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TASI lectures on String Compactification, Model Building, and Fluxes

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We review the construction of chiral four-dimensional compactifications of string theory with different systems of D-branes, including type IIA intersecting D6-branes and type IIB magnetised D-branes. Such models lead to four-dimensional theories with non-abelian gauge interactions and charged chiral fermions. We discuss the application of these techniques to building of models with spectrum as close as possible to the Standard Model, and review their main phenomenological properties. We finally describe how to implement the tecniques to construct these models in flux compactifications, leading to models with realistic gauge sectors, moduli stabilization and supersymmetry breaking soft terms.

Lecture 1. Model building in IIA: Intersecting brane worlds

1 Introduction

String theory has the remarkable property that it provides a description of gauge and gravitational interactions in a unified framework consistently at the quantum level. It is this general feature (beyond other beautiful properties of *particular* string models) that makes this theory interesting as a possible candidate to unify our description of the different particles and interactions in Nature.

Now if string theory is indeed realized in Nature, it should be able to lead not just to 'gauge interactions' in general, but rather to gauge sectors as rich and intricate as the gauge theory we know as the Standard Model of Particle Physics. In these lecture we describe compactifications of string theory where sets of D-branes lead to gauge sectors close to the Standard Model. We furthermore discuss the interplay of such D-brane systems with flux compactifications, recently introduced to address the issues of moduli stabilization and supersymmetry breaking.

Before starting, it is important to emphasize that there are other constructions in string theory which are candidates to reproduce the physics of the Standard Model at low energies, which do not involve D-branes. For instance, compactifications of heterotic string on Calabi-Yau threefolds, M-theory compactifications on G_2-holonomy spaces, etc. We emphasize D-brane models because of their simplicity, and also because they are often related to these other compactifications via string dualities. Hence, they provide a simple introduction from which the interested reader may jump onto the big picture.

This first lecture introduces D-branes and their properties, and deals with model building using intersecting D-branes. Useful reviews for this lecture are for example [1].

These lectures are organized as follows. In section 2 we quickly review properties of D-branes and their world-volume dynamics. In section 3 we describe that configurations of intersecting D6-branes naturally lead to four-dimensional chiral fermions, and discuss their spectrum and supersymmetry. In section 4 we construct compactifications of type IIA string theory to four dimensions, including configurations of intersecting D6-branes. We provide explicit descriptions of toroidal compactifications of this kind, and generalizations to more general Calabi-Yau compactifications. In section 5 we introduce

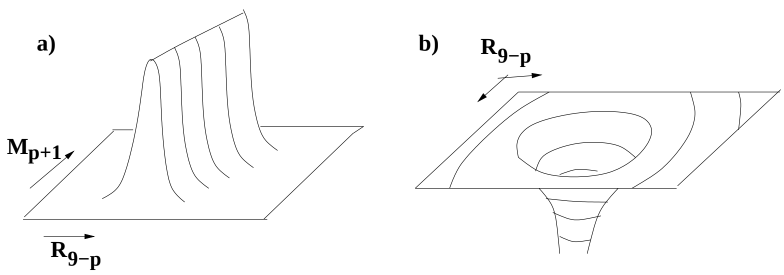

Figure 1: Two pictures of the p-brane as a lump of energy. The second picture shows only the transverse directions, where the p-brane looks like point-like.

further ingredients to improve these models, namely orientifold 6-planes. We describe their properties, discuss configurations of D6-branes and O6-planes, and describe how to include them in compactifications in section 5.3. These techniques are exploited in section 6 to construct models whose chiral spectrum is that of the standard model, and in section 7 to describe supersymmetric chiral compactifications with intersecting branes. Appendix A provides some details on the computation of open string spectra for parallel and intersecting D-branes.

2 Overview of D-branes

2.1 Properties of D-branes

The study of string theory beyond perturbation theory has led to the introduction of new objects in string theory, D-branes. For a complementary description of D-branes and their properties see [2, 3].

Type II string theories contains certain ‘soliton-like’ states in their spectrum, with $p + 1$ extended dimensions, the p-branes. They were originally found as solutions of the low-energy supergravity equations of motion. This is schematically shown in figure 1. Subsequently, it was realized [4] that certain of these objects (known as Dp-branes) admit a fully stringy description, as $(p + 1)$-dimensional subspaces on which open strings can end. Notice that these open strings are not present in the vacuum of the underlying string theory, but rather represent the fluctuations of the theory around the topological defect background. Namely, the closed string sector still describes the dynamics of the vacuum (gravitational interactions, etc), while open strings rather describe the dynamics of the object. The situation is shown in figure 2

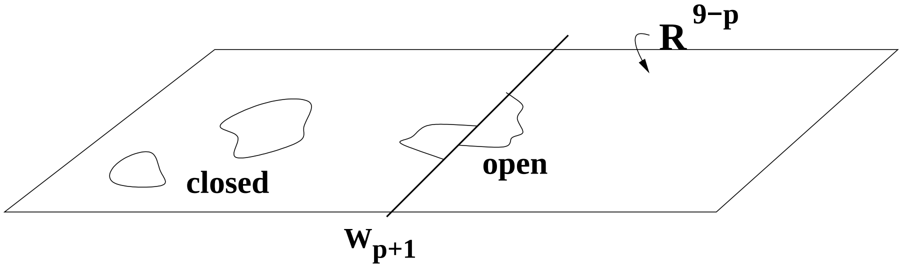

Figure 2: String theory in the presence of a Dp-brane. The closed string sector describes the fluctuations of the theory around the vacuum (gravitons, dilaton modes, etc), while the sector of open strings describes the spectrum of fluctuations of the soliton.

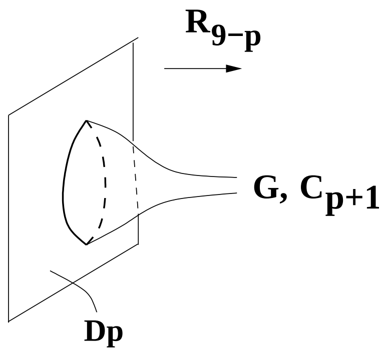

Figure 3: Disk diagram describing the interaction of a Dp brane with closed string modes.

The basic properties of Dp-branes for our purposes in these lecture are:

- Dp-branes are dynamical, and for instance have non-trivial interactions with closed string modes. Due to these couplings, they carry tension (they interact with the 10d graviton) and charge under a RR $(p+1)$-form potential C_{p+1}, see figure 3. Hence, type IIA (resp. IIB) string theory contains Dp-branes with p even (resp. odd).

- A flat Dp-brane in flat spacetime preserves half the supersymmetries of the theory. Denoting $Q_{\rm L}$, $Q_{\rm R}$ the two 16-component spinor supercharges of type II string theories, arising from the left- or right-moving world-sheet degrees of freedom, a Dp-brane with world-volume spanning the directions $012\ldots p$ preserves the linear combination

$$Q = \epsilon_{\rm R} Q_{\rm R} + \epsilon_{\rm L} Q_{\rm L} \tag{2.1}$$

where $\epsilon_{\rm L,R}$ are spinor coefficients satisfying

$$\epsilon_{\rm L} = \Gamma^{01\ldots p}\epsilon_{\rm R} \tag{2.2}$$

Thus Dp-branes are BPS states, and their charge and tension are equal.

- Dp-branes may have curved world-volumes. However, they tend to minimize the volume of the submanifold they span, hence in flat space Dp-branes tend to span flat world-volumes. In curved spaces, arising e.g. in compactifications, they may however wrap curved non-trivial homology cycles.

- As mentioned already, open string modes in the presence of D-branes are localized on the world-volume of the latter. This implies that such open strings represent the collective coordinates of the non-perturbative object, and thus their dynamics controls the dynamics of the object. In next section we will center on the zero modes, corresponding to the massless open string sector.

2.2 World-volume fields

The spectrum of fluctuations of the theory in the presence of the Dp-brane is obtained by quantizing closed strings and open strings ending on the Dp-brane. Since the open string endpoints are fixed on the D-brane, the massless modes in the latter sector yield fields propagating on the $(p+1)$-dimensional D-brane world-volume W_{p+1}.

A simplified calculation of the quantization of open strings for a configuration of a single type II Dp-brane in flat 10d is carried out in appendix A.1. The resulting set of massless particles on the Dp-brane world-volume is given by a $U(1)$ gauge boson, $9-p$ real scalars and some fermions (transforming under Lorentz as the decomposition of the 8_C of $SO(8)$ under the $(p+1)$-dimensional little group $SO(p-1)$). The scalars (resp. fermions) can be regarded as Goldstone bosons (resp. Goldstinos) of the translational symmetries (resp. supersymmetries) of the vacuum broken by the presence of the D-brane. The open string sector fills out a $U(1)$ vector multiplet with respect to the 16 supersymmetries unbroken by the D-brane.

As mentioned above, Dp-branes are charged under the corresponding RR $(p+1)$-form C_{p+1} of type II string theory, via the minimal coupling $\int_{W_{p+1}} C_{p+1}$. Since flat Dp-branes in flat space preserve 1/2 of the 32 supercharges of the type II vacuum, such D-branes are BPS states, and their RR charge is related to their tension. This implies that there is no net force among parallel branes (roughly, gravitational attraction cancels against 'Coulomb' repulsion due to their RR charge). Hence one can consider

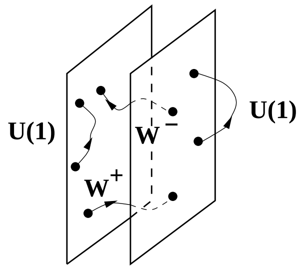

Figure 4: Non-abelian gauge bosons in a configuration of coincident D-branes.

dynamically stable configurations of several parallel Dp-branes, labeled by a so-called Chan-Paton index a, at locations x_a^i in the transverse coordinates, $i = p+1, \ldots, 9$.

We would like to consider the situation with n coincident Dp-branes, located at the same position in transverse space. In such situation there are n^2 open string sectors, labeled ab for an open string starting at the a^{th} D-brane and ending at the b^{th} D-brane. The computation for each sector ab is similar to the single brane case. Hence the spectrum of physical states contains, at the massless level, n^2 gauge bosons, $n^2 \times (9-p)$ scalars, and n^2 sets of $(p+1)$-dimensional fermions (in representations obtained from decomposing the $\mathbf{8}_{\mathrm{C}}$ of $SO(8)$).

This multiplicity renders interactions between open strings non-abelian. It is possible to see that the gauge bosons in the aa sector correspond to a $U(1)^n$ gauge symmetry, and that states in the ab sector have charges $+1$ and -1 under the a^{th} and b^{th} $U(1)$, respectively. This enhances the gauge symmetry to $U(n)$, and makes the different fields transform in the adjoint representation. The complete massless open string spectrum is given by $U(n)$ gauge bosons, $9-p$ adjoint scalars and adjoint fermions, filling out a $U(n)$ vector multiplet with respect to the 16 unbroken supersymmetries. The structure of gauge bosons for $n = 2$ is shown in figure 4.

D-branes provide a nice and simple realization of non-abelian gauge symmetry in string theory. The low-energy effective action for the massless open string modes has several pieces. One of them is the Dirac-Born-Infeld action, which has the form

$$S_{\mathrm{DBI}} = -T_p \int_{W_{p+1}} \mathrm{d}^{\mathrm{p+1}}\mathrm{x}^{\mu}\ \left[-\det(\mathrm{G} + \mathrm{B} + 2\pi\alpha'\mathrm{F})\right]^{1/2} \tag{2.3}$$

where T_p is the Dp-brane tension, and $G_{\mu\nu} = \partial_\mu \phi^i \partial_\nu \phi^j\, G_{ij}$ is the metric induced on the D-brane worldvolume, and similarly $B_{\mu\nu}$ is the induced 2-form. These terms introduce the dependence of the action on the world-volume scalars $\phi^i(x^\mu)$. Finally $F_{\mu\nu}$ is the field strength of the worldvolume gauge field.

Neglecting the dependence on the field strength, it reduces to the D-brane tension times the D-brane volume $\int(\det G)^{1/2})$. At low energies, i.e. neglecting the α' corrections, it reduces to a kinetic term for the scalars plus the $(p+1)$-dimensional Yang-Mills action for the worldvolume gauge fields, with gauge coupling given by $g_{U(n)}^2 = g_s$. Of course the above action should include superpartner fermions, etc, but we skip their discussion.

