CERN LIBRARIES, GENEVA

CM-P00057323

Archive

VERY HIGH ENERGY SCATTERING OF STRONGLY
INTERACTING PARTICLES

Lectures at 1966 Scottish Universities'
Summer School in Physics

L. Van Hove
CERN - Geneva

I. INTRODUCTION AND SURVEY OF EXPERIMENTAL DATA

These lectures will be mainly concerned with two-body collisions of hadrons at high energy. Such collisions are of the type

$$A + B \rightarrow C + D \quad (1)$$

A, B, C and D being hadrons, i.e., strongly interacting particles. They can be mesons, baryons and antibaryons. For the few choices of incident particles A, B for which actual collision experiments can be carried out, a considerable body of experimental information is now available. Depending on the case, the outgoing particles C and D can be stable, long-lived or short-lived hadrons, including mesonic, baryonic or antibaryonic resonances. In many cases, they are detected through their decay products.

It has up to now been impossible to obtain detailed information concerning two-body collisions from cosmic ray experiments. The experimental results available to date have been obtained with high energy accelerators, one of the initial particles, let us say A, being contained in a primary or secondary beam originating from the accelerator, and the other, say B, being essentially at rest in a target. Let $p^A = (p_0^A, \underline{p}^A)$ be the four-momentum of the beam particle. With accelerators operating at present the accessible energy interval is

$$p_0^A \leq 33 \text{ GeV} = 33 \times 10^9 \text{ eV} \quad (2)$$

for protons, and correspondingly lower energies for other hadrons. The target particle B is usually a proton or a neutron. Representing its four-momentum by

$$p^B = (p_0^B, \underline{p}^B), \quad p_0^B \approx m_B, \quad \underline{p}^B \approx 0$$

2.

where m_B is the mass of B, we have for the square of the c.m. energy (c.m. stands for "centre-of-mass")

$$s = (p^A + p^B)^2 = (p_O^A + p_O^B)^2 - |\underline{p}^A + \underline{p}^B|^2 \simeq 2m_B p_O^A + m_A^2 + m_B^2. \quad (3)$$

For $m_B = m_p =$ proton mass, the interval (2) becomes

$$s \lesssim 64 \text{ (GeV)}^2. \quad (4)$$

Future extensions of the intervals in Eqs. (2) and (4) can be expected to come from at least three sources : higher energy accelerators, intersecting beam devices for hadrons, and appropriate developments of cosmic ray experimentation. In a limited sense, an extension is already possible with existing accelerators, when B is a nucleon bound in a target nucleus and when one takes advantage of its Fermi motion. From recent measurements, using incident protons of 30 GeV energy on bound nucleons, one collision in about 10^7 gives a c.m. energy $s^{\frac{1}{2}}$ larger than 14 GeV¹⁾. In such cases, however, it is clear that the remaining part of the target nucleus plays an important dynamical role which must seriously affect the collision, making it more complex than a two-body collision, and our knowledge of nuclear structure is insufficient to allow for an accurate calculation of the necessary corrections.

Difficulties of similar kind but of much greater severity are encountered when one tries to extract information on collisions of virtual hadrons from collisions of real ones. Thus, one has advocated the use of incident pions hitting target protons in order to study pion-pion collisions, invoking the common assertion that the proton is surrounded by a cloud of virtual pions which are in some sense bound to it. However, the concept of virtual hadron is extremely ill-defined, and our knowledge of strong interaction dynamics is at present incapable of giving it an accurate mathematical content. It is therefore impossible to obtain anything but the crudest information concerning collisions involving virtual particles, and this information

is of little help in our attempts to extract from the experimental data an accurate picture of strong interactions at high energies. In the following considerations, A and B are supposed to be real, free hadrons undergoing no interaction before they collide with each other, and so are C, D after the collision.

I.1

KINEMATICAL VARIABLES AND CROSS-SECTION FORMULAE

In terms of the four-momenta p^A, p^B, p^C, p^D , we define as usual the kinematical invariants of reaction (1)

$$s = (p^A + p^B)^2, \quad t = (p^A - p^C)^2, \quad u = (p^A - p^D)^2 \quad (5)$$

where the notation is the same as in Eq. (3) (signature of metric + ---). As is well known the following relations are satisfied

$$\begin{aligned} (p^A)^2 &= m_A^2, \quad (p^B)^2 = m_B^2, \quad (p^C)^2 = m_C^2, \quad (p^D)^2 = m_D^2 \\ p^A + p^B &= p^C + p^D \\ s + t + u &= M^2 \equiv m_A^2 + m_B^2 + m_C^2 + m_D^2. \end{aligned} \quad (6)$$

The units used are such that $\hbar = c = 1$. The c.m. momenta in the initial (A+B) and final state (C+D) will be denoted by k and k' respectively. They satisfy

$$s^{\frac{1}{2}} = (k^2 + m_A^2)^{\frac{1}{2}} + (k^2 + m_B^2)^{\frac{1}{2}} = (k'^2 + m_C^2)^{\frac{1}{2}} + (k'^2 + m_D^2)^{\frac{1}{2}}. \quad (7)$$

The c.m. scattering angle θ , defined as the angle between the momenta of A and C in the c.m. system, is given by

4.

$$t = \left[(k^2 + m_A^2)^{\frac{1}{2}} - (k'^2 + m_C^2)^{\frac{1}{2}} \right]^2 - k^2 - k'^2 + 2kk' \cos \theta. \quad (8)$$

Equations (7) and (8) are special cases of the relativistically invariant definitions (5).

The six quantities s, t, u, k, k', θ are functions of two independent parameters, for which one may for example choose s and t , or k and θ . In addition, the collision depends on the spin orientations of those particles which have non-vanishing spin, so that the collision amplitude is a function of the form

$$T(s, t, \lambda_A, \lambda_B, \lambda_C, \lambda_D) \quad (9)$$

where the λ_K 's ($K = A, B, C, D$) label the various spin orientations of the particles. For the λ_K 's one often chooses the helicities of Jacob and Wick ²⁾. We take (9) to represent the relativistically invariant amplitude. Its relation to the S matrix in an arbitrary reference system is given by

$$\begin{aligned} & T(s, t, \lambda_A, \lambda_B, \lambda_C, \lambda_D) (p_O^A p_O^B p_O^C p_O^D)^{-\frac{1}{2}} \delta_4(p^A + p^B - p^C - p^D) = \\ & = i \langle \underline{p}^C, \lambda_C ; \underline{p}^D, \lambda_D | 1-S | \underline{p}^A, \lambda_A ; \underline{p}^B, \lambda_B \rangle \end{aligned} \quad (10)$$

with the single particle states normalized to

$$\langle \underline{p}', \lambda' | \underline{p}, \lambda \rangle = \delta_{\lambda' \lambda} \delta_3(\underline{p}' - \underline{p}). \quad (11)$$

The differential cross-section for fully polarized incident and outgoing particles is

$$\frac{d\sigma}{dt} = (4\pi^3/k^2s) |T(s, t, \lambda_A, \lambda_B, \lambda_C, \lambda_D)|^2. \quad (12)$$

The special case of elastic scattering obtains when one of the outgoing particles (let say C) is identical to A and the other (D) identical to B. All other cases of two-body collisions are called inelastic. The elastic scattering amplitude is written

$$T_{el}(s, t, \lambda_A, \lambda_B, \lambda'_A, \lambda'_B) \quad (13)$$

$\lambda_{A,B}$ referring to the incident and $\lambda'_{A,B}$ to the outgoing particles. For s, t, u we keep the definitions (5), where $p^C = p'^A$ is now the outgoing momentum of A and $p^D = p'^B$ the one of B. Through the optical theorem, which reads

$$\sigma_T(\lambda_A, \lambda_B) = (8\pi^2/ks^{\frac{1}{2}}) \text{Im } T_{el}(s, 0, \lambda_A, \lambda_B, \lambda_A, \lambda_B), \quad (14)$$

T_{el} is related to the total cross-section $\sigma_T(\lambda_A, \lambda_B)$ for A + B collisions.

We note that all our equations remain valid when the particles A, B are identical, and also when C, D are identical. The only modification then required for their validity is that the range of variation of the scattering angle θ should be restricted to $0 \leq \theta \leq \pi/2$ instead of the full range $0 \leq \theta \leq \pi$. For unpolarized incident particles, Eqs. (12) and (14) have to be averaged over λ_A, λ_B .

I.2 SUMMARY OF EXPERIMENTAL INFORMATION

We attempt in this Section a qualitative description of the experimental information on high-energy scattering, giving the most relevant references to the literature. By high-energy region we shall understand the interval starting around

$$p_0^A \gtrsim 5 \text{ GeV}, \quad s \gtrsim 12 (\text{GeV})^2 \quad (15)$$

and reaching to the highest energies available with present accelerators, Eqs. (2) and (4). All measurements done up to now for two-body collisions of hadrons in this region have given differential cross-sections $d\sigma/dt$ which, at fixed t , behave like smooth, non-oscillating functions of s ; the same is probably true for $d\sigma/dt$ as a function of s at fixed c.m. scattering angle θ . This smooth variation with s is in all cases very naturally extrapolated to represent a simple, non-oscillatory asymptotic behaviour for $s \rightarrow +\infty$, and it is under the assumption of validity of such extrapolations that the theoretical analysis of the high-energy scattering data is usually carried out. We shall adopt the same assumption here. We warn, however, that it may be wrong in several ways. Thus, oscillations in s may be present, either too slow to be visible in the limited interval $12 (\text{GeV})^2 \lesssim s \lesssim 64 (\text{GeV})^2$ now covered, or too fast to be detected with the resolution attained in available experiments; the possibility of such oscillations will be mentioned later (Section II.1). Our description of the data ignores it.

The main feature of the existing data on two-body collisions is that in all cases the differential cross-section $d\sigma/dt$ tends rapidly to zero for increasing energy, with one notable exception which is elastic scattering $A + B \rightarrow A + B$ for finite values of the momentum transfer $(-t)^{\frac{1}{2}}$. We begin by discussing the latter case, for which good data are available ³⁾⁻⁶⁾. For all choices of A, B which could be studied till now and at least in the interval

$$0 \lesssim -t = -(p^A - p'^A)^2 \lesssim 1 (\text{GeV})^2 \quad (16)$$

(p'^A denotes the final four-momentum of A) the measured values of $d\sigma_{\text{el}}/dt$ taken for fixed t vary little with s for $s \rightarrow +\infty$. The data are compatible with the assumption that, for $s \rightarrow +\infty$ at fixed t , the elastic differential cross-section approaches a non-vanishing limit, i.e., that the amplitude verifies

$$s^{-1} |T_{el}(s, t, \lambda_A, \lambda_B, \lambda'_A, \lambda'_B)| \xrightarrow{s \rightarrow \infty} f(t, \lambda_A, \lambda_B, \lambda'_A, \lambda'_B) \quad (17)$$

where f is finite for all values of its arguments, and is positive at least in the interval (16) and for at least some values of the helicities λ, λ' . Also the data on total cross-sections are compatible with the assumption that σ_T approaches a finite positive limit, which from Eq. (14) would mean that

$$s^{-1} \text{Im } T_{el}(s, 0, \lambda_A, \lambda_B, \lambda_A, \lambda_B) \xrightarrow{s \rightarrow \infty} g(\lambda_A, \lambda_B) > 0 \quad (18)$$

for at least some values of λ_A, λ_B . Of course, the data are also compatible with f or g being zero or infinity, but then the trend of $d\sigma_{el}/dt$ or σ_T toward a vanishing or infinite limit would be very slow at the energies measured.

At large energy, the shape of $d\sigma_{el}/dt$ as a function of t in the interval (16) is a very simple one, common to all cases studied. The experimental data are best parametrized in the form of the power series

$$\ln \frac{d\sigma_{el}}{dt} = a + bt + ct^2 + \dots \quad (19)$$

b and c are positive. The contribution of the second order term is very small in the interval (16); one finds

$$0 \leq c/b^2 \lesssim 0.04. \quad (20)$$

Further terms in the series (19) are negligible in the interval (16). The parameters a, b vary little with energy, their behaviour being compatible with the assumption that they approach positive limits for $s \rightarrow +\infty$. The same may hold for c , but this parameter, giving only a very small contribution to Eq. (19), is not yet accurately measured.

One has no direct information at present on the relative contributions of non-spin-flip ($\lambda'_A = \lambda_A, \lambda'_B = \lambda_B$) and spin-flip ($\lambda'_A \neq \lambda_A$ and/or $\lambda'_B \neq \lambda_B$) amplitudes to elastic scattering. It is often assumed that at very high energy the interaction becomes spin-independent, which would mean that the spin-flip amplitudes become negligible compared to the non-spin-flip ones and that the latter become all equal to each other. Except for the obvious case where one of the particles has spin zero and where the scattering angle vanishes (forward scattering), there is no good justification for this spin independence assumption, and experiment will have to decide. If one nevertheless accepts the assumption for very small t ($-t \lesssim 0.05 (\text{GeV})^2$), and if one considers the case where both A and B are charged, one can derive the phase of T_{el} from very small angle scattering data by making use of the interference between nuclear and Coulomb interactions. This has been done for pp (7)-11) and for $\pi^\pm p$ (12) collisions, with the result that the ratio

$$\rho = \text{Re } T_{el} / \text{Im } T_{el} \quad (21)$$

is of order -0.3 for pp and of order -0.2 for $\pi^\pm p$ in the energy interval

$$10 (\text{GeV})^2 \lesssim s \lesssim 50 (\text{GeV})^2 \quad (22)$$

and for $t \simeq 0.01 (\text{GeV})^2$, the region of maximum interference. The data on the ratio ρ are not yet reliable and accurate enough to draw definite conclusions as to its energy variation in the interval (22). Within the large uncertainties ρ is about constant for pp and seems to approach zero for $\pi^\pm p$.

We proceed now with a brief summary of other cases of two-body collisions, i.e., elastic scattering with large momentum transfer and all types of inelastic collisions (i.e., collisions $A + B \rightarrow C + D$ with C and/or D different from A and/or B). For $s \gtrsim 10 (\text{GeV})^2$ the most accurate experiments done so far concern pp ¹³⁾ and πp ¹⁴⁾ elastic scattering at large angles, backward πp and np elastic scattering ^{15),16),17)}, and a variety of inelastic processes: $\pi^- p \rightarrow \pi^0 n$ ^{18),19)}, $\pi^- p \rightarrow \eta n$ ²⁰⁾, $K^- p \rightarrow \bar{K}^0 n$ ²¹⁾, $\bar{p} p \rightarrow \bar{n} n$ ²²⁾ at low momentum transfer. All cross-sections $d\sigma/dt$ are found to go down rapidly with increasing energy. The decrease is of order s^{-1} to s^{-2} for $\pi^- p \rightarrow \pi^0 n$, $\pi^- p \rightarrow \eta n$, $K^- p \rightarrow \bar{K}^0 n$ and $\bar{p} p \rightarrow \bar{n} n$. One has a somewhat faster power decrease for πp and np backward elastic scattering. The decrease of $d\sigma/dt$ with s appears to be appreciably faster for large angle πp and pp elastic scattering; in fact, for c.m. scattering angles θ around 90° , the drop of $d\sigma/dt$ with s can be described by an exponential $\exp[-As^{\frac{1}{2}}]$ with $A \approx 3 (\text{GeV})^{-1}$. The slopes of the various differential cross-sections mentioned above show more structure and diversity than for elastic scattering at small t . In some cases there are pronounced dips which are of considerable interest. We must also mention that, beyond the experiments referred to before and at somewhat lower energies, numerous data on two-body inelastic reactions are available from bubble chamber work ²³⁾. Finally, high energy polarization experiments on $\pi^- p$ elastic ²⁴⁾ and charge-exchange scattering ²⁵⁾ and on pp elastic scattering ²⁶⁾ have recently been carried out and have given non-vanishing values for the polarization parameter.

I.3

GENERAL REMARKS ON THEORETICAL METHODS

The theoretical approach to very high energy scattering of hadrons is, in its essential features, rather similar to the theoretical framework of strong interaction physics at lower energies. There is no theory in the real sense of the word, which means that there is no mathematical formalism giving through well-defined and consistent

procedures numerical predictions for the important experimental quantities (the most important one in our context being of course the scattering amplitude). There is, however, an incomplete but very useful mathematical framework, which is the S matrix formalism as it can and has been abstracted from the known examples of renormalizable, relativistic quantum field theory. Despite the fact that more ambitious hopes have been expressed and that much work has been done to support them, it seems impossible to regard at present the S matrix formalism as more than a mathematical framework. Even when supplemented by other general assumptions, it has not allowed the development of reliable computational methods having the predictive power of a true theory.

The framework of the S matrix formalism, despite the limitations just mentioned, has proved to be well adapted to the task of analyzing theoretically high energy scattering phenomena. On the one hand, it allows to express very compactly the observable implications of the general symmetry and analyticity principles believed to hold true for strong interactions, which are C, P, T and Lorentz invariance, isospin and approximate unitary symmetry, as well as crossing symmetry and a certain amount of analyticity of the S matrix elements in s and t (the latter as well as CPT and Lorentz invariance are believed to hold not only for strong, but also for electromagnetic and weak interactions). On the other hand, the S matrix formalism provides the possibility of defining in a mathematically precise way various dynamical models which can then be used to analyze the experimental data in terms of the parameters or functions left undetermined in the models.

All the theoretical work on high energy scattering combines to various degrees the use of specific dynamical models with the exploitation of the general symmetry and analyticity principles. In one class of investigations, briefly mentioned in Sections II.2 and II.3, the latter principles, supplemented by a minimum of additional general assumptions, are used to derive relations between observable quantities. The most striking examples are the dispersion relation for forward

pion-nucleon scattering, and relations between the high energy behaviours of the reactions $A + B \rightarrow C + D$ and $\bar{C} + B \rightarrow \bar{A} + D$ ($\bar{C}$ is the anti-particle of C , and similarly for $\bar{A}$). The latter are extensions of the famous Pomeranchuk theorem on the equality of total cross-sections for $A + B$ and $\bar{A} + B$ ²⁷⁾. Up to now all such consequences of the symmetry and analyticity principles for which experimental tests were made have been found to agree with the facts, a circumstance which gives considerable confidence in the soundness of the general theoretical framework.

The situation is more complicated regarding the successes and failures encountered by dynamical models. The evaluation of the achievements of a model will clearly depend on the number and nature of the unspecified parameters or functions which it leaves free to fit the data. It will also depend on the simplicity and elegance of the model, in particular on its ability to incorporate in a natural way the general symmetry and analyticity principles mentioned above (thereby automatically achieving agreement with experiment on the observable consequences of these principles). Thus, field theoretical models (like for example the one-meson exchange models), which score rather well on the latter requirement, have little freedom in terms of unspecified parameters and functions. It is therefore not surprising that they encounter great difficulties in describing both the s and t dependence of the observed scattering amplitudes. In contrast, optical models have more than the freedom needed to fit the data, but they do not incorporate easily the symmetry and analyticity principles. A successful compromise is struck at present by the Regge pole model for high energy two-body collisions. It elegantly embodies all accepted symmetry and analyticity principles, it has an ample number of free functions to adjust to the data, but it makes nevertheless some definite predictions which up to now agree with the facts (details on the Regge pole model are given in Sections II.4 to II.6).

