EVENT RATE FOR EXTREME MASS RATIO BURST SIGNALS IN THE LASER INTERFEROMETER SPACE ANTENNA BAND

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ABSTRACT

Stellar mass compact objects in short period ($P < 10^3$ s) orbits about a $10^{4.5} - 10^{7.5}$ M$_\odot$ massive black hole (MBH) are thought to be a significant continuous-wave source of gravitational radiation for the ESA/NASA Laser Interferometer Space Antenna (LISA) gravitational wave detector. These extreme mass-ratio inspiral sources began in long-period, nearly parabolic orbits that have multiple close encounters with the MBH. The gravitational radiation emitted during the close encounters may be detectable by LISA as a gravitational wave burst if the characteristic passage timescale is less than $10^6$ s. Scaling a static, spherical model to the size and mass of the Milky Way bulge we estimate an event rate of $\sim 15$ yr$^{-1}$ for such burst signals, detectable by LISA with signal-to-noise ratio greater than five, originating in our Galaxy. When extended to include Virgo Cluster galaxies our estimate increases to a gravitational wave burst rate of $\sim 18$ yr$^{-1}$. We conclude that these extreme mass-ratio burst sources may be a steady and significant source of gravitational radiation in the LISA data streams.

Subject headings: black hole physics — Galaxy: nucleus — gravitational waves — stellar dynamics

1. INTRODUCTION

The inspiral of compact objects onto massive black holes (MBHs) in galactic nuclei is an important gravitational radiation source for the ESA/NASA Laser Interferometer Space Antenna (LISA) (Hils & Bender 1992; Sigurdsson & Rees 1997; Glampedakis & Kennefick 2002; Glampedakis et al. 2002; Ivanov 2002; Pretorius 2003; Gair et al. 2004; Danzmann & Rüdiger 2003; Sumner & Shah 2004; Jennrich 2002; Seto et al. 2001). These extreme mass-ratio inspiral (EMRI) sources will complete an inspiral into the MBH. In each of these inspirals, the periapsis of an EMRB orbit can be much greater than the tidal disruption radius for low-mass sequence stars. For example, an M2V star could disrupt and decayed until it entered the LISA band as a continuous source. However, if the MBH encounter timescale is too long, the orbit may become an EMRI without any significant radiation burst in the LISA band.

Not all EMRBs evolve to become EMRIs, either. An object on a highly eccentric orbit may be scattered by other stars after encountering the MBH, or may plunge directly into the MBH. In addition, while massive main sequence stars will be disrupted long before appearing as EMRIs, the periastron of an EMRB orbit can be much greater than the tidal disruption radius for low-mass main sequence stars. For example, an M2V star could pass within 0.65 AU of the Milky Way’s MBH without disruption and, if on a nearly radial orbit so that its pericenter velocity is highly relativistic, may radiate a gravitational wave burst with a characteristic frequency of several mHz.

2. INSPIRALS VS. BURSTS

To illustrate the evolution from EMRB to EMRI consider a 0.6 M$_\odot$ white dwarf in a nearly radial Keplerian orbit about a MBH of mass $M_\bullet \sim 10^6$ M$_\odot$. If the initial orbit has an apocenter of 100 pc and a pericenter of 0.2 AU (corresponding to 10 Schwarzschild radii for the MBH), then during the first pericenter pass the orbit loses enough energy via gravitational radiation that the next apocenter is $\sim 60$ pc, while the change in pericenter is negligible. With each subsequent pericenter pass, a strong burst of gravitational radiation reduces the subsequent apocenter and the orbit becomes more circular. LISA is sensitive to gravitational waves in the approximately $10^{-5}$ to $10^{-1}$ Hz band, so we classify as EMRBs those systems with orbital periods greater than $10^5$ s and with pericenter passage timescales of less than $10^5$ s. Conversely, EMRIs have orbital periods less than or on order $10^3$ s and radiate continuously in the LISA band.

EMRIs and EMRBs sample different components of a galactic nucleus. Every EMRI source was, in its past, a possible EMRB source: i.e., it was on a highly eccentric orbit that, owing to gravitational wave emission, circularized and decayed until it entered the LISA band as a continuous source. However, if the MBH encounter timescale is too long, the orbit may become an EMRI without any significant radiation burst in the LISA band.

