region.

IV. RIGOROUS TREATMENT OF THE FIXED ANGLE AMPLITUDE

In this Section we want to avoid making a specific assumption about a possible factorization of the scattering amplitude. The price we shall pay is that we shall be unable to incorporate in a really proper way the fact that the scattering amplitude has this factorable form at least for physical, finite, transfers; and so we shall get correspondingly weaker results. Here we shall essentially repeat the argument presented already by Cerulus and Martin⁴⁾ and Kinoshita³⁾, but get rid of some unnecessary assumptions on the growth of the scattering amplitude outside the physical region.

The principle is this : one imposes the growth conditions on the cuts of the scattering amplitude in the t or $\cos\theta$ complex plane. One imposes an upper bound on the scattering amplitude $\exp -\phi(s)$ from θ to $\pi - \theta$. Then by the Poisson formula one calculates an upper bound of

the scattering amplitude at $t=0$ and also at finite t . Then $\phi(s)$ will be adjusted so that these upper bounds are not in contradiction with the constancy, or near constancy, of the total cross-sections at high energy and, possibly, the absence of shrinking of the diffraction peak.

The results we shall obtain are the following :

- a) if on the cut in the t plane the scattering amplitude increases less fast than

$$S \frac{t^{1/2}}{(\log t)^{1+\varepsilon}}$$

which includes the case of a polynomial bound, then the large angle scattering amplitude cannot decrease faster than

$$\exp - C(\theta) \sqrt{s} \log s \quad (7)$$

where $C(\theta)$ behaves like $\sqrt{\sin \theta}$ at least for θ small. This is obtained by imposing that the forward scattering amplitude should not decrease faster than s^{-2} , a fact which has been proved from forward dispersion relations and unitarity ¹³⁾. This includes the case where the forward amplitude behaves like s , i.e., gives a constant high energy cross-section.

If one makes the stronger requirement that for any negative t , finite, the scattering amplitude should be larger than a constant depending on t times s , or even should not decrease faster than s^{-1} ¹⁴⁾, then one finds that

$$\exp - C(\theta) \sqrt{s} \log s$$

is too fast a decrease. However, we were unable to exclude a behaviour

$$\exp - C(\theta) \sqrt{s} \log s \psi(s)$$

where $\psi(s)$ is a function going to zero arbitrarily slowly. So the results obtained are slightly weaker than those of the last Section.

- b) if, on the cut, the scattering amplitude behaves like $s t^{\frac{1}{2}}$ then the fastest allowed decrease is

$$\exp - (\theta) \sqrt{s} (\log s)^2 \quad (8)$$

Now, before going to the details, why is it useful to worry about such anomalous growth of the scattering amplitude? Essentially because there is now some suspicion, after the work of Mandelstam in particular¹⁵⁾, that as t goes to $+\infty$ the number of subtractions in fixed t dispersion relation goes to infinity, so that the scattering amplitude would not have the polynomial boundedness property when both s and t go to infinity, which is the situation we have to consider here.

The special case $f(s, t) \sim s t^{\frac{1}{2}}$ has a particular interest because, if no unforeseen cancellation occurs, it has been shown that multi-pomeron exchanges lead to a series of cuts in the angular momentum plane located at¹⁶⁾

$$\alpha_n(t) = n \left[\alpha_1\left(\frac{t}{n^2}\right) - 1 \right] + 1$$

where $\alpha_1(t)$ is the original Pomeron trajectory. With the usual assumption that $\text{Re } \alpha_1(t)$ has a maximum at a finite t and then drops to 0 or -1 as $t \rightarrow \infty$ we easily see that the maximum, over all n of $\text{Re}(\alpha_n(t))$ cannot behave worse than $t^{\frac{1}{2}}$, so that the amplitude will behave at worst like $s t^{\frac{1}{2}}$. It is fortunate that the price we pay for this very fast increase of the amplitude outside the physical region is not very high since $\sqrt{s} \log s$ is replaced by $\sqrt{s} \log^2 s$ in the lower bound at large angles.

Let us present now the proof. (We recommend non-specialists to omit it.)

The absorptive part of scattering amplitude $A(s, t)$ is supposed to be analytic, for fixed energy, in a twice cut plane, from $t = -\infty$ to $t = -(4k^2 + 4\mu^2)$ and from $t = 4\mu^2$ to $t = +\infty$. We suppose that, on the left-hand cut $|A(s, t)|$ is bounded by $s^{N(t)}$ where $N(t)$ is a positive increasing function of t to be determined later. On the right-hand cut it is bounded by $s^{N(u)}$ where $u = 4M^2 - s - t$, and where M is the common mass of the scattering particles (should the masses be unequal, this would not make a serious difference).

