K-meson parity from dispersion relations
Selleri, Franco

Published in Nuovo Cimento 15 (1960) 986-990

**This article has been scanned with the kind permission of the
Società Italiana di Fisica.**

From: Angela Oleandri <oleandri@sif.it>
Date: October 12, 2007 4:07:47 PM GMT+02:00
To: Salvatore Mele <Salvatore.Mele@cern.ch>
Subject: Re: Request for permission to reproduce published material.

Dear Dr Mele,

further to your request, sent in July to Carmen Vasini (herewith attached)
I'm pleased to inform you that the Italian Physical Society is happy to
grant you permission of reproduction of the listed material for your
archival purposes at no charges.

K-Meson Parity from Dispersion Relations.

F. SELLERI (*)

CERN - Geneva

(ricevuto il 12 Febbraio 1960)

In recent months many efforts have been devoted to the study of the K-parity problem with the aid of dispersion relations⁽¹⁾. However no definite conclusions have been reached yet. In this paper we want to rediscuss the problem, following the lines of the work by AMATI⁽¹⁾ (hereafter referred to as A). This approach starts from an effective-range formula deduced from the dispersion relation for the forward K^+ -proton scattering amplitude. The advantages of such a method are

1) the formula used is weighted against the contributions coming from the unphysical region;

2) it does not contain the hitherto rather ambiguously known real part of the forward K^- -proton scattering amplitude at zero kinetic energy, $D^-(K)$;

3) the obtained results are rather cut-off independent.

The dispersion relations used by other authors (see for example KARPLUS *et al.*⁽¹⁾), are more convergent than those under discussion. However, under very general assumptions on the asymptotic behavior of the cross-sections it has been shown that the integrals used here are convergent⁽²⁾. Furthermore with rather general arguments it is possible to infer the effect of the very high energy contributions. We shall see that the advantages of this method shall allow us to get clearer conclusions than those obtained with other dispersion relation approaches.

As has been shown in A, if one makes the hypothesis of constant behaviour of the cross-section for K^+ -proton scattering at low energy, for which good experimental information is lacking, there is evidence for a pseudoscalar K with respect to

(*) On leave from University of Bologna, Italy.

(1) P. T. MATTHEWS and A. SALAM: *Phys. Rev.*, **110**, 569 (1958); C. GOEBEL: *Phys. Rev.*, **110**, 572 (1958); K. IGI: *Progr. Theor. Phys. (Kyoto)*, **19**, 238 (1958); D. AMATI: *Phys. Rev.*, **113**, 1692 (1959); E. GALZENATI and B. VITALE: *Phys. Rev.*, **113**, 1635 (1959); D. AMATI, E. GALZENATI and B. VITALE: *Nuovo Cimento*, **12**, 627 (1959); R. KARPLUS, L. KERTH and T. KYCIA: *Phys. Rev. Lett.*, **2**, 510 (1959); M. M. ISLAM: *Nuovo Cimento*, **13**, 224 (1959).

(2) D. AMATI, M. FIERZ and V. GLASER: *Phys. Rev. Lett.*, **4**, 86 (1960)..

both Λ and Σ hyperons. In what follows we want to rediscuss the problem using the effective range formula deduced in A without the restrictive assumption of constant σ^+ and adopting for the scattering amplitude in the unphysical region the solutions given by DALITZ and TUAN⁽³⁾. We will use the σ^+ slope in the low energy region as a parameter and obtain the K-meson parity with respect to it. A full discussion of the errors will also be carried out. Let w be the total laboratory energy of an incoming K-meson (the K-meson mass is taken as unit). Then the effective range formula is

$$(1) \quad D^+(w) = D^+(1) + (w-1)r^+(w),$$

$r^+(w)$ being given by

$$(2) \quad B(w) = r^+(w) - \frac{1}{\pi} \int_1^\infty \frac{k' \sigma^+(w') dw'}{(w'-w)(w'-1)} + \frac{1}{\pi} \int_1^\infty \frac{k' \sigma^-(w') dw'}{(w'+w)(w'+1)} + U(w),$$

where $B(w) = R_\Lambda^+(w) + R_\Sigma^+(w)$ — see A for the explicit form — is the bound states contribution and has two different expressions according to whether the K-hyperon parity is even or odd. For example at the point $w=1.22$, one has

$$(3) \quad R_\Lambda^+(1.22) = \begin{bmatrix} -7.80 \\ 0.42 \end{bmatrix} \frac{g_\Lambda^2}{4\pi}; \quad R_\Sigma^+(1.22) = \begin{bmatrix} -5.97 \\ 0.25 \end{bmatrix} \frac{g_\Sigma^2}{4\pi},$$