3. STELLAR MODEL

To evaluate the EMRB event rate we begin with a model of our Galaxy center. The Milky Way bulge consists of a $M_\bullet = 3.7 \times 10^6$ M$_\odot$ MBH (Gluz et al. 2005) embedded in a stellar ellipsoid (e.g. Binney et al. 2005).
 embedding a η model with a pre-existing central MBH leaves the spherical density profile unchanged, but changes the central potential so that \( \Phi_\eta(r) = \Phi(r) - M_\star/r \), where \( \Phi_\eta \) is the potential of the MBH-embedded model and \( \Phi \) is the stellar potential of an η model. The new potential naturally adjusts the distribution function, which can be obtained from a spherical, isotropic density profile via Eddington’s formula (Binney & Tremaine 1987), which for a MBH-embedded model can be solved analytically near the MBH:

\[
f(\epsilon) = \frac{\eta \Gamma(4-\eta)}{2^{7/2} \pi^{5/2} M_\star^{3-\eta} \Gamma(5/2 - \eta)} \epsilon^{3/2 - \eta},
\]

where \( \epsilon \) is the absolute value of the energy per unit mass. When normalized to the total mass of the model, \( f(\epsilon) d\epsilon dr dv \) is the mass contained in the local volume element centered on \( r, v \).

4. BURST WAVEFORMS

Orbits that may generate observable EMRBs are characterized by large orbital periods (\( \sim 1 - 10^9 \) yrs) and high eccentricity. Objects on these orbits may be highly relativistic during periapsis passage, with velocities an appreciable fraction of the speed of light (\( v_p \sim 0.3c \)). The gravitational wave emission from such a “gravitational bremsstrahlung” event emerges as a burst, lasting as long as the MBH encounter and beam in the direction of the secondary’s velocity at periapsis. For this preliminary exploration we treat trajectories near periapsis as Keplerian, with the same \( v_p, r_p, \) and mass enclosed within \( r_p \) as dictated by the η model, and we calculate the gravitational radiation as if we were well described by the quadrupole formula. A more detailed treatment would include the effect of beaming, the radiative contributions from terms in \( v/c \) higher than quadrupole, and take into account the actual relativistic trajectory, all of which may be important for trajectories with small \( r_p/M_\star \) or large \( v/c \) (cf. Gair et al. 2002).

From the (quadrupole) waveforms we calculate the signal-to-noise ratio (SNR) in the LISA detector assuming the source is at a distance of 8 kpc. In general, the square of the SNR is given by,

\[
\rho^2 = 4 \int_0^{\infty} \frac{\tilde{h}(f)^2}{S_n(f)} df,
\]

where \( \tilde{h}(f) \) is the Fourier transform of the gravitational wave signal projected onto the LISA detector and \( S_n(f) \) is the one-sided instrument noise power spectral density. For our purposes, the values of \( S_n(f) \) are taken from the Online Sensitivity Curve Generator (Larson et al. 2000) with the standard LISA settings and the inclusion of a white dwarf background “noise” contribution (Bender & Hils 1997).

To gain insight into the magnitude and scaling of the SNR, equation 2 can be approximated by taking advantage of the simple burst structure in the waveforms, which allows us to quickly approximate the integral and Fourier transform,

\[
\rho \approx 100 \frac{(R/8 \text{ kpc})^{-1} (M_\star/\text{M}_\odot) (v_p/0.3c)^2 (f_\text{s}/1 \text{ mHz})^{-1/2}}{4 \times 10^{-37} \text{ Hz}^{-1}},
\]

where \( R \) is the distance to the EMRB, \( M_\star \) is the mass of the secondary, and \( f_\text{s} = v_p/r_p \) is the inverse of the burst duration. The value of \( f_\text{s} \) can be viewed as a characteristic frequency for an EMRB event, but these events are extremely broadband in nature. Note that the noise spectral density scales as \( f^{-4} \) for frequencies below 1 mHz and the pre-factor accounts for the white dwarf binary background.

5. EVENT RATES

To arrive at an EMRB event rate estimate, we divided our η model into five million elements in \( (r, v) \) phase space. We chose the element centers uniformly spaced in \( \log(r) \) and \( \log(v) \) with \( r \in [7.6 \times 10^{-7}, 2 \times 10^5] \) pc and \( v \in [0, 2 \times 10^5] \) km s\(^{-1} \). For each phase-space element we calculated \( f(\epsilon) \), the average orbital period \( P_{\text{orb}} \), periapsis \( r_p \), and velocity at periapsis \( v_p \).