Now, from $t = -2k^2(1 - z_0)$ to $t = -2k^2(1 + z_0)$, ($z_0 > 0$), we assume that the amplitude is uniformly less than $\exp -\phi(s, z_0)$. Finally, it will be necessary to say something about the behaviour of the amplitude from $t = -4k^2 - 4\mu^2$ to $t = -2k^2(1 + z_0)$. We know ¹⁷⁾ that from $t = -4k^2$ to $t = -2k^2(1 + z_0)$ it is bounded by $s \log^2 s$. From $t = -4k^2 - 4\mu^2$ to $t = -4k^2$ it is, at least for an infinite sequence of energies, bounded by s^2 ¹⁸⁾.

We shall forget about the analyticity of the scattering amplitude from $t = -4k^2 - 4\mu^2$ to $t = -2k^2(1 - z_0)$ and calculate an upper bound of the fixed t (in particular $t = 0$) absorptive part of the scattering amplitude by applying the Poisson formula to the cut plane, with cuts from $t = -\infty$ to $t = -2k^2(1 - z_0)$ and from $t = 4\mu^2$ to $t = +\infty$. It is convenient to use the reduced variable

$$y = \frac{t - \frac{1}{2}(4\mu^2 - 2k^2(1 - z_0))}{\frac{1}{2}(4\mu^2 + 2k^2(1 - z_0))}$$

such that the cuts start at $y = \pm 1$. Then we have for $-1 < y < +1$

$$\log |A(s, y)| \leq \frac{\sqrt{1-y^2}}{\pi} \left[\int_{-\infty}^{-1} \frac{dy' \log |A(s, y')|}{(y' + y) \sqrt{y'^2 - 1}} + \int_1^{\infty} \frac{dy' \log |A(s, y')|}{(y' - y) \sqrt{y'^2 - 1}} \right] \quad (9)$$

Now, let us notice that for $y' > 1$ we have

$$\frac{\sqrt{1-y^2} dy'}{(y'-y)\sqrt{y'^2-1}} < \frac{\sqrt{4\mu^2-t} dt'}{(t'-t)\sqrt{t'-4\mu^2}}$$

and, for $t' < -4\mu^2 - 4k^2$

$$\frac{\sqrt{1-y^2} dy}{(y-y')\sqrt{y'^2-1}} < \frac{|du'|\sqrt{4\mu^2-t}}{|t+4k^2+u'|\sqrt{4\mu^2+4k^2+u'}}$$

So we have

$$\log |A(s,t)| \leq I_1 + I_2 + I_3 + I_4 \quad (10)$$

$$I_1 = \log s \frac{\sqrt{4\mu^2-t}}{\pi} \int_{4\mu^2}^{\infty} \frac{dt' N(t')}{(t'-t)\sqrt{t'-4\mu^2}}$$

$$I_2 = \log s \frac{\sqrt{4\mu^2-t}}{\pi} \int_{4\mu^2}^{\infty} \frac{dt' N(t')}{(t'+4k^2+t)\sqrt{4\mu^2+4k^2+t'}}$$

$$I_3 = -\phi(s, z_0) \frac{\sqrt{1-y^2}}{\pi} \int_1^{y'} \frac{dy'}{(y'+y)\sqrt{y'^2-1}}$$

where y_1 corresponds to $t_1 = -2k^2(1+z_0)$

$$I_4 = 2 \log s \frac{\sqrt{1-y^2}}{\pi} \int_{y_1}^{y_2} \frac{dy'}{(y'+y) \sqrt{y'^2-1}}$$

where y_2 corresponds to $t_2 = -4k^2 - 4\mu^2$.

First, we notice that for $t > -2k^2$, which is the case we consider, $I_1 > I_2$; in fact, if I_1 converges, $I_2/I_1 \rightarrow 0$ as $k^2 \rightarrow \infty$ for fixed t . I_4 is less than

$$2 \log s \frac{\sqrt{1-y^2}}{\pi} \left[\int_{-\infty}^{-1} \frac{dy'}{(y-y') \sqrt{y'^2-1}} + \int_{1}^{\infty} \frac{dy'}{(y'-y) \sqrt{y'^2-1}} \right] = 2 \log s$$

As for I_3 , which takes into account the large angle decrease, it is negative and we have

$$|I_3| > \frac{\phi(s, z_0)}{\pi} \sqrt{1-y^2} \sqrt{\frac{2z_0}{1+z_0 + \frac{2\mu^2}{k^2}}}$$