A second piece of the effective action is the Wess-Zumino terms, of the form

$$S_{\rm WZ} = -Q_p \int_{W_{p+1}} \mathcal{C} \wedge {\rm ch}(F)\, \hat{A}(R) \tag{2.4}$$

where $\mathcal{C} = C_{p+1} + C_{p-1} + C_{p-3} + \ldots$ is a formal sum of the RR forms of the theory, and ${\rm ch}(F)$ is the Chern character of the worldvolume gauge bundle on the D-brane volume

$${\rm ch}(F) = \exp(\frac{F}{2\pi}) = 1 + \frac{1}{2\pi}{\rm tr}\, F + \frac{1}{8\pi^2}{\rm tr}\, F^2 + \ldots \tag{2.5}$$

and $\hat{A}(R)$ is the A-roof genus, characterizing the tangent bundle of the D-brane worldvolume $\hat{A}(R) = 1 - {\rm tr}\, R^2/(2\pi^2) + \ldots$. Integration is implicitly defined to pick up the degree $(p+1)$ pieces in the formal expansion in wedge products. Hence we get terms like

$$\begin{aligned} S_{\rm WZ} = -Q_p \Bigg(& \int_{W_{p+1}} C_{p+1} + \frac{1}{2\pi} \int_{W_{p+1}} C_{p-1} \wedge {\rm tr}\, F \\ & + \frac{1}{8\pi^2} \int_{W_{p+1}} C_{p-3} \wedge ({\rm tr}\, F^2 - {\rm tr}\, R^2) + \ldots \Bigg) \end{aligned} \tag{2.6}$$

A very important property of this term is that it is topological, independent of the metric or on the particular field representatives in a given topological sector. This is related to the fact that these terms carry the information about the RR charges of the D-brane configuration.

2.3 Chirality and D-branes

We have obtained simple configurations of D-branes leading to non-abelian gauge symmetries on their world-volume. It is interesting to wonder if such configurations could be exploited to reproduce the gauge sector describing high energy particle physics, so as to embed it into a string theory model. Clearly, the main obstruction is that the standard model of particle physics is chiral in four dimensions. This property is incompatible with the large amount of supersymmetry preserved by the D-brane configurations considered.

There is an alternative heuristic way to intuitively understand the lack of chirality in our D-brane configuration. Four-dimensional chirality is a violation of four-dimensional parity. In the spectrum of open strings there is a correlation (implied by the GSO projection) between the 4d chirality and the chirality in the six extra dimensions. Hence to achieve 4d parity violation the configuration must violate 6d parity. However, the above configurations of D-branes do not violate 6d parity, do not introduce a preferred six-dimensional orientation.

The latter remark indeed suggest how to proceed to construct configurations of D-branes leading to four-dimensional chiral fermions. The requirement is that the configuration introduces a preferred orientation in the six transverse dimensions. There are several ways to achieve this, as we discuss now.

• D-branes sitting at *singular* (rather than smooth) points in transverse space can lead to chiral open string spectra. The prototypical example is given by stacks of D3-branes sitting at the singular point of orbifolds of flat space, e.g. orbifold singularities $\mathbf{C^3}/\mathbf{Z_N}$, as studied in [34]. A particularly simple and interesting case is the $\mathbf{C^3}/\mathbf{Z_3}$ orbifold, which will be studied in our second lecture. The key idea is that the discrete rotation implied by the $\mathbf{Z_3}$ action defined a preferred orientation in the 6d space, and allows for chirality on the D-branes.

• Consider a stack of D9-branes in flat 10d spacetime, split as $M_4 \times \mathbf{R^2} \times \mathbf{R^2} \times \mathbf{R^2}$. For simplicity we ignore for the moment the issue of RR tadpole cancellation. Otherwise, to make the configuration consistent it suffices to introduce orientifold 9-planes, namely consider the configuration in type I string theory. Now introduce non-trivial field strength background for the world-volume $U(1)_a$ gauge fields, F_a^i in the $i^{\rm th}$ $\mathbf{R^2}$, with $i = 1, 2, 3$, (see [5, 6] for early discussions, and [7, 8, 9] for more recent ones). The magnetic fields introduce a preferred orientation in the transverse six dimensions (obtained by using $F \wedge F \wedge F$ as the volume form, where F is the 2-form associated to the field strength). Hence the configurations lead naturally to 4d chiral fermions, as we describe in our second lecture.

• Sets of intersecting D-branes can also lead to chiral fermions in the sector of open strings stretched between different kinds of D-brane [29], and are the topic of our lecture today.

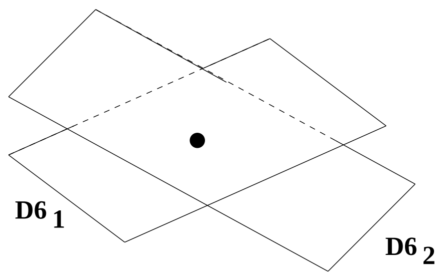

Figure 5: Picture of D6-branes intersecting over a 4d subspace of their volumes.

3 Intersecting D6-branes

3.1 Local geometry and spectrum

The basic configuration of intersecting D-branes leading to chiral 4d fermions at their intersection is two stacks of D6-branes in flat 10d intersecting over a 4d subspace of their volumes. Consider flat 10d space $M_4 \times \mathbf{R^2} \times \mathbf{R^2} \times \mathbf{R^2}$, and two stacks of D6-branes, spanning M_4 times a line in each of the three 2-planes. Figures 5, 6 provide two pictorial representations of the configurations. The local geometry is fully specified by the three angles θ_i which define the rotation between the two stacks of D6-branes. As we discuss below, the chiral fermions are localized at the intersection of the brane volumes.

The appearance of chirality can be understood from the fact that the geometry of the two D-brane introduces a preferred orientation in the transverse 6d space, namely by considering the relative rotation of the second D6-brane with respect to the first. This also explains why one should choose configurations of D6-branes. For example, two sets of D5-branes intersecting over 4d do not lead to 4d chiral fermions, since they do not have enough dimensions to define an orientation in the transverse 6d space.

A more detailed computation of the spectrum of open string models on systems of intersecting branes is provided in appendix A.2. Here it will suffice to mention the results of the spectrum for this configuration. The open string spectrum in a configuration of two stacks of n_1 and n_2 coincident D6-branes in flat 10d intersecting over a 4d subspace of their volumes consists of three open string sectors:

$\mathbf{6}_1\mathbf{6}_1$ Strings stretching between D6$_1$-branes provide $U(n_1)$ gauge bosons, three real adjoint scalars and fermion superpartners, propagating over the 7d world-volume of the D6$_1$-branes.

$\mathbf{6}_2\mathbf{6}_2$ Similarly, strings stretching between D6$_2$-branes provide $U(n_2)$ gauge bosons,

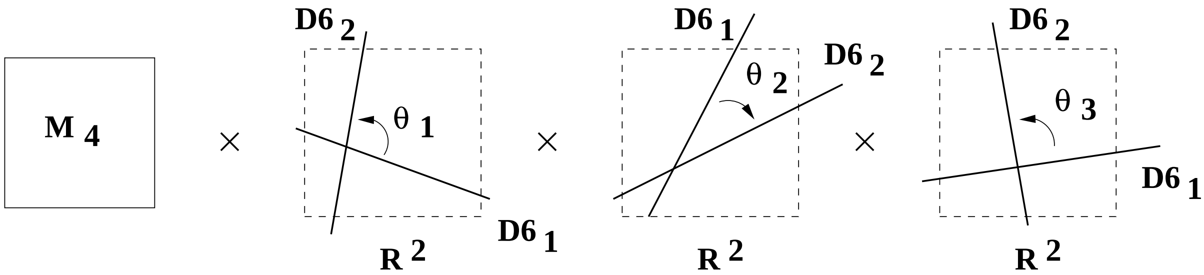

Figure 6: A more concrete picture of the configuration of two D6-branes intersecting over a 4d subspace of their volumes.

three real adjoint scalars and fermion superpartners, propagating over the $D6_2$-brane 7d world-volume.

$\mathbf{6}_1\mathbf{6}_2 + \mathbf{6}_2\mathbf{6}_1$ Strings stretching between both kinds of D6-brane lead to a 4d chiral fermion, transforming in the representation $(n_1, \overline{n}_2)$ of $U(n_1) \times U(n_2)$, and localized at the intersection. The chirality of the fermion is encoded in the orientation defined by the intersection; this will be implicitly taken into account in our discussion.

So we have succeeded in constructing a configuration of D-branes leading to 4d chiral fermions in the open string sector. Again, let us emphasize that the appearance of chiral fermions in the present system is the angles between the branes (technically, leading to the reduction of the Clifford algebra of fermion zero modes in the open strings between branes). Notice that the 4d chiral fermions lead to a localized anomaly at the intersection of the D6-branes. This anomaly is however canceled by the anomaly inflow mechanism, see [30].

In addition to the chiral fermions at intersections, there are several potentially light complex scalars at the intersection, transforming in bifundamental representations, and with masses (in α' units) given by

$$\begin{array}{cc} \tfrac{1}{2\pi}(-\theta_1+\theta_2+\theta_3) & \frac{1}{2\pi}(\theta_1-\theta_2+\theta_3) \\ \tfrac{1}{2\pi}(\theta_1+\theta_2-\theta_3) & 1-\frac{1}{2\pi}(-\theta_1-\theta_2-\theta_3) \end{array} \tag{3.1}$$

These scalars, as we further discuss in section 3.2), can be massless, massive or tachyonic.

3.2 Supersymmetry for intersecting D6-branes

It is interesting to consider if the above configurations preserve some supersymmetry. This can be analyzed following [29]. The condition that there is some supersymmetry preserved by the combined system of two D6-brane stacks is that there exist spinors $\epsilon_{\rm L}$, $\epsilon_{\rm R}$ that satisfy

$$\begin{aligned} \epsilon_{\rm L} = \Gamma_6 \epsilon_{\rm R} \quad &; \qquad \Gamma_6 = \Gamma^0 \ldots \Gamma^3 \Gamma^4 \Gamma^6 \Gamma^8 \\ \epsilon_{\rm L} = \Gamma_{6'} \epsilon_{\rm R} \quad &; \qquad \Gamma_{6'} = \Gamma^0 \ldots \Gamma^3 \Gamma^{4'} \Gamma^{6'} \Gamma^{8'} \end{aligned} \tag{3.2}$$

where 468 and $4'6'8'$ denote the directions along the two D6-branes in the six dimensions 456789. The above is simply the condition (2.2) for each of the branes.

Let R denote the $SO(6)$ rotation that takes the first D6-brane into the second, acting on the spinor representation. Then we have $\Gamma_{6'} = R\Gamma_6 R^{-1}$. A preserved spinor exists if and only if there is a 6d spinor which is invariant under R. This implies that R must belong to an $SU(3)$ subgroup of $SO(6)$. This can be more explicitly stated by rewritting R in the vector representation as

$$R = {\rm diag}\,({\rm e}^{{\rm i}\theta_1}, {\rm e}^{-{\rm i}\theta_1}, {\rm e}^{{\rm i}\theta_2}, {\rm e}^{-{\rm i}\theta_2}, {\rm e}^{{\rm i}\theta_3}, {\rm e}^{-{\rm i}\theta_3}) \tag{3.3}$$

The condition that the rotation is within $SU(3)$ is

$$\theta_1 \pm \theta_2 \pm \theta_3 = 0 \quad {\rm mod} \quad 2\pi \qquad \text{for some choice of signs} \tag{3.4}$$

Indeed, one can check that the open string spectrum computed above is boson-fermion degenerate in such cases. In the generic case, there is no supersymmetry invariant under the two stacks of branes, and the open string sector at the intersection is non-supersymmetric. However, if $\theta_1 \pm \theta_2 \pm \theta_3 = 0$ for some choice of signs, one of the scalars becomes massless, reflecting that the configuration is $\mathcal{N} = 1$ supersymmetric. $\mathcal{N} = 2$ supersymmetry arises if e.g. $\theta_3 = 0$ and $\theta_1 \pm \theta_2 = 0$, while $\mathcal{N} = 4$ arises only for parallel stacks $\theta_i = 0$.

As described above, the light scalars at intersections may be massless, massive or tachyonic. The massless case corresponds to a situation with some unbroken supersymmetry. The massless scalar is a modulus, whose vacuum expectation value (vev) parametrizes the possibility of recombining the two intersecting D-branes into a single smooth one, as pictorially shown in figure 7. That is, the intersecting geometry belongs to a one- (complex) parameter family of supersymmetry preserving configurations of D-branes. Mathematically, there is a one-parameter family of supersymmetric 3-cycles,

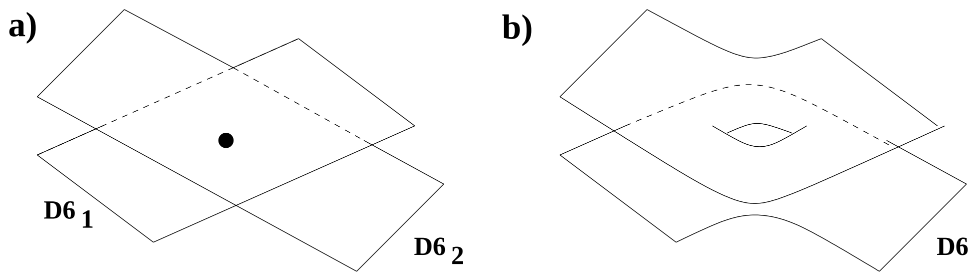

Figure 7: Recombination of two intersecting D6-branes into a single smooth one, corresponding to a vev for an scalar at the intersection.

i.e. special lagrangian submanifolds of $\mathbf{R^6}$, with the same asymptotic behaviour as the intersecting D-brane configuration. In the simpler situation of D-branes intersecting at $SU(2)$ angles (i.e. $\mathcal{N}=2$ supersymmetry), the recombination is very explicit. It is given by deforming two intersecting 2-planes, described by the complex curve $uv=0$, to the smooth 2-cycle $uv=\epsilon$, with ϵ corresponding to the vev of the scalar at the intersection.

