CM-P00057188

COULOMB CORRECTIONS CONCERNING A TEST FOR TIME REVERSAL
VIOLATION IN NEUTRINO-INDUCED REACTIONS

J.J.J. Kokkedee

CERN - Geneva

A B S T R A C T

Recently it was proposed to test time reversal invariance in weak interactions through the measurement of the transversal polarization of the muon in the reaction $\nu (\bar{\nu}) + \text{nucleus} \rightarrow \mu^- (\mu^+) + \text{all final states}$. Neglecting Coulomb corrections this transversal polarization must be zero if time reversal invariance holds, but for heavy nuclei it is not clear whether or not, in this connection, the effect of Coulomb final state interactions of the muon is negligible. In this paper an upper limit for the amount of transverse polarization due to Coulomb interactions is derived under some rather general assumptions. It is found that for muon energies of 500 MeV or higher this amount of polarization will be below 1%, the effects being smaller the higher the energy of the muon.

1. INTRODUCTION

Recently several tests have been proposed ^{1),2)} to inspect general properties (time reversal invariance, the conserved vector current and partially conserved axial vector current hypothesis) of the weak interactions in high-energy neutrino reactions. These tests have in common the feature that they are independent of the complications of nuclear structure, but in general Coulomb effects may interfere. In this paper we confine ourselves to the test concerning time reversal invariance ¹⁾, the validity of which has been questioned by a recent experiment on K_{02} decay ³⁾.

For the time reversal test one considers the polarization of the muon transversal to the muon-neutrino momentum plane. It was proved in Ref. ¹⁾ that if time reversal invariance is valid, the transverse muon polarization vanishes, provided one averages over all strongly interacting particles in the final state and uses an unpolarized target, and provided one neglects final state interactions of the muon. Conversely, under these conditions a non-zero value for the transverse muon polarization would indicate a violation of time reversal invariance. However, as is well known ⁴⁾ final state Coulomb interaction of the muon may give rise to such a polarization, even if time reversal invariance holds; the Coulomb field may turn the muon spin out of the muon-neutrino plane.

In this note we try to make an estimate of this effect. For the general process (1) we derive an upper bound for the transverse muon polarization, assuming time reversal invariance for the weak interactions; this upper bound is independent of the detailed structure of the process. Hereby it is assumed that the final state Coulomb interaction of the muon is described sufficiently accurately as the interaction of the muon with a static Coulomb field corresponding to a charge Ze , being the charge of the final nucleus. This interaction is then taken into account by using for the muon a relativistic Coulomb wave function in a high energy approximation. Of course, perturbation theory is inadequate here, since the electromagnetic coupling constant $Ze^2/\hbar c$ is not small for heavy nuclei.

For muon energies in the GeV region the upper limit obtained for the transverse polarization is only a fraction of a percent, thus justifying the conclusion that if in the future some considerable transverse muon polarization is found in high energy reactions of type (1), this would have to be interpreted as an effect of time reversal violation.

2. THE MATRIX ELEMENT

We consider processes of the type :

$$\nu + \text{nucleus} \rightarrow \mu^- + \text{all final states} \quad (1.a)$$

$$\bar{\nu} + \text{nucleus} \rightarrow \mu^+ + \text{all final states} \quad (1.b)$$

For definiteness, we shall restrict ourselves in the following to incident neutrinos [reaction (1.a)]. At the end of Section 4, however, we shall also write down the results for processes (1.b).