To arrive at the number of compact objects and low mass main sequence stars in our model we do the following. Assuming all of the mass density in our model results from a single burst of star formation 10 Gyr ago, and that the number of stars per unit mass follows a Kroupa IMF\(^2 \) (Kroupa 2001), we converted \( f(\epsilon) \) into the total number of stars per mass per phase space element. This number can be less than one per phase space element. To determine the number of main sequence stars per phase space element we calculated the maximum stellar mass that would survive tidal disruption by the MBH (Sridhar & Tremaine 1992) and integrate the IMF up to this mass, setting the main sequence stellar mass equal to the average mass that survives tidal disruption. We determined the white dwarf population by assuming white dwarfs formed from all stars between \( 1 - 8 \) M\(_\odot\) and the final white dwarf mass is 0.6 M\(_\odot\). Similarly, neutron stars were assigned masses of 1.4 M\(_\odot\) and form from stars \( 8 - 20 \) M\(_\odot\), while black holes are 10 M\(_\odot\) objects formed from \( 20 - 120 \) M\(_\odot\) stars. Our calculation neglects the effects of mass segregation, multiple episodes of star formation, and forces all stars between \( 1 - 120 \) M\(_\odot\) to be a stellar remnant.

Only a small fraction of the initial phase space contain EMRBs. To isolate the EMRB orbits we made

\(^{1}\) Although we only considered elliptical systems within the η model, it is worth noting that the Keplerian equivalent orbit near the MBH is not necessarily elliptical.

\(^{2}\) We varied the IMF choice to include Salpeter and Scalo IMFs; this did not change our rate significantly.
a series of cuts. We immediately discarded those orbits that were either unbound or had periapsis within four Schwarzschild radii. We excluded orbits with $P_{\text{orb}}$ greater than the relaxation time, $t_{\text{relax}} \equiv v^2 / D(\Delta v^2)$, where $D(\Delta v^2)$ is the Fokker-Planck diffusion coefficient [Binney & Tremaine 1987, eq. 8-68]. We also excluded orbits with $P_{\text{orb}} < 10^6$ s, which would yield a continuous signal in the LISA band and are more accurately classified as EMRIs. We also excluded orbits that radiate so strongly that they plunge into the MBH faster than a dynamical time, where the plunge timescale $T_{\text{plunge}}$ is

$$T_{\text{plunge}} \approx 3.2 \times 10^6 \, \text{yrs} \left( \frac{M_*}{10^6 M_\odot} \right)^2 \left( \frac{M_*}{1 M_\odot} \right)^{-1} \times \left( \frac{r_{p,i}}{10 R_\odot} \right)^4 \left( \frac{1 - e_i}{10^{-5}} \right)^{-1/2}. \quad (4)$$

Here $r_{p,i}$ is the initial pericenter distance and $e_i$ is the initial eccentricity. Finally, we excluded all orbits with an encounter timescale $\Delta t = r_p/v_p > 10^5$ s, as these passes emit radiation outside the LISA band. The remaining stars constituted our potential EMRB population.

Figure 1 shows the phase space remaining after these cuts; $\sim 1,800$ phase space elements remained, which does not imply 1800 distinct objects; in fact there are typically $10^{-7}$ white dwarfs per $d^3 v$ in this region. Overlaid in the same figure is the subset of the white dwarf phase space with SNR $> 5$.

The total event rate $\nu$ is the sum over all orbits with LISA SNR greater than 5:

$$\nu = \sum_{\rho > 5} \left( \frac{N_{\text{LMMS,survive}}}{P_{\text{orb}}} + \frac{N_{\text{WD}}}{P_{\text{orb}}} + \frac{N_{\text{NS}}}{P_{\text{orb}}} + \frac{N_{\text{BH}}}{P_{\text{orb}}} \right). \quad (5)$$

For our Milky Way bulge model, we find $\nu_{\text{WD}} = 3$ yr$^{-1}$, $\nu_{\text{NS}} = 0.1$ yr$^{-1}$, $\nu_{\text{BH}} = 0.06$ yr$^{-1}$, and $\nu_{\text{LMMS,survive}} = 12$ yr$^{-1}$. These event rates imply that EMRBs may be an important, heretofore unrecognized source of gravitational waves in the LISA band. The left panel of figure 2 shows the accumulative event rate as a function of the orbital period. Most EMRBs originate from orbits that would produce multiple bursts of radiation per year in the LISA band. The right panel of figure 2 shows the number of black hole EMRBs as a function of SNR; note that some have SNRs high enough to be seen out to Virgo distances. If we repeat our analysis with the EMRB distances set to 16 Mpc and multiply by the number of galaxies in the cluster with suitable mass MBHs, we find a Virgo event rate of $\sim 3$ yr$^{-1}$, all due to encounters of low mass black holes with central MBHs.