There is another general property of the S matrix which we have not mentioned yet although it is a direct consequence of the most general principles of quantum physics. It is the unitarity of the S matrix, and it provides the mathematical expression for the fact that the sum of probabilities of all possible final configurations in a collision is always equal to one. Its consequences for two-body collisions are of two types. It firstly implies inequalities involving two-body scattering amplitudes or cross-sections. Secondly, it gives relations connecting two-body scattering with certain bulk properties of the inelastic collisions (many of which have three or more hadrons in the final state). This relation is particularly important for elastic scattering at high energy if, as is suspected (Section I.2), the elastic amplitude becomes almost purely imaginary for small t . The small angle elastic scattering is then almost purely shadow scattering of the inelastic collisions, and therefore reflects in a rather direct way some bulk properties of the latter. This allows to discuss elastic scattering in terms of simple, qualitative features known or assumed to hold for the inelastic collisions (see Chapter III).

Considerable attention has been devoted lately to the consequences of the approximate SU_3 and SU_6 symmetries for high energy two-body processes. The use of group-theoretical methods for this purpose encounters considerable difficulties for SU_6 symmetry, as is easily understood from the fact that the SU_6 symmetry concept clashes with relativistic effects. Another approach, which is more naive but turns out to be more constructive, starts from the quark model, i.e., the composite model of baryons and mesons suggested by the successful aspects of SU_6 symmetry. This quark model approach to high energy scattering is in a less advanced state of development than the Regge pole model. Its results are nevertheless very suggestive and promising (see Chapter IV).

II. HIGH-ENERGY EXPANSION OF TWO PARTICLE COLLISION AMPLITUDE AT FINITE MOMENTUM TRANSFERS. REGGE POLES

II.1 FORM OF THE EXPANSION

We discuss the asymptotic behaviour of the scattering amplitude for the two-body collision

$$A + B \rightarrow C + D$$

for high energy, $s \rightarrow +\infty$, the momentum transfer

$$t = (p^A - p^C)^2$$

remaining fixed. We recall that the c.m. scattering angle θ between the directions of A and C then tends to zero. As mentioned in Section I.2, the experimental behaviour of the differential cross-section $d\sigma/dt$ under such conditions is a non-oscillatory one, with approximate constancy in s for small angle elastic scattering ($C = A, D = B$) and a pronounced decrease with s for other processes (large angle elastic scattering : $D = A \neq C = B$, as well as inelastic two-body collisions). This decrease seems to go with a power of s .

The behaviour just described is observed for momentum transfers in the interval (16) and for all energies measured above a limit as given in Eq. (15). It is customary to assume this behaviour to be characteristic of the asymptotic variation of the scattering amplitude $T(s, t, \lambda_A, \lambda_B, \lambda_C, \lambda_D)$ at high energies ($s \rightarrow +\infty$). One often represents it theoretically by an asymptotic series expansion of form

$$T(s, t, \lambda_A, \lambda_B, \lambda_C, \lambda_D) \underset{s \rightarrow +\infty}{\approx} \sum_{j=1,2,\dots} c_j (s/s_j)^{\alpha_j} [\ln(s/s_j)]^{\beta_j} \quad (23)$$

where the parameters $c_j, s_j, \alpha_j, \beta_j$ can be functions of t and the helicities λ . In fact s_j , which has dimension $[\text{energy}]^2$, is largely arbitrary because changing its value only affects c_j and adds to the series new terms with lower powers of $\ln(s/s_j)$. The parameters c_j, α_j, β_j are dimensionless. While c_j must be allowed to be complex, α_j and β_j are usually taken real, with the condition

$$\alpha_{j+1} \leq \alpha_j, \quad \beta_{j+1} < \beta_j \quad \text{if} \quad \alpha_{j+1} = \alpha_j. \quad (24)$$

This assumption is suggested by the fact that no oscillations have been found experimentally in the high-energy variation of the cross-sections. While this is undoubtedly a natural supposition, one must realize that present experiments exclude neither very fast nor very slow oscillations in some terms of Eq. (23). As an example, we take forward elastic scattering without spin flip. Oscillations in the total cross-section, Eq. (14), could be caused by having one of the α_j complex, and they would probably have remained undetected if either one of the following conditions is satisfied

$$|\text{Im } \alpha_j| \gtrsim 10^3, \quad |\text{Im } \alpha_j| \lesssim 1. \quad (25)$$

The first limit corresponds to fast oscillations in s with a period Δs of order $10^{-2}s$, the second to the case where the whole energy interval of Eq. (2) covers about a quarter oscillation. Since the actual values of $\alpha_1, \alpha_2, \dots$ are of order $1, \frac{1}{2}, \dots$ the first inequality may seem highly improbable, but the second one is in no way unreasonable. Nevertheless, unless stated otherwise, α_j and β_j will be taken real in the following.

Especially with this restriction, the ansatz (23) for the asymptotic behaviour of the amplitude T may appear much too restrictive as far as its phase is concerned, since so little is known experimentally about it. It implies that the phase of T has a well-defined limit for $s \rightarrow +\infty$. It is justified by the limited experimental evidence on the ratio ρ of Eq. (21) (see Section I.2), and by theoretical considerations presented later (Section II.3). We stress nevertheless that it is in no way well established.

The inclusion of logarithmic terms in Eq. (23) is not required by present experimental results which can all be reproduced by means of powers of s only (i.e., all $\beta_j = 0$). They have, however, considerable theoretical interest (Section II.4), and it may be hoped that in the long run experiment will be able to decide on their presence or absence. The meaning of this statement requires clarification because, evidently, it will always be possible to approximate a logarithmic term over a finite energy interval by a superposition of sufficiently many pure power terms. The point is that one wishes an asymptotic ansatz containing a small number of terms and covering with good accuracy all energies above a minimum of the order of Eq. (15). From present evidence, one might hope to reach this goal with no more than a few terms (perhaps two, three or four) for each reaction, and with a number of relations between the terms belonging to different reactions (such relations would follow from the general symmetry and analyticity principles mentioned in Section I.3, or would be characteristic of specific dynamical models). Within this context, the analytic form of the terms in s becomes important, and the presence of logarithmic factors (or other slowly varying functions, like $\ln \ln s$ for example) may be necessary.

For theoretical reasons which will be mentioned later (see Section II.3), it is more convenient to replace the expansion (23) by

$$T(s, t, \lambda) \underset{s \rightarrow +\infty}{\simeq} \sum_{j=1,2,\dots} c_j (s_t/s_j)^{\alpha_j} \left[\ln(s_t/s_j) - \frac{i\pi}{2} \right]^{\beta_j} \quad (26)$$

where s_t is defined as

$$s_t = s + \frac{t}{2} - \frac{1}{2} (m_A^2 + m_B^2 + m_C^2 + m_D^2) = s + \frac{t}{2} - \frac{M^2}{2}. \quad (27)$$

Within the present experimental accuracy (26) has the same applicability as (23), and the comments just made concerning (23) apply to (26) without change. In particular the existing high energy data can be described in the form (26) with a small number of terms and with $\beta_j = 0$, i.e., without logarithmic factors.

II.2 CONSERVATION OF INTERNAL QUANTUM NUMBERS

The conservation of isospin in the process $A + B \rightarrow C + D$ at large s and finite t is conveniently taken into account by separating the scattering amplitude $T(s, t, \lambda)$ into a sum of terms

$$T(s, t, \lambda) = \sum_I (I^A, I_3^A | I^C, I_3^C ; I, I_3^A - I_3^C) \quad (28)$$

$$T_I(s, t, \lambda) (I^B, I_3^B ; I, I_3^D - I_3^B | I^D, I_3^D).$$

Each term corresponds to exchange of isospin I between A and B , in the sense that when A becomes C it transfers isospin I to B which becomes D . The bracket symbols in (28) are Clebsch-Gordan coefficients for isospin. The exchanged isospin I as defined in (28)

is usually referred to as isospin exchanged in the t channel (see Section II.3). Because of isospin invariance, the same amplitudes T_I as occur in (28) will describe other reactions $A' + B' \rightarrow C' + D'$, where A' is a particle belonging to the same isospin multiplet as A , and similarly for B' , C' , D' . Equation (28) can be used directly for the new reaction by making the necessary changes in the Clebsch-Gordan coefficients. As an example we quote the form taken by (28) for πp scattering

$$\begin{aligned} T_{\pi^+ p \rightarrow \pi^+ p} &= T_{I=0} - \frac{1}{\sqrt{6}} T_{I=1} \\ T_{\pi^- p \rightarrow \pi^- p} &= T_{I=0} + \frac{1}{\sqrt{6}} T_{I=1} \\ T_{\pi^- p \rightarrow \pi^0 n} &= - T_{\pi^0 p \rightarrow \pi^+ n} = - \frac{1}{\sqrt{3}} T_{I=1} \end{aligned} \quad (29)$$

We note that Eq. (28) is only valid if isospin invariance holds exactly. This is supposed to be true for the strong interactions when electromagnetic and weak interaction effects are neglected. This approximation is certainly well founded for strong interaction processes having a sizable cross-section. It may become questionable for processes of very small cross-section, like, e.g., large angle scattering at very high energies (see Section I.2), a possibility which only experiment will be able to explore.

When the isospin analysis (28) of T is applied to the high energy expansion (26), it is clear that each term of (26) will make contributions to one or more of the T_I . The present evidence is that in general each term of (26) contributes to one amplitude T_I only. In other words the high energy expansions of the various T_I do not seem to contain common powers. If this is so, each term j of the asymptotic series (26) contributes to exchange of a specific isospin I_j in the t channel. (In the opposite case, some terms of (26) would split between various values of the exchanged isospin.)

The situation is even simpler for the additive internal quantum numbers N and Y , i.e., baryon number and hypercharge. Given the reaction $A + B \rightarrow C + D$, the baryon number exchanged in the t channel has the unique value $N_A - N_C = N_D - N_B$, and similarly for the hypercharge. It can therefore trivially be said that each term in the expansion (26) contributes to exchange of $N_j = N_A - N_C$ and $Y_j = Y_A - Y_C$ in the t channel. For example

$$N_j = 0, \quad Y_j = -1 \quad \text{for} \quad \pi^- p \rightarrow K^0 \Lambda^0 \quad (30)$$

$$N_j = -1, \quad Y_j = 1 \quad \text{for} \quad \pi^- p \rightarrow \Lambda^0 K^0. \quad (31)$$

Notice that, since t is kept fixed as s increases, (30) differs from (31) by the fact that in the c.m. system the Λ^0 goes backward in (30) and forward in (31).

The procedure described above to take isospin conservation into account can be generalized in a straightforward way to SU_3 symmetry if the latter is assumed to hold exactly. The experimental situation indicates, however, that the SU_3 symmetry breaking is no less important in high energy collisions at small momentum transfer than in hadron spectroscopy, particularly in the mass splittings within SU_3 multiplets (one has no experimental information on the degree of validity of unitary symmetry in large momentum transfer processes, a question which is obviously of the greatest interest). When extending (28) to SU_3 one should therefore include the first order effects of symmetry breaking. This will give a formula of type

$$T = \sum_{\xi, I, Y} \gamma(A|C; \xi, YII_3) T_{\xi, YI} (B; \xi, YIII_3|D) \quad (32)$$

where ξ labels the SU_3 representations ($\xi = 1, 8, 10, \overline{10}, \dots$) and YII_3 the elements within a representation. The γ 's are only SU_3 Clebsch-Gordan coefficients in the limit of exact SU_3 ; their

actual values will contain symmetry breaking effects. Similarly, symmetry breaking implies that the amplitude $T_{\xi, YI}$ for exchange of ξ , YI in the t channel depends not only on ξ but also on Y, I . For the example of reactions (29), $T_{I=0}$ becomes a sum $T_{1,0,0} + T_{8,0,0} + T_{27,0,0}$, whereas $T_{I=1}$ becomes $T_{8,0,1} + T_{10,0,1} + T_{\overline{10},0,1}$.

As regards the use of (32) for the high energy expansion (26) the situation is the same as described for the isospin case; in general each term of (26) contributes to a single amplitude $T_{\xi, YI}$ and therefore to exchange in the t channel of a specific element Y_j, I_j of a specific SU_3 representation ξ_j . We shall assume this to be the case and shall refer again later on to the complete set of exchanged internal quantum numbers N_j, ξ_j, Y_j, I_j belonging to each term of (26).

In our opinion, there is at present no fully satisfactory way of dealing with broken SU_6 symmetry for high energy scattering. In these lectures (see Chapter IV) we shall regard SU_6 properties as resulting from the dynamical assumption of a composite model of hadrons (quark model). This model allows also to obtain many of the broken SU_3 properties without using group theory.

II.3 ANALYTICITY AND CROSSING SYMMETRY BETWEEN s AND u CHANNELS

The most important implications of the analyticity and crossing symmetry properties of the scattering amplitude for high energy and finite momentum transfer will now be described in terms of the asymptotic expansion (26). They are generalizations of the well-known Pomeranchuk theorem on total cross-sections²⁷⁾.

The essential content of the principles of analyticity and crossing symmetry is that the scattering amplitude $T(s, t, \lambda)$ for $A + B \rightarrow C + D$ can be continued analytically in s and t along properly selected paths, and that proper linear combinations of the

continuations of T or T^* (depending on the path) become identical to the scattering amplitudes of the crossed reactions

$$\bar{C} + B \rightarrow \bar{A} + D \quad (33)$$

$$A + \bar{C} \rightarrow \bar{B} + D. \quad (34)$$

The kinematical variables of the crossed reactions are given by Eq. (5) with the substitutions

$$p^A \rightarrow -p^{\bar{A}}, \quad p^C \rightarrow -p^{\bar{C}} \quad \text{for} \quad \bar{C} + B \rightarrow \bar{A} + D \quad (35)$$

$$p^B \rightarrow -p^{\bar{B}}, \quad p^C \rightarrow -p^{\bar{C}} \quad \text{for} \quad A + \bar{C} \rightarrow \bar{B} + D \quad (36)$$

so that $u^{\frac{1}{2}}$ is the c.m. energy of (33) and $t^{\frac{1}{2}}$ the one of (34). Accordingly the reactions $A + B \rightarrow C + D$, (33) and (34) are often said to belong to the s, u and t channels respectively (this explains the term t channel used in the previous Section).

We are interested for the moment in the crossing relation between the s and u channel reactions. It is obtained by keeping t constant and continuing $T(s, t, \lambda)$ analytically in the variable s through the upper half complex s plane toward such values on the real axis that

$$u = -s - t - m_A^2 - m_B^2 - m_C^2 - m_D^2 \quad (37)$$

takes a physically realizable value for the (c.m. energy)² of the u channel reaction (33), i.e., such that one can have

$$u = (p^{\bar{C}} + p^B)^2, \quad t = (p^{\bar{C}} - p^{\bar{A}})^2 \quad (38)$$

for physical values of the four momenta involved. The complex conjugate $T^*(s, t)$ of this analytically continued $T(s, t)$ is related to the scattering amplitude $\bar{T}$ of (33) by the crossing relation

$$\bar{T}(u, t) = T^*(s, t) \quad (39)$$

when u , s and t satisfy (37). The crossing relation (39) is written down for the case of spinless particles. For particles with arbitrary spin, the crossing relation can be given a similar form

$$\bar{T}(u, t, \lambda'_C, \lambda'_B, \lambda'_A, \lambda'_D) = T^*(s, t, \lambda_A, \lambda_B, \lambda_C, \lambda_D) \quad (40)$$

with u , s , t satisfying again (37) and with

$$\lambda'_C = -\lambda_C, \quad \lambda'_B = \lambda_B, \quad \lambda'_A = -\lambda_A, \quad \lambda'_D = \lambda_D. \quad (41)$$

The simple form (40) is only obtained for some special choices of definition of the spin amplitudes. It holds for the "generalized" helicity amplitudes defined, following Trueman and Wick²⁸⁾, in an arbitrary reference frame which is not supposed to change in the analytic continuation of T in s ^{29),30)}. It holds also for the "transversity amplitudes" of Kotanski, for which the spin quantization axis is taken to be the normal to the scattering plane³¹⁾. It should be noted that Eq. (40) is valid only for specific choices of the phases of the spin states; otherwise it would contain a phase factor; the derivation makes explicit use of time-reversal invariance³⁰⁾.

For other choices of the spin amplitudes, Eq. (40) becomes more complicated in that the right-hand side is replaced by a linear combination of spin amplitudes. This is the case for the usual helicity amplitudes of Jacob and Wick³²⁾ where the helicities are taken by definition to refer to the c.m. system of the collision. The crossing relation for this case, originally derived by Trueman and Wick²⁸⁾ for another path of analytic continuation, has been given for continuation at fixed t in the upper half s plane by Biaľas and Svensson³³⁾, to whose paper we refer for the explicit form of the relation. For the very simple and common case of spin 0 for A , C and spin $\frac{1}{2}$ for B , D , one can also write the crossing relations in the form

$$\bar{g}(u,t) = g^*(s,t), \quad \bar{h}(u,t) = h^*(s,t). \quad (42)$$

The functions g and h are related to the amplitude T of the s channel reaction by

$$T = \chi_D \left[g + i \frac{\vec{\sigma} \cdot (\vec{k}_A \times \vec{k}_C)}{|\vec{k}_A \times \vec{k}_C|} h \right] \chi_B \quad (43)$$

where $\vec{k}_A, \vec{k}_C$ are the three-momenta in the c.m. system, whereas χ_B, χ_D are the Pauli two-component spinors representing the spin orientations of particles B and D in their respective rest frames ³⁴).

The definition of $\bar{g}, \bar{h}$ is the same for the u channel reaction (33).

After this description of the crossing relation between the s and u channels, we return to high energy scattering. From (39), (40) or (42), and the analogous relation obtained by permuting T and $\bar{T}$ (which is valid since the crossing relations are symmetric in T and $\bar{T}$), one can write T as a sum

$$T = T_+ + T_-, \quad T_{\pm} = \frac{1}{2} [T(s,t,\lambda) \pm \bar{T}(s,t,\lambda')] \quad (44)$$

where the notation λ, λ' of Eqs. (40) and (41) is used. The new amplitude $T_+(T_-)$ is even (odd) under crossing, i.e., it verifies the identity

$$T_{\pm}(u,t,\lambda') = \pm T_{\pm}^*(s,t,\lambda) \quad (45)$$

$T_+(T_-)$ is said to be the positive (negative) signature part of T .

To apply the subdivision (44) to the high energy expansion (26), one can again assume for simplicity that each term of (26) makes a contribution either to T_+ , or T_- . If a term would contribute both to T_+ and T_- , it could be split accordingly in its two contributions; experiment up to now does not require such degeneracies. We can therefore

say that each term j of the expansion (26) has a definite signature $\varepsilon_j = \pm 1$ (+1 for positive, -1 for negative signature). Using the usual assumptions of analyticity and slower than exponential growth of T in the upper half s plane one can then demonstrate that the phase of the coefficient c_j is determined up to π by the exponent α_j and the signature ε_j . It is given by the formula

$$c_j = \pm |c_j| \exp\left[-\frac{i\pi}{2} \left(\alpha_j + \frac{1}{2} \varepsilon_j - \frac{1}{2}\right)\right]. \quad (46)$$

This important consequence of analyticity and crossing symmetry is a far reaching generalization of the Pomeranchuk theorem on total cross-sections²⁷⁾. It was derived in various stages of generality in a number of papers³⁵⁾⁻³⁹⁾. Its validity requires the special form adopted in (26) for the high energy expansion, which uses s_t instead of s and $(\ln s_t - \frac{i\pi}{2})$ instead of $\ln s$ as expansion parameters.