The matrix element for reaction (1.a) can be written as follows :

$$M(q, p, w, P, K, \lambda, \lambda') = C \int d_4 x \langle p, w | j_\mu(x) | q \rangle \langle K, \lambda' | J_\mu(x) | P, \lambda \rangle. \quad (2)$$

Here p and q are the four-momenta of the muon and neutrino, respectively ^{*)}, while w is a polarization four-vector for the muon (see Section 3). The dependence of M on w indicates that we wish to observe muon states corresponding to definite spin directions. In the

^{*)} We use natural units such that $\hbar = c = 1$. Our metric is $a_\mu b_\mu = a \cdot b - a_4 b_4$ and $a_4 = i a_0$. Greek indices run from 1 to 4, latin indices from 1 to 3.

following, it is essential that the location of the target nucleus is correlated with the location of the charges that generate the Coulomb field; thus the initial nucleus is described by a wave packet, with a certain average four momentum P . Further, K is the four-momentum of the system of final strongly interacting particles (hadrons) and λ and λ' respectively stand for the internal variables of the initial and final hadron systems, such as spin, which further characterize these states. Finally, $j_\mu = \bar{\psi}^{(m)} \gamma_\mu (1 + \gamma_5) \psi^{(n)}$ is the lepton current, and J_μ represents the hadronic weak current. In Eq. (2), and in all equations to follow, C always denotes for our purpose irrelevant proportionality constants. Equation (2) is supposed to include all possible strong or electromagnetic interaction effects among the hadrons.

Now, since the muon moves in the Coulomb field of the final nucleus, we have to insert for $|p, w\rangle$ a wave function describing a Dirac particle in a Coulomb field. This Coulomb wave function for the muon has to be found by solving the Dirac equation

$$\left(\gamma_\mu \frac{\partial}{\partial x_\mu} + \gamma_4 V(r) - m \right) \psi(x) = 0, \quad (3)$$

where m is the muon mass and $V(r)$ is the central Coulomb potential, centered at the origin. Here we have assumed that the Coulomb field generated by the system of final hadrons (with total charge Ze) may be approximated by a static Coulomb field corresponding to a charge Ze . Thus with respect to the Coulomb interaction, we neglect recoil and also incoherence effects, i.e., collisions between the muon and individual hadrons.

Equation (3) cannot be solved in closed form. However, for our purpose, a suitable approximate solution is provided by the so-called Sommerfeld-Maue wave function⁵⁾, which is of the form (we specialize to the case of an outgoing particle) :

$$\bar{\Psi}^{(m)} = N e^{-i\vec{p}_0 \cdot \vec{r} + i p_0 t} \bar{u}^{(m)}(\vec{p}, w) \left[1 + \frac{1}{2p_0} \gamma_4 \gamma_i \frac{\partial}{\partial x_i} \right] F(\vec{r}, \vec{p}) . \quad (4)$$

In this expression $\bar{u}^{(m)}(\vec{p}, w)$ is the usual four-component Dirac spinor corresponding to a given spin direction w , N is a normalization constant and $N \bar{u}^{(m)}(\vec{p}, w) e^{-i\vec{p}_0 \cdot \vec{r}} F(\vec{r}, \vec{p})$ is the solution of the second order equation (r is the length of the vector $\vec{r}$):

$$\left[\nabla^2 + p_0^2 - m^2 - 2p_0 V(r) \right] \varphi = 0 . \quad (5)$$

For the case of a point charge distribution, corresponding to $V(r) = -Ze^2/r$, Eq. (5) can be solved in terms of a confluent hypergeometric function, with the result

$$F(\vec{r}, \vec{p}) = F_1(a, 1, i\vec{p}_0 \cdot \vec{r} + i|\vec{p}|r) . \quad (6)$$

$$a = \frac{Ze^2 p_0}{|\vec{p}|} \quad (7)$$

The Sommerfeld-Maue wave function, as given by (4) and (6), has been used for calculating cross-sections for bremsstrahlung and external pair production ⁶⁾, and for production of intermediate vector bosons ⁷⁾. It is essentially a high-energy approximation, in the sense that the momentum transfer to the Coulomb field is assumed to be small with respect to the momentum of the particle.