6. DISCUSSION

EMRBs may comprise a significant new low frequency gravitational wave source, rivaling the total number of EMRI events visible in any year of LISA observation. For example, Freitag (2001) estimates that $O(0.1)$ white dwarfs per year are in the EMRI phase in the Milky Way. We find $O(1)$ EMRBs per year from close white dwarf encounters. The relaxation requirements on EMRBs are much less strict: EMRIs require thousands of undisturbed orbits to accumulate a SNR large enough to be detected ($T_{\text{plunge}} > (1 - e)T_{\text{relax}}$), while EMRBs merely need to pass by the MBH before scattering via 2-body relaxation.

Our estimated Milky Way EMRB event rate of $\sim 15$ yr$^{-1}$ may appear high in light of other processes, such as the white dwarf capture rate, which estimates place $O(10^{-7})$ per year [Freitag 2001]; however, the greatest contribution to the event rate we calculate arises from sources that burst multiple times a year, yet individually have a small phase space density. In other words, our rate does not imply that 15 $M_\odot$ of material interact with the black hole per year; rather, given these EMRB orbits we calculate that Milky Way black hole grows by less than $10^4 M_\odot$ over a Hubble time via EMRB decay, with many EMRBs bursting hundreds of times before falling into the MBH. Our simple stellar model passes other sanity-checks as well: given the capture definition of Freitag (2001), we expect a white dwarf capture rate of $O(10^{-6})$ per year and find, in our calculations, a tidal disruption rate of $O(10^{-5})$ per year, which is consistent with observational estimates [Donley et al. 2002].

We have introduced EMRBs as the evolutionary precursor to EMRIs; many EMRBs will inspiral and eventually evolve into EMRIs as gravitational radiation carries away energy and angular momentum. This implies that the lifespan of the typical EMRB phase may only be $O(10^5) - O(10^6)$ years after galaxy formation or a star formation episode. We have assumed, however, that
EMRB phase space is always occupied. For there to be a significant population of EMRBs in the present-day galaxy sample, there must be a refilling mechanism. There are several mechanisms that may refill the burst reservoir. One such mechanism, the dynamical migration of a stellar cluster, may have recently left its mark in the inner 0.04 pc of our galaxy in the form of the young S stars [Kim et al. 2004]. Other mechanisms, such as triaxiality [Merritt & Poon 2004], Holley-Bockelmann & Sigurdsson 2000 or resonant relaxation [Hopman & Alexander 2006], can act to refill the loss cone on timescales much shorter than the two-body relaxation time. Since we were primarily concerned with determining whether EMRBs were an overlooked class of LISA sources, we considered the simplest possible model for the Milky Way bulge, the radiation properties, and the ability of gravitational wave analysis techniques to detect the signal. Under these approximations the predicted event rate implies that bright EMRBs are so numerous that the local universe will generate a background of gravitational wave bursts with a mean rate of once per three weeks. Changing the assumptions, however, can alter this rate by several orders of magnitude; modeling the bulge as a bar, for example, can increase the capture rate by two orders of magnitude [Holley-Bockelmann & Sigurdsson 2000], while mass-segregation can decrease the EMRB rate from low mass main sequence stars by two orders of magnitude [Freitag 2003].

The relatively large event rate calculated here suggests that EMRBs may in fact be a significant source for the LISA detector and indicates that further, more accurate modeling needs to be undertaken. To better characterize EMRBs and study how they can constrain models of galactic nuclei, future work needs to include a more accurate model for the gravitational wave emission, a better treatment of the orbit in the relativistic regime, more realistic time-dependent galaxy and star formation models, and a proper treatment of stellar dynamics, including mass segregation. In addition, the potential presence of these burst gravitational wave sources introduces a new set of data analysis challenges, associated with the detection and characterization of the EMRB signal from LISA data. As daunting as the prospect of these challenges are their potential payoff: the signal from EMRBs probe the central region of our Galaxy, and galaxies in the Virgo Cluster, at scales less than $10^{-5}$ pc, far greater resolution than we can ever hope to achieve electromagnetically.

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