de Sitter String Vacua from Perturbative Kähler
Corrections and Consistent D-terms

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ABSTRACT: We present a new way to construct de Sitter vacua in type IIB flux compactifications, in which moduli stabilization and D-term uplifting can be combined in a manner consistent with the supergravity constraints. Here, the closed string fluxes fix the dilaton and the complex structure moduli while perturbative quantum corrections to the Kähler potential stabilize the volume Kähler modulus in an AdS$_4$-vacuum. Then, the presence of magnetized $D7$-branes in this setup provides supersymmetric D-terms in a fully consistent way which uplift the AdS$_4$-vacuum to a metastable dS-minimum.

KEYWORDS: D-branes, Supergravity Models, dS vacua in string theory, Flux compactifications.
1. Introduction

The last few years have seen the discovery of a vast 'landscape' \[1, 2, 3, 4\] of stable and meta-
stable 4d vacua of string theory. This marks remarkable progress in the formidable task
of constructing realistic 4d string vacua. In particular, the most pressing issues have been
how to stabilize the geometrical moduli of a compactification, and at the same time address
the tiny, positive cosmological constant that is inferred from the present-day accelerated
expansion of the universe. Recently, the use of closed string background fluxes in string
compactifications has been studied in this context \[5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24\]. Such flux compactifications can stabilize the dilaton and
the complex structure moduli in type IIB string theory. Non-perturbative effects such as
the presence of $Dp$-branes \[25\] and gaugino condensation were then used by Kachru et
al (henceforth KKLT) \[2\] to stabilize the remaining Kähler moduli in such type IIB flux
compactifications (for related earlier work in heterotic M-theory see \[26\]). Simultaneously
these vacua allow for SUSY breaking and thus the appearance of metastable $dS_4$-minima
with a small positive cosmological constant fine-tuned in discrete steps. KKLT \[2\] used the
SUSY breaking effects of an $\overline{D3}$-brane to achieve this. Alternatively the effect of D-terms
on $D7$-branes has been considered in this context \[27\].

Bearing in mind the importance of constructing 4d de Sitter string vacua in a reliable
way, one should note the problems of using $\overline{D3}$-branes as uplifts for given volume-stabilizing
AdS minima. The SUSY breaking introduced by an $\overline{D3}$-brane is explicit and the uplifting
term it generates in the scalar potential cannot be cast into the form of a 4d $\mathcal{N} = 1$ supergravity analysis. Thus, the control that we have on possible corrections in supergravity is lost once we use $D3$-branes for SUSY breaking. Replacing the $D3$-branes by D-terms driven by gauge fluxes on $D7$-branes \cite{27} is a way to alleviate this problem because then the SUSY breaking is only spontaneous. In this case the requirements of both 4d supergravity and the $U(1)$ gauge invariance necessary for the appearance of a D-term place consistency conditions on the implementation of a D-term (noted in \cite{27}, and emphasised in \cite{28, 29, 30}). These conditions have not yet been met by any concrete stringy realisation of \cite{27}, where the proposal was made in the context of KKLT. A consistent mechanism of stabilizing a modulus via D-terms and uplifting its minimum to a metastable dS vacuum has been constructed within the context of 4d supergravity by \cite{30} without, however, having a viable string embedding - a more stringy and consistent model can be found in \cite{29}.

In view of these difficulties it is appealing that recently the possibility of stabilizing the remaining Kähler volume modulus of type IIB flux compactifications purely by perturbative corrections to the Kähler potential has been studied \cite{31, 32}. The leading corrections which the Kähler potential receives are given by an $O(\alpha'^3)$-correction \cite{33} and string loop corrections \cite{34}. The $\alpha'$-corrections have recently been used to provide a realization of the simplest KKLT $dS$-vacua without the need for $D3$-branes as the source of uplifting \cite{35, 36, 37}. Under certain conditions the interplay of both the $\alpha'$-correction and the loop corrections leads to a stabilization of the volume modulus by the perturbative corrections alone \cite{32}. The corrections to the Kähler potential do not break the shift symmetry of the volume modulus. Therefore, in the present note, we show that such a Kähler stabilization mechanism allows for a consistent D-term uplift, by gauging this shift symmetry with world-volume gauge fluxes on a single $D7$-brane. Moreover, from simple scaling arguments one can conclude that the resulting vacuum does not suffer from any tachyonic directions.