The configuration with tachyonic scalars corresponds to situations where this recombination is triggered dynamically. Namely, the recombination process correspond to condensation of the tachyon at the intersection. It is interesting to point out that in the degenerated case where the intersecting brane system becomes a brane-antibrane system (e.g. $\theta_1=\theta_2=0$, $\theta_3=1$), the tachyons are mapped to the well-studied tachyon of brane-antibrane systems. The situation where all light scalars have positive squared masses correspond to a non-supersymmetric intersection, which is nevertheless dynamically stable against recombination. Namely, the recombined 3-cycle has volume larger than the sum of the volumes of the intersecting 3-cycles.

Indeed, the different regimes of dynamics of scalars at intersections have a one-to-one mapping with the different relations between the volumes of intersecting and recombined 3-cycles [10]. Namely, the conditions to have or not tachyons are related to the angle criterion [11] determining which the particular 3-cycle having smaller volume. The supersymmetric situation corresponds to both the intersecting and recombined configurations having the same volume; the tachyonic situation corresponds to the recombined 3-cycle having smaller volume; the massive situation corresponds to the intersecting 3-cycle having smaller volume.

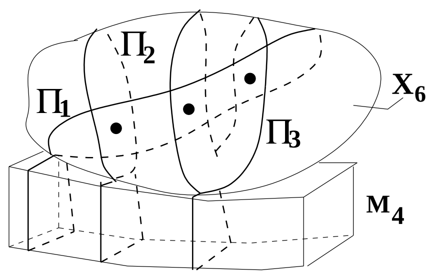

Figure 8: Compactification with intersecting D6-branes wrapped on 3-cycles.

4 Compact four-dimensional models

Once we have succeeded in describing configurations of D-branes leading to charged chiral fermions, in this section we employ them in building models with 4d gravity and gauge interactions. Although intersecting D6-branes provide 4d chiral fermions already in flat 10d space, gauge interactions remain 7d and gravity interactions remain 10d unless we consider compactification of spacetime.

The general kind of configurations we are to consider (see figure 8) is type IIA string theory on a spacetime of the form $\mathbf{M_4} \times \mathbf{X_6}$ with compact $\mathbf{X_6}$, and with stacks of N_a D6_a-branes with volumes of the form $M_4 \times \Pi_a$, with $\Pi_a \subset \mathbf{X_6}$ a 3-cycles. It is important to realize that generically 3-cycles in a 6d compact space intersect at points, so the corresponding wrapped D6-branes will intersect at M_4 subspaces of their volumes. Hence, compactification reduces the 10d and 7d gravitational and gauge interactions to 4d, and intersections lead to charged 4d chiral fermions. Also, generically two 3-cycles in a 6d space intersect several times, therefore leading to a replicated sector of opens strings at intersections. This is a natural mechanism to explain/reproduce the appearance of replicated families of chiral fermions in Nature!

4.1 Toroidal models

4.1.1 Construction

In this section we mainly follow [9], see also [8]. To start with the simplest configurations, consider compactifying on a six-torus factorized as $\mathbf{T^6} = \mathbf{T^2} \times \mathbf{T^2} \times \mathbf{T^2}$. Now we consider stacks of D6_a-branes (with a an index labeling the stack), spanning M_4 and wrapping a 1-cycle (n_a^i, m_a^i) in the i^{th} 2-torus. Namely, the a^{th} D6-brane wraps n_a^i, m_a^i times along the horizontal and vertical directions in the i^{th} two-torus, see figure 9 for

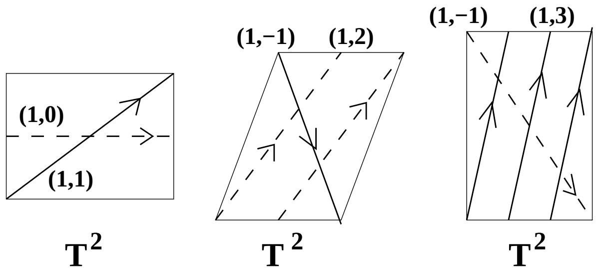

Figure 9: Examples of intersecting 3-cycles in $\mathbf{T^6}$.

examples [1].

The general kind of configurations we are to consider (see figure 8) is thus type IIA string theory on a spacetime of the form $\mathbf{M_4} \times \mathbf{T^6}$, and with stacks of N_a D6$_a$-branes with volumes of the form $M_4 \times \Pi_a$, with $\Pi_a \subset \mathbf{X_6}$ a 3-cycle as described above. It is important to realize that generically 3-cycles in a 6d compact space intersect at points, so the corresponding wrapped D6-branes will intersect at M_4 subspaces of their volumes. Hence, compactification reduces the 10d and 7d gravitational and gauge interactions to 4d, and intersections lead to charged 4d chiral fermions.

Also, generically two 3-cycles in a 6d space intersect several times. Locally, each intersection is exactly of the form studied in section 3.1, therefore the construction leads to a replicated sector of open strings at intersections. This is a natural mechanism to explain/reproduce the appearance of replicated families of chiral fermions in Nature, as we show below. It is also important to notice that in compactifications, the angles between branes are derived quantities, and depend on the closed string moduli controlling the torus geometry. For instance, for a rectangular torus of radii R_1, R_2 along the horizontal and vertical directions, the angle between the 1-cycle $(1,0)$ and (n,m) is

$$\tan\theta = \frac{mR_2}{nR_1} \tag{4.1}$$

In this toroidal case, the intersection number is given by the product of the number of intersections in each 2-torus, and reads

$$I_{ab} \, = \, (n_a^1 m_b^1 - m_a^1 n_b^1) \times (n_a^2 m_b^2 - m_a^2 n_b^2) \times (n_a^3 m_b^3 - m_a^3 n_b^3) \tag{4.2}$$

It is useful to introduce the 3-homology class $[\Pi_a]$ of the 3-cycle Π_a, which can be

[1] These factorizable branes are not the most general possibility. Branes wrapped on non-factorizable cycles exist, and can be obtained e.g. by recombination of factorized branes. For simplicity, we will not use them in these lectures.

thought of as a vector of RR charges of the corresponding D6-brane. The 1-homology class of an (n,m) 1-cycle in a 2-torus is $n[a]+m[b]$, with $[a],[b]$ the basic homology cycles in $\mathbf{T^2}$. For a 3-cycle with wrapping numbers (n_a^i, m_a^i) we have

$$[\Pi_a] \;=\; \otimes_{i=1}^3 \left(n_a^i\,[a_i] \,+\, m_a^i\,[b_i] \right) \qquad (4.3)$$

The intersection number (4.2) is intersection number in homology, denoted $I_{ab} = [\Pi_a] \cdot [\Pi_b]$. This is easily shown using $[a_i] \cdot [b_j] = \delta_{ij}$ and linearity and antisymmetry of the intersection pairing.

With the basic data defining the configuration, namely $\mathbf{N_a}$ D6$_a$-branes wrapped on 3-cycles $[\mathbf{\Pi_a}]$, with wrapping numbers $(\mathbf{n_a^i}, \mathbf{m_a^i})$ on each $\mathbf{T^2}$ and intersection numbers $\mathbf{I_{ab}}$, we can compute the spectrum of the model.

The closed string sector produces 4d $\mathcal{N}=8$ supergravity. There exist different open string sectors:

$\mathbf{6}_a\mathbf{6}_a$ String stretched among D6-branes in the a^{th} stack produce 4d $U(N_a)$ gauge bosons, 6 real adjoint scalars and 4 adjoint Majorana fermions, filling out a vector multiplet of the 4d $\mathcal{N}=4$ supersymmetry preserved by the corresponding brane.

$\mathbf{6}_a\mathbf{6}_b$ + $\mathbf{6}_b\mathbf{6}_a$ Strings stretched between the a^{th} and b^{th} stack lead to I_{ab} replicated chiral left-handed fermions in the bifundamental representation $(N_a, \overline{N}_b)$. Negative intersection numbers lead to a positive number of chiral fermions with right-handed chirality. Additional light scalars may be present, with masses determined by the wrapping numbers and the $\mathbf{T^2}$ moduli.

Generalization for compact spaces more general than the 6-torus will be discussed in section 4.2. We have therefore obtained a large class of four-dimensional theories with interesting non-abelian gauge symmetries and replicated charged chiral fermions. Hence compactifications with intersecting D6-branes provide a natural setup in which string theory can produce gauge sectors with the same rough features of the Standard Model. In coming sections we explore them further as possible phenomenological models, and construct explicit examples with spectrum as close as possible to the Standard Model.

4.1.2 RR tadpole cancellation

String theories with open string sectors must satisfy a crucial consistency condition, known as cancellation of RR tadpoles. As mentioned above, D-branes act as sources for RR p-forms via the disk coupling $\int_{W_{p+1}} C_p$, see fig 3. The consistency condition amounts to requiring the total RR charge of D-branes to vanish, as implied by Gauss

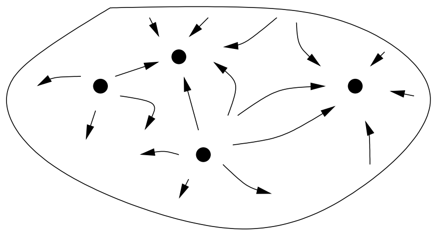

Figure 10: In a compact space, fluxlines cannot escape and the total charge must vanish.

law in a compact space (since RR field fluxlines cannot escape, figure 10). In our setup, the 3-cycle homology classes are vectors of RR charges, hence the condition reads

$$[\Pi_{\rm tot}] \,=\, \sum_a N_a\, [\Pi_a] \,=\, 0 \tag{4.4}$$

Equivalently, the condition of RR tadpole cancellation can be expressed as the requirement of consistency of the equations of motion for RR fields. In our situation, the terms of the spacetime action depending on the RR 7-form C_7 are

$$\begin{aligned} S_{C7} &= \int_{\mathbf{M_4}\times\mathbf{X_6}} H_8\wedge *H_8 \,+\, \sum_a N_a \int_{\mathbf{M_4}\times\mathbf{\Pi_a}} C_7 \\ &= \int_{\mathbf{M_4}\times\mathbf{X_6}} C_7\wedge dH_2 \,+\, \sum_a N_a \int_{\mathbf{M_4}\times\mathbf{X_6}} C_7\wedge\delta(\Pi_a) \end{aligned} \tag{4.5}$$

where H_8 is the 8-form field strength, H_2 its Hodge dual, and $\delta(\Pi_a)$ is a bump 3-form localized on Π_a in $\mathbf{X_6}$. The equations of motion read

$$dH_2 \,=\, \sum_a N_a\, \delta(\Pi_a) \tag{4.6}$$

The integrability condition (4.4) is obtained by taking this equation in homology.

In the toroidal setup the RR tadpole conditions provide a set of constraints, given by

$$\begin{aligned} \textstyle\sum_a N_a n_a^1 n_a^2 n_a^3 &= 0 \\ \textstyle\sum_a N_a n_a^1 n_a^2 m_a^3 &= 0 \text{ and permutations} \\ \textstyle\sum_a N_a n_a^1 m_a^2 m_a^3 &= 0 \text{ and permutations} \\ \textstyle\sum_a N_a m_a^1 m_a^2 m_a^3 &= 0 \end{aligned} \tag{4.7}$$

4.1.3 Anomaly cancellation

Cancellation of RR tadpoles in the underlying string theory configuration implies cancellation of four-dimensional chiral anomalies in the effective field theory in our con-

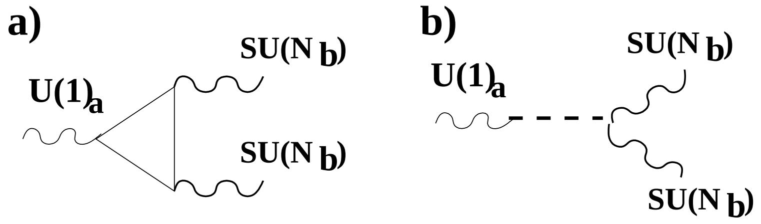

Figure 11: Triangle and Green-Schwarz diagrams contributing to the mixed $U(1)$ - non-abelian anomalies.

figurations. Recall that the chiral piece of the spectrum is given by I_{ab} chiral fermions in the representation $(N_a, \overline{N}_b)$ of the gauge group $\prod_a U(N_a)$.