To give an example of application of the phase rule (46), we consider the case of elastic scattering. Experiment suggests that at very high energy the differential cross-section $d\sigma_{el}/dt$ becomes approximately independent of s at fixed t . From Eq. (12) this implies that for T_{el} the leading term $j = 1$ of the expansion (26) has $\alpha_j \approx 1$ for $0 \leq -t \lesssim 1(\text{GeV})^2$. Applying (46), one concludes that c_j is purely imaginary if the signature ε_1 of the leading term is +1, whereas it would be purely real if $\varepsilon_1 = -1$. Experiment suggests that the former case is by far the most likely to occur. As another example we take $\pi^- p \rightarrow \pi^0 n$. The crossed reaction (33) is $\pi^0 p \rightarrow \pi^+ n$. From isospin invariance, Eq. (29), we have

$$T_{\pi^0 p \rightarrow \pi^+ n} = - T_{\pi^- p \rightarrow \pi^0 n}. \quad (47)$$

Hence $T = T_{\pi^- p \rightarrow \pi^0 n}$ is odd under crossing, $T = T_-$, the signature is odd, and (46) applies with the negative sign to each term of the expansion of the charge exchange amplitude $T_{\pi^- p \rightarrow \pi^0 n}$.

II.4 INTERPRETATION OF THE HIGH-ENERGY EXPANSION IN TERMS OF THE t CHANNEL

In the preceding paragraph we used crossing between the s and u channels. Let us consider now crossing between the s and t channels, i.e., between the original reaction $A + \bar{C} \rightarrow \bar{B} + D$, Eq. (34). The corresponding substitution of momenta is given by Eq. (36). The scattering amplitude of (34) is denoted by

$$\tilde{T}(t, s, \tilde{\lambda}_A, \tilde{\lambda}_{\bar{C}}, \tilde{\lambda}_{\bar{B}}, \tilde{\lambda}_D), \quad t = (p^A + p^{\bar{C}})^2, \quad s = (p^A - p^{\bar{B}})^2. \quad (48)$$

We continue it analytically in t and s , in the upper half t plane, so as to keep

$$u = (p^A - p^D)^2 = M^2 - (t+s), \quad M^2 = m_A^2 + m_B^2 + m_C^2 + m_D^2 \quad (49)$$

constant and so as to reach values of s larger than $(m_A + m_B)^2$. In full analogy with (39), (except that $\bar{C}$ plays the role of A , A of B and $\bar{B}$ of C), we obtain the crossing relation

$$T(s, t, \lambda_A, \lambda_B, \lambda_C, \lambda_D) = \tilde{T}^*(t, s, \tilde{\lambda}_A, \tilde{\lambda}_{\bar{C}}, \tilde{\lambda}_{\bar{B}}, \tilde{\lambda}_D) \quad (50)$$

where $\tilde{T}^*$ is the complex conjugate of the analytically continued $\tilde{T}$, and where the spin orientations are related by

$$\tilde{\lambda}_A = \lambda_A, \quad \tilde{\lambda}_{\bar{C}} = -\lambda_C, \quad \tilde{\lambda}_{\bar{B}} = -\lambda_B, \quad \tilde{\lambda}_D = \lambda_D. \quad (51)$$

We have assumed the definition of spin amplitudes to be such as to give to the crossing relation this simple form, as discussed after Eqs. (40) - (41); the best choice consists in taking the transversity amplitudes of Kotansky⁴⁰⁾.

In Section II.2 we discussed for the reaction $A + B \rightarrow C + D$ the internal quantum numbers exchanged in the t channel. It is clear that for the reaction $A + \bar{C} \rightarrow \bar{B} + D$ these internal quantum numbers are simply those of the $A + \bar{C}$ and $\bar{B} + D$ systems themselves. Thus, if $\tilde{T}_I$ is the scattering amplitude for the isospin I component of the $A + \bar{C} \rightarrow \bar{B} + D$ reaction, so that

$$\begin{aligned} \tilde{T}(t, s, \tilde{\lambda}) &= \sum_I (I^A, I_3^A ; I^{\bar{C}}, I_3^{\bar{C}} | I, I_3^A + I_3^{\bar{C}}) \\ \tilde{T}_I(t, s, \tilde{\lambda}) &= (I, I_3^{\bar{B}} + I_3^D | I^{\bar{B}}, I_3^{\bar{B}} ; I^D, I_3^D) \end{aligned} \quad (52)$$

the analytic continuation of $\tilde{T}_I$, carried out as in (50), will give through

$$T_I = \tilde{T}_I^* \quad (53)$$

the amplitude T_I of Eq. (28).

Also the signature concept, introduced for reaction $A + B \rightarrow C + D$ in Eq. (44), has a simple and important interpretation in terms of the t channel reaction $A + \bar{C} \rightarrow \bar{B} + D$. It can be obtained as follows. Equation (50) relates the scattering amplitude T of the s channel reaction $A + B \rightarrow C + D$ to the analytic continuation of the amplitude $\tilde{T}$ of $A + \bar{C} \rightarrow \bar{B} + D$. We write down the analogous equation for the scattering amplitude $\bar{T}$ of the u channel reaction $\bar{C} + B \rightarrow \bar{A} + D$. It is

$$\bar{T}(u, t, \lambda_{\bar{C}}', \lambda_B', \lambda_{\bar{A}}', \lambda_D') = \tilde{T}^*(t, u, \tilde{\lambda}_{\bar{C}}', \tilde{\lambda}_A', \tilde{\lambda}_{\bar{B}}', \tilde{\lambda}_D'). \quad (54)$$

The arguments in $\bar{T}$ are

$$u = (p_{\bar{C}} + p_B)^2, \quad t = (p_{\bar{C}} - p_{\bar{A}})^2.$$

The right-hand side of (54) is the complex conjugate of the analytic continuation, at constant $t + u$ and in the upper half t plane, of the scattering amplitude $\tilde{T}(t, u, \tilde{\lambda}'_{\bar{C}}, \tilde{\lambda}'_A, \tilde{\lambda}'_{\bar{B}}, \tilde{\lambda}'_D)$ of the reaction $\bar{C} + A \rightarrow \bar{B} + D$. The arguments in the amplitude $\tilde{T}$ are

$$t = (p_{\bar{C}} + p_A)^2, \quad u = (p_{\bar{C}} - p_{\bar{B}})^2.$$

The relation between the λ 's is similar to Eq. (51)

$$\tilde{\lambda}'_{\bar{C}} = \lambda'_{\bar{C}}, \quad \tilde{\lambda}'_A = -\lambda'_A, \quad \tilde{\lambda}'_{\bar{B}} = -\lambda'_{\bar{B}}, \quad \tilde{\lambda}'_D = \lambda'_D. \quad (55)$$

We now remark that the reaction $\bar{C} + A \rightarrow \bar{B} + D$ is identical to the t channel reaction $A + \bar{C} \rightarrow \bar{B} + D$, except that the roles of the two incident particles are interchanged. Hence $\tilde{T}$ is given in terms of $\tilde{T}$ by

$$\tilde{T}(t, u, \tilde{\lambda}'_{\bar{C}}, \tilde{\lambda}'_A, \tilde{\lambda}'_{\bar{B}}, \tilde{\lambda}'_D) = \tilde{T}(t, M^2 - t - u, \tilde{\lambda}'_A, \tilde{\lambda}'_{\bar{C}}, \tilde{\lambda}'_{\bar{B}}, \tilde{\lambda}'_D). \quad (56)$$

We proceed by considering the positive and negative signature parts $T_{\pm}$ of T , as defined in Eq. (44), and apply (50) and (54) to them. Notice first that (41), (51) and (55) give

$$\tilde{\lambda}'_{\bar{C}} = \tilde{\lambda}_{\bar{C}}, \quad \tilde{\lambda}'_A = \tilde{\lambda}_A, \quad \tilde{\lambda}'_{\bar{B}} = \tilde{\lambda}_{\bar{B}}, \quad \tilde{\lambda}'_D = \tilde{\lambda}_D. \quad (57)$$

Hence we obtain from (44), (50), (54) and (56)

$$T_{\pm}(s, t, \lambda_A, \lambda_{\bar{B}}, \lambda_{\bar{C}}, \lambda_D) = \tilde{T}_{\pm}^*(t, s, \tilde{\lambda}_A, \tilde{\lambda}_{\bar{C}}, \tilde{\lambda}_{\bar{B}}, \tilde{\lambda}_D) \quad (58)$$

where the right-hand side is the analytic continuation, at constant $t + s$ and in the upper half t plane, of

$$\begin{aligned} \tilde{T}_{\pm}(t, s, \tilde{\lambda}_A, \tilde{\lambda}_{\bar{C}}, \tilde{\lambda}_{\bar{B}}, \tilde{\lambda}_D) = & \frac{1}{2} [\tilde{T}(t, s, \tilde{\lambda}_A, \tilde{\lambda}_{\bar{C}}, \tilde{\lambda}_{\bar{B}}, \tilde{\lambda}_D) \\ & \pm \tilde{T}(t, M^2 - t - s, \tilde{\lambda}_A, \tilde{\lambda}_{\bar{C}}, \tilde{\lambda}_{\bar{B}}, \tilde{\lambda}_D)] \end{aligned} \quad (59)$$

$\tilde{T}_+$ and $\tilde{T}_-$ are the even and odd parts of the amplitude $\tilde{T}$ for exchange of the incident particles A and $\bar{C}$ in the t channel, i.e., for exchange of s and $u = M^2 - t - s$.

Consider further the partial wave expansion of $\tilde{T}$ for the t channel reaction $A + \bar{C} \rightarrow \bar{B} + D$, which we write in the form

$$\tilde{T}(t, s, \tilde{\lambda}) = \sum_{\ell=0,1,2,\dots} (2\ell+1) \zeta_{\ell}(t, \tilde{\lambda}) D_{\ell}(\tilde{\lambda}, \cos\theta_t) \quad (60)$$

D_{ℓ} is a rotation function depending on the definition of the spin amplitudes; it is a linear superposition of Legendre polynomials $P_{\ell}(\cos\theta_t)$ and of their derivatives. The angle θ_t is the scattering angle of A to $\bar{B}$ in the c.m. system. It is related to s by the equivalent equations

$$s = \left[(k_t^2 + m_A^2)^{\frac{1}{2}} - (k_t'^2 + m_B^2)^{\frac{1}{2}} \right]^2 - k_t^2 - k_t'^2 + 2k_t k_t' \cos\theta_t \quad (61)$$

$$u = M^2 - t - s = \left[(k_t^2 + m_C^2)^{\frac{1}{2}} - (k_t'^2 + m_D^2)^{\frac{1}{2}} \right]^2 - k_t^2 - k_t'^2 - 2k_t k_t' \cos\theta_t \quad (62)$$

k_t, k_t' being the c.m. momenta of initial and final states :

$$t = \left[(k_t^2 + m_A^2)^{\frac{1}{2}} + (k_t^2 + m_C^2)^{\frac{1}{2}} \right]^2 = \left[(k_t'^2 + m_B^2)^{\frac{1}{2}} + (k_t'^2 + m_D^2)^{\frac{1}{2}} \right]^2. \quad (63)$$

Assume that either $m_A = m_C$, or $m_B = m_D$ ⁴¹⁾. Exchange of s and u then simply means replacement of θ_t by $\pi - \theta_t$, so that $\tilde{T}_{\pm}$ is given by

$$\tilde{T}_{\pm}(t, s, \tilde{\lambda}) = \frac{1}{2} \sum_{\ell=0,1,\dots} (2\ell+1) \zeta_{\ell}^{\pm}(t, \tilde{\lambda}) \left[D_{\ell}(\tilde{\lambda}, \cos\theta_t) \pm D_{\ell}(\tilde{\lambda}, -\cos\theta_t) \right] \quad (64)$$

where we have written $\xi_{\ell}^{\pm}$ for the values of ξ_{ℓ} entering $\tilde{T}_{\pm}$. Because of the fact that the forces in the s and u channels (e.g., the forces due to exchange of particles in these channels) are in general independent of each other, it is indeed natural to regard $\tilde{T}_{+}$ and $\tilde{T}_{-}$, and hence ξ_{ℓ}^{+} and ξ_{ℓ}^{-} , as independent of each other. Because of parity invariance, one can assume, that D_{ℓ} is even (odd) in $\cos \theta_t$ for ℓ even (odd), as is the case for spinless particles when D_{ℓ} is simply the Legendre polynomial P_{ℓ} . For positive (negative) signature the sum (64) then contains only terms with even (odd) ℓ , and $\xi_{\ell}^{+}(\xi_{\ell}^{-})$ is only defined for even (odd) ℓ .

One now introduces the essential concept of continuation of the orbital momentum ℓ to complex values. Under proper assumptions, (e.g., when the Mandelstam representation holds for the scattering amplitude), one can follow the procedure developed by Regge⁴²⁾ in non-relativistic potential scattering and obtain one and only one continuation of $\xi_{\ell}^{\pm}(t, \tilde{\lambda})$ into an analytic function of the complex variable ℓ , t and $\tilde{\lambda}$ being regarded as fixed parameters⁴³⁾. One then applies to (64) the so-called "Sommerfeld-Watson transformation", transforming the sum in a contour integral⁴⁴⁾

$$\tilde{T}_{\pm}(t, s, \tilde{\lambda}) = \int_C (2\ell + 1) \xi_{\ell}^{\pm}(t, \tilde{\lambda}) \frac{D_{\ell}(\tilde{\lambda}, -\cos \theta_t) \pm D_{\ell}(\tilde{\lambda}, \cos \theta_t)}{4i \sin \pi \ell} d\ell \quad (65)$$

C is the contour encircling counter-clockwise the positive real semi-axis of the ℓ plane :

$$\ell = +\infty + i\varepsilon \rightarrow i\varepsilon ; \ell = \varepsilon e^{i\varphi}, \varphi = \frac{\pi}{2} \rightarrow \frac{3\pi}{2} ; \ell = -i\varepsilon \rightarrow +\infty - i\varepsilon$$

with ε positive infinitesimal. In (65) the even (odd) character of D_{ℓ} in $\cos \theta_t$ for ℓ even (odd) has been used. The analytic continuation of D_{ℓ} in ℓ can be obtained from the standard analytic continuation of P_{ℓ} ⁴⁵⁾.

The next step is to assume that the analytic continuation in s and t leading from $\tilde{T}$ to T , Eqs. (50) and (58), can be carried out in the right-hand side of (65), where (61) and (63) allow to eliminate $\cos \theta_t$ in terms of s and t . One can then discuss the asymptotic behaviour of T_+ , T_- and $T = T_+ + T_-$ for $s \rightarrow +\infty$ at fixed t by making use of the integral representation (65). Indeed, for $s \rightarrow +\infty$, t fixed, one has $\cos \theta_t \rightarrow -\infty$ (remember that the analytic continuation in s and t must go through the upper half t plane with $u = \text{real}$ constant, so that $k_t k'_t$ becomes negative). Furthermore, one knows that for ℓ real or complex,

$$P_\ell(z) \underset{z \rightarrow \infty}{\sim} \frac{\Gamma(\ell + \frac{1}{2})}{\Gamma(\ell + 1)} \frac{(2z)^\ell}{\pi^{\frac{1}{2}}} F(-\frac{\ell}{2}, -\frac{\ell-1}{2}, -\ell + \frac{1}{2}, z^{-2}) + \left. \begin{aligned} &+ \frac{\Gamma(-\ell - \frac{1}{2})}{\Gamma(-\ell)} \frac{(2z)^{-\ell-1}}{\pi^{\frac{1}{2}}} F(\frac{\ell+1}{2}, \frac{\ell}{2} + 1, \ell + \frac{3}{2}, z^{-2}) \end{aligned} \right\} \quad (66)$$

and that the hypergeometric function F in (66) tends to unity for $z \rightarrow \infty$. The asymptotic behaviour is therefore obtained by opening up the contour $\mathcal{C}$ to the left until one reaches the line $\text{Re } \ell = -\frac{1}{2}$, and it is controlled by those singularities of $\xi_\ell^\pm(t, \tilde{\lambda})$ in the ℓ plane which have the largest values for $\text{Re } \ell$ and satisfy $\text{Re } \ell \geq -\frac{1}{2}$. In succession of decreasing $\text{Re } \ell$, they give rise to the successive terms of the asymptotic expansion (26).

If these singularities are simple poles, they give for $s \rightarrow +\infty$ terms of type (26) without logarithmic factors. This follows from (66). Thus, in the special case where $D_\ell = P_\ell$, a simple pole of $\xi_\ell^\pm$ at $\ell = \alpha_j$ with $\text{Re } \alpha_j > -\frac{1}{2}$ gives a term in (26) with the power $s_t^{\alpha_j}$, plus other terms with powers $s_t^{\alpha_j-1}$, $s_t^{\alpha_j-2}$, The term in $s_t^{\alpha_j}$ originates from the occurrence, in the pole contribution to (65), of the combination

$$P_{\alpha_j}(-\cos\theta_t) \pm P_{\alpha_j}(\cos\theta_t) \approx \frac{\Gamma(\alpha_j + \frac{1}{2})}{\Gamma(\alpha_j + 1)} \frac{|2\cos\theta_t|^{\alpha_j}}{\pi^{\frac{1}{2}}} (1 \pm e^{i\pi\alpha_j}) \quad (67)$$

for $\cos\theta_t \rightarrow -\infty$. The sign of the exponent in $e^{i\pi\alpha_j}$ is obtained from the fact that the analytic continuation of (61) in s and t , for large s , leads to the value

$$\cos\theta_t = -|\cos\theta_t| + i\varepsilon, \quad \varepsilon > 0.$$

After insertion of $\tilde{T}_{\pm}$ in (58), the resulting contribution to $T_{\pm}$ contains

$$\frac{\Gamma(\alpha_j + \frac{1}{2})}{\Gamma(\alpha_j + 1)} \frac{|2\cos\theta_t|^{\alpha_j}}{\pi^{\frac{1}{2}}} \frac{1 \pm e^{-i\pi\alpha_j}}{\sin\pi\alpha_j}. \quad (68)$$

We note that the phase agrees with the one given in (46) on the basis of much more general arguments, so that we conclude that the residue of $\zeta_{\ell}^{\pm}$ in the pole $\ell = \alpha_j$ is real. From Eqs. (61) - (62), under the restrictive assumption already mentioned that $m_A = m_C$ or $m_B = m_D$, we note that

$$s_t = \frac{s-u}{2} = 2k_t k_t' \cos\theta_t \quad (69)$$

which explains again the particular role played by s_t in asymptotic expansions.