In the limit of high energy, finite t , finite $z_0 < 1$, we get

$$|I_3| > \phi(s, z_0) \frac{2\sqrt{2}}{\pi} \sqrt{\frac{2z_0}{(1-z_0^2)s}} \sqrt{4\mu^2 - t} \quad (11)$$

Let us now consider first the situation for $t=0$. We have

$$\log |A(s, 0)| < \log s \left[\frac{4\mu}{\pi} \int_{4\mu^2}^{\infty} \frac{dt' N(t')}{t' \sqrt{t'-4\mu^2}} + 2 \right] - \phi(s, z_0) \frac{2\sqrt{2}\mu}{\pi} \sqrt{\frac{2z_0}{(1-z_0^2)s}}$$

provided the integral

$$J = \frac{4\mu}{\pi} \int_{4\mu^2}^{\infty} \frac{dt' N(t')}{t' \sqrt{t' - 4\mu^2}}$$

converges, which is the case if $A(s, t)$ does not grow faster than

$$s^{t'^{1/2}/(\log t)^{1+\varepsilon}}$$

We can now impose

$$\log |A(s, 0)| > C + \log s$$

which corresponds to a constant high energy cross-section. Then we see that $\phi(s, z_0)$ is submitted to the condition

$$\phi(s, z_0) < \frac{\pi}{2r_2 \mu} \sqrt{\frac{(1-z_0^2)s}{2z_0}} (J+1) \log s \quad (12)$$

so that the fastest allowed large angle decrease is

$$\exp - C \frac{k_T \log s}{\sqrt{\cos \theta}} \quad (13)$$

Notice that this is not as good as the expression given in Ref. 3). This is due to the fact that the analyticity of $A(s, t)$ from $t = -4k^2 - 4\mu^2$ to $t = -2k^2(1+z_0)$ has been disregarded.

If we prefer not to impose to $A(s, 0)$ to behave like s at high energies, we can use $A(s, 0) > s^{-5} \log s^{-4}$ which follows from forward dispersion relations, unitarity and analyticity in the region $|t| < 4\mu^2$ 13). Then we get exactly the same result as (13) except that $J+1$ is replaced by $(J+7)$.

Now if in addition to the requirement that $A(s,0)$ should not decrease too fast, we add the requirement that for arbitrarily large, but finite t , $A(s,t)$ should be larger than $sf(t)$ or than $s^{-2}f(t)$, then we notice that since for given $\phi(s, z_0)$ $|I_3|/I_1+I_2+I_4 \rightarrow \infty$ as $t \rightarrow -\infty$, necessarily

$$\phi(s, z_0) / \sqrt{s} \log s \rightarrow 0 \quad (14)$$

So we are now closer to the answer obtained in the preceding Section. However, we were not able to give in a more precise way a measure of the improvement.

Another disturbing thing is that if formula (13) seems relatively adequate for not too large angles, it becomes meaningless as $\cos\theta \rightarrow 0$. This is not a surprise because, in our approach, we cannot prevent the scattering amplitude from being infinitely small in an infinitely small angular interval. Here we certainly meet a deficiency of the present approach. The only remedy to this would be to separate the left-hand cut and the right-hand cut contributions in the scattering amplitude and to assume that they give rise, separately, to positive monotonous contributions.

Next, let us say what happens if the integral

$$\int_{4\mu^2}^{\infty} \frac{dt' N(t')}{t' \sqrt{t' - 4\mu^2}}$$

diverges logarithmically. I.e., if $N(t') \sim t'^{\frac{1}{2}}$. As we said already, this case has some importance because if the Regge cuts found by Mandelstam are really there, this is the behaviour one expects along the cut. Then one must evaluate directly

$$I_1 = \sqrt{1-y^2} \int_1^{\infty} \frac{dy' \log |A(s, y')|}{(y' - y) \sqrt{y'^2 - 1}}$$

which we can rewrite as

$$I_1 = \sqrt{1+y} \int_{4\mu^2}^{\infty} \frac{dt' N(t') \log s \sqrt{t-4\mu^2}}{(t'-t) \sqrt{t'-4\mu^2} \sqrt{y'+1}}$$

and which we will majorize as follows

$$I_1 \leq \sqrt{4\mu^2-t} \int_{4\mu^2}^s \frac{dt' N(t') \log s}{(t'-t) \sqrt{t'-4\mu^2}} + C(z_0) \sqrt{4\mu^2-t} \int_s^{\infty} \frac{dt' N(t') \sqrt{s} \log s}{(t'-t) (t'-4\mu^2)}$$

if $N(t')$ is bounded by $t'^{\frac{1}{2}}$ the second integral behaves like $\sqrt{4\mu^2-t} \times C'(z_0) \times \log s$ while the first behaves like $\sqrt{4\mu^2-t} \times C'(z_0) (\log s)^2$ and is therefore the dominant contribution.