Cubic non-abelian anomalies

The $SU(N_a)^3$ cubic anomaly is proportional to the number of fundamental minus antifundamental representations of $SU(N_a)$, hence it is proportional to

$$A_a = \sum_b I_{ab} N_b. \tag{4.8}$$

It is easy to check this vanishes due to RR tadpole cancellation: Starting with (4.4), we consider the intersection of $[\Pi_{\rm tot}]$ with any $[\Pi]$ to get

$$0 \;=\; [\Pi_a] \cdot \sum_b \, N_b \, [\Pi_b] \;=\; \sum_b N_b I_{ab} \tag{4.9}$$

as claimed [2].

Cancellation of mixed anomalies

The $U(1)_a$-$SU(N_b)^2$ mixed anomalies also cancel as a consequence of RR tadpole cancellation. They do so in a trickier way, namely the anomaly receives two non-zero contributions which cancel each other, see fig 11. Mixed gravitational triangle anomalies cancel automatically, without Green-Schwarz contributions.

The familiar field theory triangle diagrams give a contribution which, even after using RR tadpole conditions, is non-zero and reads

$$A_{ab} \;\simeq\; N_a \, I_{ab} \tag{4.10}$$

[2] It is interesting to notice that RR tadpole cancellation is slightly stronger than cancellation of cubic non-abelian anomalies. In fact, the former requires that the number of fundamental minus antifundamentals vanishes even for the cases $N_a = 1, 2$, where no gauge theory anomaly exists. This observation will turn out relevant in phenomenological model building in section 6.

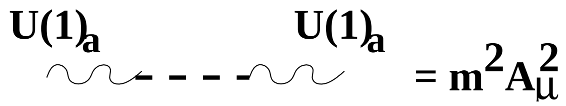

Figure 12: The $B \wedge F$ couplings lead to a $U(1)$ gauge boson mass term.

On the other hand, the theory contains contributions from Green-Schwarz diagrams, where the gauge boson of $U(1)_a$ mixes with a 2-form which subsequently couples to two gauge bosons of $SU(N_b)$, see figure 11. These couplings arise in the KK reduction of the D6-brane world-volume couplings $N_a \int_{\mathrm{D6}_a} C_5 \wedge \mathrm{tr}\, F_a$ and $\int_{\mathrm{D6}_b} C_3 \wedge \mathrm{tr}\, F_b^2$, as follows.

Introducing a basis $[\Lambda_k]$ and its dual $[\Lambda_{\tilde{l}}]$, we can define the KK reduced 4d fields

$$(B_2)_k = \int_{[\Lambda_k]} C_5 \quad , \quad \phi_{\tilde{l}} = \int_{[\Lambda_{\tilde{l}}]} C_3 \qquad \text{with } d\phi_{\tilde{l}} = -\delta_{k\tilde{l}} *_{4d} (B_2)_k \tag{4.11}$$

The KK reduced 4d couplings read

$$N_a q_{ak} \int_{4d} (B_2)_k \mathrm{tr}\, F_a \quad , \quad q_{b\tilde{l}} \int_{4d} \phi_{\tilde{l}} \mathrm{tr}\, F_b^2 \tag{4.12}$$

with $q_{ak} = [\Pi_a] \cdot [\Lambda_k]$, and similarly for $q_{b\tilde{l}}$. The total amplitude is proportional to

$$A_{ab}^{\mathrm{GS}} = -N_a \sum_k q_{ak} q_{b\tilde{l}} \delta_{k\tilde{l}} = \ \ldots \ = -N_a I_{ab} \tag{4.13}$$

leading to a cancellation between both kinds of contributions.

An important observation is that any $U(1)$ gauge boson with $B \wedge F$ couplings gets massive, with mass roughly of the order of the string scale, see fig 12. Such $U(1)$'s disappear as gauge symmetries from the low-energy effective field theory, but remain as global symmetries, unbroken in perturbation theory. Introducing the generators Q_a of the $U(1)$ inside $U(N_a)$, the condition that a $U(1)$ with generator $\sum_a c_a Q_a$ remains massless is

$$\sum_a N_a q_{ak} c_a = 0 \qquad \text{for all } k \tag{4.14}$$

Such $U(1)$ factors remain as gauge symmmetries of the low energy theory.

4.2 Generalization beyond torus: Model building with A-type branes

Clearly the above setup is not restricted to toroidal compactifications. Indeed one may take take any compact 6-manifold as internal space, for instance a Calabi-Yau threefold,

which would lead to 4d $\mathcal{N}=2$ supersymmetry in the closed string sector. In this situation we should pick a set of 3-cycles Π_a on which we wrap N_a D6-branes (for instance special lagrangian 3-cycles of $\mathbf{X_6}$ if we are interested in preserving supersymmetry), making sure they satisfy the RR tadpole cancellation condition $\sum_a N_a[\Pi_a]=0$.

The final open string spectrum (for instance, in the case of supersymmetric wrapped D6-branes) arises in two kinds of sectors

6_a-6_a Leads to $U(N_a)$ gauge bosons ($\mathcal{N}=1$ vector multiplets in the supersymmetric case) and $b_1(\Pi_a)$ real adjoint scalars (chiral multiplets in susy case).

6_a-6_b+6_b-6_a We obtain I_{ab} chiral fermions in the representation $(N_a,\overline{N}_b)$ (plus light scalars, massless in supersymmetry preserving intersections). Here $I_{ab}=[\Pi_a]\cdot[\Pi_b]$.

Notice that the chiral spectrum is obtained in terms of purely topological information of the configuration, as should be the case.

Our whole discussion up to this point has simply been a pedagogical way of describing a general class of string compactifications. Namely, compactifications of type IIA string theory on Calabi-Yau threefolds with A-type D-branes. In the geometrical large volume regime, these are described as D-branes wrapped on special lagrangian 3-cycles, and reproduce the structures we have been discussing. A-type branes are extensively studied from the point of view of topological strings, with results of immediate application to our models. To name a few, the fact that such D-brane states do not have lines of marginal stability in Kahler moduli space, that their world-volume superpotential arises exclusively from worldsheet instantons, and their nice relation via mirror symmetry with type IIB compactifications with B-type D-branes, see next lecture. It is very satisfactory that a phenomenological motivation has driven us to consider a kind of configurations so interesting from the theoretical viewpoint as well.

The phenomenology of toroidal and non-toroidal models is quite similar to that of toroidal compactifications with D-branes, see next subsection. Thus, the later are in any event good toy model for many features of general compactifications with intersecting branes. This is particularly interesting since it is relatively difficult to construct explicit configurations of intersecting D6-branes in Calabi-Yau models (although some explicit examples have been discussed in [12, 13]).

4.3 Phenomenological features

We now turn to a brief discussion of the phenomenological properties natural in this setup [9].

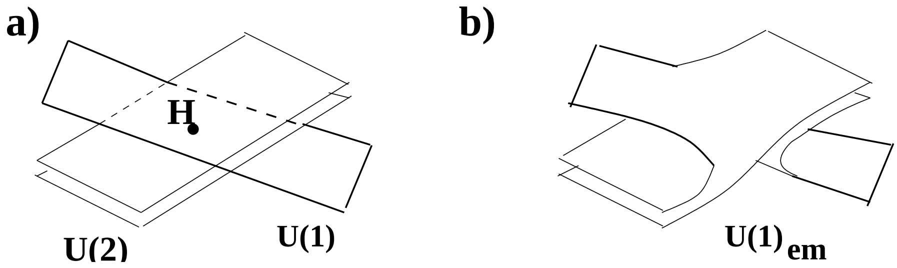

Figure 13: Higgs mechanism as recombination of intersecting branes.

• Most models constructed in the literature are non-supersymmmetric. It is however possible to construct fully $\mathcal{N} = 1$ supersymmetric models, see section 7. For non-supersymmetric models, unless alternative solutions to the hierarchy model are provided, the best proposal low string scale $M_s \simeq$ TeV to avoid hierarchy, along the lines of [14].

• The proton is stable in these models, since the $U(1)$ within the $U(3)$ color factor plays the role of baryon number, and is preserved as a global symmetry, exactly unbroken in perturbation theory. Non-perturbative effects breaking it arise from euclidean D2-branes wrapped on 3-cycles, and have the interpretation of spacetime gauge theory instantons, hence reproducing the non-perturbative breaking of baryon number in the Standard Model.

• These models do *not* have a natural gauge coupling unification, even at the string scale. Each gauge factor has a gauge coupling controlled by the volume of the wrapped 3-cycle. Gauge couplings are related to geometric volumes, hence their experimental values can be adjusted/reproduced in concrete models, rather than predicted by the general setup.

• There exists a geometric interpretation for the spontaneous electroweak symmetry breaking. In explicit models, the Higgs scalar multiplet arises from the light scalars at intersections, and parametrizes the possibility of recombining two intersecting cycles into a single smooth one, as shown in figure 13. In the process, the gauge symmetry is reduced, corresponding to a Higgs mechanism in the effective field theory. See [15] for further discussion.

• There is a natural exponential hierarchy of the Yukawa couplings. Yukawa couplings among the scalar Higgs and chiral fermions at intersections arise at tree level in the string coupling from open string worldsheet instantons; namely from string worldsheets spanning the triangle with vertices at the intersections and sides on the

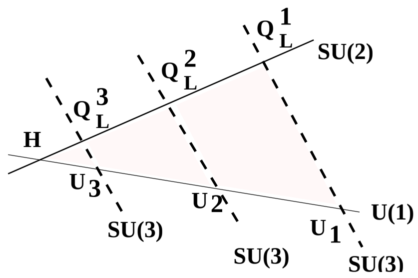

Figure 14: Geometric origin of the hierarchy of Yukawa couplings for different generations.

D-branes. Their value is roughly given by e^{-A}, with A the triangle area in string units. Since different families are located at different intersections, their triangles have areas increasing linearly with the family index, leading to an exponential Yukawa hierarchy, see fig 14. See e.g. [16] for further analysis of yukawa couplings in explicit models.

5 Orientifold models

The above constructed models are non-supersymmetric. One simple way to see it is that we start with type IIA string theory compactified on $\mathbf{X_6}$, and introduce D6-branes. Since RR tadpole cancellation requires that the total RR charge vanishes, we are forced to introduce objects with opposite RR charges, in a sense branes and antibranes, a notoriously non-supersymmetric combination.

An equivalent derivation of the result is as follows: If we would succeed in constructing a supersymmetric configuration of D6-branes, the system as a whole would be a supersymmetric BPS state of type IIA on $\mathbf{X_6}$. Since for a BPS state the tension is proportional to the RR charge, and the latter vanishes due to RR tadpole cancellation, the tension of the state must vanish. The only D6-brane configuration with zero tension is having no D6-brane at all. Hence the only supersymmetric configuration would be just type IIA on $\mathbf{X_6}$, with no brane at all.

These arguments suggest a way out of the impasse. In order to obtain $\mathcal{N} = 1$ supersymmetric compactifications we need to introduce objects with negative tension and negative RR charge, and which preserve the same supersymmetry as the D6-branes. Such objects exist in string theory and are orientifold 6-planes, O6-planes. Introduction of these objects leads to an interesting extension of the configurations above constructed, and will be studied in section 5.3. In particular we will use them to construct supersymmetric compactifications with intersecting D6-branes.

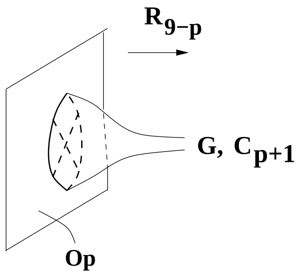

Figure 15: Diagram describing the interaction of an Op brane with closed string modes. The dashed cross denotes a crosscap, namely a disk with an identification of antipodal points in the boundary, so that the world-sheet is closed and unoriented.

5.1 Properties of O6-planes

To start, consider type IIA string theory on 10d flat space M_{10}, and mod it out by the so-called orientifold action $\Omega R(-)^{F_L}$. Here Ω is world-sheet parity, which flips the orientation of the fundamental strings; R is a $\mathbf{Z_2}$ geometric action, acting locally as $(x^5, x^7, x^9) \to (-x^5, -x^7, -x^9)$; finally $(-)^{F_L}$ is left-moving world-sheet fermion number, introduced for technical reasons.

The quotient theory contains a special subspace in spacetime, fixed under the geometric part R of the above action. Namely, it is a 7d plane defined by $x^5 = x^7 = x^9 = 0$, and spanned by the coordinates 0123468. This set of points fixed under the orientifold action is called an orientifold 6-plane (O6-plane), since it has six spatial dimensions (in general one can define other orientifold quotients of type II string theories, containing Op-planes of p spatial dimensions). Physically, it corresponds to a region of spacetime where the orientation of a string can flip (since a string at the O6-plane is identified, by the orientifold action, with itself with the opposite orientation). The description of string theory is the presence of orientifold planes is modified only by the inclusion of unoriented world-sheets, for instance with the topology of the Klein bottle.