If some singularities of $\zeta_{\ell}^{\pm}$ in the complex plane for $\text{Re } \ell \geq -\frac{1}{2}$ are more complicated than simple poles, e.g., if they are branch cuts, the asymptotic expansion of T will contain terms of type (26) with logarithmic factors. Such terms are also likely to arise as a consequence of the integration along the line $\text{Re } \ell = -\frac{1}{2}$. The theoretical situation concerning branch cuts is rather unclear. From the

work of Amati, Fubini and Stanghellini⁴⁶⁾ and of Mandelstam⁴⁷⁾, one expects that poles cannot be the only singularities of $\xi_{\ell}^{\pm}$ in ℓ and that branch cuts must exist. One has also a tentative mechanism through which, if poles exist, cuts are generated in positions determined by the poles⁴⁸⁾. All such considerations are uncertain, however, because they could well be affected by unforeseen cancellations between singularities in various contributions to the partial wave amplitudes. As mentioned in Section II.1, the matter is also open from the experimental side, in the sense that there is as yet no experimental indication for or against logarithmic factors in the asymptotic expansion (26).

The present attitude consists in assuming that the leading terms of the expansion (26) originate from simple poles of the partial wave amplitudes $\xi_{\ell}^{\pm}(t, \tilde{\lambda})$ in the $\text{Re } \ell > -\frac{1}{2}$ region of the complex ℓ plane, and to assume that these poles have the properties which can be inferred from consideration of the likely behaviour of the t channel reaction⁴⁹⁾. This gives the Regge pole model description of high-energy scattering, as proposed by several authors in 1962, especially Chew and Frautschi⁵⁰⁾, and Gribov and Pomeranchuk^{51), 52)}. The main features of this model will be reviewed in the next Section, a brief comparison with experiment being given in Section II.6.

II.5 THE REGGE POLE MODEL

The assumption formulated at the end of the previous Section leads to an asymptotic expansion of form

$$T(s, t, \lambda) \underset{s \rightarrow +\infty}{\approx} \sum_{j=1, 2, \dots} c_j (s_t/s_j)^{\alpha_j} \quad (70)$$

for the scattering amplitude T of the s channel reaction $A + B \rightarrow C + D$, the α_j being given by simple poles in ℓ of the partial wave amplitude $\xi_{\ell}^{\pm}$ of the t channel reaction $A + \bar{C} \rightarrow \bar{B} + D$ in the way described above, Eqs. (67) - (69). This assumption leads to a series of very interesting predictions which we now list :

- a) Universality property of the exponents α_j , and
- b) Factorization property of residues.

A priori, the α_j could depend on all parameters determining $\xi_\ell^\pm(t, \tilde{\lambda})$ as a function of ℓ , i.e., on t , the four particles $A, \bar{C}, \bar{B}, D$, the signature $\pm$ and the spin orientations $\tilde{\lambda}$. In the t channel, it is however possible to regard $\xi_\ell^\pm$ as a matrix element

$$\xi_\ell^\pm = \langle A, \tilde{\lambda}_A; \bar{C}, \tilde{\lambda}_{\bar{C}} | Z_J^\pm(t) | \bar{B}, \tilde{\lambda}_{\bar{B}}; D, \tilde{\lambda}_D \rangle \quad (71)$$

of a matrix $Z_J^\pm(t)$, where J is the total angular momentum, which in each particular case can be expressed in function of ℓ and $\tilde{\lambda}$. For the particles $A, \bar{C}, \bar{B}, D$ and their spin orientations $\tilde{\lambda}$, all choices compatible with the signature and the internal quantum numbers in the t channel can be taken. Poles of $\xi_\ell^\pm$ in the complex ℓ plane correspond to poles of the matrix $Z_J^\pm(t)$ in the complex J plane, and the residue of $\xi_\ell^\pm$ at a pole in ℓ will be given by the residue of the matrix element (71) at the corresponding pole in J . Concerning the poles $J_j^\pm(t)$ of $Z_J^\pm(t)$ in J (the so-called Regge poles) the following predictions can be made immediately: the position of the poles depends exclusively on the value of t , on the signature and on the internal quantum numbers in the t channel. Since a simple pole of a matrix is usually a pole of only one of its eigenvalues, the residues of the matrix element (71) have the form of a product ⁵³⁾

$$R_j^\pm(t) = [\chi_j^\pm(t; A, \tilde{\lambda}_A; \bar{C}, \tilde{\lambda}_{\bar{C}})]^* [\chi_j^\pm(t; \bar{B}, \tilde{\lambda}_{\bar{B}}; D, \tilde{\lambda}_D)]. \quad (72)$$

Except for inessential complications due to the fact that J, ℓ and α may differ in the case of particles with spin, the above properties of the poles $J_j^\pm$ of $Z_J^\pm$ apply to the poles of $\xi_\ell^\pm$, and hence to the

exponents α_j in (70). In fact, even in presence of spin, and with the normalization of amplitude here used, see Eq. (12), one finds $J_j^\pm(t) = \alpha_j$. We therefore conclude that the exponents α_j in (70) depend only on the value of t , on the signature and on the internal quantum numbers in the t channel (universality property); the residues of $J_j^\pm$ have the product form (72) (factorization property). From now on we shall write $\alpha_j^\pm(t)$ instead of α_j , thereby stressing that α_j should in general depend on t .

c) Relation of $\alpha_j^\pm(t)$ to particles.

A pole of the matrix $Z_J^\pm(t)$ in the J plane at fixed t , if its position depends on t , will in general also be a pole of $Z_J^\pm(t)$ in the t plane at fixed J . If J has a non-negative integral or half-integral value which is physically compatible with the signature and the internal quantum numbers in the t channel ⁵⁴⁾, a pole of $Z_J^\pm(t)$ at $t = t_0$ in the t plane will correspond to a particle of mass $t_0^{\frac{1}{2}}$, with spin J and the same quantum numbers, where one should suppose

$$\text{Re } t_0 > 0, \quad |\text{Im } t_0| \ll \text{Re } t_0.$$

If $\text{Im } t_0 = 0$ the particle is stable, otherwise it is a resonance. The parity of this particle or resonance is related to the signature ⁵⁴⁾.

Conversely, as advocated by Chew, Frautschi ⁵⁰⁾, Blankenbecler and Goldberger ⁵⁵⁾, for every particle X one can assume that the matrix $Z_J^\pm(t)$ with the same internal quantum numbers and with the signature allowed by the parity of the particle ⁵⁴⁾ has a Regge pole $J_j^\pm(t)$ such that

$$J_j^\pm(m_X^2) = J_X \quad (73)$$

m_X , J_X being the mass and spin of X . The function $J_j^\pm(t)$ is also called Regge trajectory of X . Several particles can fall on the same trajectory by satisfying (73) for the same $J_j^\pm(t)$. In this case, because the signature is fixed, their spins J will differ by 2, 4..., while their parities and internal quantum numbers must be the same. Such "Regge recurrences" are now well known for baryons and are likely to exist also for mesons. For baryons we quote as a single example the nucleon resonances of isospin $\frac{3}{2}$ with masses 1236 MeV and 1924 MeV. Their spin and parities are $\frac{3}{2}^+$ and $\frac{7}{2}^+$ (the positive parity corresponding to negative signature in the pion-nucleon system).

We therefore conclude that the same Regge trajectories $J_j^\pm(t)$ can be expected to pass through the known particles, as in Eq. (73), and to determine the exponents $\alpha_j^\pm(t) = J_j^\pm(t)$ of the asymptotic expansion (70).

d) Reality and positive slope
of $\alpha_j^\pm(t)$.

As long as t is positive one can regard $Z_j^\pm(t)$ as describing a system of energy $t^{\frac{1}{2}}$ and angular momentum J . Hence, if $J_0 = J_j^\pm(t_0)$ with t_0 positive, $t_0^{\frac{1}{2}}$ is an eigenvalue of the Hamiltonian generalized to the value J_0 of the angular momentum, a value which will in general be non-physical. Conversely one can regard $t_0^{\frac{1}{2}}$ as given and determine J_0 as an eigenvalue of the angular momentum variable J entering the generalized Hamiltonian⁵⁶⁾. As long as $t_0^{\frac{1}{2}}$ is below the threshold of the continuum of scattering states, the eigenvalue $t_0^{\frac{1}{2}}$ is real if J_0 is real, because real J_0 makes the generalized Hamiltonian Hermitian. Conversely, if $t_0^{\frac{1}{2}}$ is a given real energy below the continuum threshold, the eigenvalue J_0 of J will also be expected to be real.

This leads to the expectation that $J_j^\pm(t)$, and therefore $\alpha_j^\pm(t)$, is real for t positive below the continuum threshold. It is tempting to assume that the same property would continue to hold for negative values of t , as are relevant for the $A + B \rightarrow C + D$ reaction.

While this assumption seems reasonable when the t channel system $A + \bar{C}$ behaves as a boson⁵⁷⁾, it was shown by Gribov to be wrong when $A + \bar{C}$ behaves like a fermion⁵⁸⁾: the $J_j^\pm(t)$ for $t < 0$ become complex and occur in pairs which are complex conjugate to each other. Hence, if the reaction $A + B \rightarrow C + D$ is such that bosons are exchanged in the t channel, the Regge pole model predicts reality of the exponents α_j in the expansion (70). If fermions are exchanged, the exponents become complex for $t < 0$, so that oscillations could occur in the asymptotic behaviour. According to Kinoshita⁵⁷⁾ such oscillations are not expected in the cross-section but in the polarization.

A word of caution is needed for the case of fermion exchange. All simple reactions of this type, like

$$\pi^\pm + p \rightarrow p + \pi^\pm, \quad \pi^- + p \rightarrow n + \pi^0, \quad \pi^- + p \rightarrow \bar{p} + d, \quad p + p \rightarrow \pi^+ + d$$

correspond to the unequal mass case ($m_A \neq m_{\bar{C}}$ and $m_B \neq m_D$ in our general notation) for which a correct treatment of fermions is not yet available⁵⁹⁾. This, however, in no way decreases the interest of their careful experimental investigation.

Returning to our previous discussion of the Regge poles $J_j^\pm(t)$ for t positive below the continuum threshold, it is clear that one there expects the slope $dJ_j^\pm/dt$ to be positive. Indeed, $t_0^{\frac{1}{2}}$ can be regarded as an energy eigenvalue of a generalized Hamiltonian with angular momentum $J_0 = J_j^\pm(t_0)$. Increasing J_0 means increasing the centrifugal potential, and this will increase the energy eigenvalue $t_0^{\frac{1}{2}}$, thereby showing that $dJ_j^\pm/dt$, and hence $d\alpha_j^\pm/dt$ must be positive. These properties are likely to remain valid for $t < 0$, at least for boson trajectories. As we mentioned before fermion trajectories become complex for $t < 0$, but the imaginary part will be small for small t .

We finally note that all considerations under c) and d) only apply to moving poles, i.e., when $J_j^\pm(t)$ effectively varies with t . There has been much argument in the literature on whether fixed poles or more generally fixed singularities (i.e., singularities independent of t) can exist or not. The matter is essentially undecided in view of the difficulty that, if there exist branch cuts moving with t (which is likely), they can move across a fixed singularity and force the latter to move from one sheet to another of the Riemann surface in ℓ .

e) Reality and zeros of residues.

It is generally believed that the residue of the partial wave amplitude (71) at a Regge pole $J_j^\pm(t)$ is a real quantity for values of $t = (p^A - p^C)^2$ as are taken in the $A + B \rightarrow C + D$ reaction. In the simple case of $D_\ell = P_\ell$ considered to derive Eq. (68), we have already concluded to the reality of the residue. The argument there used applies generally when $J_j^\pm(t)$ and $\alpha_j^\pm(t)$ are real, and shows that reality of the residue then follows from analyticity of T at fixed t (Section II.3). For the fermion exchange case the same is probably true to the approximation that the $\alpha_j^\pm(t)$ would be almost real as long as $|t|$ is small.

Another interesting property of residues is that they may be expected to vanish for special values of $t = (p^A - p^C)^2$ in the scattering process $A + B \rightarrow C + D$ at high energy. Consider the residue $R_j^\pm(t)$ of the partial wave amplitude (71) at $J = J_j^\pm(t)$, and let $\alpha_j^\pm(t) = \alpha_j$ be the exponent of the corresponding term in the expansion (70). The coefficient c_j of this term is essentially the product of R_j and of an expression of type (68) resulting from the last factor in the integrand of (65). Now (68) has a pole for $\alpha_j = \alpha_j^\pm(t)$ equal to an even (odd) integer when the signature is even (odd). If the asymptotic expansion (70) is valid, the residue $R_j^\pm(t)$ must then vanish for the corresponding value of t so that the scattering amplitude T remains finite. It has been conjectured (60)-(62) that $R_j^\pm(t)$ would also vanish for some

values of t for which $\alpha_j^\pm(t)$ equals an odd (even) integer when the signature is even (odd). Since (68) is then finite, this would mean that the j -th term of the asymptotic expansion would vanish for such t , an effect which would be observable. The present version of this conjecture is that $R_j^\pm(t)$ would only vanish if, in addition to the above circumstance, either one of the following conditions would be satisfied :

- i) the value J' of $J_j^\pm(t)$ at the t value in question ⁶³⁾ is negative, or
- ii) J' being non-negative, a Feynman diagram for one-particle exchange in $A + B \rightarrow C + D$, consisting solely of exchange of a particle of spin J' in the t channel, gives contribution zero to the partial amplitude $T(s, t, \lambda_A, \lambda_B, \lambda_C, \lambda_D)$ considered.

f) Analyticity properties of poles and residues.

In addition to the above properties or conjectures, one has some information on the analytic properties in t of the Regge poles $J_j^\pm(t)$ and residues $R_j^\pm(t)$ of the partial wave amplitude (71). Either from analogy with non-relativistic Schrödinger theory, or from consideration of the Mandelstam representation for the relativistic amplitude, one expects that $J_j^\pm(t)$ and $R_j^\pm(t)$ are real analytic functions of t if $A + \bar{C}$ is a boson, or of $t^{\frac{1}{2}}$ if $A + \bar{C}$ is a fermion, with a cut from the lowest threshold in the t channel to $+\infty$ ⁶⁴⁾.

II.6

COMPARISON WITH EXPERIMENT

We end this chapter with a brief discussion of the experimental situation in terms of the asymptotic expansion (70) and of the Regge pole model ⁶⁵⁾.

For elastic scattering, the experimental facts suggest that the leading term of the expansion has, for all reactions, the exponent

$$\alpha_1 \approx 1,$$

leading to constant σ_T and $d\sigma_{el}/dt$ in the high-energy limit, that its signature is positive and that its t channel quantum numbers are those of the vacuum. In the Regge pole model, $\alpha_1 = \alpha_1^+(t) \approx 1$ is referred to as the Pomerenchuk trajectory. It appears to be universal, its slope $d\alpha_1^+/dt$ is vanishing or small, it can be associated with either of the 2^+ mesons having $I = Y = 0$ (f_0 with mass 1250 MeV, f'_0 with 1525 MeV).

The deviations of σ_T and $d\sigma_{el}/dt$ from constancy can be explained by introducing further terms of the expansion, which have the t channel quantum numbers of an SU_3 nonet of mesons of odd signature and of an octet or a nonet of mesons of even signature. If the slope $d\alpha_j^\pm/dt$ for these terms is given a value of order $1(\text{GeV}/c)^{-2}$, the corresponding Regge trajectories can be extrapolated as approximately straight lines through the well-known nonet of 1^- mesons (for the odd signature ones) and the suspected nonet of 2^+ mesons (for the even signature ones) ⁶⁶.

Charge exchange reactions give information on the terms of the expansion with charge ± 1 and zero hypercharge in the t channel. Among the trajectories mentioned, only the odd signature term, i.e., the ρ trajectory $\alpha_\rho(t)$, contributes to $\pi^- p \rightarrow \pi^0 n$. This has allowed the experimental determination of its shape, which was indeed found to be very straight for $-1(\text{GeV}/c)^2 \lesssim t \leq 0$ and to have the slope $\sim 1(\text{GeV}/c)^{-2}$ which makes it pass through the spin 1 of the ρ meson when extrapolated as a straight line to $t = m^2$. This corresponds to an appreciable shrinkage of the peak and is the most remarkable success of the Regge pole model to date. The prediction on vanishing residues described in the previous Section (end of point d) applies to the spin-

flip part of the $\pi^- p \rightarrow \pi^0 n$ amplitude. It is predicted to vanish at the t value where $\alpha \bar{\rho}(t)$ vanishes, which is $t \approx -0.6 (\text{GeV}/c)^2$. It is striking that the cross-section has a marked minimum for this value of t . Only one aspect of $\pi^- p \rightarrow \pi^0 n$ is not explained in terms of the ρ trajectory alone. It is the non-vanishing polarization at 6 GeV²⁵⁾, which must be explained by interference between the ρ trajectory term and another term in the amplitude. At present, many choices are possible for this term, but it cannot belong to the two nonets mentioned above.

The reaction $K^- p \rightarrow \bar{K}^0 n$ is dominated by the ρ trajectory and its positive signature counterpart, which is supposed to belong to the A_2 meson of $I = 1$, $Y = 0$ with mass 1290 MeV (this trajectory assignment requires the spin parity of A_2 to be 2^+ , a point which is not completely settled). The A_2 trajectory passes through the A_2 meson at spin $J = 2$ if it is given the slope of $1(\text{GeV}/c)^{-2}$ mentioned above. This allowed the theoretical prediction of the reaction $\pi^- p \rightarrow \eta n$, which is entirely dominated by the A_2 trajectory⁶⁷⁾, in good agreement with the later experimental findings²⁰⁾.

On the whole the Regge pole analysis of meson-nucleon forward reactions has been extremely successful and it will be of great importance to carry it through to a higher degree of precision ($\pi^- p \rightarrow \pi^0 n$ is the only reaction where the tests of the Regge model have reached the truly quantitative stage). Backward $\pi^\pm p$ scattering can also be successfully described in terms of two very straight trajectories of opposite signature, one belonging to the nucleon and one to $N_{\frac{33}{22}}^*(1236 \text{ MeV})$. The data^{15), 16)} show that the latter is more weakly coupled than the former. Furthermore, a dip in $d\sigma/dt$ for $\pi^+ p \rightarrow p \pi^+$ can be accounted for in terms of the prediction in the previous Section (end of point d), which is here applicable to the residue of the nucleon trajectory⁶⁵⁾.

The situation for charge-exchange reactions with nucleons and antinucleons is less satisfactory. The results available on $np \rightarrow pn$, Refs. 15)-17), and $\bar{p}p \rightarrow \bar{n}n$ ^{22), 68)} cannot be explained in terms of the two nonets of trajectories mentioned above^{69), 70)}. At least one further trajectory is needed⁶⁹⁾, or one has to resort to other explanations, like the long-range part of the one-pion exchange proposed by Byers⁷¹⁾ in the framework of the coherent droplet model⁷²⁾.

III. DIFFRACTION AS SHADOW SCATTERING

III.1 UNITARITY CONDITION AND OVERLAP FUNCTION

It is likely that the scattering amplitude T_{el} for elastic scattering $A + B \rightarrow A + B$ becomes almost purely imaginary at high energy for small momentum transfers. Since the differential cross-section $d\sigma_{el}/dt$ becomes extremely small for all but small values of t , one concludes that elastic scattering, whenever appreciable, is essentially shadow scattering caused by the existence of inelastic processes, and it can therefore be approximately expressed in terms of bulk properties of the inelastic processes. The present Chapter discusses some implications of this general property, which is an immediate consequence of the unitarity of the S matrix.

