$|\Delta \lambda| = \frac{1}{2}$ rule with one intermediary charged.

Meson field.

E. d'Espagnat

Centre de Physique Theorique d'Orsay and CERN - Geneva.

The leptonic decays of baryons and mesons are most economically described in terms of a product of one charged current with its own hermitian conjugate, $J^+_{\mu} J^\dagger_{\mu}$. On the other hand, non-leptonic decays obey the $|\Delta \lambda| = \frac{1}{2}$ rule. A satisfactory universal theory of weak interactions should therefore lead - at least in some approximation - to the $|\Delta \lambda| = \frac{1}{2}$ rule and still be expressible in terms of a single product $J^+_{\mu} J^\dagger_{\mu}$ (or, what amounts to the same, in terms of interactions with a single complex boson field $X_{\mu}^{(t)}$).

The purpose of this note is to point out that these requirements can indeed be satisfied simultaneously. More precisely, it will be shown that, as long as the $N - \Xi$ mass splitting due to strong interactions is ignored, the Hamiltonian

$$H = \frac{1}{2} \left[ J^+_{\mu}^{(B)} + J^\dagger_{\mu}^{(L)} \right] X^{(t)}_{\mu} + h.c. \quad (1)$$

leads to the $|\Delta \lambda| = \frac{1}{2}$ rule. In (1), $J^+_{\mu}^{(L)}$ is of course the conventional leptonic current

$$J^+_{\mu}^{(L)} = \left( \bar{e}_- + \bar{\mu}_- \right) i \gamma^\mu \left( 1 + \gamma_5 \right) \nu$$

8723/TH96
and the baryonic current \( J^{(B)}_\mu \), involving \( |\Delta I| = 1, 0, 2 \) terms, has the form

\[
J^{(B)}_\mu = F' i \gamma_\mu (1 + \gamma_5) p - \bar{\Xi}_o i \gamma_\mu (1 + \gamma_5) F'
\]

where \( a, b, c \) are arbitrary real coefficients satisfying

\[
a + b = 1
\]

For simplicity, the operators \( i \gamma_\mu (1 + \gamma_5) \) will no more be written in what follows; they are however understood to be present everywhere in the obvious way.

In order to derive (1) we look for combinations of \( |\Delta I| = \frac{1}{2} \) interactions that involve only charged currents. We begin by noticing that the two expressions

\[
U = (\bar{N} \cdot \Xi) (\Xi \cdot \Lambda) = (\bar{p} \Xi_o + \bar{n} \Xi_o)/(\Xi \cdot \Lambda)
\]

\[
V = (\Xi \cdot N) (\bar{p} \cdot \Lambda) = (\Xi_o p + \Xi_o n)/(\bar{p} \cdot \Lambda)
\]

both lead to the \( |\Delta I| = \frac{1}{2} \) rule, their first bracket being an iso-scalar (they are therefore components of an iso-spinor). They both are products of charged currents but cannot however be used directly for our purpose since they involve no strangeness-conserving current (required for \( \rho \) and \( \pi \) decays). Similarly

\[
A = (\bar{N} \cdot N) (\bar{\Xi}_o \cdot \Lambda) \quad \quad \quad \quad \quad \quad B = (\Xi \cdot \Xi) (\bar{n} \cdot \Lambda)
\]

obviously lead to the \( |\Delta I| = \frac{1}{2} \) rule. Finally

\[
C = (\bar{p} \cdot \Xi + \bar{\Xi}_o \cdot n - \bar{n} \cdot \Xi_o) (\bar{\Xi}_o + \bar{n} \cdot \Lambda)
\]

also leads to the \( |\Delta I| = \frac{1}{2} \) rule but only in the approximation, stated above, that the part of the strong interactions which splits the \( N \) and \( \Xi \) masses can be neglected. This can be shown as follows: in this approximation the strong interactions are invariant in M space\(^{(1)}\); \( C \) on the other hand can be expressed as

\[
C = \sqrt{2} \left( \bar{B}_+ B_- - B_0 \bar{B}_0 \right) \left( B_0 \cdot \Lambda \right)
\]

with

\[
\bar{B}_+ = \rho, \quad \bar{B}_- = \Xi, \quad \bar{B}_0 = 2^{-\frac{1}{2}} (\Xi_o - n), \quad \bar{B}_0 = 2^{-\frac{1}{2}} (\Xi_o + n)
\]

and it also, therefore, an invariant in \( M \) space; it then follows from a general theorem\(^{(2)}\) that the decays induced by \( C \) obey the \( |\Delta I| = \frac{1}{2} \) rule to any order in the \( M \) conserving strong interactions.
Using the identity (due to the unwritten $i \gamma^\mu (1 + \gamma_5)$ inside each bracket)

$$(\overline{U}_1 \gamma_2) \overline{V}_3 \gamma_4 = (\overline{U}_3 \gamma_2) \overline{V}_1 \gamma_4$$