Orientifold planes have some features similar to D-branes of the same dimension. For instance, Op-planes carry tension and are are charged under the RR $(p+1)$-form C_{p+1}. The diagram responsible for these couplings is shown in figure 15. For instance, and O6-plane is charged under the RR 7-form, and its charge is given by $Q_{\rm O6} = \pm 4$, in units where the D6-brane charge is $+1$. Here the two possible signs correspond to two different kinds of O6-planes; we will center on the negatively charged O6-plane in what follows. Also, O-planes preserve the same supersymmetry as a D-brane. This implies that there is a relation between the tension and charge of O-planes.

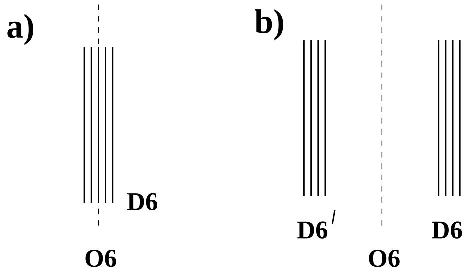

Figure 16: Configurations of D6-brane stacks parallel to an O6-plane. Figure a) show the situation where the branes are on top of the O6-plane, while figure b) corresponds to branes separated from it. Although the branes within a stack are coincident, they are shown slightly separated, for clarity.

There are however some important differences between O-planes and D-branes, the main one being that O-planes do not carry world-volume degrees of freedom. Hence, they are better regarded as part of the spacetime geometrical data, rather than dynamical objects.

5.2 O6-planes and D6-branes

It is interesting to include orientifold planes in compactifications or configurations with D-branes. These configurations are most simply described in the covering space of the orientifold quotient. Here we must include the images of the D-branes under the orientifold action, denoted by primed indices. The spectrum of open strings in the orientifold quotient theory is obtained by simply computing the spectrum in the covering space, and then imposing the identifications implied by the orientifold action (taking into account the flip in the open string orientation implied by the latter).

• Let us start by considering the simple situation of configurations of parallel D6-branes and O6-planes. Consider first a stack of n D6-branes on top of an O6-plane, see figure 16a. The open string spectrum before the orientifold action is given by the n^2 open string sectors, giving rise to an $U(n)$ vector multiplet. The orientifold action implies the following identification among the ab open strings

$$|ab\rangle \leftrightarrow \pm|ba\rangle \tag{5.1}$$

with the negative (positive) sign corresponding to the choice of negatively (positively) charged O6-plane. Centering on the former case, the physical states in the quotient correspond to the $n(n-1)/2$ antisymmetric linear combinations $(|ab\rangle - |ba\rangle)/2$. The

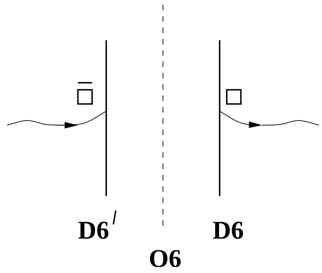

Figure 17: The orientifold projection relates the gauge groups on D-branes and images such that open string endpoint in the fundamental representation of one map to endpoints in the antifundamental of the other.

massless modes correspond to an $SO(n)$ vector multiplet with respect to the 16 supercharges unbroken by the O6/D6 configuration.

• Consider now a configuration of n coincident D6-branes, parallel but separated from the O6-plane. The configuration must include an orientifold image of the D-brane stack, namely a set of n D6'-branes, see figure 16b. The massless open string spectrum before the orientifold projection is given by a $U(n) \times U(n)'$ gauge group plus superpartners. The orientifold action implies and identification of the degrees of freedom in both $U(n)$ factors, so that only a linear combination survives. In the quotient, we just obtain an $U(n)$ vector multiplet (which agrees with the intuition that massless modes on D-branes are not sensitive to distant objects, hence the n D6-branes in the quotient do not notice, at the level of the massless spectrum, the distant O6-plane).

An important observation in the identification of the $U(n)$ factors is that, due to the orientation reversal, and open string starting on the D6-brane stack is mapped to an open string ending on the D6'-brane stack, and viceversa, see figure 17. This implies that the $U(n)$ is identified with $U(n')$ with the fundamental $\square$ mapping to the anti-fundamental $\overline{\square}'$, and viceversa. This will be important in the computation of open string massless spectra in more involved configurations (or in these simple ones, if one is interested in computing the massive spectrum).

Let us consider another local geometry similar to the above. Let us consider configurations of D6-branes orthogonal to the O6-plane in some of the directions, so that the D6-brane stack is still mapped to itself under the orientifold action. A configuration which appears often (since it preserves 8 supercharges) is when there are four dimensions not commonly along or commonly transverse to the objects. For instance,

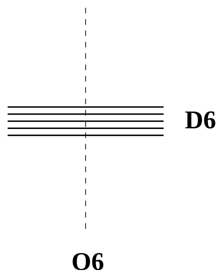

Figure 18: Configurations of D6-brane stacks with some directions orthogonal to an O6-plane.

consider an O6-plane along 0123456 and a D6-brane along 0123478, see figure 18. For one such stack of n D6-branes, the final gauge group (for a negatively charged O6-plane) is $USp(n)$ (hence n must be even) and fills out a vector multiplet with respect to the eight unbroken supersymmetries. In addition there is a hypermultiplet in the two-index symmetric (reducible) representation. The change of gauge group with respect to the case of the parallel O6/D6 system is due to an additional sign in the orientifold action [18].

Let us now consider situations with intersecting D6-branes (and their images) in the presence of O6-planes. All the D6-branes and O6-planes are taken to be parallel in four of their common dimensions, so that the intersections are geometrically of the kind studied above. There are several different situations to be considered, depending on the relative geometry of the intersection and the O6-plane. To simplify the discussion, we center on describing the gauge group and chiral fermions at intersections.

• Consider two stacks of D6-branes, labeled a and b, intersecting away from the O6-plane. The configuration also includes the image D6'-branes, labeled a', b', see figure 19a. Before the orientifold projection, the gauge group is $U(N_a) \times U(N_b) \times U(N_a)' \times U(N_b)'$. Also, the intersections in the figure (ignoring other possible intersections of the branes) provide 4d chiral fermions in the representation $(\square_a, \overline{\square}_b)$ and $(\square'_b, \overline{\square}'_a)$, due to the different relative orientation of the branes and their images. After the identification implied by the orientifold action (recalling the effect on fundamental representations and their images), we are left with a gauge group $U(N_a) \times U(N_b)$ and a 4d chiral fermion in the $(\square_a, \overline{\square}_b)$.

• Consider now the intersection of a stack of D6$_a$-branes with D6$'_b$-branes, namely the orientifold image of a stack of D6$_b$-branes, see figure 19b. Before the orientifold projection, the gauge group is $U(N_a) \times U(N_b)' \times U(N_a)' \times U(N_b)$, with 4d chiral fermions

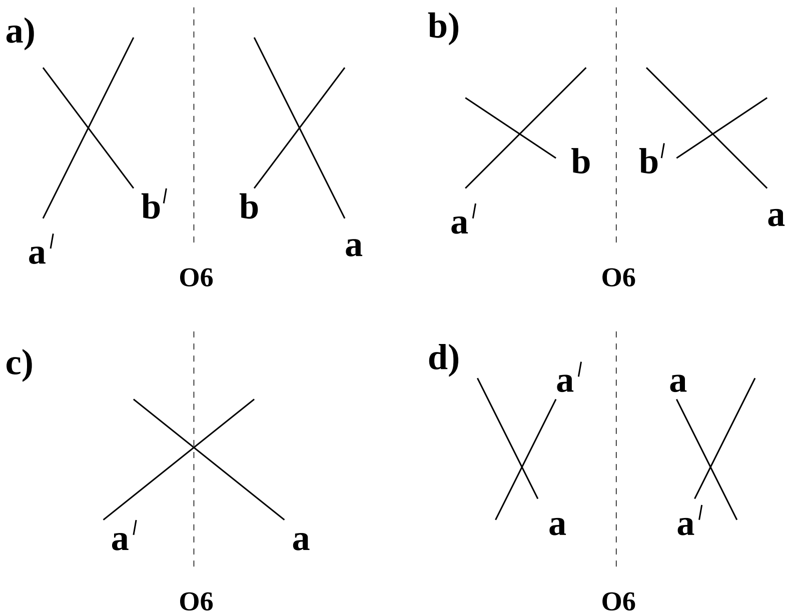

Figure 19: Configurations of intersecting D6-brane stacks in the presence of an O6-plane. Figure a) shows the intersections of two stacks a and b, away from the O6-plane. Figure b) shows the intersection of a stack a with the image of another b'. Figures c) and d) show the intersection of the stacks a and its image a' on top of the O6-plane and away from it, respectively.

in the representation $(\square_a, \overline{\square}'_b)$ and $(\square_b, \overline{\square}'_a)$. After the orientifold action, we have a gauge group $U(N_a) \times U(N_b)$ and a 4d chiral fermion in the $(\square_a, \square_b)$.

• Consider the intersection of a stack of D6_a-branes, with its own image, on top of the O6-plane, see figure 19c. Before the orientifold action, the gauge group is $U(N_a) \times U(N_a)'$ and there is a 4d chiral fermion in the $(\square_a, \overline{\square}'_a)$. The orientifold action reduces the gauge group to $U(N_a)$. The initial 4d chiral fermions thus transform under this as the tensor product of $\square_a$ and $\overline{\square}'_a = \square_a$, namely $\begin{smallmatrix}\square\\\square\end{smallmatrix}_a + \square\!\square_a$. After the orientifold projection (for a negatively charged O6-plane), however, only 4d fermions in the $\begin{smallmatrix}\square\\\square\end{smallmatrix}_a$ component survive.

• Consider finally the intersection of a stack of D6_a-branes, with D$6'_a$-branes, away from the O6-plane, see figure 19d. Before the orientifold action, the gauge group is $U(N_a) \times U(N_a)'$ and there are 4d chiral fermions in the $2(\square_a, \overline{\square}'_a)$, due to the two intersections. The orientifold action reduces the gauge group to $U(N_a)$, and identifies both intersections. Thus, the 4d chiral fermions in the quotient transform in the representation $\begin{smallmatrix}\square\\\square\end{smallmatrix}_a + \square\!\square_a$.

It is easy to derive the spectra for intersections of generic D6-brane stack with stacks overlapping or orthogonal to the O6-plane. With these ingredients, we have enough information to describe compactifications with O6-planes and intersecting D6-branes.

5.3 Orientifold compactifications with intersecting D6-branes

5.3.1 Construction

Consider type IIA theory on e.g. a Calabi-Yau $\mathbf{X_6}$, and mod out the configuration by $\Omega R(-)^{F_L}$, where R is an antiholomorphic $\mathbf{Z_2}$ symmetry of $\mathbf{X_6}$. Hence it locally acts as $(z_1, z_2, z_3) \to (\overline{z}_1, \overline{z}_2, \overline{z}_3)$ on the CY complex coordinates, or as $(x^5, x^7, x^9) \to (-x^5, -x^7, -x^9)$ in suitable real ones. The set of fixed points of R are O6-planes, similar to those introduced above, with the difference that they are not flat in general, but rather wrap a (special lagrangian) 3-cycle in $\mathbf{X_6}$. Let us denote $\Pi_{\rm O6}$ the total 3-cycle spanned by the set of O6-planes in the configuration.

We now introduce stacks of N_a D6_a-branes, and their image D$6'_a$-branes, in the above orientifold quotient, see figure 20. They are wrapped on 3-cycles, denoted Π_a and Π'_a, respectively. The model is $\mathcal{N} = 1$ supersymmetric if all the D6-branes are wrapped on special lagrangian 3-cycles, see section 7 for concrete examples.

Taking into account the different sources of RR 7-form in the configuration, the RR

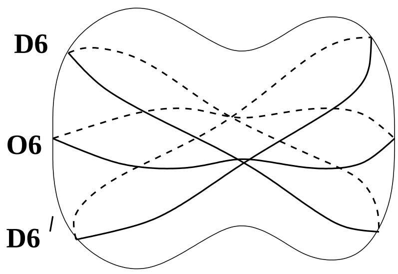

Figure 20: D6-branes and their images in an orientifold compactification.

tadpole cancellation conditions read [3]

$$\sum_a N_a\,[\Pi_a] \,+\, \sum_a N_a\,[\Pi_{a'}] \,-\, 4\times[\Pi_{\rm O6}] \,=\, 0 \tag{5.2}$$

The open string spectrum in orientifolded models can be easily computed. It only requires computing the relevant numbers of intersections, and if required how many lie on top of the orientifold planes. The results for the different sectors and the corresponding chiral spectra, assuming that no D6-branes are mapped to themselves under the orientifold action, are as follows

aa+a'a' Contains $U(N_a)$ gauge bosons and superpartners

ab+ba+b'a'+a'b' Contains I_{ab} chiral fermions in the representation $(N_a,\overline{N}_b)$, plus light scalars.

ab'+b'a+ba'+a'b Contains $I_{ab'}$ chiral fermions in the representation (N_a,N_b), plus light scalars.

aa'+a'a Contains $n_{\Box\!\Box_a}$ 4d chiral fermions in the representation $\Box\!\Box_a$ and $n_{\overset{\Box}{\Box}_a}$ in the $\overset{\Box}{\Box}_a$, with

$$n_{\Box\!\Box_a} = \frac{1}{2}(I_{aa'} - I_{a,{\rm O6}}) \quad , \quad n_{\overset{\Box}{\Box}_a} = \frac{1}{2}(I_{aa'} + I_{a,{\rm O6}}) \tag{5.3}$$

where $I_{a,{\rm O6}} = [\Pi_a]\cdot[\Pi_{\rm O6}]$ is the number of aa' intersections on top of O6-planes.