The paper is organized as follows. Section 2 reviews the D-term uplifting procedure. Further, it summarizes the known constraints from 4d $\mathcal{N} = 1$ supergravity on the implementation of so-called field dependent Fayet-Iliopoulos (FI) D-terms by gauging a $U(1)$ shift symmetry. In Section 3 we review the mechanism of stabilizing the volume modulus in an $AdS_4$-minimum by using the perturbative corrections to the Kähler potential, whose structure is summarized. These results are then used in Section 4 to gauge the $U(1)$ shift symmetry of the volume modulus by turning on gauge flux on a single $D7$-brane. Then, for an illustrative - if incomplete - example, we calculate the full scalar potential resulting from the F-terms and the D-term and show that by an appropriate tuning of the fluxes we can obtain a metastable $dS$-vacuum for the volume modulus with all the other moduli also...
fixed. Finally, we summarize our results in the Conclusion.

2. D-terms uplifts and consistency conditions from 4d $\mathcal{N} = 1$ supergravity

The proposal to use a field dependent FI D-term as a source of uplifting $AdS$- to $dS$-vacua was constructed in [27]. Consider a 4d $\mathcal{N} = 1$ compactification of type IIB string theory on an orientifolded Calabi-Yau 3-fold in the presence of closed string fluxes. The $G_{(3)}$-flux fixes the dilaton $S$ and the complex structure moduli $U^I$. Generically, this procedure leaves the Kähler moduli unfixed and in particular the universal Kähler volume modulus $T$. Now, the volume modulus enjoys a Peccei-Quinn type symmetry: $T \rightarrow T + i\alpha$. In the presence of a background 2-form gauge field strength $F_{mn}$, threading the world-volume of a D7-brane wrapped on a 4-cycle $\Gamma$ of the compact internal manifold, this symmetry is gauged. The corresponding gauge covariant derivative acts on $b = \text{Im}~T$ as $D_\mu b = \partial_\mu b + iqA_\mu$, with $q$ the charge. The necessary coupling, $qA^\mu \partial_\mu b$, arises from the $a_{(2)} \wedge F_{(2)}$-coupling contained in the world volume action of the D7-brane, where $b$ and $a_{(2)}$ are dual fields. Here $a_{(2)}$ denotes the 2-form potential contained in the closed string 4-form $C_{(4)}$ which has the world volume coupling $C_{(4)} \wedge F_{(2)} \wedge F_{(2)}$ to the $U(1)$-gauge field strength $F_{(2)} = dA_{(1)}$ on the D7-brane. As long as we assume just one D7-brane its $U(1)$ world volume gauge theory has no local anomalies.\footnote{As expected, the gauging goes hand in hand with a D-term potential, and specifically a contribution to the scalar potential for the volume modulus $T$. This arises from the world-volume action of the wrapped D7-brane, as:}

$$V_D(T) \sim T_7 \cdot \int_\Gamma d^4y \sqrt{g_5} F_{mn} F^{mn} \sim \frac{q^2}{(T + \bar{T})^3} , \tag{2.1}$$

where $T_7$ is the D7-brane tension. For simplicity we are assuming a single Kähler modulus, and also the absence of matter fields charged under the $U(1)$ gauge group. This latter assumption may be justified in a model with a single isolated D7-brane: The matter fields arising from open strings stretching between the D7- and other branes would then become very massive thus driving their VEVs to zero. In the presence of light charged matter fields, one must consider whether their dynamics are such as to minimise the D-term potential at $V_D = 0$, or to allow this supersymmetry breaking contribution.\footnote{Since it is a shift symmetry of the chiral superfield $T$ that is gauged by the D7-brane gauge flux, we will summarize now the requirements of supergravity for the case where the}

\footnote{In case of a stack of coincident D7-branes there will be in general an anomaly whose cancellation arises via a generalized Green-Schwarz mechanism in a model-dependent way. The additional Green-Schwarz coupling, $q' A^\nu \partial_\nu b'$, may be small enough compared to the coupling induced by the world volume gauge fluxes to be neglected [28, 29].}