g_Λ and g_Σ being the K-hyperon-nucleon coupling constants. In R_Λ^+ the upper (lower) value must be taken if the K- Λ parity is even (odd). Similarly for R_Σ^+ . Thus from the sign and magnitude of $B(w)$ it is possible to obtain information about the K-hyperon parities and coupling constants respectively. $U(w)$, unphysical region contribution, is given by

$$(4) \quad U(w) = \frac{1}{\pi} \int_{w_{\Delta\pi}}^1 \frac{k' \sigma^-(w') dw'}{(w'+w)(w'+1)},$$

with $w_{\Delta\pi} \simeq 0.48$. As previously stated formula (2) does not contain $D^-(K)$. In the calculation of the first and the second integral on the r.h.s. of (2), which we will call I^+ and I^- respectively, we will use a cut-off of 5, corresponding to $T_{\text{lab}}^{\text{max}} \simeq 2$ GeV. However we shall discuss how our conclusions can be changed from the neglected parts of the integrals. Furthermore we will take $w=1.22$ (as in A), corresponding to $T^{\text{lab}} \simeq 110$ MeV. Of the two integrals I^+ and I^- only the former has singularities and these are confined to the low energy region. Thus a priori we can expect a rather strong dependence of I^+ from the slope of σ^+ in this region. To introduce the parameter which represents this slope we will use for σ^+ the expression

$$(5) \quad \sigma^+(w') = \sigma^+(1) + b(w'-1),$$

in the $1 \leq w' \leq 1.4$ region, while both for $\sigma^+(w')$ in the $w' > 1.4$ region and for $\sigma^-(w')$ we will use the experimental data. We note that the point $w'=1.4$ lies

(3) R. H. DALITZ and S. F. TUAN: *Ann. Phys.*, **8**, 100 (1959).

in a region where three good experimental values for σ^+ have been obtained⁽⁴⁾. From these it is easy to obtain $\sigma^+(1.4) = (16 \pm 1) \text{ mb} = (10.0 \pm 0.6) K^{-2}$ which introduced in (5) permits the elimination of one of the two parameters $\sigma^+(1)$, b . By elimination of $\sigma^+(1)$ one obtains

$$(6) \quad \sigma^+(w') = (10.0 \pm 0.6) + b(w' - 1.4).$$

Formula (5) is only the simplest way to introduce a parameter which represents the mean slope of σ^+ in the region $w' \leq 1.4$ and the obtained results are not very dependent upon the explicit w dependence of $\sigma^+(w)$. Only a pathological behaviour of σ^- with strong oscillations could make our results meaningless.

To make the discussion easier it is better to consider both integrals on the r.h.s. of (2) divided in two parts; from 1 to 1.4 and from 1.4 to 5. The b -dependent quantities in the expression that can be obtained from (2) taking $w=1.22$ are $r^+(1.22)$ and $I^+|_1^{1.4}$. Indeed from (1), using the experimental isotropy of the differential cross-section, the optical theorem and the fact that the K^+ -proton potential is repulsive, which gives a minus sign to D^+ , one can obtain

$$(7) \quad r^+(1.22) = -4.55 \left[\frac{k_l}{k_b} \sqrt{4\pi\sigma^+(1.22) - k_b^2\sigma^{+2}(1.22)} - \frac{N+K}{N} \sqrt{4\pi\sigma^+(1)} \right].$$

Here k_l and k_b are, respectively, laboratory and baricentric momenta corresponding to $w=1.22$; N and K are the nucleon and K -meson masses. Thus the b -dependent terms contributing to $B(1.22)$, which are obviously those containing σ^+ , are given by

$$(8) \quad f(b) = r^+(1.22) - \frac{1}{\pi} \cdot I^+ \Big|_1^{1.4}.$$

Upon insertion in (8) of σ^+ , as obtained by (6), one has

$$(9) \quad f(b) \simeq -24.5 [\sqrt{8.32 - 0.12b} - \sqrt{10.0 - 0.4b}] + 17.8 - 0.91b.$$

In the approximation (5), (6), the errors on $f(b)$ come out only from $\sigma^+(1.4)$. They can easily be calculated using $\sigma^+(1.4) = 10.6 K^{-2}$ and $\sigma^+(1.4) = 9.4 K^{-2}$ instead of $\sigma^+(1.4) = 10.0 K^{-2}$.

Next let us discuss the b -independent part of the r.h.s. of (2). As we have already said, we have calculated the integrals $I^+|_{1.4}^5$ and $I^-|_1^5$ taking the experimental values of the cross-sections; more precisely we have used the fit to the experimental points given by A. SALAM at the Kiev Conference⁽⁵⁾. The results were

$$(10) \quad \frac{1}{\pi} I^+ \Big|_{1.4}^5 = (18.2 \pm 0.7) K^{-2},$$

$$(11) \quad \frac{1}{\pi} I^- \Big|_1^5 = (6.4 \pm 0.6) K^{-2}.$$

⁽⁴⁾ L. T. KERTH, T. F. KYCIA and R. G. BAENDER: *Bull. Am. Phys. Soc.*, **4**, 25 (1959).