Inserting (4) into (2), together with a plane wave for the neutrino, we can write in the system of reference where the origin of the Coulomb field is at rest :

$$M(q, p, w, P, K, \lambda, \lambda') = C \int d_4 x e^{i(q-p)x} \langle K, \lambda' | J_\mu(x) | P, \lambda \rangle.$$

$$\left(\bar{u}^{(m)}(p, w) \left\{ 1 + \frac{1}{2p_0} \gamma_4 \gamma_i \frac{\partial}{\partial x_i} \right\} F(z, A) \gamma_\mu (1 + \gamma_5) u^{(n)}(q) \right). \quad (8)$$

In order to avoid a detailed description of the form of the wave packet of the initial target, we will make the unessential assumption that in the system of reference used here also the initial nucleus is at rest, and is then of course located at the centre of the Coulomb field. This means that we take it that most of the electric charge carried by the final hadron system remains in or around the location of the original nucleus. It follows that the space part of the integral in Eq. (3) extends only over a finite region in space, roughly over a sphere of radius R , the nuclear radius, around the centre of the Coulomb field.

Furthermore, it follows from (8) that in the absence of a Coulomb field, $M(q, p, \dots)$ can be written as the product of a leptonic part $(\bar{u}^{(m)}(p, w) \gamma_\mu (1 + \gamma_5) u^{(n)}(q))$ and a hadronic part h_μ , given by

$$h_\mu = C \int d_4 x e^{-ikx} \langle K, \lambda' | J_\mu(x) | P, \lambda \rangle. \quad (9)$$

Apart from the internal variables λ and λ' , h_μ depends only on the four-vectors K , P and k , where $k = p - q$ is the momentum transfer from the leptons to the hadrons.

When the muon interacts with the nuclear Coulomb field, which - as is said before - is considered static, the momentum transfer from leptons to hadrons is no longer exactly equal to $p - q$, because momentum is being transferred from muon to Coulomb field. In this case, a factorization of the right-hand side of (8) into a leptonic part and a hadronic part is not possible in an exact way. For high-energy muons, however,

such a factorization can be obtained to very good approximation. This is very desirable for our purpose of trying to find an upper limit for the transverse muon polarization which does not involve strong interaction effects.

In order to achieve this result, we observe that in the case of a point charge the functions $F(\underline{r}, \underline{p})$ and $(\partial/\partial x_i)F(\underline{r}, \underline{p})$ are very smooth functions in space, apart from some oscillations around $r=0$ over a region $r(1+\cos\theta) \lesssim |\underline{p}|^{-1} \ll R^*$). For high muon energies this latter region is very small compared to the nuclear volume. Moreover, in practice we have a smeared out charge distribution (and a smeared out nucleus) and no special fluctuations around $r=0$ occur. Thus, if $R|\underline{p}| \ll 1$, it is reasonable to replace the functions $F(\underline{r}, \underline{p})$ and $(\partial/\partial x_i)F(\underline{r}, \underline{p})$ in Eq. (8) by their average values over the nuclear region. In case we use a wave function corresponding to a point charge, this averaging will eliminate the undesired variations around $r=0$. We thus set :

$$\begin{aligned} G(\underline{p}) &= \frac{1}{\Omega} \int d\underline{r} F(\underline{r}, \underline{p}) , \\ H_i(\underline{p}) &= \frac{1}{\Omega} \int d\underline{r} \frac{1}{2p_i} \frac{\partial}{\partial x_i} F(\underline{r}, \underline{p}) , \end{aligned} \quad (10)$$

where the integrations extend over the nucleus, Ω is the nuclear volume. This leads us to the following expression for the matrix element

$$\begin{aligned} M(q, p, \omega, P, K, \lambda, \lambda') = \\ h_\mu(P, K, k, \lambda, \lambda') \left[G(\underline{p}) \left(\bar{u}^{(m)}(\underline{p}, \omega) \gamma_\mu (1 + \gamma_5) u^{(n)}(\underline{q}) \right)_{(11)} \right. \\ \left. + H_i(\underline{p}) \left(\bar{u}^{(m)}(\underline{p}, \omega) \gamma_4 \gamma_i \gamma_\mu (1 + \gamma_5) u^{(n)}(\underline{q}) \right) \right] . \end{aligned}$$