As expected, the new RR tadpole conditions in the presence of O6-planes guarantee the cancellation of 4d anomalies of the new chiral spectrum, in analogy with the toroidal case. (In the orientifold case, mixed gravitational anomalies may receive Green-Schwarz contributions, see appendix in the first reference in [19]). The condition that a $U(1)$ remains massless is given by the orientifold version of (4.14)

$$\sum_a N_a(q_{ak} - q_{a'k})c_a = 0 \qquad \text{for all } k \tag{5.4}$$

[3]There are additional discrete constraints arising from cancellation of $\mathbf{Z_2}$-valued K-theory charges. We skip their discussion for the moment.

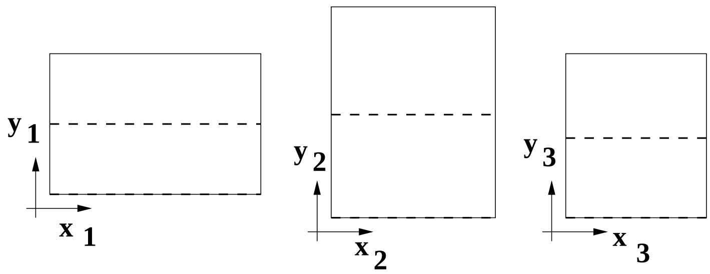

Figure 21: Orientifold 6-planes in the orientifold quotient of IIA on $\mathbf{T^6}$ by $\Omega R(-)^{F_L}$, with $R: y_i \to -y_i$.

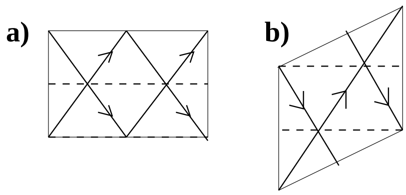

Figure 22: Cycles and their orientifold images in a rectangular and tilted 2-tori.

5.3.2 Toroidal orientifold models

A simple class of examples is provided by compactifications on $\mathbf{X_6} = \mathbf{T^6}$, with factorized $\mathbf{T^6}$, and with R given by the action $y_i \to -y_i$, where y_i are the vertical direction on each $\mathbf{T^2}$. This is a symmetry for rectangular two-tori or for two-tori tilted by a specific angle [20], see figure 22. Let us introduce a quantity $\beta = 0, \frac{1}{2}$, corresponding to the rectangular and tilted cases.

For a geometry with rectangular two-tori, as in figure 21, the set of fixed points is given by x_i arbitrary, $y_i = 0, R_{y_i}/2$, hence it has 8 components. They correspond to O6-planes wrapped on the 3-cycle with wrapping numbers $(n_i, m_i) = (1, 0)$, so that $[\Pi_{\rm O6}] = 8[a_1][a_2][a_3]$.

We now introduce D6$_a$-branes, with multiplicities N_a and wrapping numbers (n_a^i, m_a^i) of the D6-brane stacks. We also introduce their orientifold images, with wrapping numbers $(n_a^i, -m_a^i)$ for rectangular 2-tori, or $(n_a^i, -n_a^i - m_a^i)$ for tilted tori, see fig 22. To unify their description, we introduce $\tilde{m}_a = m_a + \beta n_a$, so that branes and images have wrapping numbers $(n_a, \tilde{m}_a)$ and $(n_a, -\tilde{m}_a)$ respectively.

The RR tadpole conditions are simple to obtain. In the case of rectangular two-tori,

they are explicitly given by

$$\begin{aligned} \textstyle\sum_a N_a n_a^1 n_a^2 n_a^3 = & \quad 16 \\ \textstyle\sum_a N_a n_a^1 m_a^2 m_a^3 = & \quad 0 \text{ and permutations} \end{aligned} \tag{5.5}$$

The spectrum is as discussed above, with the specific intersection numbers computed using (4.2). Namely

Explicit examples are discussed in further sections.

6 Getting just the standard model

In this section we consider some of the phenomenologically most interesting constructions, where the chiral part of the low-energy spectrum is given by that of the Standard Model (SM). The models are based on brane configurations first discussed in [21, 16].

As we have discussed above, models without orientifold planes do not lead to the chiral spectrum of the Standard Model. In fact, there is a general argument [21] showing that any such construction always contains additional chiral fermions in $SU(2)$ doublets, beyond those in the SM, as follows. First notice that in such models, the gauge group is a product of unitary factors, so the electroweak $SU(2)$ must belong to a $U(2)$ factor in the gauge group. As mentioned in section 4.1.3, the RR tadpole cancellation conditions imply that the number of fundamentals and antifundamentals for each $U(N)$ factor must be equal, even for $U(2)$ (where the 2 and the $\overline{2}$ are distinguished by their $U(1)$ charge). Now in any such model with SM gauge group containing $SU(3) \times SU(2)$, the left-handed quarks must belong to a representation $3(3,\overline{2})$, contributing nine antifundamentals of $SU(2)$. The complete spectrum must necessarily contain nine fundamentals of $SU(2)$, three of which may be interpreted as left-handed leptons; the remaining six doublets are however exotic chiral fermions, beyond the spectrum of the SM.

The introduction of orientifold planes in the construction allows to avoid this issue in several ways, as we describe in this section. In fact, as a consequence, they allow to construct string compactifications with the chiral spectrum of just the SM. This is a remarkable achievement.

6.1 The $U(2)$ class

One possibility [21] is to exploit the fact that in orientifold models there are two different kinds of bifundamental representations that arise in the spectrum, namely

$(\square, \overline{\square})$ and $(\square, \square)$. This allows an alternative construction of the SM chiral fermion spectrum, satisfying the RR tadpole constraint on the spectrum without exotics, as follows. Consider realizing the three families of left-handed quarks as $(3, \overline{2}) + 2(3, 2)$. This contributes three net $SU(2)$ doublets, hence the three $SU(2)$ doublets required in the model correspond simply to the three left-handed leptons.

Indeed, it is possible to propose a set of intersection numbers, such that any configuration of D6-branes wrapped on 3-cycles with those intersections numbers reproduces the chiral spectrum of the SM. Consider [21] four stacks of D6-branes, denoted a, b, c, d (and their images), giving rise to a gauge group $U(3)_a \times U(2)_b \times U(2)_c \times U(1)_d$. If the intersections numbers of the corresponding 3-cycles are given by

$$\begin{array}{llll} I_{ab} = 1 & I_{ab'} = 2 & I_{ac} = -3 & I_{ac'} = -3 \\ I_{bd} = 0 & I_{bd'} = -3 & I_{cd} = -3 & I_{cd'} = 3 \end{array} \tag{6.1}$$

then the chiral spectrum of the model has the non-abelian quantum numbers of the chiral fermions in the SM (plus right-handed neutrinos). In order to reproduce exactly the SM spectrum, one also needs to require that the linear combination of $U(1)$'s

$$Q_y = \frac{1}{6}Q_a - \frac{1}{2}Q_c + \frac{1}{2}Q_d \tag{6.2}$$

which reproduces the hypercharge quantum numbers, remains as the only massless $U(1)$ in the model.

It is important to emphasize that at this level, we have not constructed any explicit model. Rather, we have made a general proposal of what kind of structure one must implement in concrete examples to lead to the SM chiral spectrum. This is however a very useful step.

In [21] there is a large class of examples of models of this kind, constructed explicitly in terms of D6-branes on factorized 3-cycles in the orientifold of $\mathbf{T^6}$ discussed in section 5.3.2. To illustrate the discussion with an example, consider the model in [21] corresponding to the parameters

$$\beta^1 = \beta^2 = 1 \quad ; \quad \epsilon = \rho = 1 \quad ; \quad n_a^2 = 4 \quad ; \quad n_b^1 = 1 \quad ; \quad n_c^1 = 5 \quad ; \quad n_d^2 = 2 \tag{6.3}$$

The D6-brane configuration (without specifying the images) is given by

	N	(n^1, m^1)	(n^2, m^2)	$(n^3, \tilde{m}^3)$
a	3	$(1, 0)$	$(4, 1)$	$(1, \frac{1}{2})$
b	2	$(1, 1)$	$(1, 0)$	$(1, \frac{3}{2})$
c	1	$(5, 3)$	$(1, 0)$	$(0, 1)$
d	1	$(1, 0)$	$(2, -1)$	$(1, \frac{3}{2})$

Let us emphasize again that the proposal to obtain the SM from models with the intersection numbers above is not restricted to the toroidal orientifold setup. Indeed, they have been discussed in [12, 13] in the large volume regime of geometric compactifications, and in [22] a large class of models has been constructed in Gepner constructions. The latter models are fully supersymmetric, leading to almost MSSM spectra (differing from it in the structure of the non-chiral Higgs sector), showing that the proposal can be exploited to construct supersymmetric models as well.

6.2 The $USp(2)$ class

Another possible way to avoid the problem of the extra $SU(2)$ doublets, is to exploit the fact that D6-branes in the presence of orientifold planes may contain $USp(N)$ gauge factors (see section 5.2). For the latter, all representations are real, and RR tadpole conditions do not impose any constraint on the matter content. Since $USp(2) \equiv SU(2)$, it is possible to realize the electroweak $SU(2)$ in terms of such D6-brane with $USp(2)$ gauge group, and thus circumvent the constraints on the number of doublets.

Indeed such a construction is proposed in [16]. The SM spectrum would arise in terms of a configurations of four stacks of D6-branes, leading to a gauge group $U(3)_a \times USp(2)_b \times U(1)_c \times U(1)_d$, with intersection numbers

$$\begin{gathered} I_{ab} = 3 \qquad I_{ab'} = 3 \qquad I_{ac} = -3 \qquad I_{ac'} = -3 \\ I_{db} = 3 \qquad I_{db'} = 3 \qquad I_{dc} = -3 \qquad I_{dc'} = 3 \quad I_{bc} = -1 \qquad I_{bc'} = 1 \end{gathered} \tag{6.4}$$

the $U(1)$ that needs to be massless in order to reproduce the SM hypercharge is

$$Q_Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c - \frac{1}{2}Q_d \tag{6.5}$$

Moreover, explicit realizations of D6-branes on 3-cycles with those intersection numbers (and with massless hypercharge) have been constructed in toroidal orientifolds [16]. Let us consider an illustrative example, corresponding to $\rho = 1$ in that reference. The set of D6-branes (to which we should add the images) is specified by

	N	(n^1, m^1)	(n^2, m^2)	(n^3, m^3)
a	3	$(1, 0)$	$(1, 3)$	$(1, -3)$
b	1	$(0, 1)$	$(1, 0)$	$(0, -1)$
c	1	$(0, 1)$	$(0, -1)$	$(1, 0)$
d	1	$(1, 0)$	$(1, 3)$	$(1, -3)$

One needs to add additional branes to satisfy the RR tadpole condition, but this may be done with the latter having no intersection with the above one. Hence the additional D6-branes are decoupled, and we do not discuss them for simplicity. The above D6-brane configuration can preserve supersymmetry locally, but with supersymmetry is eventually broken by the additional decoupled D6-brane sector.

Notice that in this realization, the $USp(2)$ factor arises from the D6-brane b and its image, when they are coincident. Notice also that the D6-brane d and its image, and the a and c stacks, can be taken to coincide. Thus the above standard model configuration can be considered a spontaneously broken Pati-Salam theory, with original gauge group $U(4) \times USp(2)_L \times USp(2)_R$.

As emphasized above, the proposed intersection numbers may be realized in other contexts, also with or without supersymmetry. Explicit constructions of models with those intersection numbers, with supersymmetry will be studied in next section (see also [23], and [22] for Gepner model constructions).

7 Supersymmetric models

In this section we review some simple supersymmetric 4d chiral models of intersecting D6-branes, in [19] to which we refer the reader for additional details (see e.g. [24, 25, 26] for additional models in other orbifolds, see also [27] for early work on diverse non-chiral supersymmetric orbifolds with intersecting branes). For a more geometric description of the model, adapting the recipe in section 4.2, see [12].