\footnote{See [27] for more discussion on this important point.}

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U(1) shift symmetry of a general modulus $\Phi$ is gauged, and gives rise to a field dependent FI D-term \cite{28,30}. Consider a typical tree level Kähler potential of the form:

$$K = -p \cdot \ln(\Phi + \bar{\Phi}) \quad (2.2)$$

where $p$ is a constant prefactor, which is 3 for the volume modulus $T$ and 1 for the dilaton $S$. The shift symmetry of $\Phi$ is assumed to be an isometry of its Kähler potential: Under

$$\Phi \rightarrow \Phi + i\alpha \quad (2.3)$$

the Kähler potential is invariant up to a Kähler transformation. Defining now the function

$$G = K + \ln |W|^2 \quad (2.4)$$

with $W$ denoting the superpotential, the scalar potential reads

$$V = V_F + V_D = e^{G(G\Phi G\Phi G\Phi \bar{G}\Phi - 3)} + \frac{1}{2} (\text{Re } f_\Phi)^{-1} D^2_\Phi \quad (2.5)$$

where $G_\Phi = \partial G / \partial \Phi$, and $f_\Phi$ is the gauge kinetic function. Here the D-term is given as a solution to the Killing equation of the isometry eq. (2.3) by

$$D_\Phi = i X^\Phi G_\Phi = -q \cdot \left( \frac{\partial K}{\partial \Phi} + \frac{1}{W} \cdot \frac{\partial W}{\partial \Phi} \right) = q \cdot \left( \frac{p}{\Phi + \bar{\Phi}} - \frac{W_\Phi}{W} \right) \quad (2.6)$$

where $X^\Phi = iq$ denotes the Killing vector of the isometry eq. (2.3). The requirement for this D-term to exist is that the shift symmetry of $\Phi$ is promoted to a $U(1)$ gauge symmetry of the full supergravity. Gauging of the shift symmetry requires $G$ to be invariant under the isometry. This results in the most general form of the superpotential $W$ consistent with gauge invariance being

$$W = A \cdot e^{a\Phi} \quad (2.7)$$

for some constants $A$, $a$. If $W$ is independent of $\Phi$ and the gauge kinetic function has a typical stringy modular dependence $f_\Phi = \Phi$ then the D-term generates a potential $V_D \sim (\Phi + \bar{\Phi})^{-3}$ of the type of eq. (2.1).

In type IIB string theory compactified to 4d the role of $\Phi$ is played by the universal Kähler volume modulus $T$. However, the superpotential in type IIB flux compactifications

\footnote{In terms of the notation in \cite{40}, the action of 4D $\mathcal{N} = 1$ supergravity is specified by the functions $K(T, \bar{T}), W(T), \Gamma(T, \bar{T}, V)$ and $f(T)$. Some formulations of the gauged supergravity absorb the function $\Gamma$ into a modified Kähler potential $K'$, which - in the case at hand - is defined as $K = -3 \cdot \ln(T + \bar{T}) \rightarrow K' = -3 \cdot \ln(T + \bar{T} + cV)$ (see e.g. \cite{27}). In the latter formulation, we have then $D = \frac{\partial K'}{\partial V} |_{V=0}$, which yields the same as below.}
for the case of just the one universal Kähler modulus $T$, i.e. the volume) including non-perturbative effects generically takes the form

\[ W = W_{\text{flux}}(S, U^I) + A \cdot e^{aT}. \] (2.8)

This form of the superpotential is guaranteed to all orders in perturbation theory by a non-renormalization theorem \[41\]. Unless the non-perturbative corrections in $T$ are absent, this superpotential breaks the invariance under an isometry of the type of eq. (2.3) which the tree level Kähler potential of $T$ has. Thus, if background fluxes are used to stabilize $S$ and the $U^I$ and non-perturbative effects are used to stabilize $T$, as in KKL T \[2\], then the shift symmetry that the Kähler potential has cannot be gauged to yield a D-term uplift \[30\].