*) Remember that for a point charge $F(\underline{r}, \underline{p})$ is given by [see Eq. (6)]
 $F(\underline{r}, \underline{p}) = {}_1F_1(ia, 1, i|\underline{p}|r + i\underline{p} \cdot \underline{r}) = {}_1F_1(ia, 1, i|\underline{p}|r(1+\cos\theta))$, where θ is the angle between $\underline{r}$ and $\underline{p}$. Furthermore, note that $R \simeq 30(\text{GeV})^{-1}$ [see Section 4].

We see that in this expression we have obtained the desired factorization of M into a hadronic part h_μ and a leptonic part, the factor within square brackets. The hadronic part is given by (9) with the momentum transfer $k=p-q$, i.e., it is the same as for the case without Coulomb field. In other words, the approximation made above is such that the deviation of the momentum transfer from the value $p-q$ is neglected.

Squaring (11), we find, after summing over final states $|K, \lambda\rangle$ and averaging over initial states $|P, \lambda\rangle$ for given P , which we indicate by the symbol $\sum_{K, \lambda, \lambda'}$:

$$|M(q, p, \omega, P)|^2 = N_{\mu\nu}(P, k) L_{\mu\nu}(p, q, \omega), \quad (12)$$

$$N_{\mu\nu} = \sum_{K, \lambda, \lambda'} h_\mu(P, K, k, \lambda, \lambda') h_\nu^\dagger(P, K, k, \lambda, \lambda') \quad (13)$$

$$L_{\mu\nu} = a_\mu a_\nu^\dagger |G(p)|^2 + a_\mu b_{\nu i}^\dagger G(p) H_i^*(p) + \\ + b_{\mu i} a_\nu^\dagger G^*(p) H_i(p) + b_{\mu i} b_{\nu j}^\dagger H_i(p) H_j^*(p) \quad (14)$$

$$a_\mu = (\bar{u}^{(m)}(p, \omega) \gamma_\mu (1 + \gamma_5) u^{(n)}(q)) \quad (15)$$

$$b_{\mu i} = (\bar{u}^{(m)}(p, \omega) \gamma_\mu \gamma_i \gamma_5 (1 + \gamma_5) u^{(n)}(q)) \quad (16)$$

As a consequence of Lorentz invariance, hermiticity and time reversal invariance, the most general expression for $N_{\mu\nu}$ is :

8.

$$N_{\mu\nu} = A_1 \delta_{\mu\nu} + A_2 P_\mu P_\nu + A_3 k_\mu k_\nu + A_4 (P_\mu k_\nu + k_\mu P_\nu) + A_5 \varepsilon_{\mu\nu\sigma\tau} P_\sigma k_\tau, \quad (17)$$

in which the real scalar quantities $A_1, \dots, A_5$ are functions of the invariants k^2, P^2 and (Pk) ; $\varepsilon_{\mu\nu\sigma\tau}$ is the completely antisymmetric tensor with $\varepsilon_{1234} = 1$. If time reversal invariance is not valid, we would have to add an additional term ¹⁾ $iA_6(P_\mu k_\nu - k_\mu P_\nu)$.