7.1 Orientifold of $T^6/Z_2 \times Z_2$

In order to obtain supersymmetric models, one needs a sufficient number of O6-planes in the construction. One of the simplest possibilities is the $\Omega R(-)^{F_L}$ orientifold of the $\mathbf{T^6}/(\mathbf{Z_2} \times \mathbf{Z_2})$ orbifold.

We consider type IIA theory on $\mathbf{T^6}/(\mathbf{Z_2} \times \mathbf{Z_2})$, with generators θ, ω associated to the twists $v = (\frac{1}{2}, -\frac{1}{2}, 0)$ and $w = (0, \frac{1}{2}, -\frac{1}{2})$, hence acting as

$$\begin{aligned} \theta : &\quad (z_1, z_2, z_3) \to (-z_1, -z_2, z_3) \\ \omega : &\quad (z_1, z_2, z_3) \to (z_1, -z_2, -z_3) \end{aligned} \tag{7.1}$$

where z_i are complex coordinates in the $\mathbf{T^6}$. The action projects out some of the moduli, in particular implies the $\mathbf{T^6}$ is factorizable. We mod out this theory by $\Omega R(-)^{F_L}$, where

$$R : (z_1, z_2, z_3) \to (\overline{z}_1, \overline{z}_2, \overline{z}_3) \tag{7.2}$$

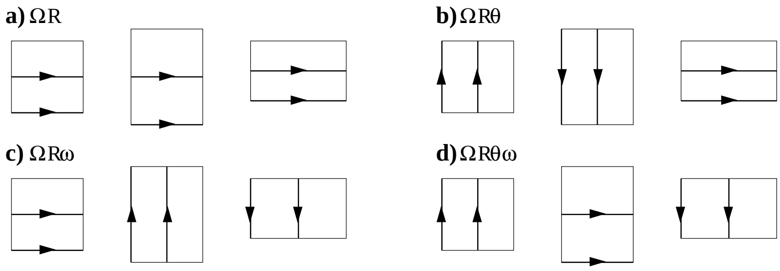

Figure 23: O6-planes in the orientifold of $\mathbf{T^6}/(\mathbf{Z_2}\times\mathbf{Z_2})$.

The model contains four kinds of O6-planes, associated to the actions of ΩR, $\Omega R\theta$, $\Omega R\omega$, $\Omega R\theta\omega$, as shown in Figure 23 (for rectangular 2-tori). For simplicity we henceforth center on rectangular two-tori.

In order to cancel the corresponding RR tadpoles, we introduce D6-branes wrapped on three-cycles as in previous discussions. Also for simplicity we assume that each stack of D6-branes is passing through $\mathbf{Z_2}\times\mathbf{Z_2}$ fixed points. These extra projections are responsible for the fact that N D6-branes lead to an $U(N/2)$ gauge symmetry. The RR tadpole conditions have the familiar form

$$\sum_a N_a\,[\Pi_a] + \sum_a N_a\,[\Pi_{a'}] - 4[\Pi_{O6}] = 0 \tag{7.3}$$

where $[\Pi_{O6}]$ is the homology charge of the complete set of O6-planes. More explicitly, for instance for rectangular tori we have

$$\begin{aligned} \sum_a N_a n_a^1 n_a^2 n_a^3 - 16 &= 0 \\ \sum_a N_a n_a^1 m_a^2 m_a^3 + 16 &= 0 \\ \sum_a N_a m_a^1 n_a^2 m_a^3 + 16 &= 0 \\ \sum_a N_a m_a^1 m_a^2 n_a^3 + 16 &= 0 \end{aligned} \tag{7.4}$$

Skipping the details, the chiral spectrum is given in table 1

The condition that the system of branes preserves $N=1$ supersymmetry is that each stack of D6-branes is related to the O6-planes by a rotation in $SU(3)$, see section 3.2. More specifically, denoting by θ_i the angles the D6-brane forms with the horizontal direction in the i^{th}two-torus, supersymmetry preserving configurations must satisfy

$$\theta_1 + \theta_2 + \theta_3 = 0 \tag{7.5}$$

Sector	Representation
aa	$U(N_a/2)$ vector multiplet 3 Adj. chiral multiplets
$ab+ba$	I_{ab} $(\square_a,\overline{\square}_b)$ fermions
$ab'+b'a$	$I_{ab'}$ $(\square_a,\square_b)$ fermions
$aa'+a'a$	$\frac{1}{2}(I_{aa'}-4I_{a,Op})$ $\square\square$ fermions $\frac{1}{2}(I_{aa'}+4I_{a,Op})$ $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ fermions

Table 1: General chiral spectrum on generic D6_a-branes in the ΩR orientifold of $\mathbf{T^6}/(\mathbf{Z_2}\times\mathbf{Z_2})$. The models may contain additional non-chiral pieces which we ingnore here. In supersymmetric situations, scalars combine with the fermions given above to form chiral supermultiplets.

For fixed wrapping numbers (n^i,m^i), the condition translates into a constraint on the ratio of the two radii on each torus. For rectangular tori, denoting $\chi_i=(R_2/R_1)_i$, with R_2,R_1 the vertical resp. horizontal directions, the constraint is

$$\arctan(\chi_1\frac{m_1}{n_1})\ +\ \arctan(\chi_2\frac{m_2}{n_2})\ +\ \arctan(\chi_3\frac{m_3}{n_3})\ =\ 0 \tag{7.6}$$

To provide an illustrative example, we consider a model [25] containing a sector of branes leading to the SM fields (belonging to the $USp(2)$ class in section 6.2, plus an additional set of branes required for RR tadpole cancellation (and contributing vector-like exotic matter in the spectrum). The set of branes is given in table 7.1

The addition of the D-brane sectors h_1, h_2 and f, which are necessary to embed the MSSM local model into a global $\mathcal{N}=1$ compactification, add new gauge groups as well as chiral matter. In general, some of this additional chiral matter will be charged under the MSSM gauge group, and hence will introduce chiral exotics in our spectrum. Nevertheless, we can get rid of most of these exotics by taking appropriate scalar flat directions. In the present context, such flat directions can be engineered from the D-brane perspective by the process of D-brane recombination. We now consider one such example. Consider that stacks a and d are on top of each other and hence we have a Pati-Salam gauge group. This will hardly affect the discussion, but will render our expressions more compact. Also assume that all the D3-branes are at the origin, and hence our gauge group includes a $USp(40)$ factor.

The total chiral spectrum of this model is displayed in table 7.1, including the

N_α	(n_α^1, m_α^1)	(n_α^2, m_α^2)	(n_α^3, m_α^3)
$N_a = 6$	$(1, 0)$	$(3, 1)$	$(3, -1)$
$N_b = 2$	$(0, 1)$	$(1, 0)$	$(0, -1)$
$N_c = 2$	$(0, 1)$	$(0, -1)$	$(1, 0)$
$N_d = 2$	$(1, 0)$	$(3, 1)$	$(3, -1)$
$N_{h_1} = 2$	$(-2, 1)$	$(-3, 1)$	$(-4, 1)$
$N_{h_2} = 2$	$(-2, 1)$	$(-4, 1)$	$(-3, 1)$
40	$(1, 0)$	$(1, 0)$	$(1, 0)$

Table 2: D-brane magnetic numbers giving rise to an $\mathcal{N} = 1$ MSSM like model, in the $\mathbf{T^6}/(\mathbf{Z_2} \times \mathbf{Z_2})$ orientifold.

charges of the chiral matter under the only $U(1)$ factor which is massless. This $U(1)$ is given by the combination $U(1)' = \frac{1}{3}U(1)_a - 2\,[U(1)_{h_1} - U(1)_{h_2}]$, and almost all the chiral matter is charged under it. We have also displayed the Higgs multiplet and the 196 singlets in the $h_1 h_2'$ sector of the theory. The latter are of particular interest, since they parametrise a subspace of flat directions in the $\mathcal{N} = 1$ effective theory. Indeed, we can give a non-vanishing v.e.v. to a particular combination of the scalar fields in the 196 chiral multiplets without breaking supersymmetry. In terms of D-brane physics, this is nothing but the D9-brane recombination

$$h_1 + h_2' \to h. \tag{7.7}$$

This amounts to deforming the two intersecting D6-branes into a single smooth one, with homology class given by $[\Pi_h] = [\Pi_{h_1}] + [\Pi_{h_2'}]$.

This Higgsing does not affect the Pati-Salam gauge group. It does, however, have an important effect on the chiral spectrum of the theory. Indeed, we can compute the chiral spectrum after (7.7) with the 3-cycle $[\Pi_h]$ and the topological formulae of table 5, finding that the final theory has the extremely simple chiral content of Table 7.1.

We hope that these examples suffice to illustrate the flexibility of the techniques we have discussed and allows the reader to safely jump into the literature for further details.

Sector	Matter	$SU(4) \times SU(2) \times SU(2) \times [USp(40)]$	Q_a	Q_{h_1}	Q_{h_2}	Q'
(ab)	F_L	$3(4,2,1)$	1	0	0	$1/3$
(ac)	F_R	$3(\bar{4},1,2)$	-1	0	0	$-1/3$
(bc)	H	$(1,2,2)$	0	0	0	0
(ah_1')		$6(\bar{4},1,1)$	-1	-1	0	$5/3$
(ah_2)		$6(4,1,1)$	1	0	-1	$-5/3$
(bh_1)		$8(1,2,1)$	0	-1	0	2
(bh_2)		$6(1,2,1)$	0	0	-1	-2
(ch_1)		$6(1,1,2)$	0	-1	0	2
(ch_2)		$8(1,1,2)$	0	0	-1	-2
$(h_1 h_1')$		$23(1,1,1)$	0	-2	0	4
$(h_2 h_2')$		$23(1,1,1)$	0	0	-2	-4
$(h_1 h_2')$		$196(1,1,1)$	0	1	1	0
(fh_1)		$(1,1,1) \times [40]$	0	-1	0	2
(fh_2)		$(1,1,1) \times [40]$	0	0	-1	-2

Table 3: Chiral spectrum of the three generation Pati-Salam $\mathcal{N}=1$ chiral model of table 7.1. The Abelian generator of the unique massless $U(1)$ is given by $Q' = \frac{1}{3}Q_a - 2(Q_{h_1} - Q_{h_2})$.

Sector	Matter	$SU(4) \times SU(2) \times SU(2) \times [USp(40)]$	Q_a	Q_h	Q'
(ab)	F_L	$3(4,2,1)$	1	0	$1/3$
(ac)	F_R	$3(\bar{4},1,2)$	-1	0	$-1/3$
(bc)	H	$(1,2,2)$	0	0	0
(bh)		$2(1,2,1)$	0	-1	2
(ch)		$2(1,1,2)$	0	$+1$	-2

Table 4: $\mathcal{N}=1$ spectrum derived from the D-brane content of table 7.1 after D-brane recombination. There is no chiral matter arising from ah, ah', hh' or charged under $USp(40)$. The generator of $U(1)'$ is now given by $Q' = \frac{1}{3}Q_a - 2Q_h$.

Lecture 2. Model building in type IIB string theory

8 Introduction

In the previous lecture we have studied interesting compactifications of type IIA string theory on Calabi-Yau threefolds, with A-type D-branes, namely D6-branes wrapped on intersecting 3-cycles. Mirror symmetry exchanges type IIA and type IIB string compactifications, and maps A-type branes to B-type branes. Hence, it should be possible to obtain interesting compactifications in type IIB string theory on Calabi-Yau threefolds with B-type branes. This lecture is devoted to studying these compatifications (and their mirror relation to intersecting brane models).

B-type branes correspond to D-branes wrapped on holomorphic cycles of the Calabi-Yau threefold, and carrying holomorphic (and stable) world-volume gauge bundles. Namely, we should consider D3-brane sitting at points, D5- and D7- branes wrapped on 2- and 4-cycles, respectively, and D9-branes wrapped on the entire Calabi-Yau. On the wrapped volumes, one can turn on a topologically non-trivial background for the world-volume gauge field.

As a jargon-related comments, in the literature the different branes are described in a unified language, by describing them as 'coherent sheaves' on the CY threefold. Sheaves are generalizations of gauge bundles, which generalize to backgrounds with support on lower-dimensional subspaces. Thus once can describe a D7-brane wrapped on a 4-cycle as a sheaf on a CY, with support on the corresponding 4-cycle. We will see a more explicit realization of this idea in a class of examples below.

Although the discussion of the construction could be carried out quite far in this general language, it is more pedagogical to center on particular simple classes of this general formalism. We consider two such classes. The first is magnetised D-branes on toroidal compactifications (and quotients thereof). Namely, D-branes wrapped on products of 2-tori in $\mathbf{T^6}$, carrying constant $U(1)$ magnetic fields on their world-volume. The simple properties of the torus make the description of the bundles very easy. Regarding the models as containing B-type D9-branes, they can be described as a toroidal compactification of a 10d theory with a non-trivial gauge background. This description, reminiscent of heterotic compactifications, suggests that 4d chiral matter arises form a non-trivial index of the Dirac operator for 10d fermions charged under the gauge background. We will see how this arises in detail. Also, the mirror relation to toroidal intersecting brane models is manifest, and helps in understanding the construction.