In the absence of non-perturbative effects, the shift symmetry of $T$ can be promoted to a gauge symmetry, thus providing a D-term uplift of precisely the form of eq. (2.1). Here we are assuming, for simplicity, that the $U(1)$ gauging the shift symmetry lives on a single $D7$-brane wrapped on a 4-cycle $\Gamma$ with $\text{Re} \ T = \text{Vol}(\Gamma)$, which implies $f_T = T$. Indeed, requiring that none of the wrapped D-branes are stacked guarantees that there is no gaugino condensation in non-Abelian gauge sectors with gauge coupling $T$. This restriction in brane distributions amounts to a choice of flux. Alternatively one can hope to avoid gaugino condensation by noting that, in fact, world-volume gauge theories usually have too much matter to generate superpotentials \[12\]. Similarly, Euclidean $D3$-brane instantons - which would also break the shift symmetry - are not a generic phenomenon \[13\]. Since both effects are exponentially suppressed they would anyway break the shift symmetry only on a much lower scale than the typical scale of perturbative corrections.\(^5\) Keeping the invariance under shifts (to allow the D-term) while stabilizing $T$ demands that $T$ has to be stabilized by corrections depending solely on $T + \bar{T}$. By holomorphy of the superpotential we are then led to consider stabilization of $T$ by perturbative corrections to the Kähler potential, which depend only on $T + \bar{T}$.

3. Perturbative corrections to the Kähler potential and volume stabilization

Recently the possibility of stabilizing the volume modulus of type IIB flux compactifications

\(^4\)See, however, \[53\].

\(^5\)In any case, gaugino condensation on stacks of D7’s should not pose any problem if we have more than one Kähler modulus. Rather it could just serve to lift the additional Kähler moduli, which correspond to the size of their wrapped 4-cycles $\Gamma^i$, $T_i$. This can be seen from the effective 4D gauge coupling, which descends from the world-volume coupling as $\int_{\mathbb{R}^4 \times \Gamma^i} \sqrt{g} \cdot F_{\mu \nu} F^{\mu \nu} \sim \int_{\mathbb{R}^4 \times (T_i + \bar{T}_i)} F_{\mu \nu} F^{\mu \nu}$. Here a tilde denotes the use of a rescaled metric without dependence on the warp factor. Meanwhile, world-volume background fluxes could still gauge the (thus far unbroken) shift symmetry in the volume modulus, as seen from $a_{(2)} \wedge F_{(2)} \int_{\Gamma^i} J_{(2)} \wedge F_{(2)}$. 

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soley by perturbative corrections to the Kähler potential has received some attention \cite{31, 32}. This is due to the fact that the two leading corrections have been derived in type IIB string theory explicitly (for a few concrete examples, at least).

Firstly, one has in type IIB compactified on an orientifolded Calabi-Yau threefold an $O(\alpha'^3)$ $R^4$-correction to the 10d type IIB supergravity action \cite{33, 44} (see below for a discussion of other corrections at $O(\alpha'^3)$)

\begin{equation}
S_{\text{IIB}} = -\frac{1}{2\kappa_0^2} \int d^{10}x \sqrt{-g} \, e^{-2\phi} \left[ R + 4 (\partial\phi)^2 + \alpha' \frac{\zeta(3)}{3 \cdot 21^1} J_0 + \ldots \right].
\end{equation}

Here $J_0$ denotes the higher-derivative interaction

\[ J_0 = \left( t^{M_1 N_1 \ldots M_4 N_4} t^{M'_1 N'_1 \ldots M'_4 N'_4} + \frac{1}{8} \epsilon^{ABM_1 N_1 \ldots M_4 N_4} \epsilon_{ABM'_1 N'_1 \ldots M'_4 N'_4} R^{M'_1 N'_1 \ldots M'_4 N'_4} \right) R^{M_1 N_1 \ldots M_4 N_4}, \]

and the tensor $t$ is defined in \cite{33}. This generates a correction to the Kähler potential

\[ \Delta K^R_{\alpha'} = -2 \cdot \ln \left( 1 + (2\pi\alpha')^3 \frac{\hat{\xi}}{2\sqrt{2}} \right), \quad \hat{\xi} = -\frac{1}{4\sqrt{2}} \zeta(3) \cdot \chi \cdot (S + \bar{S})^{3/2} =: \xi \cdot (S + \bar{S})^{3/2} \]

\begin{align}
&= -2 \cdot \ln \left( 1 + (2\pi\alpha')^3 \frac{\hat{\xi}}{2(T + \bar{T})^{3/2}} \right) \\
&= -(2\pi\alpha')^3 \frac{\hat{\xi}}{(T + \bar{T})^{3/2}} + O(\alpha'^6). \quad (3.2)
\end{align}

Here $V = (T + \bar{T})^{3/2}$ denotes the Calabi-Yau volume and from now on we set $2\pi\alpha' = 1$ (of course one can put the appropriate powers of $2\pi\alpha'$ into the final results by using dimensional analysis).