The unitarity condition in operator form is

$$S^\dagger S = 1, \quad \text{or} \quad i(T^\dagger - T) = T^\dagger T \quad \text{for} \quad T = i(1-S). \quad (74)$$

We take its matrix element between the states $\langle \underline{p}^A, \underline{p}^B |$ and $| \underline{p}^A, \underline{p}^B \rangle$ of the $A + B$ system, neglecting spins altogether as would be allowed if the spin-flip amplitudes become negligible compared to the non-spin-flip ones at very high energy. In the matrix element of $T^\dagger T$ we separate the elastic and the inelastic intermediate states. The formulae become particularly simple in terms of the partial wave amplitudes $i\eta_\ell$, which are defined by

$$T_{el}(s, t) = \frac{is^{\frac{1}{2}}}{4\pi k} \sum_{\ell} (2\ell + 1) \eta_{\ell} P_{\ell}(\cos\theta) \quad (75)$$

with the same notation as in Section (I.1). One finds

$$\text{Re} \eta_{\ell} - \frac{1}{2} |\eta_{\ell}|^2 = f_{\ell} \quad (76)$$

where f_ℓ is determined by the inelastic collisions according to the following rule. For initial state $|\underline{p}^A, \underline{p}^B\rangle$ of the $A + B$ system, consider the inelastic part of the final state

$$|\psi(\underline{p}^A, \underline{p}^B)\rangle = PS|\underline{p}^A, \underline{p}^B\rangle \quad (77)$$

where S is the S matrix and P the projection operator on all inelastic states, i.e., on all states other than $A + B$. We then have

$$\langle \psi(\underline{p}'^A, \underline{p}'^B) | \psi(\underline{p}^A, \underline{p}^B) \rangle = (p_O^A p_O^B)^{-1} \int_4 (\underline{p}^A + \underline{p}^B - \underline{p}'^A - \underline{p}'^B) F(s, t) \quad (78)$$

$$F(s, t) = \frac{s^{\frac{1}{2}}}{2\pi k} \sum_{\ell} (2\ell + 1) f_{\ell} P_{\ell}(\cos\theta) \quad (79)$$

$\underline{p}^A, \underline{p}^B$ are as usual the four momenta of A and B , $\underline{p}'^A$ and $\underline{p}'^B$ are other possible values of these variables, with

$$(\underline{p}^A)^2 = (\underline{p}'^A)^2 = m_A^2, \quad (\underline{p}^B)^2 = (\underline{p}'^B)^2 = m_B^2$$

$$s = (\underline{p}^A + \underline{p}^B)^2, \quad t = (\underline{p}^A - \underline{p}'^A)^2 \quad (80)$$

$$s^{\frac{1}{2}} = (k^2 + m_A^2)^{\frac{1}{2}} + (k^2 + m_B^2)^{\frac{1}{2}}$$

$$t = -2k^2 (1 - \cos\theta).$$

The relativistically invariant function $F(s, t)$ has a simple significance. Taking two initial states $|\underline{p}^A, \underline{p}^B\rangle$, $|\underline{p}'^A, \underline{p}'^B\rangle$ with $\underline{p}^A + \underline{p}^B = \underline{p}'^A + \underline{p}'^B$, i.e., with same c.m. system and same c.m. energy, $F(s, t)$ measures the amount of overlap between the wave functions $|\psi(\underline{p}^A, \underline{p}^B)\rangle$ and $|\psi(\underline{p}'^A, \underline{p}'^B)\rangle$ of the corresponding inelastic final states. This function, which is called the overlap function ⁷³⁾, describes the bulk properties of inelastic collisions which are relevant for determining the resulting shadow. Indeed, F fixes the f_{ℓ} through Eq. (79), and f_{ℓ} determines η_{ℓ} through Eq. (76) when T_{el} is purely imaginary, i.e., η_{ℓ} purely real ⁷⁴⁾. Equation (76) then gives two possible values of η_{ℓ}

$$\eta_\ell = 1 \pm (1 - 2f_\ell)^{\frac{1}{2}} \quad (81)$$

which are real because unitarity of the S matrix applied to the ℓ -th partial wave for elastic $A + B$ scattering gives

$$|1 - \eta_\ell| \leq 1 \quad \text{or} \quad 0 \leq f_\ell \leq \frac{1}{2}. \quad (82)$$

The sign ambiguity in (81) is easily lifted by a continuity argument : experiment suggests that at high energy η_ℓ and f_ℓ tend to continuous functions of the impact parameter $b = \ell k^{-1}$, that η_ℓ never exceeds 1, and that $\eta_\ell \rightarrow 0$ for $\ell \rightarrow \infty$. Continuity then requires to choose the negative sign

$$\eta_\ell = 1 - (1 - 2f_\ell)^{\frac{1}{2}}. \quad (83)$$

For qualitative considerations one can approximate (83) by the first term of its power expansion :

$$\eta_\ell \simeq f_\ell, \quad \text{or} \quad T_{el}(s, t) \simeq \frac{i}{2} F(s, t) \quad (84)$$

because experiment shows that, except at small ℓ , higher order corrections are small. This is due to the almost exponential shape of the diffraction peak, see Eqs. (19) and (20), which implies that the ratio σ_{el}/σ_T of integrated elastic to total cross-sections is always small ($\lesssim 0.2$). One therefore concludes that in first approximation the shape of the diffraction peak, i.e., of $d\sigma_{el}/dt$ at small $|t|$, is directly given by the t dependence of the overlap function $F(s, t)$ defined in (78). Experimentally, the most common inelastic collisions at high energy appear to produce an appreciable number of secondaries with small transverse momentum but rather large longitudinal momentum in the c.m. system. This is often referred to as the "jet" structure of common high-energy collisions. The jet structure immediately implies that $F(s, t)$

should drop steeply with t and therefore predicts a similar steep drop for $d\sigma_{el}/dt$. It is difficult to put such considerations in more quantitative terms before more detailed information becomes available on common inelastic collisions. The work done in this direction will be briefly reviewed in Section III.3. It is clear, however, that the problem is of considerable interest, because it is on the t dependence of differential cross-sections at high energy that existing theoretical models have made the least progress⁷⁵⁾.

We end this Section with a few cross-section formulae

$$\left. \begin{aligned} \sigma_T &= 2\pi k^{-2} \sum_{\ell} (2\ell+1) \operatorname{Re} \eta_{\ell}, & \sigma_{el} &= \pi k^{-2} \sum_{\ell} (2\ell+1) |\eta_{\ell}|^2 \\ \sigma_{in} &= \sigma_T - \sigma_{el} = 2\pi k^{-2} \sum_{\ell} (2\ell+1) f_{\ell} & &= 4\pi^2 k^{-1} s^{-\frac{1}{2}} F(s, 0) \end{aligned} \right\} (85)$$

III.2 INEQUALITIES BETWEEN CROSS-SECTIONS

In the same qualitative vein as in the discussion after Eq. (84), consideration of the overlap function allows to predict on simple, intuitive grounds a number of inequalities between cross-sections which are in agreement with experiment. Consider first as an example pp inelastic collisions at high energy. Their main characteristic is the jet structure mentioned above, especially the small values of all transverse momenta. It suggests the assumption that the outgoing particles can be roughly divided into two groups, one containing the particles which go forward in the c.m. system (forward group) and one containing the particles going backward (backward group). In each of these groups the leading particle, i.e., the particle with largest c.m. momentum, is almost always a nucleon or a nucleon isobar⁷⁶⁾. In addition, antinucleon and strange particle production is rare. Hence, both the forward and the backward groups

have most of the time their total baryon number and strangeness equal to those of a proton. In fact, although few measurements are available, it seems likely that also the total charge of each group would be mostly equal to the proton charge.

These properties can be formulated more generally in terms of exchange. Call A the incident particle, B the target particle, A' the forward group and B' the backward group (A' and B' are in general many-particle systems, i.e., neither single particles nor resonances). In analogy with two-body reactions, the process $A + B \rightarrow A' + B'$ can be called the s channel reaction, and $A + \bar{A}' \rightarrow \bar{B} + B'$ the t channel one. The properties mentioned above then mean that t channel exchange of baryon number, strangeness and charge is very rare in the bulk of inelastic collisions at high energy, as we know it to be also in two-body collisions. This would probably hold also for isospin, i.e., for all internal quantum numbers conserved by strong interactions. The complete inelastic final state ψ_{AB} of an $A + B$ collision is thereby split approximately as

$$\psi_{AB} \approx \psi_{AB}^{(0)} + \psi_{AB}^{(1)} \quad (86)$$

where $\psi_{AB}^{(0)}$ refers to those final states for which vanishing internal quantum numbers are exchanged in the t channel, and $\psi_{AB}^{(1)}$ to the final states where non-vanishing exchange took place. Using the definition (78) one obtains a corresponding separation of the overlap function into two parts

$$F_{AB}(s, t) \approx F_{AB}^{(0)}(s, t) + F_{AB}^{(1)}(s, t). \quad (87)$$

The expected absence of t channel exchange of non-vanishing quantum numbers at high energy then implies

$$F_{AB}^{(1)}/F_{AB}^{(0)} \rightarrow 0 \quad \text{for } s \rightarrow \infty. \quad (88)$$

Compare now e.g., pp and np collisions. Any final state $p' + p''$ belonging to $\psi_{pp}^{(0)}$ will correspond to a final state $n' + p''$ belonging to $\psi_{np}^{(0)}$, simply by relating n' to p' through the same isospin rotation as turns n into p , and vice-versa. Hence

$$F_{pp}^{(0)}(s,t) = F_{np}^{(0)}(s,t). \quad (89)$$

The same does not hold, however, for the final states $p' + p''$, $n' + p''$ contained in $\psi_{pp}^{(1)}$, $\psi_{np}^{(1)}$. There will be more such states of the $n' + p''$ type because charge can be exchanged in more ways between n and p than between p and p . Since each particular final state contributes a positive number to the overlap function at $t = 0$, see Eq. (78), we conclude

$$F_{pp}^{(1)}(s,0) < F_{np}^{(1)}(s,0). \quad (90)$$

Remembering (85), this gives an inequality between inelastic cross-sections ⁷⁷⁾

$$\sigma_{in}(pp) < \sigma_{in}(np). \quad (91)$$

Similarly, considering $\bar{p}p$ and $\bar{n}p$, the more numerous possibilities of charge exchange in the first than in the second reaction gives the prediction

$$\sigma_{in}(\bar{p}p) > \sigma_{in}(\bar{n}p) \quad (92)$$

whereas the abundant possibility of nucleon exchange (mainly annihilation) in $\bar{n}p$ means that

$$\sigma_{in}(\bar{n}p) > \sigma_{in}(np). \quad (93)$$

In deriving the last inequality, we have assumed that

$$F_{\bar{n}p}^{(0)}(s,t) = F_{np}^{(0)}(s,t).$$

One expects indeed in general that

$$F_{\bar{A}B}^{(0)}(s,t) = F_{AB}^{(0)}(s,t) \quad (94)$$

corresponding to the fact that only the charge conjugation number $C = +1$ would be exchanged in the t channel at very high energy ⁷⁸.

Entirely similar considerations apply to meson-baryon collisions. They give the following inequalities

$$\sigma_{in}(\pi^- p) > \sigma_{in}(\pi^+ p) \quad (95)$$

$$\sigma_{in}(K^- p) > \sigma_{in}(\bar{K}^0 p) \quad (96)$$

$$\sigma_{in}(\bar{K}^0 p) > \sigma_{in}(K^0 p) \quad (97)$$

$$\sigma_{in}(K^0 p) > \sigma_{in}(K^+ p). \quad (98)$$

All except (97) follow from considerations of charge exchange. Equation (97) uses again (94) and relies on the fact that strangeness exchange is possible in many more inelastic states for $\bar{K}^0 p$ than for $K^0 p$, because hypercharge $Y = 0$ hyperons are numerous whereas no $Y = 2$ baryon is known so far with certainty.

All inequalities above are due to the presence of t channel exchange of non-vanishing internal quantum numbers. As expressed in Eq. (88) this exchange becomes rare at high energy, so that in each inequality (91) - (98) the two inelastic cross-sections become equal for $s \rightarrow +\infty$, just as the corresponding elastic scattering cross-sections $d\sigma_{el}/dt$ have the same asymptotic property for the same reasons.

To present the above considerations, we have assumed that the outgoing particles in most inelastic collisions can be subdivided into a forward and a backward group. Our conclusions, however, have a somewhat greater validity. What is required in order to compare $\sigma_{\text{in}}(A_1 B)$ with $\sigma_{\text{in}}(A_2 B)$ is that the final states of, say, inelastic $A_1 + B$ collisions have analogues of equal amplitude in $A_2 + B$, whereas some final states of $A_2 + B$ have no analogue in $A_1 + B$. This property is clearly more general than what was assumed in our discussion. How exactly it would be implemented in nature is impossible to say at present, because our experimental knowledge of inelastic final states containing more than two particles or resonances is extremely limited. In fact it is bound to progress very slowly in view of the great complications inherent in the analysis of many-particle systems. It would be desirable, however, to exhibit in some way the type of properties described in the present paragraph.

III.3 THE t DEPENDENCE OF THE OVERLAP FUNCTION

When the real part of the elastic scattering amplitude is neglected, the t dependence of the overlap function $F(s, t)$ can be deduced from the observed differential cross-section, Eq. (19). If one goes beyond the approximation (84) and uses Eq. (83), one finds that the t dependence of F is closer to an exact exponential

$$F(s, t) = F(s, 0) e^{b't} \quad (99)$$

than is the case for $d\sigma_{\text{el}}/dt$. In fact, using (99), one calculates a differential cross-section having for small $|t|$ the shape (19) with $c/b^2 \simeq 0.02$, whereas at large t it behaves as $\exp(-b''\sqrt{-t})$ (79). It appears therefore that the simple t dependence (99) is a good approximation to the facts. It probably holds over a range of t such that F decreases by more than an order of magnitude.

One can ask whether the simple exponential shape of F can be given a qualitative explanation. A possibility is that the most common inelastic final states might have many degrees of freedom excited, with weak correlations between them. In the extreme case where even the correlations imposed by the conservation laws would be neglected, the inelastic final state (77) would then have the form of a product

$$\psi(\underline{p}^A, \underline{p}^B) = \prod_{i=1}^N \psi_i(\underline{p}^A, \underline{p}^B) \quad (100)$$

where $\psi_1, \dots, \psi_N$ would refer to various groups of degrees of freedom, N being a rather large number at high energy. Equation (78) then implies

$$F(s, t) = \prod_{i=1}^N F_i(s, t), \quad (101)$$

$$F_i(s, t) \propto \langle \psi_i(\underline{p}'^A, \underline{p}'^B) | \psi_i(\underline{p}^A, \underline{p}^B) \rangle$$

where the constraint of energy momentum has been dropped from the formulae (the effect of this constraint will of course be the smaller, the larger the value of N). All $F_i(s, t)$ are real functions decreasing with t . If the number N of factors is large the t decrease of the product is necessarily of exponential type

$$F(s, t) \approx \prod_i F_i(s, 0) (1 + a_i(s)t) \approx \left[\prod_i F_i(s, 0) \right] \exp \left[t \sum_i a_i(s) \right]. \quad (102)$$

This type of consideration has been carried out in more detail, taking into account momentum and energy conservation, for a special "uncorrelated jet" model⁸⁰⁾. It is impossible to say at present whether it provides the correct explanation for the exponential shape of F . While the multiplicity of particles produced in high-energy collisions seems appreciable, information is lacking completely on possible correlations among them.

We mention in closing this Chapter an investigation by Kokkedee, Ref. 81). He studies $\bar{p}p$ collisions by considering the overlap function as composed of two parts, one referring to inelastic collisions with annihilation and one without, corresponding roughly to the terms $F^{(1)}$ and $F^{(0)}$ in Eq. (87). The rapid energy variation still shown by the $\bar{p}p$ system at the highest energies mentioned, compared with the approximate constancy of the pp and np systems, suggests that the contribution of $F^{(1)}$ is sizable but decreases rapidly with energy. Kokkedee proposes a model for $F^{(1)}$ which corroborates this fact and accounts well for the data.

IV. QUARK MODEL OF HIGH ENERGY PROCESSESIV.1 SU₆ SYMMETRY, CURRENT ALGEBRAS AND QUARK MODEL

Whereas there is no particular difficulty in using the methods of Chapter II to analyze the effect of approximate SU₃ symmetry on high energy reactions, it is more difficult to adapt them to the discussion of SU₆ symmetry which involves spin in an essential way ; one then encounters all the complications of relativistic spin-orbit coupling. Still, very soon after its introduction, SU₆ led to a remarkable prediction concerning differences between total cross-sections, the Johnson-Treiman relations⁸²⁾

$$\frac{1}{2}[\sigma_T(K^+p) - \sigma_T(K^-p)] = \sigma_T(\pi^+p) - \sigma_T(\pi^-p) = \sigma_T(K^+n) - \sigma_T(K^-n), \quad (103)$$

which are in reasonable agreement with experiment. On the other hand, the agreement is very poor for other relations like for example the triangular inequality for forward scattering⁸³⁾

$$\left[\frac{d\sigma}{dt}(K^-p \rightarrow \pi^- \Sigma^+) \right]^{\frac{1}{2}} \geq \left[\frac{d\sigma_{el}}{dt}(\pi^-p) \right]^{\frac{1}{2}} - \left[\frac{d\sigma_{el}}{dt}(K^-p) \right]^{\frac{1}{2}} \quad (104)$$

although it follows from SU₃ symmetry alone and does not require SU₆. In fact, the left-hand side of (104) is found experimentally to decrease rapidly with energy, whereas the right-hand side stays positive and almost constant. Clearly, symmetry breaking is very important in some cross-sections or combinations of cross-sections while others are hardly affected by it. This type of difficulty affects of course also the theoretical analysis of SU₃ and SU₆ symmetry in high energy reactions. The problem is usually, in deriving successful relations between cross-sections, not to use arguments which would lead to further relations in disagreement with experiment.

This situation is not very different from the one encountered in the SU_6 symmetry analysis of electromagnetic and weak interactions of hadrons. Also here a consistent and successful treatment is lacking by group-theoretical methods, and one has therefore developed other approaches, mainly those based on current algebras⁸⁴⁾ and on the quark model⁸⁵⁾. The latter is a dynamical model treating the hadrons as composite particles formed of SU_3 triplets called quarks. It gives rise to very successful but admittedly very crude calculations, based on a non-relativistic treatment of the quarks inside the hadrons but by-passing and leaving unanswered the problems connected with the binding force between quarks. The current algebra method makes a less explicit use of quarks. It postulates for the currents the commutation rules which hold when the current operators are given the simplest form in terms of quark fields. These commutation rules are then used to derive sum rules for observable properties of hadrons, without any further reference to quarks. Also the current algebra method has achieved many successes, the most remarkable being the Adler-Weisberger sum rule which gives in terms of pion-nucleon total cross-sections the absolute value of the ratio between axial-vector and vector coupling constants of beta decay⁸⁶⁾.