3. EXPRESSIONS FOR THE LONGITUDINAL AND TRANSVERSE POLARIZATION OF THE MUON

The quantities involving a_{μ} and $b_{\mu,i}$ in Eq. (14) can, in the usual way, be reduced to traces of γ matrices, making use of energy projection operators and the projection operator for a given spin direction ⁵⁾. In this way one finds for $a_{\mu} a_{\nu}^+$:

$$a_{\mu} a_{\nu}^+ = \frac{1}{p_0 q_0} \left[q_{\mu} p_{\nu} + p_{\mu} q_{\nu} - \delta_{\mu\nu} (pq) + p_{\alpha} q_{\beta} \varepsilon_{\alpha\mu\beta\nu} - m \{ q_{\mu} w_{\nu} + w_{\mu} q_{\nu} - \delta_{\mu\nu} (qw) + w_{\alpha} q_{\beta} \varepsilon_{\alpha\mu\beta\nu} \} \right], \quad (18)$$

where w is a unit spacelike vector, which in the system in which we work points into the direction of the muon polarization. It satisfies $w_{\mu} w_{\mu} = 1$ and $p_{\mu} w_{\mu} = 0$. It is related to w_0 , the polarization vector in the rest system of the muon, by means of a Lorentz transformation, as follows ($\hat{p}$ is a unit vector in the direction of $\underline{p}$):

$$\underline{\omega} = \underline{\omega}_0 + \left(\frac{p_0}{m} - 1 \right) (\underline{\omega}_0 \cdot \hat{\underline{p}}) \hat{\underline{p}},$$

$$\omega_4 = \frac{i}{m} \underline{p} \cdot \underline{\omega}_0 \quad (19)$$

The last three terms in Eq. (14) produce the Coulomb corrections. Here we are only interested in those contributions which give rise to a transverse polarization of the muon. It can be shown that only the second and third term in (14) contribute to the transverse polarization. The result of the calculation is

$$\begin{aligned} \left(a_\mu b_{\nu,i}^+ \right)_{t.p.} &= \left(b_{\mu,i} a_\nu^+ \right)_{t.p.} = \frac{m}{p_0 q_0} \left[q_\alpha \omega_\beta \delta_{\mu\nu} \varepsilon_{i4\beta\alpha} \right. \\ &\quad \left. - \omega_\beta (q_\mu \varepsilon_{i4\beta\nu} + q_\nu \varepsilon_{i4\beta\mu}) - q_i (\omega_\nu \delta_{\mu 4} - \omega_\mu \delta_{\nu 4}) \right. \\ &\quad \left. - q_4 (\omega_\mu \delta_{\nu i} - \omega_\nu \delta_{\mu i}) \right], \end{aligned} \quad (20)$$

where the subscript t.p. indicates that only the terms giving rise to transverse polarization are written down.

We now first observe that $\underline{H}(\underline{p})$ is a vector in the direction of $\underline{p}$, so that we can write

$$\underline{G}(\underline{p}) \underline{H}^*(\underline{p}) = T \underline{p}, \quad (21)$$

with

$$T = \frac{1}{|\underline{p}|} \underline{G}(\underline{p}) \hat{\underline{p}}_i H_i^*(\underline{p}) \quad (22)$$

Using the above results, we are led to the following expression for the squared matrix element

$$|M(q, p, w, P)|^2 = \frac{I}{p_0 q_0} \left[\lambda_\mu p_\mu - m \lambda_\mu w_\mu + m \Lambda_{\underline{w}} \cdot (\underline{\lambda} \times \underline{p}) \right] , \quad (23)$$

with

$$\lambda_\mu = \alpha q_\mu + \beta p_\mu + \gamma P_\mu , \quad (24)$$

$$\alpha = -\beta - 2A_1 - A_2 P^2 - A_3 k^2 - 2A_4 (kP) , \quad (25)$$

$$\beta = 2A_3 (pq) + 2A_4 (Pq) + 2A_5 (Pq) , \quad (26)$$

$$\gamma = 2A_2 (Pq) + 2A_4 (pq) - 2A_5 (pq) , \quad (27)$$

$$I = |G(\underline{p})|^2 , \quad (28)$$

and

$$\Lambda = \frac{T + T^*}{I} . \quad (29)$$

We remark once more that in (23) and in the equations to follow, Coulomb effects are only taken into account in the transverse polarization terms. In terms of the polarization vector $\underline{w}_0$ in the muon rest system, Eq. (23) becomes, noting that in the laboratory system $\underline{P} = 0$:

$$|M(\underline{q}, \underline{p}, \omega)|^2 = \frac{I}{p_0 q_0} \left[\lambda_{\mu} p_{\mu} - \left\{ m \alpha \underline{q}_{\underline{m}} + \beta' \hat{\underline{p}}_{\underline{m}} \right\} \cdot \underline{\omega}_{\underline{m}0} + m \alpha \wedge \underline{\omega}_{\underline{m}0} \cdot (\underline{q}_{\underline{m}} \times \underline{p}_{\underline{m}}) \right], \quad (30)$$

with

$$\beta' = \alpha (p_0 \underline{q}_{\underline{m}} \cdot \hat{\underline{p}}_{\underline{m}} - q_0 |\underline{p}_{\underline{m}}| - m \underline{q}_{\underline{m}} \cdot \hat{\underline{p}}_{\underline{m}}) - \gamma P_0 |\underline{p}_{\underline{m}}|. \quad (31)$$

From Eq. (30) we obtain for the longitudinal polarization, i.e., the polarization in the plane of $\underline{p}_{\underline{m}}$ and $\underline{q}_{\underline{m}}$ in the absence of Coulomb forces :

$$\underline{l}_{\underline{m}} = l \hat{\underline{l}}_{\underline{m}}, \quad l = \frac{|m \alpha \underline{q}_{\underline{m}} + \beta' \hat{\underline{p}}_{\underline{m}}|}{\lambda_{\mu} p_{\mu}}. \quad (32)$$

In the following we will essentially use the fact that $|\underline{l}| \leq 1$. For the transverse polarization, perpendicular to the decay plane, θ being the angle between $\underline{p}_{\underline{m}}$ and $\underline{q}_{\underline{m}}$ we have

$$\underline{t}_{\underline{m}} = t \hat{\underline{t}}_{\underline{m}}, \quad t = \frac{m \alpha \wedge |\underline{p}_{\underline{m}}| q_0 \sin \theta}{\lambda_{\mu} p_{\mu}}. \quad (33)$$

4. AN UPPER LIMIT FOR THE TRANSVERSE POLARIZATION

Both (32) and (33) are complicated expressions, involving, by means of the quantities $A_1, \dots, A_5$, the coupling constants of the hadronic current. For the general processes (1), these coupling constants and their

dependence on the momentum transfer are completely unknown. However, because the numerators of the expressions for ℓ and t depend on the various coupling constants only by means of the combinations α and β' , we may obtain an upper limit for t , which is independent of the coupling constants, by combining (32) and (33). Eliminating $\lambda_{\mu p}$ in these formulae, we get

$$t = \frac{\Lambda |p| q_0 \sin \theta}{|q - g p|} \ell, \quad (34)$$

with

$$g = \frac{-\beta'}{m \alpha |p|}$$

In the forward direction ($\theta = 0$), $t = 0$, as follows from (33). For $\theta \neq 0$, one has $|q - g p| \neq 0$. An upper limit for $|t|$ is obtained when $|q - g p|$ for given p and q attains its minimum value $q_0 \sin \theta$ and $|\ell|$ its maximum value 1, giving the final result

$$|t| \leq t_{\max} = |\Lambda| |p| \quad (35)$$

This result is completely independent of the details of the reaction, like nuclear structure, hadronic form factors, etc.

In order to find the value of the right-hand side of (35) we have to calculate Λ . As a first approximation we do this for the case of a point charge.