Next, there exist string loop corrections to the Kähler potential. Ref. \cite{33} studied field theory loop corrections arising in the 4d $\mathcal{N} = 1$ supergravity after compactification of type IIB string theory, which by dimensional analysis start with a correction to the Kähler potential $\sim (T + \bar{T})^{-2}$. The string loop corrections have been calculated explicitly by \cite{34} for compactification of type IIB string theory on the $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold with Hodge numbers $(h_{11}, h_{21}) = (3, 51)$, and for the $\mathcal{N} = 2$ sector contribution in the $T^6/\mathbb{Z}_6'$ orientifold. Here, the induced corrections to the Kähler potential have the form

\[ \Delta K^{(g_s)} = -\beta_1 \cdot \frac{E_2^{D3}(A, U)}{(S + \bar{S})(T + \bar{T})} - \beta_2 \cdot \frac{E_2^{D7}(0, U)}{(T + \bar{T})^2}, \quad (3.3) \]

where $E_2^{D3}(A, U)$ and $E_2^{D7}(0, U)$ are string loop functions, depending on the complex structure moduli, collectively denoted with $U$, and D3-brane position open string moduli, $A$.\footnote{The D7-brane scalars and the twisted moduli have been neglected.}$\beta_1$ and $\beta_2$ are constants which read $\beta_2 = \beta_1 = 3/256\pi^6$ on the $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ while for the...
$T^6/\mathbb{Z}_6'$ they have not been determined \[34\]. Assuming that the dilaton and complex structure are stabilised, $D_S W = 0$ and $D_U W = 0$, then the large volume expansion of the scalar potential induced by all the above corrections starts with \[31, 32\]

$$V = e^{K^{(0)}} \cdot |W|^2 \cdot \left( \frac{c_1}{(T+\bar{T})^{3/2}} + \frac{c_2}{(T+\bar{T})^2} + \cdots \right)$$

(3.4)

where the first contribution comes from the $\alpha'$ correction, and the second from the string loop correction. Here, the constants $c_1$ and $c_2$ are given by:

$$c_1 = 3/4 \cdot \dot{\xi},$$

(3.5)

$$c_2 = 2 \beta_2 \cdot E_{D7}^2(0, U).$$

(3.6)

Note that it is again the piece $\sim (T+\bar{T})^{-2}$ in $\Delta K^{(g_s)}$ which is the relevant loop correction.$^7$

Now we can see that when $c_2 > 0$ and $c_1 < 0$ (which corresponds to $\chi > 0$) and $|c_2/c_1| \gg 1$ there is inevitably a non-supersymmetric $AdS_4$-minimum for the scalar potential of $Re T$ containing both corrections at large volume \[31\] (see also \[32\]). $^8$

Unfortunately, in the only fully calculated example, $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$, we have $\chi = 2 \cdot (h_{11} - h_{21}) < 0$, for which there is no minimum. We may however look to the orientifold $T^6/\mathbb{Z}_6'$ \[34, 32\] as a promising candidate for the implementation of our scenario. There, $\chi > 0$ and the known $\mathcal{N} = 2$ part of the loop corrections takes the same form as the $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ corrections (the inequivalent $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$-orientifold also has $\chi > 0$ but there the requirement of exotic $O$-planes \[47\] may complicate the loop corrections, which are presently unknown).

We therefore review the semi-explicit example of type IIB on the $T^6/\mathbb{Z}_6'$-orientifold discussed in \[34, 32\], neglecting the unknown $\mathcal{N} = 1$ sector contribution for the moment. This example will serve to illustrate the dynamics, although numerical results may change in a complete calculation of this - or any other - model. This orientifold has besides the volume modulus $T$ one untwisted complex structure modulus $U$ and the dilaton $S$. The Kähler potential including the corrections reads (we drop the $D3$-brane contribution - see footnote $\text{7}$)

$$K = -3 \cdot \ln(T + \bar{T}) - \ln(U + \bar{U}) - \ln(S + \bar{S}) - \frac{\dot{\xi}}{(T + \bar{T})^{3/2}} - \beta_2 \cdot \frac{E_{D7}^2(0, U)}{(T + \bar{T})^2}$$

(3.7)

$^7$Terms in $\Delta K^{(g_s)}$ scaling as $(T+\bar{T})^{-1}$ cancel out to leading order in the scalar potential. Here, their contribution at $O((T+\bar{T})^{-2})$ (in the bracket of eq. (3.4)) is suppressed by $(\text{Re} S)^{-2}$ which allows us to consistently drop it.