One of the most attractive features of the quark model is that it predicts in a very direct way relations between certain properties of nucleons and analogous properties of mesons. The best known example is the correct relation between the proton magnetic moment and the partial width of the decay $\omega^0 \rightarrow \pi^0 + \gamma$ ⁸⁷⁾. As we shall see later on, an even more striking prediction is obtained for high energy scattering, where the quark model gives for total cross-sections :

$$\lim_{s \rightarrow +\infty} \sigma_T(\pi p) / \sigma_T(pp) = \frac{2}{3} \quad (105)$$

Extrapolating measured πp and pp cross-sections to constant values at infinite energy gives $\sigma_T(\pi p) \simeq 22$ mb, $\sigma_T(pp) \simeq 36$ mb, in close agreement with (105). It is the qualitative success of such simple predictions and the fact that they do not presuppose exact SU_3 symmetry

which make the quark model so attractive and fruitful in the present stage of theoretical developments. Most of its predictions can be obtained in slightly modified form from less radical assumptions by means of the current algebra method, but the derivation is then more complex and the input depends more on the particular property considered ⁸⁸⁾. Thus we mention a very recent extension of the current algebra method to high energy scattering by Cabibbo, Horwitz, Kokkedee and Ne'eman ⁸⁹⁾. These authors set out to obtain Eq. (105) and other consequences of the quark model for high energy scattering by introducing scalar and vector currents for the coupling of the particles to two nonets of Regge poles, the scalar (vector) currents being coupled to the even (odd) signature poles. The commutation rules of these "strong currents" are derived from the quark model but no further use is made of quarks. One obtains the remarkable results that, while Eq. (105) and many other successful relations at $t = 0$ are derived, a fit to the s dependence of total cross-sections requires all σ_T to approach zero as s^a , $a \approx -0.07$ for $s \rightarrow \infty$ ⁹⁰⁾.

The subsequent discussion will be devoted to the quark model approach to high energy scattering. While we shall review only a few among the many relations which have been derived for the forward scattering amplitude and its imaginary part, i.e., the total cross-section ⁹¹⁾, we shall pay more attention to the t dependence of the amplitude at small t and to the consistency of the basic additivity assumption ⁹²⁾.

IV.2 ADDITIVITY ASSUMPTION FOR SCATTERING AMPLITUDE

The basic assumption underlying all applications of the quark model ⁸⁵⁾ is one of additivity, stating that some particular property of a hadron is a sum of terms belonging to the quarks and antiquarks composing that hadron. In weak or electromagnetic interactions, the matrix element of the (weak or electromagnetic) current operator J_μ between two hadronic states A, A' is written as a sum of terms :

$$\langle A' | J_\mu^i | A \rangle = \sum_i \langle A' | J_\mu^i | A \rangle \quad (106)$$

where J_μ^i acts on the i th quark⁹³⁾ in A and leaves the other quarks of A unaffected. The structure of $\langle A' | J_\mu^i | A \rangle$ is always assumed to be of the form

$$\langle A' | J_\mu^i | A \rangle = \langle Q_i^{A'} | J_\mu^i | Q_i^A \rangle f_i^{A'A}(\Delta p_\mu) \quad (107)$$

$\langle Q_i^{A'} | J_\mu^i | Q_i^A \rangle$ represents the current matrix element between quark states $Q_i^A, Q_i^{A'}$ calculated for the same value of the momentum transfer $p_\mu^{A'} - p_\mu^A = \Delta p_\mu$ as occurs in $\langle A' | J_\mu | A \rangle$. It depends on A, A' only through the spin and SU_3 variables of $Q_i^A, Q_i^{A'}$. The form factor $f_i^{A'A}$ represents the overlap between the quark wave function of hadron A' and the wave function generated by action of J_μ^i on the quark wave function of A . When A' is identical to A and J_μ^i does not flip either spin or SU_3 state of Q_i one has

$$f_i^{AA}(0) = 1 \quad (108)$$

Equations (106) and (107) describe the contents of the additivity assumption. Equation (106) states the additivity proper, and Eq. (107) describes the approximation which allows to compare properties of, say, baryons and mesons. Indeed $\langle Q_i^{A'} | J_\mu^i | Q_i^A \rangle$ is supposed to depend only on Δp_μ , on the spin and on the SU_3 states of quark Q_i in A and A' , whereas $f_i^{A'A}$ describes the effect of the quark wave functions of A and A' 94).

We now generalize the additivity assumption to the two-body scattering process

$$A+B \rightarrow A'+B' \quad (109)$$

A and A' are supposed to have the same quark composition, i.e., to be both baryons or both mesons; the same holds for B and B' . As usual s will denote the square of the c.m. energy. We write Δp_μ for the four-momentum transfer

$$\Delta p_\mu = p_\mu^{A'} - p_\mu^A = p_\mu^B - p_\mu^{B'} \quad (110)$$

Let Q_i denote the quarks in A ($i=1,2,3$ if A is a baryon ; $i=1,2$ if A is a meson : as before we use the term quark both for quarks and antiquarks). Similarly, let Q_j denote the quarks in B . The additivity assumption now means that the process (109) takes place through scattering of a single quark Q_i in A on a single quark Q_j in B , the remaining quarks in A and B remaining unaffected. In terms of the collision matrix

$$T = i(1-S) \quad (111)$$

this can be written

$$\langle A'B' | T | AB \rangle = \delta_4(p^{A'} + p^{B'} - p^A - p^B) \sum_{ij} \langle A'B' | T_{ij} | AB \rangle \quad (112)$$

where the operator T_{ij} acts on the i^{th} quark of A and the j^{th} quark of B , but leaves the other quarks unaffected. Equation (112), which is analogous to (106), must be supplemented by an equation analogous to (107). We write it in the form

$$\langle A'B' | T_{ij} | AB \rangle = \langle Q_i^{A'} Q_j^{B'} | T_{ij} | Q_i^A Q_j^B \rangle f_i^{A'A}(\Delta p_\mu) f_j^{B'B}(-\Delta p_\mu) \quad (113)$$

$\langle Q_i^{A'} Q_j^{B'} | T_{ij} | Q_i^A Q_j^B \rangle$ describes the collision of quarks Q_i, Q_j with momentum transfer Δp_μ , bringing them from the spin and SU_3 states Q_i^A, Q_j^B which they have in A, B to the spin and SU_3 states $Q_i^{A'}, Q_j^{B'}$. As in (107) the form factors $f_i^{A'A}$ represent the overlap between the quark wave function of A' and the wave function generated by the action of T_{ij} on Q_i , the remaining quarks of A being unaffected, and similarly for $f_j^{B'B}$ (94). The form factors satisfy Eq. (108) for $A'=A, B'=B$ and those forward collisions where Q_i and Q_j keep their spin and SU_3 states unchanged.

The dependence of the various factors in Eq. (113) on the incident energy $s^{\frac{1}{2}} = p_0^A + p_0^B$ deserves special attention. It is normal to consider that for given Δp_μ the form factors $f_i^{A'A}, f_j^{B'B}$ are

independent of s . The same can be assumed reasonably for $\langle Q_i^A Q_j^B | T_{ij} | Q_i^A Q_j^B \rangle$ in the case of elastic scattering at very high energy ($A'=A, B'=B, s \rightarrow +\infty$) if all cross-sections become constant. In that case, indeed, one can assume that the covariant amplitude (13) becomes proportional to s , so that, from the normalization (10), the matrix element (112) becomes s independent. The same can then naturally be assumed for (113). In other words, the fact that the $A+B$ collision becomes energy independent at very high energy is accounted for by the assumption that the same holds for the Q_i+Q_j collisions. In this simple case one can avoid the question of which energy to attribute to the Q_i+Q_j collision when the energy $s^{\frac{1}{2}}$ of the $A+B$ collision is given.

This question must be faced, however, for elastic scattering at finite values of s , and at all s for inelastic processes ($A' \neq A$ and/or $B' \neq B$) with decreasing cross-sections. Since the quark model is regarded as mainly characterized by additivity, it seems reasonable to say that the properties carried by the hadrons are the sum of those carried by the constituting quarks, and that this holds not only for baryon number, isospin, hypercharge and spin, but also for momentum and energy. Thus if A has four momentum p_μ^A , its constituting quarks Q_i will have four momenta

$$p_\mu^i = c_i^A p_\mu^A, \quad \sum_i c_i^A = 1 \quad (114)$$

and it is natural to assume all c_i^A to be approximately equal so that

$$c_i^A \simeq c_A, \quad c_A = \frac{1}{2} \text{ for mesons, } c_A = \frac{1}{3} \text{ for baryons} \quad (115)$$

It is clear that (114) is not compatible with the relation

$$(p^i)^2 = m_Q^2 \gg m_A^2 \quad (116)$$

which would hold for a free quark of large mass m_Q . The properties discussed here are those of quarks as they are supposed to behave when bound together in the hadrons, and these quarks have then an effective mass m_Q^{eff} which (114) and (115) suggest to be of order

$$m_Q^{\text{eff}} \simeq \frac{1}{3} m_B \simeq \frac{1}{2} m_M \quad (117)$$

where m_B, m_M are average values of the baryon and meson masses respectively. Such a low effective mass for the quark is also suggested by the value of the baryon magnetic moments under the additivity hypothesis, Refs. 87), 95). In fact, all successful applications of the quark model use the additivity property for bound quark properties, and are quite independent of what a free quark would be if it existed. The essential assumption, however, is that the bound quark properties are the same in baryons and mesons, and it is this assumption which is tested when comparing baryons with mesons in the framework of the quark model.

We remark also in this connection that all applications of the model refer to small moment transfer properties, so that they deal with bound quark properties averaged over lengths and times $\gtrsim m_B^{-1}$. On a shorter time scale, quarks may have other properties of oscillating nature 96). This may hold in particular for the Fermi momentum of the quarks inside the hadrons ; they may be appreciable but give a small contribution to the time-averaged quantities p_μ^i of Eq. (114).

We shall take Eqs. (112) and (113) as the explicit formulation of the additivity assumption for high energy scattering with the understanding that the $\langle Q_i^{A'} Q_j^{B'} | T_{ij} | Q_i^A Q_j^B \rangle$ are the same for different hadron collisions when the resulting quark-quark collision energy given by Eqs. (114)-(115) is the same. This energy is given by

$$s_{ij} = (p_i^i + p_j^j)^2 \simeq (c_A m_A)^2 + (c_B m_B)^2 + 2c_A c_B (p^A p^B) \quad (118)$$

At high energy it is well approximated by

$$s_{ij} \simeq c_A c_B (s - m_A^2 - m_B^2) \simeq c_A c_B s_0 \quad (119)$$

where s_0 is the value at $t=0$ of s_t , Eq. (27). This means that to compare baryon-baryon, meson-baryon and meson-meson collisions, they must be taken at energies in the following proportions

$$\frac{s_o^{BB}}{9} = \frac{s_o^{MB}}{6} = \frac{s_o^{MM}}{4} \quad (120)$$

The agreement with experiment is indeed better if the comparison between baryon-baryon and meson-baryon cross-sections is done in this way.

IV.3 RELATIONS BETWEEN TOTAL CROSS-SECTIONS

The additivity assumption, Eqs. (112)-(113), implies many relations between scattering amplitudes. We shall present a few of them here in the form in which they have been first derived⁹¹⁾, i.e., under neglect of the spin variables. As was shown recently by Itzykson and Jacob⁹⁷⁾, spin variables can be taken into account, several relations derived with neglect of spin are recovered for the spin average quantities, and new relations can be obtained. In the few cases where experimental tests are possible with present data, the agreement is surprisingly good in view of the expected crudeness of the model.

Consider first forward elastic processes, i.e.,

$$A' = A, \quad B' = B, \quad \Delta_{p\mu} = 0. \quad (121)$$

All form factors reduce to 1, as shown by Eq. (108). We use the quark composition of baryons and mesons, with spins neglected

$$\left. \begin{aligned} \pi^+ &= Q_p \bar{Q}_n, & K^+ &= Q_p \bar{Q}_\lambda, \\ p &= Q_p Q_p Q_n, & n &= Q_p Q_n Q_n \end{aligned} \right\} \quad (122)$$

where Q_p, Q_n, Q_λ denote the $I = \frac{1}{2}, I = 0$ quarks respectively. Using the abbreviations

$$\left. \begin{aligned} \langle AB | T | AB \rangle &= (AB), \\ \langle Q_i Q_j | T_{ij} | Q_i Q_j \rangle &= (Q_i Q_j), \end{aligned} \right\} \quad (123)$$

we obtain immediately

$$\begin{aligned}
 (pp) &= 4(Q_p Q_p) + 4(Q_p Q_n) + (Q_n Q_n) \\
 (np) &= 2(Q_p Q_p) + 5(Q_p Q_n) + 2(Q_n Q_n) \\
 (\bar{n}p) &= 2(\bar{Q}_p Q_p) + 4(\bar{Q}_n Q_p) + (\bar{Q}_p Q_n) + 2(\bar{Q}_n Q_n) \\
 (\bar{p}p) &= 4(\bar{Q}_p Q_p) + 2(\bar{Q}_p Q_n) + 2(\bar{Q}_n Q_p) + (\bar{Q}_n Q_n) \\
 (\pi^+ p) &= 2(Q_p Q_p) + (Q_p Q_n) + 2(\bar{Q}_n Q_p) + (\bar{Q}_n Q_n) \\
 (\pi^- p) &= 2(\bar{Q}_p Q_p) + (\bar{Q}_p Q_n) + 2(Q_n Q_p) + (Q_n Q_n) \\
 (K^+ p) &= 2(Q_p Q_p) + (Q_p Q_n) + 2(\bar{Q}_\lambda Q_p) + (\bar{Q}_\lambda Q_n) \\
 (K^+ n) &= (Q_p Q_p) + 2(Q_p Q_n) + (\bar{Q}_\lambda Q_p) + 2(\bar{Q}_\lambda Q_n) \\
 (K^- p) &= 2(\bar{Q}_p Q_p) + (\bar{Q}_p Q_n) + 2(Q_\lambda Q_p) + (Q_\lambda Q_n) \\
 (K^- n) &= (\bar{Q}_p Q_p) + 2(\bar{Q}_p Q_n) + (Q_\lambda Q_p) + 2(Q_\lambda Q_n)
 \end{aligned} \tag{124}$$

A simple consequence is

$$\begin{aligned}
 \frac{1}{3} [(pp) + (np) + (\bar{n}p) + (\bar{p}p)] &= [(\pi^+ p) + (\pi^- p)] = \\
 &= [2(Q_p Q_p) + 3(Q_p Q_n) + (Q_n Q_n) + 2(\bar{Q}_p Q_p) + (\bar{Q}_p Q_n) + 2(\bar{Q}_n Q_p) + (\bar{Q}_n Q_n)]
 \end{aligned} \tag{125}$$

At very high energy, the absence of exchange scattering and annihilation implies that all amplitudes in the same square bracket become equal, so that

$$\frac{4}{3}(pp) = 2(\pi^+ p) = 12 (Q_p Q_p) . \tag{126}$$

Through the optical theorem, Eq. (105) follows. Applying the optical theorem directly to Eq. (125) one obtains :

$$\frac{2[\sigma_{\pi^+p} + \sigma_{\pi^-p}]}{\sigma_{pp} + \sigma_{np} + \sigma_{\bar{n}p} + \sigma_{\bar{p}p}} = \frac{2}{3} \quad (127)$$

Taking the πp cross-sections at 13 GeV and the nucleon-nucleon and antinucleon-nucleon ones at 20 GeV, as proposed by the rule (120), the experimental ratio is 0.59 ± 0.05 . It would be 0.54 ± 0.05 if all cross-sections were taken at the same energy, 13 GeV or 20 GeV.

As another example we consider the Johnson-Treiman relations (103). From (124) one obtains

$$\begin{aligned} (K^+p) - (K^-n) &= 2(\delta_{pp} - \delta_{\lambda p}) + \delta_{pn} - \delta_{\lambda n} \\ (\pi^+p) - (\pi^-p) &= 2(\delta_{pp} - \delta_{np}) + \delta_{pn} - \delta_{nn} \\ (K^+n) - (K^-n) &= 2(\delta_{pn} - \delta_{\lambda n}) + \delta_{pp} - \delta_{\lambda p} \end{aligned} \quad (128)$$

with the notation

$$\delta_{ab} = (Q_a Q_b) - (\bar{Q}_a Q_b) ; \quad a, b = p, n, \lambda \quad (129)$$

Isospin invariance gives

$$\delta_{pp} = \delta_{nn}, \quad \delta_{pn} = \delta_{np}, \quad \delta_{\lambda p} = \delta_{\lambda n} \quad (130)$$

Hence

$$(K^+p) - (K^-p) = (\pi^+p) - (\pi^-p) + (K^+n) - (K^-n) \quad (131)$$

As shown by Lipkin and Scheck⁹¹⁾, the corresponding relation for total cross-sections is in very good agreement with experiment for $s \gtrsim 15(\text{GeV})^2$. To obtain the Johnson-Treiman relations (103) one needs further

$$\delta_{\lambda p} = \delta_{np} \quad (132)$$

which holds if the quark-quark and quark-antiquark amplitudes obey SU_3 symmetry. It is therefore satisfactory that Eq. (131) for σ_T , which is not affected by symmetry breaking, is better satisfied by the data than the full Johnson-Treiman relations.

As a last remark on total cross-sections, we note that the relations discussed above begin to disagree with the facts when s decreases below a value of order $10(\text{GeV})^2$. This shows that the additivity assumption for scattering amplitudes holds only in the high energy region.