From (22), (28) and (29) we have :

$$\Lambda = \frac{2 \operatorname{Re} G(p) \hat{p}_i H_i^*(p)}{|p| |G(p)|^2} \quad (36)$$

By using (10), (6) and (7), and noting that for high energy muons $|p| \simeq p_0$, equation (36) simplifies to :

$$\Lambda = \frac{\operatorname{Re} I_3 (I_2^* - p_0 R I_1^*)}{p_0 (p_0 R)^2 |I_3|^2} \quad (37)$$

with

$$I_1 = \int_0^{2p_0 R} dx \, F_1(ia, 1, ix) \quad (38)$$

$$I_2 = \int_0^{2p_0 R} dx \, x \, F_1(ia, 1, ix) \quad (39)$$

$$I_3 = \int_0^{2p_0 R} dx \int_0^1 dy \, y^2 \, F_1(ia, 1, ixy) \quad (40)$$

$$a = \frac{Ze^2 p_0}{|p|} \simeq Ze^2$$

The above integrals are computed for $Z=80$, or $a=0.6$, and for various values of $p_0 R$. For R we have taken $R = 1.2 A^{1/3} 10^{-13}$ cm, with A the atomic number. For $Z=80$, and in natural units, this becomes $R \simeq 30 \text{ GeV}^{-1}$. The results of the calculation for three values of the muon energy p_0 are collected in the Table. In this Table one also finds the results for processes of type (1.b), i.e., for positive muons. The corresponding values of Λ for this case are obtained from (37) by replacing a by $-a$.

From this Table we see that even for $p_0 = \frac{1}{2} \text{ GeV}$ $t_{\max}$ does not exceed 1%. It decreases with increasing muon energy and for $p_0 \sim 1 \text{ GeV}$ is only a fraction of a percent. For very low energies (limit $|p| \rightarrow 0$),

p_0 (GeV)	μ^-		μ^+	
	$ \Lambda $	$t_{\max}(\%)$	$ \Lambda $	$t_{\max}(\%)$
$\frac{1}{2}$	$6 \cdot 10^{-3}$	0.30	$17 \cdot 10^{-3}$	0.85
1	$1.6 \cdot 10^{-3}$	0.16	$3.5 \cdot 10^{-3}$	0.35
1.67	$0.6 \cdot 10^{-3}$	0.10	$1.2 \cdot 10^{-3}$	0.20

T A B L E

$t_{\max}$ for the case of a point charge with $Z = 80$

on the contrary, $t_{\max}$ will be quite high, of the order Ze^2 . The actual values of the transverse polarization, as they follow from Eq. (33), will of course be lower than the corresponding values for $t_{\max}$. Rough calculations for the elastic process

$$\nu + n \rightarrow \mu^- + p$$

and N^* production

$$\nu + n \rightarrow \mu^- + N^*$$

indicate that for these processes t is at least a factor of three lower than $t_{\max}$ (for energies in the GeV region).

The low values predicted here for the transverse polarization of high-energy muons due to Coulomb final state interaction should be compared to the values for the transverse polarization which one expects when time reversal invariance is violated. In that case one has to add a sixth term to the general expression (17) for $N_{\mu\nu}$. Calculations¹⁾ on the basis of Cabibbo's scheme for time reversal violation⁸⁾ show that even for muon energies of several GeV the transverse polarization can be of the order of 30%.

It is interesting to apply our results to the case of β decay. Here we have that the nuclear radius is small with respect to the electron momentum, and instead of averaging one may directly take (for allowed transitions) the electron wave function for $r=0$. Taking a point charge distribution, one thus replaces ${}_1F_1(ia, 1, ip \cdot r + i|p|r)$ by 1, and $(\partial/\partial x_i) {}_1F_1(ia, 1, ip \cdot r + i|p|r)$ by $-a(p_i + p(x_i/r))$. This amounts to taking the limit $R \rightarrow 0$ in Eq. (37). One then easily finds (p is now the momentum of the electron)

$$\Lambda = \frac{Ze^2}{|p|} \quad , \quad (41)$$

and as a consequence

$$t_{\max} = Ze^2 \quad . \quad (42)$$

For pure Fermi transitions one furthermore has $A_1 = 2k^2$, $A_2 = 8$, $A_3 = 0$, $A_4 = -4$, $A_5 = 0$. Treating as usual the nucleons non-relativistically (nucleon mass $\rightarrow \infty$), one finds for the transverse polarization t of the electron in this case