$^8$For an alternative approach to Kähler stabilization without turning to non-perturbative effects see \[46\], where, however, the gauge consistency conditions for D-terms from 4d $\mathcal{N} = 1$ supergravity were not discussed.
where for \( \text{Re}\, U \gg 1 \) we have \( \mathcal{E}_2^{DT}(0, U) = 1/\beta \cdot (U + \bar{U})^2 \) with \( \beta \) a constant, assumed positive. Combined with the flux superpotential

\[
W = \frac{1}{2\pi} \int_M G_{(3)} \wedge \Omega
\]

(3.8)

this fixes \( U \) and \( S \) in minima given by the conditions \( D_U W = 0 \) and \( D_S W = 0 \).\(^9\) We then arrive at a scalar potential for \( T \) given by

\[
V_F(T) = e^{K^{(0)}(0)} |W|^2 \cdot \left( \frac{3}{4} \frac{\xi}{(T + \bar{T})^{3/2}} + \frac{2\beta_2}{\beta} \frac{(U + \bar{U})^2}{(T + \bar{T})^2} \right) .
\]

(3.9)

Since \( \chi = 48 > 0 \) on the \( T^6/\mathbb{Z}_6' \) this leads (\( c_1 < 0 \) and \( c_2 > 0 \) here) to an \( AdS_4 \)-minimum for \( \text{Re}\, T \) at

\[
\text{Re}\, T = \left( \frac{80\beta_2}{27\beta} \right)^2 \frac{1}{\xi^2} \cdot \frac{(\text{Re}\, U)^4}{(\text{Re}\, S)^3} .
\]

(3.10)

From this expression it is clear that tuning the \( AdS_4 \)-minimum for \( \text{Re}\, T \) to moderately large volume needs a tuning of \( \text{Re}\, U \) to large values \(^{32}\).

Let us now comment on the large volume expansion in presence of the other higher-dimension operators appearing in the type IIB effective action at \( \mathcal{O}(\alpha'^3) \). The full supersymmetric form of the \( \alpha' \)-correction in 10d is interpreted in \(^{33, 48}\) as arising at the bosonic level from both the \( R^4 \)-term and \( R^3G^2 \) - and \( R^2(DG)^2 \)-corrections. The form of the contribution to the scalar potential induced by eq. (3.2) shows a volume scaling

\[
\Delta V_{\alpha'^3}^{R^4} \sim V^{-3}
\]

(3.11)

compared with the scaling of the tree level flux contribution

\[
V_{\text{flux}} = e^{K^{(0)}(0)} K^{IJ}_{cs} D_I W \overline{D_J W} \sim V^{-2} .
\]

(3.12)

Here \( K_{cs} \) denotes the tree level Kähler potential for the complex structure moduli. All other competing higher-dimension operators at \( \mathcal{O}(\alpha'^3) \) (such as \( G^4R^2 \), \( \ldots \), \( G^8 \)) have a subdominant volume scaling \( \sim V^{-s} \), \( s \geq 11/3 \) with respect to the leading correction discussed above \(^{48}\). Note that this is also subdominant with respect to the string theoretic 1-loop corrections given above since they induce a correction to the scalar potential scaling as

\[
\Delta V_{gs} \sim V^{-10/3} .
\]

(3.13)

\(^9\)Note that the string loop corrections of \(^{34}\) have been calculated in the absence of bulk fluxes. As usual, we assume that the back reaction of the \( G_{(3)} \)-flux on the geometry is weak in the large volume limit, which leaves the corrections unchanged to leading order. Possible additional corrections from the open string/D-brane sector have also been neglected in \(^{33, 34}\).
Furthermore, within our assumptions, higher-order $\alpha'$-corrections are subleading to the above string loop corrections in the region of the scalar potential around the minimum.