Eq. (12)

we obtain easily the relation

$$C = (\overline{\tilde{\rho}} \tilde{\rho} + \tilde{\Xi} \cdot \tilde{\Xi} - \tilde{\Xi}_0 \tilde{\Xi}_0)(\tilde{n} \Lambda) + (\overline{\tilde{\rho}} \tilde{\rho} - \tilde{n} \tilde{n} + \tilde{\Xi} \cdot \tilde{\Xi})(\tilde{\Xi}_0 \Lambda) + D$$

with

$$D = (\overline{\tilde{\Xi}_0} \tilde{\Xi}_0)(\tilde{n} \Lambda) - (\tilde{\Xi}_0 \tilde{n})(\tilde{\Xi}_0 \Lambda)$$

Eq. (13)

Thus

$$A + B + C = (\overline{\tilde{\rho}} \tilde{\rho} + 2 \tilde{\Xi} \cdot \tilde{\Xi})(\tilde{n} \Lambda) + (\overline{\tilde{\rho}} \tilde{\rho} + \tilde{\Xi} \cdot \tilde{\Xi})(\tilde{\Xi}_0 \Lambda) + D$$

Eq. (14)

or, using (12) again

$$A + B + C = [\overline{\tilde{n}} \tilde{n} + 2 \tilde{\Xi}_0 \tilde{\rho}](\tilde{\rho} \Lambda) + [\tilde{\Xi}_0 \tilde{\Xi}_0 + 2 \tilde{n} \tilde{n}](\tilde{\Xi}_0 \Lambda) + D$$

Eq. (15)

This is still essentially a sum of the products of charged currents in order to get only one product we must still add some terms obeying $|A'_{\tilde{l}}| \leq \frac{1}{2}$ in such a way that the modified square brackets become, up to some factor, hermitian conjugates to each other. This can be achieved by adding $\alpha U + \beta V$, $\alpha$ and $\beta$ being, for the time being, unknown coefficients. Then

$$H = A' + B + C + \alpha U + \beta V = \left[\overline{\tilde{n}} \tilde{n} + (2 + \rho) \tilde{\Xi}_0 \tilde{\rho} + \rho \tilde{\Xi} \cdot \tilde{\Xi} \Lambda\right] +$$

$$+ \left[\tilde{\Xi}_0 \tilde{\Xi}_0 + (2 + \rho) \tilde{n} \tilde{n} + \rho \tilde{\Xi} \cdot \tilde{\Xi} \Lambda\right] + D$$

Eq. (16)

A part of the last term in each square bracket can now, using (15), be transformed to the other square bracket, as shown in eq. (10) and (20) below:

$$H' = \left[\overline{\tilde{n}} \tilde{n} + (2 + \rho) \tilde{\Xi}_0 \tilde{\rho} - (2 + \rho) \tilde{\Xi} \cdot \tilde{n} \Lambda\right] + (2 + \rho) \tilde{\Xi} \cdot \tilde{n} \Lambda +$$

$$+ \left[\tilde{\Xi}_0 \tilde{\Xi}_0 + (2 + \rho) \tilde{n} \tilde{n} - (2 + \rho) \tilde{\Xi} \cdot \tilde{\Xi} \Lambda\right] + (2 + \rho) \tilde{\Xi} \cdot \tilde{\Xi} \Lambda + D$$

Eq. (17)

$$H' = \left[\overline{\tilde{n}} \tilde{n} + (2 + \rho) \tilde{\Xi}_0 \tilde{\rho} + (2 + \rho) \tilde{\Xi}_0 \tilde{\rho} - (2 + \rho) \tilde{\Xi} \cdot \tilde{n} \Lambda\right] +$$

$$+ \left[\tilde{\Xi}_0 \tilde{\Xi}_0 + (2 + \rho) \tilde{n} \tilde{n} + (2 + \rho) \tilde{\Xi} \cdot \tilde{\Xi} \Lambda\right] + D$$