$$t = \frac{m_e \frac{Ze^2}{p_0} \sin \Theta}{1 + \frac{p_0 q}{p_0 I_0}} \quad , \quad (43)$$

which is identical to the expression derived earlier by Jackson et al. ⁹⁾. For $Z=30$, one has $t_{\max} = 22\%$, while the actual value of t for $\Theta = \pi/2$ and $p_0 = 1$ MeV, calculated from (43), is $t = 11\%$.

The results in the Table, which hold for a point charge, should be corrected by taking into account the fact that the nuclear charge has certain spatial dependence, i.e., that the function $F(\underline{r}, p)$ occurring in the expression for Λ is not exactly equal to ${}_1F_1(ia, 1, ip \cdot r + i|p|r)$, the wave function for a point charge distribution. However, since in the framework of the basic approximation we have made, the relevant quantities are the averages of $F(\underline{r}, p)$ and $(\partial/\partial x_i)F(\underline{r}, p)$ over the nucleus (compare the discussion preceding Eq. (10)), we do not expect that the smeared out character of the charge distribution will change our results for $t_{\max}$ in the Table appreciably.

We make one final remark concerning the influence of incoherence effects. Up to now we have assumed that the muon interacts with the Coulomb field of the final nucleus as a whole, i.e., with a Coulomb field generated by a charge Ze , thereby neglecting special correlations between the muon and individual hadrons. However, incoherence effects, whereby the muon interacts with one individual nucleon in the final hadron system, might be of importance. There is, in particular, one important correlation, viz. in elastic neutrino processes, where the neutrino interacts with an individual neutron and where the resulting proton will be created at the same point as the muon. We can get some idea about $t_{\max}$ for this special situation by applying the foregoing results to the case where the final hadron system consists of a single point particle with charge e . The corresponding value for Λ is obtained from (37) by putting $Z=1$ and letting R go to zero (compare the above discussion for β decay). This gives $\Lambda = e^2/|p|$ and $t_{\max} = e^2 = 0.7\%$. This result is independent of the muon energy up to energies where the wavelength of the muon becomes comparable to the charge radius of the proton, i.e., up to about $\frac{1}{2}$ GeV. Beyond that, $t_{\max}$ will decrease rapidly.

ACKNOWLEDGEMENTS

It is a pleasure to thank Dr. M. Veltman for suggesting this investigation and for numerous helpful discussions.

R E F E R E N C E S

- 1) S.M. Berman and M. Veltman, Phys.Letters 12, 275 (1964);
see also, R.R. Silbar and H. Überall (to be published).
- 2) S.L. Adler, Phys.Rev. 135, B963 (1964).
- 3) J.H. Christenson, J.W. Cronin, V.L. Fitch and R. Turlay, Phys.Rev. Letters 13, 138 (1964).
- 4) See, for instance, T.D. Lee and C.N. Yang, "Elementary Particles and Weak Interactions", Brookhaven National Laboratory, Report 443 (T-91), (1957).
- 5) See, for instance, M.E. Rose, "Relativistic Electron Theory", John Wiley and Sons, Inc., New York-London (1961).
- 6) H.A. Bethe and L.C. Maximon, Phys.Rev. 93, 768 (1954).
- 7) M. Veltman, Physica 29, 161 (1963).
In this reference a correction to the Sommerfeld-Maue wave function is made by adding a third term to the right-hand side of (4). We will not take this extra term into account since its influence on the transverse muon polarization is expected to be small.
- 8) N. Cabibbo, Phys.Letters 12, 137 (1964).
- 9) J.D. Jackson, S.B. Treiman and H.W. Wyld Jr., Nuclear Phys. 4, 206 (1957); Zeitschr.f.Phys. 150, 640 (1958).