Finally, we should comment on the stability of the minimum found above with respect to the minima for the complex structure moduli and the dilaton. Prior to the introduction of the perturbative corrections to the Kähler potential, the complex structure moduli and the dilaton were fixed by background fluxes through the conditions $D_U W = 0$ and $D_S W = 0$. Consider now the case that the Kähler corrections, which are negative near to the minimum of $T$, try to drive away $S$ and/or $U$ from their minima $D_S W = D_U W = 0$. Then the tree level flux potential yields a contribution $V_{\text{flux}} \sim O(+1/V^2)$, while the Kähler corrections contribute at $O(-1/V^3)$, which is subleading at large volumes. Thus, the corrections cannot destabilize the original minima of $S$ and $U$ and these remain minima of the full theory including the Kähler corrections which stabilize $T$.\(^{10}\) Moreover, similar arguments can of course be used after including the uplifting, to which we now turn.

4. de Sitter vacua from a consistent D-term

It is now easy to see that the perturbative $AdS_4$-minimum for $T$ discussed in the last Section can be uplifted to a $dS_4$-minimum with a consistent D-term. The full theory including the perturbative corrections to the Kähler potential is a function of $T + \bar{T}$ alone. Thus it is fully invariant under the shift $T \rightarrow T + i\alpha$ and in particular we have invariance of $G = K + \ln |W|^2$ under this shift symmetry. Therefore, the mechanism of Kähler stabilization of the volume modulus $T$ fulfills the consistency constraints of Sect. 2. This allows us to gauge the shift symmetry, using world-volume fluxes on a $D7$-brane, as described in that section.

The full scalar potential will now contain a D-term piece in addition to the F-term contributions from the Kähler corrections. For the $T^6/Z'_6$ example the potential, expanded

\(^{10}\)This argument follows a similar discussion in \cite{49, 50} where the interplay of the $O(\alpha'^3)$-correction with the potential induced by gaugino condensation was studied at large volume. Notice that this is an improvement on KKLT \cite{50}, where the stabilizing F-term potential from gaugino condensation is $O(-1/V^2)$.\]
up to $\mathcal{O}(\alpha'^3/(T + \bar{T})^{3/2})$ and to leading order in the string loop corrections, reads

$$V = V_F + V_D = e^{K(0)}|W|^2 \left( \frac{3}{4} \xi \cdot \frac{(S_0 + \bar{S}_0)^{3/2}}{(T + \bar{T})^{3/2}} + \frac{2\beta_2}{\beta} \frac{(U_0 + \bar{U}_0)^2}{(T + \bar{T})^2} \right)$$

$$+ \frac{1}{2} \left( \text{Re} f_{T7}^{-1} D_T^2 \right),$$

$$f_{T7} = T + kS \text{ and } D_T = \frac{3q}{T + \bar{T}} \cdot \left( 1 + \mathcal{O}(\alpha'^3, g_s^2) \right), \quad k \geq 0$$

$$= \frac{|W|^2}{(T + \bar{T})^3} \cdot \left( \frac{3}{4} \xi \cdot \frac{(S_0 + \bar{S}_0)^{3/2}}{(T + \bar{T})^{3/2}} + \frac{2\beta_2}{\beta} \frac{(U_0 + \bar{U}_0)^2}{(T + \bar{T})^2} \right)$$

$$+ \frac{1}{1 + k \cdot \frac{S_0 + \bar{S}_0}{T + \bar{T}}} \cdot \frac{9q^2}{(T + \bar{T})^3}$$

(4.1)

where $U_0$ and $S_0$ are constants given by the values of $U$ and $S$ in the minima determined by $D_U W = D_S W = 0$. Note that $V_D$ has been expanded only to leading order since later tuning will require $V_D$ to cancel $V_F$ to leading order. Taking into account the higher orders in $D_T$ would require to write the higher orders in $V_F$ as well for consistency. The D-term contribution is the one coming from the $D7$-brane and comparison with its DBI action allows for the identification

$$q^2 \sim \int \Gamma d^4 \xi \sqrt{g_8} F_{mn} F^{mn}. \quad (4.2)$$

Here a tilde denotes the use of the Weyl rescaled metric showing the dependence on the radial (volume) modulus $T = e^{4u} + ib$

$$ds^2 = e^{-6u} \tilde{g}_{\mu\nu} dx^\mu dx^\nu + e^{2u} \tilde{g}_{mn} dy^m dy^n. \quad (4.3)$$

This identification implies that the values of $q$ are quantized since the $D7$-brane carries an effective $D3$-charge as seen from

$$S_{D7} \supset \mu_7 \cdot (2\pi \alpha')^2 \int_{R^4 \times \Gamma} C_{(4)} \wedge F_{(2)} \wedge F_{(2)} \sim \mu_7 \cdot (2\pi \alpha')^2 \cdot \int_{\Gamma} F_{(2)} \wedge F_{(2)} \int_{R^4} C_{(4)}. \quad (4.4)$$

Since any change in the $D3$-charge has to be compensated by either changes in the number of $D3$-branes or discrete flux units this implies the discreteness of $q$.