⁽⁵⁾ A. SALAM: *Kiev Conference Report* (unpublished).

The errors indicated in (10) and (11) have been estimated as follows: for the error on I^+ we have taken 4% due to the fact that the value of σ^+ is known with about such a precision in every point. As for the error on I^- we have divided its origin into two parts: in the region $1 \leq w' \leq 1.4$ we have taken for σ^- the best fit to the three good experimental points existing in this region, which is essentially the fit given by Salam, and have estimated the errors from the extreme fits possible to these points. In the region $1.4 \leq w' \leq 5$ the function multiplying σ^- in the integral is very slowly varying. Thus what $I^-|_{1.4}^5$ is really sensitive to, is the mean value of $\sigma^-(w')$. We have estimated this mean value to be known with a precision of about 10%. In this way one obtains the value (11) for $I^-|_1^5$. Collecting the results (9), (10) and (11) we obtain from (2)

$$(12) \quad B(1.22) = f(b) - (11.8 \pm 1.3) + U(1.22).$$

There remains to be calculated the contribution given by the unphysical region term U . Theoretically four solutions are possible for the analytic extension of the imaginary part of the forward amplitude for K^- proton scattering in the unphysical region. The larger contribution comes from the curve indicated with (a-) in (3), which gives $U(1.22) \simeq 1.6K^{-2}$. The other solutions give smaller contributions.

The bound-states contribution $B(1.22)$ as a function of b is drawn in Fig. 1

Fig. 1. — Continuous lines: bound states term $B(w)$ calculated at the point $w=1.22 K$ as a function of the slope b of σ^+ in the low energy region (higher; units K^{-3}) and of $\sigma^+(1)$ (lower; units mb) for two extreme values of the unphysical region term U . The dashed lines represent the errors on the curve with $U=0$.

with $U(1.22)=0$. In Fig. 1 also the corresponding values of $\sigma^+(1)$ are given in abscissa. The broken lines give, for every b , the errors on $B(1.22)$. As it is easily seen $B(1.22)$ is positive (negative) for $b \geq 7$ ($b \leq 7$). Looking at formula (3) we see immediately that in the first (second) region the possibility of a scalar (pseudo-scalar) K-meson with respect to both Λ and Σ hyperons, is to be excluded.

Thus a future experimental determination of b will be decisive between these two possibilities. Taking for the unphysical region the solution (a-) that gives $U(1.22)=1.6K^{-2}$ instead of $U(1.22)=0$, one has the effect of shifting the whole curve upwards. However, the new curve — see Fig. 1 — remains inside the experimental errors. Inclusion of other possible solutions gives for $B(1.22)$ curves which also lie nearer to the one with $U=0$. Thus we see that in any case our conclusions are rather independent of the unphysical region contributions. Furthermore, by inspection of (2), it is possible to understand the effect of including the high energy parts of the integrals. Beyond the cut-off 5 one easily sees that the denominator in I^- is about twice that in I^+ and that they tend to become equal with increasing energy. This is also the behaviour expected for the numerators, due to the fact that values of σ^- about twice those of σ^+ are suggested by the existing experimental points and that asymptotically the two cross-sections must become equal⁽⁶⁾. That σ^- remains higher as σ^+ above 2 GeV is, in our opinion, a rather plausible hypothesis, because such high energy cross-sections are expected to be predominantly inelastic and the number of open channels for K^-p inelastic scattering is larger than the same number for K^+p at the same energy (possibility of hyperon formation). Thus the two contributions should cancel in (2), showing the cut-off independence of the dispersion relations used.

In conclusion the results of this note may be considered to be the following:

The present evidence for a weak energy dependence of σ^+ at low energy is consistent only with a pseudo-scalar K meson (pseudo-scalar at least with respect to one of the hyperons). This conclusion is based on the fact that a drop of more than ~ 4 mb in σ^+ from its characteristic constant behaviour at (100–200) MeV, when the kinetic energy decreases down to zero, seems to be a rather unlikely possibility. To ensure our conclusion then, it would be extremely interesting to have a direct experimental determination of σ^+ in the very low energy region.

The author wishes to express his deep gratitude to Professor D. AMATI for having suggested the problem and for many useful discussions.

⁽⁶⁾ I. IA. POMERANČUK: *Žurn. Èksp. Teor. Fiz.*, 34, 725 (1958) [translation: *Sov. Phys. Journ. Expt. Theor. Phys.*, 7 (34), 499 (1958)].