Since we still have $I_2 < I_1$, I_2 and I_3 unchanged, we can very quickly adapt the argument to this case and see that the large angle lower limit will be

$$\exp - C \frac{k_T (\log s)^2}{\sqrt{\cos \theta}}$$

It is fortunate that the damage is not worse. In fact, we could extend the argument to any growth of $N(t)$, as long as $N(t)$ increases less fast than $t^{1-\varepsilon}$, but then the large angle decrease of the amplitude would be too fast. If $N(t) \sim t$ it becomes definitely hopeless to find any large angle lower limit because then $N(t) \sim t$ means that the scattering amplitude increases at infinity like $\exp|t|$, so that one has the freedom to add to the scattering amplitude any entire function of order 1.

V. CONCLUSIONS AND SPECULATIONS

If one accepts to believe that the fits proposed by Orear for large angle scattering and by Yang and Wu for high transfer form factors really work up to infinite energies or transfers, then it appears that the concept of minimal interaction at very high transfers works rather successfully. This is not a theory because there is so far no argument to show that nature should have this property, but it fits nicely in the familiar framework of dispersion theory. In addition, in the present work, the most doubtful assumptions made in dispersion theory, namely the existence of a polynomial bound on form factors and scattering amplitudes outside the physical region have been replaced by much weaker assumptions, which again cannot be proved, but for instance come out naturally of recent work on Regge cuts.

On the other hand, if one accepts a statistical description of high transfer processes, again both large angle scattering and high transfer form factors can be explained. In fact, there is no incompatibility between the two explanations. One may try to say the following.

In the statistical approach particles are created freely and move freely inside a certain "volume of interaction". This volume of interaction is what remains of that force that you cannot suppress unless the scattering amplitude, or the form factor is identically zero. Now, if particles move freely and therefore obey the simple laws of statistical mechanics, this means the interaction is minimal.

Another attempt to unify both approaches goes like this : a form factor, or a scattering amplitude, in dispersion theory is dominated by the spectral function along the cut from $t = 4\mu^2$ to $t = \infty$. Now, this spectral function comes from various intermediate states. If the number of these intermediate states is finite (say a few poles) then there is no hope of observing an exponential decrease for $\sqrt{|t|} \rightarrow +\infty$. Similarly, statistical considerations show that if the spectrum of the various states does not extend to infinity, one cannot get the exponential decrease. To get the exponential decrease in statistical theory, the spectrum of the

various different states must be exponential ¹⁹⁾. Now, this is just the limiting case in the dispersion approach beyond which we no longer get the exponential law. Also, the exponential decrease requires oscillations of the spectral function; oscillations also make us think of statistical theory because then the phases of the various intermediate states should be randomly distributed ²⁰⁾. However, there is a serious difference between the two approaches : namely, in the statistical approach the lower masses are irrelevant; what matters really is the behaviour of the mass spectrum for large m . On the contrary, even the low mass region in the spectral function of the form factor or the scattering amplitude must fit with the high mass region, in order to produce the cancellation which gives rise to the exponential decrease. However, let us not forget that we are dealing with two very different objects. The spectral function of the scattering amplitude certainly contains as a factor the density of states, but this is not the only factor which enters. The very delicate cancellation which is necessary to obtain the large angle exponential decrease is not so hard to understand if one remembers, for instance, that the knowledge of the spectral function $\rho(s, t)$ in an arbitrarily strip $4\mu^2 < t < 4\mu^2 + \varepsilon$ fixes the spectral function for arbitrarily large t ²¹⁾.

Let us discuss now the universality of the prediction. First it is clear that any reaction having the desired fixed energy analytic properties will have a minimum reaction amplitude $\exp -C(\theta) \sqrt{s} \log s$. However, will $C(\theta)$ be universal or vary from process to process ? Well, it seems that for positive or zero t universality will survive even if the simple Regge pole picture is wrong, in particular if the dominant mechanism is the exchange of several Pomeranchons. So we should expect some sort of universality for all elastic processes in the forward hemisphere. For inelastic processes, the situation is less clear and we do not want to make definite statements at the moment.

It is not clear, not even likely, that the predicted large angle distribution will be symmetric around 90° for the scattering of non-identical particles. It would certainly be interesting to have accurate measurements of π p scattering at the highest possible energy and see what happens. It seems that one might expect a displacement of the minimum towards backward angles since the left-hand cut singularities are further away.

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