Given that $\text{Re} T$ is stabilized at large volume $V = (T + \bar{T})^{3/2}$, and assuming that $q^2$ is $\mathcal{O}(1)$, we can arrange for a situation where

$$\left. V_F \right|_{\text{min}} \sim \frac{|W|^2}{V^{3/2}} \sim V_D \left|_{\text{min}} \right. \sim \frac{q^2}{V^2}$$

(4.5)

$k$ is a function of the gauge flux on the $D7$-brane and vanishes for zero flux [51]. We thank D. Krefl for bringing this to our attention.
Figure 1: Dotted: The F-term scalar potential $V_F(T)$ leading to perturbative Kähler stabilization of $T$. Dashed: The uplifting D-term scalar potential $V_D(T)$. Both graphs have been rescaled by $10^{-2}$ for display reasons. Solid: The scalar potential eq. (4.1) after uplifting by switching on a gauge field background on a single $D7$-brane. The numbers are chosen in this example as $W_0 = 25.5$, $q = 1$, $\text{Re} \, U = 242$, $\text{Re} \, S = 10$, $\chi = 48$ and $k = 0$ (for simplicity since $V_D$ would get corrected by only about 5% for a $k = \mathcal{O}(1)$). Also $\beta_2 = \beta_1 = 3/256 \pi^6$ and $\beta = 4 \pi^2 \beta_2$ are taken from the $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ as a guiding example.

holds by tuning $W$ to larger values such that we get $V_F + V_D \approx 0$ in the minimum.

This situation is displayed in Fig. 1 using the potential given in eq. (4.1) for the semi-explicit $T^6/\mathbb{Z}_6'$-example. This serves as an indication of how we expect the behavior to be in a fully explicit model. However, we should keep in mind that there may be unknown contributions in this example and the numerical results may change. Moreover, on the level of the present discussion we have omitted the charged matter fields of the open string sector which may contribute to the Kähler potential via loops, too, in a complete model. In any case, given the vast landscape of type IIB flux compactifications, we expect that there should be many models in type IIB string theory where our uplifting scenario yields qualitatively the same results as discussed here.

5. Conclusion

In this paper we discussed a mechanism for generating de Sitter vacua in string theory by spontaneously breaking supersymmetry with consistent D-terms. This proposal has proven difficult to consistently embed in a stringy scenario. We find that type IIB flux compactifications, with volume stabilization via perturbative corrections to the Kähler potential, provide such a scenario. As discussed in the literature, $\alpha'$- and string loop
corrections allow for stabilization of the $T$ modulus by purely perturbative means without turning to non-perturbative effects such as gaugino condensation. Unlike non-perturbative effects, the Kähler corrections preserve the invariance of the theory under a shift symmetry of the $T$ modulus. In the presence of a magnetised $D7$-brane this unbroken shift invariance is gauged which leads to supersymmetric D-terms from string theory which fulfill all the known consistency requirements of 4d $\mathcal{N} = 1$ supergravity. These D-terms then provide a parametrically small and tunable uplift of the perturbatively stabilized $AdS_4$-minimum towards a metastable $dS$-minimum. In view of the desire to search the ‘landscape’ of string theory vacua for those regions where spontaneously broken supersymmetry allows for certain control of the low-energy effective theory, the discussed mechanism of a consistent D-term uplift in string theory promises access to a new class of metastable $dS$-vacua. In a fully explicit model, it would be necessary to calculate the string loop corrections in the presence of gauged symmetries and magnetised D-branes, for example along the lines of [52]. It would also be interesting to study the consequences of this uplifting mechanism for possible realizations of inflation in string theory, as well as the low-energy phenomenology of this type of spontaneous SUSY breaking.

While this manuscript was being prepared another paper [53] appeared which studies the possibility of consistently realizing the original proposal of [27] in a 4d KKLT inspired setup.

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