IV.4 DIFFERENTIAL CROSS-SECTIONS AND CONSISTENCY OF THE ADDITIVITY ASSUMPTION

On differential cross-sections we shall discuss the implications of the quark model with additive scattering amplitudes for the case of elastic scattering at very high energy⁹⁸⁾. This will allow us to obtain, at least in two limiting cases, an understanding of why the additivity assumption can be a good approximation. At very high energy, it is natural to assume that the quark-quark elastic scattering becomes purely diffractive and has an energy independent cross-section. Hence in Eq. (113) applied to elastic scattering ($A'=A$ and $B'=B$) we put

$$\langle Q_i^A Q_j^B | T_{ij} | Q_i^A Q_j^B \rangle = i g_{ij}(t) = i g_{ji}(t), \quad t = (\Delta p)^2 \quad (133)$$

with $g_{ij}(t)$ real. Isospin symmetry gives

$$g_{pp}(t) = g_{nn}(t), \quad g_{\lambda p}(t) = g_{\lambda n}(t) \quad (134)$$

where the indices p, n, λ refer to the quarks Q_p, Q_n, Q_λ . Neglecting SU_3 symmetry breaking effects would give in addition

$$g_{\lambda\lambda}(t) = g_{pp}(t), \quad g_{pn}(t) = g_{\lambda n}(t) \quad (135)$$

The absence of charge exchange at very high energy implies the further relation

$$g_{pp}(t) = g_{pn}(t) \quad (136)$$

Similar relations hold for antiquark-quark scattering. We list them with the necessary assumptions ($\bar{p}, \bar{n}, \bar{\lambda}$ refer to $\bar{Q}_p, \bar{Q}_n, \bar{Q}_\lambda$):

$$\begin{aligned} \text{isospin and} & \quad \left\{ \begin{aligned} g_{\bar{p}p} &= g_{\bar{n}n}, & g_{\bar{p}n} &= g_{\bar{n}p}, & g_{\bar{\lambda}p} &= g_{\bar{\lambda}n}, & g_{\bar{p}\lambda} &= g_{\bar{n}\lambda}, \\ \text{charge conjugation : } & g_{\bar{\lambda}p} &= g_{\bar{p}\lambda} \end{aligned} \right\} \quad (137) \end{aligned}$$

$$SU_3 : \quad g_{\bar{\lambda}\lambda} = g_{\bar{p}p}, \quad g_{\bar{p}n} = g_{\bar{\lambda}n} \quad (138)$$

$$\text{no charge exchange : } g_{\bar{p}p} = g_{\bar{n}p} \quad (139)$$

Finally, $\bar{Q}_i Q_j$ and $Q_i Q_j$ elastic scattering become identical at very high energy (99), which means

$$g_{\bar{p}p}(t) = g_{pp}(t), \quad g_{\bar{\lambda}p}(t) = g_{\lambda p}(t), \quad g_{\bar{\lambda}\lambda}(t) = g_{\lambda\lambda}(t) \quad (140)$$

Using these various relations, except the ones requiring SU_3 symmetry, we obtain for very high energy

$$\langle AB|T|AB \rangle = ig_{AB}(t)f^A(t)f^B(t) \quad (141)$$

with 100)

$$f^A(t) = \sum_i f_i^{AA}(\Delta p_\mu), \quad f^B(t) = \sum_j f_j^{BB}(-\Delta p_\mu) \quad (142)$$

$$\left. \begin{aligned} g_{AB}(t) &= g_{pp}(t) & \text{for } A, B &= \pi, p, n, \bar{p}, \bar{n} \\ g_{AB}(t) &= \frac{1}{2} [g_{\lambda p}(t) + g_{pp}(t)] & \text{for } A &= K, \bar{K} \text{ and } B = p, n, \pi \\ g_{AB}(t) &= \frac{1}{3} [g_{\lambda p}(t) + 2g_{pp}(t)] & \text{for } A &= \Lambda, \Sigma \text{ and } B = p, n, \pi \\ g_{AB}(t) &= \frac{1}{3} [2g_{\lambda p}(t) + g_{pp}(t)] & \text{for } A &= \Xi \text{ and } B = p, n, \pi \\ g_{AB}(t) &= g_{\lambda p}(t) & \text{for } A &= \Omega^- \text{ and } B = p, n, \pi \end{aligned} \right\} \quad (143)$$

With SU_3 symmetry all $g_{AB}(t)$ reduce to $g_{pp}(t)$.

A number of consequences of the above relations have been discussed elsewhere by Kokkedee and the present author^{91),92)}. No accurate experimental tests are available with present data, but the model predicts a great degree of universality for high energy diffraction of all hadrons, which is indeed strongly suggested by existing measurements. In our further considerations we limit ourselves to scattering of pions (π) and nucleons (N). With SU_3 symmetry, our formulae would apply to all mesons and baryons. With the notation

$$g(t) = g_{pp}(t) \quad (144)$$

we have instead of (141)

$$\langle AB|T|AB \rangle = ig(t)f^A(t)f^B(t). \quad (145)$$

This gives the factorization property

$$\langle AB|T|AB \rangle^2 = \langle AA|T|AA \rangle \cdot \langle BB|T|BB \rangle \quad (146)$$

which determines for example the $\pi\pi$ diffraction scattering from known data

$$\langle \pi\pi|T|\pi\pi \rangle = \frac{f^\pi(t)}{f^N(t)} \langle \pi N|T|\pi N \rangle = \frac{\langle \pi N|T|\pi N \rangle^2}{\langle NN|T|NN \rangle}. \quad (147)$$

Equation (145) shows that the t dependence of hadron-hadron diffraction scattering is controlled by two types of functions, the hadron form factors $f^A(t)$, $f^B(t)$ and the asymptotic quark-quark amplitude $g(t)$. The effects of these functions cannot be separated on the basis of hadron-hadron diffraction data alone. We therefore discuss two extreme cases between which a continuum of intermediate situations can be imagined. They will allow a better understanding of the validity of the additivity assumption.

In the first extreme case, the t dependence of the diffraction peak is supposed to be mainly determined by the form factors, in the sense that $g(t)$ remains practically equal to $g(0)$ over the whole width of the diffraction peak, the slope of which is determined by the factor $f^A(t)f^B(t)$ in Eq. (145). In other words, the size of quarks as it manifests itself in their high energy diffraction is supposed smaller than the measured size of hadrons. The minimum size of quarks is obtained by assuming that they behave as black spheres. Their radius R_Q is then given by

$$\sigma_T(QQ) = 2\pi R_Q^2 \quad (148)$$

where $\sigma_T(QQ)$, the total quark-quark cross-section, is in our normalization ¹⁰¹⁾

$$\sigma_T(QQ) = 4\pi^2 g(0) . \quad (149)$$

But from (145), (142) and (108), one has

$$\sigma_T(pp) = 36\pi^2 g(0) = 9\sigma_T(QQ) \quad (150)$$

so that we find

$$R_Q^2 = \frac{\sigma_T(pp)}{18\pi} \quad (151)$$

This is indeed much smaller than the size of proton (or pion) as determined from their diffraction, not only because of the factor 9 in (150), but also because the exponential shape of the former corresponds to an absorption which is only strong at zero impact parameter and decreases as a Gaussian. In the notation of Section III.1, one has in fact approximately

$$\eta_\ell \simeq \eta_0 \exp\left(-\frac{\ell^2}{2bk^2}\right) , \quad (152)$$

$b \simeq 9(\text{GeV}/c)^2$ being the width of the high energy pp or πp diffraction peaks as defined in Eq. (19). The parameter η_0 is close to 1 for pp, of order 0.7 for πp .

Our first extreme case leads us to a picture of hadrons composed of small size quarks separated from each other by distances large compared to their size. In this picture the success of the additivity assumption for quark scattering amplitudes becomes a consequence of the small size of the quarks which makes multiple scattering effects rare. The picture also suggests another interesting property. The hadron-hadron elastic scattering has now a t dependence given by

$$\frac{d\sigma_{el}}{dt} = \left(\frac{d\sigma_{el}}{dt} \right)_{t=0} [f^A(t)f^B(t)]^2 \quad (153)$$

But in our picture of small quarks forming large hadrons, the form factor $f^A(t)$ should apply to all hadron properties carried by the quarks, including the electromagnetic and weak ones. For f^A, f^B in (153) one could then take the electromagnetic form factors. It is remarkable that this conjecture is in excellent agreement with experiment for the case $A=B=\text{proton}$, the only one where it can be tested at present⁹²⁾. It holds in the interval $0 \leq -t \lesssim 1(\text{GeV}/c)^2$ where $d\sigma_{el}/dt$ drops by a factor 10^{-3} . The conjecture allows to derive from πp diffraction scattering the electromagnetic form factor of the pion, which is found to be very similar to the proton one.

We consider now the second extreme case where the t dependence of the diffraction peak is supposed to be mainly determined by the quark-quark amplitude $g(t)$, the hadron form factors $f^{A,B}(t)$ in Eq. (145) remaining approximately equal to $f^{A,B}(0)$ over the whole diffraction peak. The resulting picture is opposite to the previous one. It is now the quark itself which has a large size for diffraction, while the various quarks inside one hadron overlap almost exactly with each other in space so as to give a weak t dependence to the form factors $f^{A,B}(t)$. All hadrons have then, except for a constant factor, the same diffraction scattering in the limit of very high energy, namely

$$\langle AB|T|AB \rangle = \ln_A \ln_B g(t) \quad (154)$$

where $n_{A,B} = f^{A,B}(0)$ is the number of quarks composing A, B. This picture is compatible with the data because an extrapolation of the pp, $\bar{p}p$, $\pi^\pm p$ diffraction peaks to infinite energy indeed indicates that their shapes become remarkably close to each other. The quark-quark diffraction scattering is now entirely determined by the equations

$$\sigma_T(QQ) = \frac{1}{9} \sigma_T(pp), \quad \frac{d\sigma_{el}}{dt}(QQ) = \frac{1}{81} \frac{d\sigma_{el}}{dt}(pp) \quad (155)$$

and the corresponding absorption coefficient η_ℓ is given approximately by Eq. (152) with $\eta_0 \simeq 1/9$. While the quark has a large size for diffraction, it has only very weak absorption, reaching a maximum of about 10% at zero impact parameter. In the present picture the success of the additivity assumption for scattering amplitudes is a consequence of this low absorption or high transparency. While the quarks overlap in space, their diffractive interaction is so weak that again multiple processes involving scattering on more than one quark in the same hadron are very unlikely. This would not remain true, however, for scattering at large t because the fast drop of $g(t)$ with t would then favour multiple processes.

The weak nature of the diffractive interaction between quarks obtained in the present case suggests that it can be treated in first Born approximation, each small volume in one quark interacting with each small volume in the other one, and the total scattering amplitude being the sum over all such contributions. This suggests

$$g(t) = g(0) [F(t)]^2 \quad (156)$$

where $F(t)$ is the form factor of the quark describing how its diffractive interaction strength is distributed over space¹⁰²⁾. If the same form factor $F(t)$ describes the distribution of electromagnetic and weak interactions in the quark, it must be identified with the electromagnetic form factors of the proton and the pion because the $f^{A,B}(t)$ do not contribute to the latter in our approximation. As mentioned after

Eq. (153) the experimental facts are compatible with this identification. Needless to say, in the picture just discussed, the quark is even more difficult to visualize as a separate particle than was the case with the previous picture of small-size quarks.

The previous considerations have shown how the additivity assumption can be understood to work in two extreme cases. It is reasonable to expect that intermediate cases would combine elements of the two extremes but would again lead to the conclusion that multiple scattering on various quarks in the same hadron would only contribute small effects in hadron-hadron collisions at small momentum transfer.

IV.5 RELATION TO REGGE POLE MODEL AND INELASTIC COLLISIONS IN QUARK MODEL

We end with a few remarks comparing the quark model approach to high energy scattering with the Regge pole model (Chapter II) and the considerations of Chapter III on shadow scattering.

A few of the quark model predictions are qualitatively similar to features found in the Regge pole model. Thus, the factorization property contained in Eqs. (145) and (146) for high energy diffraction scattering and total cross-sections would also be obtained from Regge pole theory, because of the factorization property [Section II.5, point b] applied to the residue of the Pomeron pole ($\alpha_1 \simeq 1$) which dominates elastic scattering. A more direct and more general consequence of the additivity assumption (112)-(113) is that for $s \rightarrow \infty$, t finite, the quantum numbers exchanged in the t channel are those of the quark-antiquark system with positive and negative signatures, corresponding to the nonets required in Regge pole theory to describe the meson-nucleon reactions. In fact, one can say that the quark-antiquark system taken for positive and negative values of $t = m^2$ (m = mass of the system) provides a dynamical model for the Regge trajectories

of the mesons, and that the matrix $Z_J^+(t)$ of Eq. (71) refers entirely to this $Q\bar{Q}$ system. This accounts then directly for the observed universality of the exponents α_j and for their relations to particles [Section II.5, points a) and c)]. The analogies mentioned are of course only partial, and, while the Regge pole and quark models of high energy scattering have these points in common, their full implications would be rather different in many respects. It is very likely, however, that both are no more than approximate descriptions of a truly complex class of strong interaction phenomena.

The principal ideas of the quark model for high energy scattering at small momentum transfer can be extended to a large class of inelastic phenomena by allowing A' , B' in Eqs. (112) and (113) to be all states obtained by transferring a not too large amount Δp_μ of energy momentum, and possibly some isospin or hypercharge from a quark Q_i of A to a quark Q_j of B . The states A' , B' would then not necessarily be resonances but would be excited systems which would decay through virtual creation of quark-antiquark pairs liberated as mesons¹⁰³⁾. The inelastic final states $A'+B'$ could coincide with the ones discussed in Section III.2, A' and B' forming a more precise characterization of the "forward" and "backward" groups of particles there considered. The occurrence of the "leading" particle there described would be due to the fact that the excitation energy and momentum of A' (or B') radiated away in the form of mesons would have a preferential direction in space, namely the direction of motion of the particle B (or A) which has hit A (or B) and excited it to A' (or B'). This direction is of course backward for the incident particle A , and forward for the target particle B . The effect discussed is rather easily visualized when A (B) is a baryon. The quark-quark collision then slows down one quark only in A (B), the two others tending to continue with their initial momentum. The latter two quarks will keep the first one bound to them with a rather high forward (backward) momentum, the remaining energy of A' (B') being radiated away as mesons with much lower momentum. We also remark that this type of directional excitation is common in many-particle systems. It occurs for example when a crystal

is excited by a fast particle incident on it ; the phonons excited in the crystal will have a direction of propagation strongly correlated to the direction of motion of the incident particle.

Also the separation (86) of the final state in non-exchange and exchange parts $\psi_{AB}^{(0)}$, $\psi_{AB}^{(1)}$ is easily connected to Eqs. (112) and (113). It results from the fact that the $Q_i Q_j$ collision can transfer vanishing or non-vanishing values of isospin and hypercharge. The only states missing are the ones contributed to $\psi_{AB}^{(1)}$ by annihilation when $A = \text{antibaryon}$, $B = \text{baryon}$. They cannot be generated by a single quark-quark collision but require triple quark exchange. As a consequence, their possible separation into forward and backward groups cannot be interpreted in the same terms as before. More generally all processes involving baryon exchange will require a separate treatment in the quark model, also for example backward meson-nucleon scattering.

The further analysis of the annihilation effects in antibaryon-baryon collisions throws in fact some doubt on whether the quark model with additive amplitudes, when applied to total cross-sections and elastic scattering [Sections IV.3 and IV.4], treats properly that contribution to the antibaryon-baryon elastic scattering amplitude which is due to the shadow of annihilation. In Section IV.4 we have treated hadron-hadron and quark-quark elastic scattering as purely diffractive, as indicated by the reality of g_{ij} in Eq. (133). But the shadow of antibaryon-baryon annihilation cannot be obtained in an additive way from the shadow of antiquark-quark annihilation, whereas additivity can hold for the shadow of the non-annihilation part of $\psi_{AB} = \psi_{AB}^{(0)} + \psi_{AB}^{(1)}$. It may therefore be that our considerations require a further term in the antibaryon-baryon elastic amplitudes and total cross-sections. This is also suggested by the data, because Eq. (127) is in closer agreement with the data if the annihilation contribution to $\sigma_T(\bar{n}p)$ and $\sigma_T(\bar{p}p)$ is removed.

A last remark concerns the shape of the diffraction peak. In Chapter III we have related it through unitarity to the overlap function of inelastic collisions, whereas in Section IV.4 it was suggested to be given in terms of the hadronic form factor. This then implies a relation between form factor and overlap function. We do not know at present whether it can be given a particular significance, nor whether the approximate exponential shape of the overlap function is better understood in terms of the inelastic collisions, as proposed in Section III.3, or in terms of the form factor.

FOOTNOTES AND REFERENCES

- 1) D.E. Dorfan, J. Eades, L.M. Lederman, W. Lee, C.C. Ting, P. Piroué, Stuart Smith, J.L. Brown, J.A. Kadyk and G.H. Trilling, Phys.Rev.Letters 14, 995 (1965).
- 2) M. Jacob and G.C. Wick, Ann.Phys. 7, 404 (1959).
- 3) A.N. Diddens, E. Lillethun, G. Manning, A.E. Taylor, T.G. Walker and A.M. Wetherell, Phys.Rev.Letters 9, 108, 111 (1962).
- 4) K.J. Foley, S.J. Lindenbaum, W.A. Love, S. Ozaki, J.J. Russel and L.C.L. Yuan, Phys.Rev.Letters 10, 376, 543 (1963) ; 11, 425, 503 (1963).
- 5) D. Harting, P. Blackhall, B. Elsner, A.C. Helmholtz, W.C. Middelkoop, B. Powell, B. Zacharov, P. Zanella, P. Dalpiaz, M.N. Focacci, S. Focardi, G. Giacomelli, L. Monari, J.A. Beaney, R.A. Donald, P. Mason, L.W. Jones and D.O. Caldwell, Phys.Letters 8, 288 (1964), and Nuovo Cimento 38, 60 (1965).
- 6) K.J. Foley, R.S. Gilmore, S.J. Lindenbaum, W.A. Love, S. Ozaki, E.H. Willen, R. Yamada and L.C.L. Yuan, Phys.Rev.Letters 15, 45 (1965).
- 7) A.E. Taylor, A. Ashmore, W.S. Chapman, D.F. Falla, W.H. Range, D.B. Scott, A. Astbury, F. Capocci and T.G. Walker, Phys.Letters 14, 54 (1965).
- 8) K.J. Foley, R.S. Gilmore, R.S. Jones, S.J. Lindenbaum, W.A. Love, S. Ozaki, E.H. Willen, R. Yamada and L.C.L. Yuan, Phys.Rev. Letters 14, 74 (1965).
- 9) L. Kirillova, V. Nikitin, V. Pantuev, V. Sviridov, L. Strunov, M. Khachatryan, L. Khristov, M. Shafranov, Z. Korbel, L. Rob, S. Damyanov, A. Zlateva, Z. Zlatanov, V. Iordanov, Kh. Kanazirsky, P. Markov, T. Todorov, Kh. Chernev, N. Dalkhazhav and D. Tuvdendorzh, Soviet Journal of Nuclear Physics 1, 379 (1965).

- 10) G. Bellettini, G. Cocconi, A.N. Diddens, E. Lillethun, J.P. Scanlon, J. Walkers, A.M. Wetherrell and P. Zanella, *Phys.Letters* 14, 164 (1965).
- 11) G. Bellettini, G. Cocconi, A.N. Diddens, E. Lillethun, J.P. Scanlon and A.M. Wetherrell, *Phys.Letters* 19, 341, 705 (1965).
- 12) K.J. Foley, R.S. Gilmore, R.S. Jones, S.J. Lindenbaum, W.A. Love, S. Ozaki, E.H. Willen, R. Yamada and L.C.L. Yuan, *Phys.Rev. Letters* 14, 862 (1965).
- 13) G. Cocconi, V.T. Cocconi, A.D. Krisch, J. Orear, R. Rubinstein, D.B. Scarl, B.T. Ulrich, W.F. Baker, E.W. Jenkins and A.L. Read, *Phys.Rev.* 138, B165 (1965).
- 14) J. Orear, R. Rubinstein, D.B. Scarl, D.H. White, A.D. Krisch, W.R. Frisken, A.L. Read and H. Ruderman, *Phys.Rev.Letters* 15, 309 (1965).
- 15) W.R. Frisken, A.L. Read, H. Ruderman, A.D. Krisch, J. Orear, R. Rubinstein, D.B. Scarl and D.H. White, *Phys.Rev.Letters* 15, 313 (1965).
- 16) S.W. Kormanyos, A.D. Krisch, J.R. O'Fallon, K. Ruddick and L.G. Ratner, *Phys.Rev.Letters* 16, 709 (1966) ;
H. Brody, R. Lanza, R. Marshall, J. Niederer, W. Selove, M. Shocket and R. Van Berg, *Phys.Rev.Letters* 16, 828, 968 (1966).
- 17) J.L. Friedes, H. Palevsky, R.L. Stearns and R.J. Sutter, *Phys.Rev. Letters* 15, 38 (1965) ;
G. Manning, A.G. Parham, J.D. Jafar, H.B. van der Raay, D.H. Reading, D.G. Ryan, B.D. Jones, J. Malos and N.H. Lipman, *Nuovo Cimento* 41, 167 (1966).
- 18) I. Mannelli, A. Bigi, R. Carrara, M. Wahlig and L. Sodickson, *Phys.Rev.Letters* 14, 408 (1965).
- 19) A.V. Stirling, P. Sonderegger, J. Kirz, P. Falk-Vairant, O. Guisan, C. Bruneton, P. Borgeaud, M. Yvert, J.P. Guillaud, C. Caversasio and B. Amblard, *Phys.Rev.Letters* 14, 763 (1965), and *Phys.Letters* 20, 75 (1966).