Eq. (18)

with the choice $\alpha + \beta = -3$ the second square bracket is now equal, up to the sign, to the hermitian conjugate of the first one. Thus with

$$K = \overline{\tilde{n}} \tilde{n} - \tilde{\Xi} \cdot \tilde{\Xi}_0 + a \tilde{\Xi}_0 \tilde{\rho} - b \tilde{\Xi} \cdot \tilde{n} \; ; \; \; a + b = \alpha + \beta + \gamma = 1$$

Eq. (19)

$$H' = K (\tilde{\rho} \Lambda) - K^+(\tilde{\Xi} \cdot \tilde{\Xi} \Lambda) + D$$

Eq. (20)

Therefore

$$H + h.c = K (\tilde{\rho} \Lambda - \tilde{\Xi} \cdot \tilde{\Xi} \Lambda) + D + O^+$$

Eq. (21)

Being constructed (first eq. (10)) with the help of expressions $A, B, C, U, V$
which exactly the $|\Delta \xi| = \frac{1}{2}$ rule, $\mathcal{H'}$ necessarily satisfies if one also.

On the other hand, (eq. (15)) implies $\Delta S = \pm 3$ and therefore does not
operate in any known decays; one can therefore draw the conclusion that

$$\mathcal{K} \left( \frac{1}{2} \mathcal{R} - \Lambda \right) + h, c$$

leads to the $|\Delta \xi| = \frac{1}{2}$ rule for all known decays ($\Delta S = \pm 1$) and within the limits
of the approximations stated at the beginning of this note. For these known decays
$\Delta S = \pm 1$, (14) can also be written

$$c^{-1} \left[ \mathcal{K} + c \mathcal{R} - c \Lambda \right] \left[ \mathcal{K} + c \mathcal{R} - c \Lambda \right]$$

which leads obviously to (1) if the intermediate meson assumption
is made.

The theory can easily be generalized so as to include $\Sigma$ particles. The
procedure follows, with some minor modifications, the same lines as above. The
result is eq (1) again, with now

$$J_B^{(\beta)} = \left( \mathcal{N} + a \mathcal{E}_o + (\Lambda + d \mathcal{E}_o) \right) \rho - \mathcal{E}_o \left( \mathcal{N} + b \mathcal{E}_o + (\Lambda - d \mathcal{E}_o) \right) + d \sqrt{2} \left( \mathcal{N} - \mathcal{E}_o \right)$$

where again $i \gamma_4 (i \gamma_5)$ has been omitted everywhere, with the special choice $c = d$.

(26) can be expressed by means of the usual $Y : \left( \begin{array}{c} E_o \\ \mathcal{N} \end{array} \right)$ ket as

$$J_B^{(\beta)} = \left( \mathcal{N} + a \mathcal{E}_o \right) \rho - \mathcal{E}_o \left( \mathcal{N} + b \mathcal{E}_o \right) + c \sqrt{2} \left( \mathcal{N} - \mathcal{E}_o \right)$$

An interesting feature of eq. (26) is that leptonic decays of hyperons
and $K^*$'s are now predicted to obey the $\Delta Q/\Delta S = 1$ rule postulated by Feynman &
Colladay (3). More specifically, the strangeness-non-conserving current in (26)
is itself a $|\Delta \xi| = \frac{1}{2}$ current. The implications of this fact as regards leptonic
decays are well known (see e.g. Hambach's report, CERN 1958 (4)).
An alternative approach is the one recently proposed by Lee and Yang\textsuperscript{5).} At the price of introducing two complex boson fields (charged and neutral) instead of the single charged one of Eq.(1) above, these authors are able to achieve a more stringent $|\Delta S| = \frac{1}{2}$ rule than ours and at the same time to avoid $|\Delta S| = 2$ in non-leptonic decays. In this approach, however, the non-observation of any neutral leptonic current effect makes it necessary to reduce in a somewhat artificial way the couplings of the neutral boson field to the leptons, as compared to the corresponding charged boson field couplings. We feel that the most critical test for a discrimination between these two theories will probably be the neutral $K$-experiments on the presence or absence of $|\Delta S| = 2$ in weak interactions.

\textbf{REFERENCES}


5) T.D. Lee and C.N. Yang, to be published.