ELECTROMAGNETIC MASS SPLITTINGS AND THE BARYON OCTET MASS FORMULA

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ABSTRACT

It is shown that in order to take into account the electromagnetic mass shifts of the baryons in testing the baryon octet mass formula, the mean mass of each baryon isospin multiplet may be used.

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One of the most impressive predictions of the approximate $SU_3$ symmetry of strong interactions is the Gell-Mann-Okubo formula\(^1\),\(^2\) relating the masses of particles within an $SU_3$ multiplet. In particular, the baryon octet mass formula is valid to the same order as the observed electromagnetic splittings in each isotopic spin multiplet. This raises the problem of how to test correctly the accuracy of this formula, because it is obtained in terms of baryon masses without including the electromagnetic contributions. Let us define a characteristic deviation $\delta m$ to the baryon octet mass formulae by

$$\delta m = N^+ \equiv \frac{1}{2}(3\Lambda + \Sigma)$$  \(1\)

where $N, \Sigma, \Lambda$ and $\Sigma$ are the masses of the corresponding baryons when the electromagnetic interactions are switched off. We shall derive below two relations, Eqs. (3) and (4), for $\delta m$ in terms of baryon masses which include the electromagnetic shifts obtained in the limit of exact $SU_3$ symmetry. The accuracy of the Coleman-Weinberg relation\(^3\) for the electromagnetic mass splittings among the baryon isospin multiplets justifies the substitution of the observed baryon masses in relations (3) or (4) to obtain $\delta m$.

Since the electromagnetic interaction transforms like a scalar under $U$-spin\(^4\), the electromagnetic mass shifts $\delta B$ of the baryons in a $U$-spin multiplet are all equal in the limit of exact $SU_3$ symmetry. Furthermore, there is no electromagnetic mixing among states with the same charge but different $U$-spin. Setting $Y_0 = \frac{\sqrt{3}}{2} \Lambda + 1/2 \Sigma^0$ and $Z^0 = 1/2 \Lambda - \frac{\sqrt{3}}{2} \Sigma^0$ for the $U = 1$, $U \bar{2} = 0$ and $U = 0$, $U \bar{3} = 0$ states respectively, we have\(^3\),\(^5\)

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\[ \delta \Sigma^+ = \delta p \]
\[ \delta \Sigma^- = \delta \Xi^- \]
\[ \delta \Xi^0 = \frac{1}{2} \left( \Sigma \mid \delta \Lambda \right) + \frac{1}{4} \left( 3 \delta \Lambda - \delta \Xi^0 \right) = \delta \Xi^0 \]
\[ (\gamma^0(\delta \mid \Xi^0) = -\frac{1}{2} \left( \Sigma \mid \delta \Lambda \right) + \frac{1}{2} \left( \delta \Lambda - \delta \Xi^0 \right) = 0 \]

(2)

where \((\Sigma \mid \delta \Lambda)\) denotes the \(\Sigma - \Lambda\) electromagnetic mass mixing \(^6\). If we set \(B^q = B + \delta B^q\) for the corresponding baryon mass, where \(B\) is the mass without electromagnetic interaction, and the superscript \(q\) is the charge, we obtain by combining Eqs. (1) and (2).

\[ \delta m = \Sigma^0 - \Sigma^+ + \Xi^0 - \frac{1}{2} \left( 3 \Lambda^0 + \Sigma^0 \right) \]

(3)

or alternatively

\[ \delta m = \frac{1}{2} (p^+ N^0) + \frac{1}{2} (\Xi^- + \Xi^0) - \frac{1}{2} \left( 3 \Lambda^0 + \Sigma^+ + \Sigma^- \right) \]

(4)

Substituting the observed mean baryon masses \(^7\), we find \(\delta m = -12.4\) MeV and \(-12.9\) from Eqs. (3) and (4) respectively \(^8\). Note that Eq. (4) is nearly equivalent to substituting the mean mass of each isospin baryon multiplet in Eq. (1) because the \(\Sigma\) hyperon mass relation

\[ \Sigma^0 = \frac{1}{2} (\Sigma^+ + \Sigma^-) \]

(5)
is very well satisfied. However, Eq. (5) cannot be derived on symmetry arguments alone \(^9\).

Similar considerations can be applied to include the effect of the electromagnetic interaction in the GMO mass formula for other multiplets of \(SU_3\). For triangular representations (for example the decuplet of baryon resonances) the equal mass spacing rule should simply be applied to states with the same electric charge.

**ACKNOWLEDGEMENTS**

I would like to thank B. Zumino for very stimulating discussions and critical comments, and Professors L. Van Hove and V.T. Weisskopf for their hospitality.
REFERENCES

2) S. Okubo, Progress of Theoretical Physics 27, 959 (1961).
3) S. Coleman and S.L. Glashow, Phys. Rev. Letters 2, 423 (1961);
5) S. Okubo, Phys. Letters 4, 14 (1963);
6) R.H. Dalitz and F. von Hippel (Oxford preprint) have discussed the evidence
   for the $\Sigma^+\Lambda$ electromagnetic mixing.
7) We have used baryon mass values given by N. Roos, Phys. Letters 2, 1 (1964).
8) S. Okubo (Rochester preprint) has concluded that the baryon octet mass
   formula ought to be tested among the neutral members of the
   octet, in disagreement with our result.
9) A very crude argument for this relation is obtained from the two triplet
   Letters 12, 237 (1963), F. Gursey, T.D. Lee and M. Nauenberg,
   Phys. Rev. (to be published). Let the $I = 1/2$ component of
   the fundamental fermion triplet have electromagnetic mass shifts
   $\delta m_1$ and $\delta m_2$ for $I = 1/2$ and $-1/2$ respectively, and
   correspondingly $\delta \mu_1$ and $\delta \mu_2$ for the boson triplet.
   If we assume that the baryon electromagnetic mass shifts are simply
   the sum of the shifts of their components (justified in the case
   of weak binding) we obtain $\delta \Sigma^+ = \delta m_1 + \delta \mu_2$, $\delta \Sigma^- = \delta m_2 + \delta \mu_1$
   and $\delta \Sigma^0 = 1/2(\delta m_1 + \delta m_2 + \delta \mu_1 + \delta \mu_2)$ leading to Eq. (5).
   The relations given in Eq. (2) are also readily obtained in this
   manner, if we note that the electromagnetic mass shift $\delta m_0$ and
   $\delta \mu_0$ of the $I = 0$ member of the fermion and boson triplets
   satisfies the condition $\delta m_0 = \delta m_2$ and $\delta \mu_0 = \delta \mu_2$ respectively. A similar argument has been proposed by G. Zweig (CERN preprint).