- 20) O. Guisan, J. Kirz, P. Sonderegger, A.V. Stirling, P. Borgeaud, C. Bruneton, P. Falk-Vairant, B. Amblard, C. Caversasio, J.P. Guillaud and M. Yvert, Phys.Letters 18, 200 (1965).
- 21) P. Astbury, G. Finocchiaro, A. Michelini, C. Verkerk, C.H. West, W. Beusch, B. Gobbi, M. Pepin, M.A. Pouchon and E. Polgar, Phys.Letters 16, 328 (1965).
- 22) G. Finocchiaro, A. Michelini, W. Beusch, W.E. Fisher, B. Gobbi and E. Polgar, Paper presented at the Stony Brook Conference on High-Energy Two-Body Reactions, Stony Brook, N.Y., April (1966).
- 23) For a recent review, see D.R.O. Morrison, Paper presented at the Stony Brook Conference on High-Energy Two-Body Reactions, Stony Brook, N.Y., April (1966).
- 24) M. Borghini, G. Coignet, L. Dick, L. di Lella, A. Michalowicz, P.C. Macq and J.C. Oliver, Phys.Letters 21, 114 (1966).
- 25) P. Sonderegger, Paper presented at the Stony Brook Conference on High-Energy Two-Body Reactions, Stony Brook, N.Y., April (1966).
- 26) L. Holloway, Paper presented at the Stony Brook Conference on High-Energy Two-Body Reactions, Stony Brook, N.Y., April (1966).
- 27) I.Ia. Pomeranchuk, Zhur.Eksp.i Teoret.Fiz. 34, 725 (1958) ;
[English translation : Soviet Phys. - JETP 7, 499 (1958)].
- 28) T.L. Trueman and G.C. Wick, Ann.Phys. 26, 322 (1964).
- 29) The generalized helicity amplitude depends on the reference frame and should therefore be written $T(p^A p^B p^C p^D, \lambda_A \lambda_B \lambda_C \lambda_D)$, with explicit reference to the four momenta. One writes similarly for the u channel amplitude $\bar{T}(p^{\bar{C}} p^B p^{\bar{A}} p^D, \lambda_{\bar{C}}' \lambda_B' \lambda_{\bar{A}}' \lambda_D')$. The crossing relation (40) is then

$$\bar{T}(p^{\bar{C}} p^B p^{\bar{A}} p^D, \lambda_{\bar{C}}' \lambda_B' \lambda_{\bar{A}}' \lambda_D') = T^*(-p^{\bar{A}}, p^B, -p^{\bar{C}}, p^D, -\lambda_{\bar{A}}', \lambda_B', -\lambda_{\bar{C}}', \lambda_D') \quad (40')$$

where $T(p^A p^B p^C p^D, \lambda_A \lambda_B \lambda_C \lambda_D)$ has been continued analytically in p^A and p^C , the energies p_0^A, p_0^C retaining always a non-negative imaginary part. For a straightforward derivation of (40') and (40), using the reduction technique of local field theory, see B.E.Y. Svensson, Ref. ³⁰).

- 30) B.E.Y. Svensson, Time-reversal invariance and crossing relations, in particular for helicity amplitudes, University of Lund preprint, March (1966), to be published in Arkiv för Fysik.
- 31) A. Kotanski, Jagellonian University, Cracow, preprints TPJU-21/65 [1965], to appear in Acta Physica Polonica, May (1966) and TPJU-4/66 (1966). The normal to the scattering plane is defined for example in the c.m. system; it remains unchanged when going over to any other reference system moving with respect to the c.m. system in a direction lying in the scattering plane, e.g., when going over to the rest system of an incoming or outgoing particle.
- 32) M. Jacob and G.C. Wick, Ann.Phys. 7, 404 (1959).
- 33) A. Białas and B.E.Y. Svensson, Nuovo Cimento 42A, 672 (1966).
- 34) For unpolarized B, the cross-section and polarization formulae are then

$$\frac{d\sigma}{dt} = \frac{4\pi^3}{k^2 s} (|g|^2 + |h|^2), \quad \sigma_T = \frac{8\pi^2}{ks^{\frac{1}{2}}} \text{Im } g(s, 0),$$

$$P \equiv \langle \chi_D | \frac{\sigma \cdot (\underline{k}_A \times \underline{k}_C)}{|\underline{k}_A \times \underline{k}_C|} | \chi_D \rangle = \frac{2 \text{Im}(gh^*)}{|g|^2 + |h|^2}$$

- 35) L. Van Hove, Phys.Letters 5, 252 (1963).
- 36) A.A. Logunov, Nguyen Van Hieu, I.T. Todorov and O.A. Khrustalev, Phys.Letters 7, 69 (1963).
- 37) A. Białas and E. Białas, Nuovo Cimento 37, 1686 (1965).
- 38) L. Van Hove, High-Energy Physics and Elementary Particles, International Atomic Energy Agency, Vienna (1965), p. 179.
- 39) L. Van Hove, 1965 Tokyo Summer Lectures in Theoretical Physics, Syokabo, Tokyo and Benjamin, New York (1966). Part II, p. 17.

- 40) They have over the generalized helicity amplitudes the advantage that the momenta enter only through the relativistic invariants s, t (see footnote ²⁹). The normal to the scattering plane is an invariant concept (see footnote ³¹) and can be kept the same when crossing from the s to the u and t channels.
- 41) The correct treatment of the limit $s \rightarrow +\infty$, t fixed when $m_A \neq m_C$ and $m_B \neq m_D$ is much more complicated, although the main results may be the same. See M.L. Goldberger and C.E. Jones, Princeton Univ. preprint (1966) and Phys.Rev.Letters 17, 105 (1966).
- 42) T. Regge, Nuovo Cimento 14, 951 (1959) and 18, 947 (1960) ;
A. Bottino, A.M. Longoni and T. Regge, Nuovo Cimento 23, 954 (1962).
- 43) The continuations of ζ_ℓ^+ and ζ_ℓ^- will in general be different functions of ℓ . For details, see, e.g., E.J. Squires, Complex angular momenta and particle physics, W.A. Benjamin, New York and Amsterdam (1963) ;
S.C. Frautschi, Regge poles and S matrix theory, W.A. Benjamin, New York and Amsterdam (1963) ;
R. Omnès and M. Froissart, Mandelstam theory and Regge poles, W.A. Benjamin, New York and Amsterdam (1963).
- 44) For its history see S.C. Frautschi, Ref. ⁴³), p. 107.
- 45) E.J. Squires, Ref. ⁴³), p. 4 and fol. .
- 46) D. Amati, S. Fubini and A. Stanghellini, Phys.Letters 1, 29 (1962).
- 47) S. Mandelstam, Nuovo Cimento 30, 1127 and 1148 (1963).
- 48) See, e.g., the Addendum II in S.C. Frautschi, Ref. ⁴³), p. 185.
- 49) This behaviour is usually assumed to be a reasonable relativistic generalization of the behaviour established by Regge to hold for the non-relativistic Schrödinger equation with a superposition of Yukawa potentials (Ref. ⁴²)).
- 50) G.F. Chew and S.C. Frautschi, Phys.Rev.Letters 8, 41 (1962) ;
see also, S.C. Frautschi, M. Gell-Mann and F. Zachariasen, Phys.Rev. 126, 2204 (1962).

- 51) V.N. Gribov and I.Ia. Pomeranchuk, Phys.Rev.Letters 8, 343 and 412 (1962).
- 52) For a more detailed historical survey, see S.C. Frautschi, Ref. 42), p. 144.
- 53) That $Z_J^\pm$ has a complete set of eigenvectors and eigenvalues follows from unitarity of the S matrix in the t channel. The factorization property (72) was first recognized by M. Gell-Mann, Phys.Rev.Letters 8, 263 (1962) and V.N. Gribov and I.Ia. Pomeranchuk, Ref. 51).
- 54) Considering as in Eq. (64) the equal mass case, one notes that the signature is related to the space parity in the c.m. system of the t channel, because the replacement $\cos \theta_t \rightarrow -\cos \theta_t$ corresponds to inversion of the space co-ordinates of the incident particles A and $\bar{C}$ in that system. This is particularly simple to see on the example $A = \pi^+$, $\bar{C} = \pi^-$. Positive signature means even parity and $J = 0, 2, \dots$, negative signature giving odd parity and $J = 1, 3, \dots$.
- 55) R. Blankenbecler and M.L. Goldberger, Phys.Rev. 126, 766 (1962).
- 56) This generalized Hamiltonian is very simple to write down in non-relativistic Schrödinger theory with a central potential. One takes the radial wave equation and introduces a general value of J in the centrifugal potential term $J(J+1)r^{-2}$.
- 57) See Appendix A of T. Kinoshita, CERN Report 62-33 (1962),
- 58) V.N. Gribov, Proceedings of the 1962 International Conference on High-Energy Physics at CERN, Ed. by J. Prentki [CERN, Geneva (1962)], p. 547.
- 59) The recent discussion by Goldberger and Jones, Ref. 41), is carried out for spinless particles.
- 60) R.J.N. Phillips and W. Rarita, Phys.Rev. 139, B1336 (1965).

- 61) G. Höhler, J. Baacke, H. Schlaile and P. Sonderegger, Phys. Letters 20, 79 (1966).
- 62) F. Arbab and C.B. Chiu, LRL preprint, Berkeley (1966) ;
L.L. Wang, Phys.Rev.Letters 16, 756 (1966) ;
G.F. Chew, Contribution to the Stony Brook Conference on High-Energy Two-Body Reactions, State University, Stony Brook, N.Y., April (1966).
- 63) Since the corresponding value of ℓ is integral, J' is an integer (half-integer) for boson (fermion) exchange in the t channel.
- 64) See E.J. Squires, Ref. ⁴³⁾, Section 4.1, and also S.C. Frautschi, M. Gell-Mann and F. Zachariasen, Ref. ⁵⁰⁾ ;
A.O. Barut and D.E. Zwanziger, Phys.Rev. 127, 974 (1962).
- 65) A more detailed review and references will be found in the various contributions to the Stony Brook Conference on High-Energy Two-Body Reactions, State University, Stony Brook, N.Y., April (1966).
- 66) R.J.N. Phillips and W. Rarita, Ref. ⁶⁰⁾ ;
V. Barger and M. Olsson, Phys.Rev.Letters 16, 545 and 952 (1966), and Phys.Rev. (to be published).
- 67) R.J.N. Phillips and W. Rarita, Phys.Rev. 140, B200 (1965), and erratum to be published.
- 68) P. Astbury, G. Brautti, G. Finocchiaro, A. Michelini, D. Websdale, C.H. West, E. Polgar, W. Beusch, W.E. Fischer, B. Gobbi and M. Pepin, to be published in Phys.Letters.
- 69) H. Högaasen and W. Fischer, CERN preprint TH. 672, June (1966) and Phys.Letters.
- 70) J.L. Friedes, private communication.
- 71) N. Byers, UCLA preprint, June (1966).
- 72) N. Byers and C.N. Yang, Phys.Rev. 142, B976 (1966).

- 73) L. Van Hove, Physikertagung Stuttgart (1962) [Mosbach-Baden Physik-Verlag (1963)] p. 84, Nuovo Cimento 28, 798 (1963), Revs.Modern Phys. 36, 655 (1964), Strong, electromagnetic and weak interactions, Ed. by A. Zichichi [W.A. Benjamin, New York and Amsterdam (1964)] p. 84.
- 74) This remains true when T_{el} has a non-negligible real part if the phase of T_{el} can be determined on independent theoretical grounds, e.g., from the asymptotic expansion (26) and the phase rule (46) as discussed in Chapter II.
- 75) Thus, the Regge pole model makes only very few predictions on the t dependence of the overlap functions.
- 76) A. Bia~~l~~as, Nuovo Cimento 33, 972 (1964).
- 77) Such inequalities are more readily compared with experiment for total cross-sections σ_T , but the smallness of σ_{el}/σ_T and the similarity of shape of all diffraction peaks (they differ mainly in width and height, not so much in shape) implies that σ_T will obey the same inequalities as σ_{in} .
- 78) The analogous property for two-body processes is that the leading term of the expansion (26) for elastic scattering has positive signature, which implies that the corresponding value of C exchanged in the t channel is $+1$.
- 79) See the two last papers quoted in Ref. ⁷³⁾ and W.N. Cottingham and R.F. Peierls, Phys.Rev. 137, B147 (1965), where actual πp and pp data are fitted.
- 80) L. Van Hove, Revs.Modern Phys. 36, 655 (1964).
- 81) J.J.J. Kokkedee, Nuovo Cimento 43A, 919 (1966).

- 82) K. Johnson and S.B. Treiman, Phys.Rev.Letters 14, 189 (1965).
- 83) H. Harari and H.J. Lipkin, Phys.Rev.Letters 13, 208 (1964).
- 84) M. Gell-Mann, Phys.Rev. 125, 1067 (1962) and Physics 1, 63 (1964).
- 85) M. Gell-Mann, Phys.Letters 8, 214 (1964) ;
G. Zweig, CERN preprints TH.401 and TH.402 (1964), unpublished.
- 86) S.L. Adler, Phys.Rev.Letters 14, 1051 (1965) and Phys.Rev. 140,
B736 (1965) ;
W.I. Weisberger, Phys.Rev.Letters 14, 1047 (1965).
- 87) W.E. Thirring, Phys.Letters 16, 335 (1965) ; Acta Phys.Austr.
Suppl.II, 205 (1966) and Suppl.III, to be published ;
C. Becchi and G. Morpurgo, Phys.Letters 17, 352 (1965) and
Phys.Rev. 140, B687 (1965).
- 88) Another method to derive some of the consequences of the quark
model without assuming quarks is provided by the concept
of universality, which is an assumption that all relevant
currents or couplings are dominated by the known mesons.
This has been studied in detail by Freund. For application
of universality to high energy scattering, see P.G.O. Freund,
Phys.Rev.Letters 15, 929 (1965) ; Nuovo Cimento 43A, 1171 (1966).
- 89) N. Cabibbo, L. Horwitz and Y. Ne'eman, CERN preprint TH.680 (1966)
and N. Cabibbo, L. Horwitz, J.J.J. Kokkedee and Y. Ne'eman,
CERN preprint TH.685 (1965).
- 90) It should be said that the model of Cabibbo et al. does not
reproduce Eq. (105) when SU_3 breaking is taken into
account. The asymptotic ratio $\sigma_T(\pi p)/\sigma_T(pp)$ can then
take any value depending on the F/D ratio for the baryon
coupling to the even signature octet, which is a free para-
meter ; see the second paper of Ref. ⁸⁹⁾. The situation in
the quark model is different because πp and pp scattering,
which involves only the $I=\frac{1}{2}$ quarks, is not influenced by
 SU_3 breaking which first appears when the $I=0$ quark is
also involved.

- 91) E.M. Levin and L.L. Frankfurt, Zhur.Eksp.i Teoret.Fiz. Pisma v Redak. 2, 105 (1965) - [English translation : Soviet Phys. - JETP Letters 2, 65 (1965)] ;
 H.J. Lipkin and F. Scheck, Phys.Rev. Letters 16, 71 (1966) and many later papers by H.J. Lipkin and collaborators ;
 J.J.J. Kokkedee and L. Van Hove, Nuovo Cimento 42, 711 (1966) ;
 J.J.J. Kokkedee, Phys.Letters 22, 88 (1966).
- 92) J.J.J. Kokkedee and L. Van Hove, Ref. ⁹¹⁾ ;
 L. Van Hove, Proceedings of the Stony Brook Conference on High Energy Two-Body Reactions, State University, Stony Brook, N.Y. (April 1966).
- 93) To be more precise one should say here the i^{th} quark or anti-quark, but we shall usually omit the explicit distinction between quark and antiquark. For mesons the index i runs over one quark and one antiquark, for baryons over three quarks.
- 94) The separation between spin and space co-ordinates of quarks is assumed possible as is the case when the quark states in hadrons are non-relativistic and Δp is not too large. The non-relativistic assumption is desirable anyway if the quark model is to reproduce the SU_6 multiplet structure of hadrons, especially their spin values.
- 95) N.N. Bogoliubov et al., Dubna preprint D-1968 (1965).
- 96) One can speculate that the fast oscillations would be due to the strong binding force to which quarks are subjected in the hadrons. Also charge and baryon number can oscillate, so that their fractional values $\pm\frac{1}{2}, \pm\frac{2}{3}$ could be averages over integral values.
- 97) C. Itzykson and M. Jacob, Saclay preprint (July 1966).
- 98) Many other cases of differential cross-sections, including spin phenomena, are studied by Itzykson and Jacob, Ref. ⁹⁷⁾.

- 99) This is the generalized Pomerenchuk theorem for energy-independent diffraction scattering ; see L. Van Hove, Phys.Letters 5, 252 (1963), or in Chapter II of these lectures, the properties of the leading term of the asymptotic expansion (26) for elastic scattering, especially the fact that it has positive signature.
- 100) To separate the form factors f from the scattering amplitudes g when strange particles are involved one relies on the fact that $f_i^{AA}(t)$ is independent of i . This is obvious if A is a meson because it contains two quarks only, and hence one relative momentum. It is valid for hyperons to the degree that their wave function is symmetric in the three quarks.
- 101) We notice that at very high energy, t being finite, the matrix elements $\langle A'B' | T | AB \rangle$, $\langle Q_i^{A'} Q_j^{B'} | T_{ij} | Q_i^A Q_j^B \rangle$ used in Eqs. (112) and (113) are the same for all co-ordinate systems in which the incident momenta $\underline{p}$, $\underline{p}'$ are collinear, have directions opposite to each other and are both very large in absolute value. This is seen from Eq. (10) by remarking that in these systems one has

$$(p_0^A p_0^B p_0^C p_0^D)^{\frac{1}{2}} \simeq \frac{1}{2}(p_0 p'_0 - \underline{p} \cdot \underline{p}') = \text{invariant}$$

where p_0 , p'_0 are the incident energies. The c.m. system of the $A+B$ collision is such a system both for the $A+B$ collision and for the resulting Q_i+Q_j collisions. The cross-sections for the latter can therefore be calculated by direct application of Eqs. (10), (12) and (13), the matrix elements (10) of $T=i(1-S)$ being the ones used throughout in Chapter IV.

- 102) If one quark would be point-like and the other one spread out, the Born approximation would give $g(t)=g(0)F(t)$. The additional factor $F(t)$ in Eq. (156) accounts for the fact that the two quarks are spread out.
- 103) We do not consider Δp_μ to become large enough for ejection of free quarks, although this could be allowed to happen if the latter had integral charge and baryon number ⁹⁶).