The second class corresponds to D-branes at singularities, which can be regarded as a limit where the B-bype branes are wrapping cycles which are collapsed at a singularity of the CY threefold. The connection can be carried out quite explicitly by blowing up the singularity and taking the large volume limit of the cycles. Nevertheless, for orbifold singularities, there is a particularly simple and practical way to describe the system directly at the orbifold configuration. In this description, 4d chiral fermions arise from the orbifold projection.

Clearly, more general constructions are possible, on more general Calabi-Yau threefolds. In fact, the recent developments on the construction of holomorphic stable bundles (usually applied for heterotic models) could be exploited in B-type model building. Nevertheless, we prefer to skip these more technical constructions, and hope that the two above classes suffice to illustrate the conceptual issues on this kind of construction.

This lecture is organized as follows. In section 9 we describe magnetised D-branes and their physics in compactifications, both in toroidal models and orientifold and $\mathbf{Z_2} \times \mathbf{Z_2}$ quotients thereof. In section 10 we describe the application to building MSSM-like models. In section 11 we construct models of branes at singularities. Finally, section 12 contains our final remarks. Appendix B provides some details on the quantization of open strings in magnetised D-brane configurations.

9 Magnetised D-branes

In this section we review configurations of magnetised D9-branes in toroidal models. Useful references for this discussion are [5, 6] and [7, 8, 9, 28]. We first consider the case of toroidal compactifications, and subsequently incorporate orientifold projections and orbifold projections. The models are T-dual/mirror to the models of intersecting D6-branes in the previous lecture, in toroidal compactifications [9], toroidal orientifolds [8] and orbifolds [19].

9.1 Magnetised branes on $\mathbf{T^2}$

The computation of the boundary conditions for open strings stretched between D-branes with constant world-volume magnetic fields in flat 10d space is carried out in appendix B. The open string spectrum is easily obtained by relating the question to the T-dual side, where it is mapped to the spectrum of open strings between two D-branes at relative angle $\theta_{ab} = \tan^{-1} F_b - \tan^{-1} F_a$.

There are some additional features when considering the D-branes to wrap on a

$\mathbf{T^2}$. These are manifest when regarded in the T-dual picture, of D-branes wrapped on 1-cycles on the dual $\mathbf{T^2}$. For simplicty we may center on rectangular 2-tori, with vanishing NSNS 2-form, generalization to tilted tori and non-zero B-field are easy, but not essential.

The general configuration we are interested in consists of IIB D($2p$)-branes (labeled by an index a) multiply wrapped (with multiplicity m_a) on the $\mathbf{T^2}$, and carrying n_a units of world-volume $U(1)$ magnetic flux. Namely, we have

$$m_a \frac{1}{2\pi} \int_{\mathbf{T^2}} F_a = n_a \tag{9.1}$$

Notice that the magnetic flux is quantized in order to have a well-defined path integral for charged states. Taking $m = 1$ for simplicity, the argument is as follows. Consider the contribution to the path integral of an open string endpoint charged under the corresponding $U(1)$, running around a small topologically trivial closed loop C in the $\mathbf{T^2}$. The contribution is roughly $e^{i\int_C A_1}$. However, the gauge potential A_1 is not globally well-defined, so it is more appropriate to define the contribution as follows. Picking a 2d surface Σ whose boundary is C, e.g. the small 'inside' of C, the contribution can be written $Z = e^{i\int_\Sigma F_2}$. Now, there is another possible choice of surface Σ' with boundary C, namely the 'outside' of C, leading to a contribution $Z' e^{i\int_\Sigma' F_2}$. Since $\Sigma - \Sigma' = \mathbf{T^2}$ in homology (where the minus sign is due to a change of orientation to allow for the glueing), the result is independent of the choice if $\int_{\mathbf{T^2}} \in 2\pi\mathbf{Z}$ (since then the ratio of both contributions $Z/Z' = e^{i\int_{\Sigma-\Sigma'} F}$ is 1). For $m \neq 1$, the result follows from realizing that the gauge group is $U(m)$, broken to the diagonal $U(m)$ by the gauge background, and that this results in a effective charge of $1/n$ for the open string endpoints.

Notice that because of the CS couplings of the D($2p$)-branes, the worldvolume magnetic field induces m units of D($2p-2$)-brane charge. This is an alternative way to understand quantization of world-volume magnetic fluxes. Hence our configuration is a bound state of n units of D($2p$)-brane charge and m units of D($2p-2$)-brane charge.

Upon T-duality in the vertical direction, the configuration maps to a IIA D(2p-1)-brane wrapped on the (n, m) 1-cycle of the dual $\mathbf{T^2}$. That is, it wraps n times in the horizontal direction and m times in the vertical one. Notice that in the T-dual picture it is possible to consider the case of branes with numbers $(0, 1)$, namely wrapping just in the vertical direction. Its interpretation in the original picture of IIB magnetised D-branes deserves some discussion. Carrying out the T-duality directly, we obtain a IIB D($2p-2$)-brane sitting at a point in the $\mathbf{T^2}$. Applying the general language IIB description for (n, m)-branes, we can regard of the $(0, 1)$ D-branes (namely D($2p-2$)-branes) as gauge bundles with support just at a point in $\mathbf{T^2}$ (and hence with

zero wrapping). This is a layman's description of the mathematical objects know as sheaves, mentioned above. The bottomline is that one can work with labels (n, m) even in these extreme case.

9.2 Magnetised D-branes in toroidal compactifications

We start with the simple case of toroidal compactification, with no orientifold projection. Consider the compactification of type IIB theory on $\mathbf{T^6}$, assumed factorizable [4]

We consider sets of N_a D9-branes, labelled D9_a-branes, wrapped m_a^i times on the i^{th} 2-torus $(\mathbf{T}^2)_i$ in $\mathbf{T^6}$, and with n_a^i units of magnetic flux on $(\mathbf{T^2})_\mathbf{i}$. Namely, we turn on a world-volume magnetic field F_a for the center of mass $U(1)_a$ gauge factor, such that

$$m_a^i \frac{1}{2\pi} \int_{\mathbf{T^2_i}} F_a^i = n_a^i \tag{9.2}$$

Hence the topological information about the D-branes is encoded in the numbers N_a and the pairs (m_a^i, n_a^i) [5]

We can include other kinds of lower dimensional D-branes using this description. For instance, a D7-brane (denoted D$7_{(i)}$) sitting and a point in $\mathbf{T^2_i}$ and wrapped on the two remaining two-tori (with generic wrapping and magnetic flux quanta) is described by $(m^i, n^i) = (0, 1)$ (and arbitrary (m^j, n^j) for $j \neq i$); similarly, a D5-brane (denoted D$5_{(i)}$) wrapped on $\mathbf{T^2_i}$ (with generic wrapping and magnetic flux quanta) and at a point in the remaining two 2-tori is described by $(m^j, n^j) = (0, 1)$ for $j \neq i$; finally, a D3-brane sitting at a point in $\mathbf{T^6}$ is described by $(m^i, n^i) = (0, 1)$ for $i = 1, 2, 3$. This is easily derived by noticing that the boundary conditions for an open string ending on a D-brane wrapped on a two-torus with magnetic flux become Dirichlet for (formally) infinite magnetic field.

D9-branes with world-volume magnetic fluxes are sources for the RR even-degree forms, due to their worldvolume couplings

$$\int_{D9_a} C_{10} \quad ; \quad \int_{D9_a} C_8 \wedge \mathrm{tr}\, F_a \quad ; \quad \int_{D9_a} C_6 \wedge \mathrm{tr}\, F_a^2 \quad ; \quad \int_{D9_a} C_4 \wedge \mathrm{tr}\, F_a^3 \tag{9.3}$$

[4] Without orbifold projections, this requires a constrained choice of fluxes, stabilizing moduli at values corresponding to a factorized geometry. In our orbifolds below, such moduli are projected out by the orbifold, and hence are simply absent.

[5] Notice the change of roles of n and m as compared with other references. This however facilitates the translation of models in the literature to our language.

Consistency of the configuration requires RR tadpoles to cancel. Following the discussion in [9], leads to the conditions

$$
\begin{aligned}
\textstyle\sum_a N_a n_a^1 n_a^2 n_a^3 &= 0 \\
\textstyle\sum_a N_a m_a^1 n_a^2 n_a^3 &= 0 \quad \text{and permutations of } 1,2,3 \\
\textstyle\sum_a N_a m_a^1 m_a^2 n_a^3 &= 0 \quad \text{and permutations of } 1,2,3 \\
\textstyle\sum_a N_a m_a^1 m_a^2 m_a^3 &= 0
\end{aligned}
\tag{9.4}
$$

Which amounts to cancelling the D9-brane charge as well as the induced D7-, D5- and D3-brane charges.

Introducing for the i^{th} 2-torus the even homology classes $[\mathbf{0}]_i$ and $[\mathbf{T^2}]_i$ of the point and the two-torus, the vector of RR charges of the one D9-brane in the a^{th} stack is

$$
[\mathbf{Q}_a] \;=\; \prod_{i=1}^{3} \left(m_a^i[\mathbf{T}^2]_i + n_a^i[\mathbf{0}]_i\right) \tag{9.5}
$$

The RR tadpole cancellation conditions read

$$
\sum_a N_a[\mathbf{Q}_a] \;=\; 0 \tag{9.6}
$$

The conditions that two sets of D9-branes with worldvolume magnetic fields F_a^i, F_b^i preserve some common supersymmetry can be derived from [29]. Indeed, it is possible to compute the spectrum of open strings stretched between them and verify that it is supersymmetric if

$$
\Delta_{ab}^1 \pm \Delta_{ab}^2 \pm \Delta_{ab}^3 = 0 \tag{9.7}
$$

for some choice of signs. Here

$$
\Delta_i = \arctan\left[(F_a^i)^{-1}\right] - \arctan\left[(F_b^i)^{-1}\right] \tag{9.8}
$$

and

$$
F_a^i = \frac{n_a^i}{m_a^i R_{x_i} R_{y_i}} \tag{9.9}
$$

which follows from (9.2).

The spectrum of massless states is easy to obtain. The sector of open strings in the aa sector leads to $U(N_a)$ gauge bosons and superpartners with respect to the 16 supersymmetries unbroken by the D-branes. In the $ab+ba$ sector, the spectrum is given by I_{ab} chiral fermions in the representation $(N_a, \overline{N}_b)$, where

$$
I_{ab} = [\mathbf{Q}_a]\cdot[\mathbf{Q}_b] = \prod_{i=1}^{3}(n_a^i m_b^i - m_a^i n_b^i) \tag{9.10}
$$

is the intersection product of the charge classes, which on the basic classes $[\mathbf{0}]_i$ and $[\mathbf{T}^2]_i$ is given by the bilinear form

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{9.11}$$

The above multiplicity can be computed using the α'-exact boundary states for these D-branes [8], or from T-duality with configurations of intersecting D6-branes. We now provide an alternative derivation which remains valid in more complicated situations where the worldsheet theory is not exactly solvable. Consider for simplicity a single two-torus. We consider two stacks of N_a and N_b branes wrapped m_a and m_b times, and with n_a, n_b monopole quanta. Consider the regime where the two-torus is large, so that the magnetic fields are diluted and can be considered a small perturbation around the vacuum configuration. In the vacuum configuration, open strings within each stack lead to a gauge group $U(N_a m_a)$ and $U(N_b m_b)$ respectively, which is subsequently broken down to $U(N_a) \times U(N_b)$ by the monopole background, via the branching

$$U(N_a m_a) \times U(N_b m_b) \to U(N_a)^{m_a} \times U(N_b)^{m_b} \to U(N_a) \times U(N_b) \tag{9.12}$$

Open ab strings lead to a chiral 10d fermion transforming in the bifundamental $(\square_a, \overline{\square}_b)$ of the original $U(N_a m_a) \times U(N_b m_b)$ group. Under the decomposition (9.12) the representation splits as

$$(\square_a, \overline{\square}_b) \to (\underline{\square_a, \ldots}; \underline{\overline{\square}_b, \ldots}) \to m_a m_b (\square_a, \overline{\square}_b) \tag{9.13}$$

The 8d theory contains chiral fermions arising from these, because of the existence of a nonzero index for the internal Dirac operator (coupled to the magnetic field background). The index is given by the first Chern class of the gauge bundle to which the corresponding fermions couples. Since it has charges $(+1, -1)$ under the a^{th} and b^{th} $U(1)$'s, the index is

$$\mathrm{ind}\, \not{D}_{ab} = \int_{\mathbf{T^2}} (F_a - F_b) = \frac{n_a}{m_a} - \frac{n_b}{m_b} \tag{9.14}$$

Because of the branching (9.13), a single zero mode of the Dirac operator gives rise to $m_a m_b$ 8d chiral fermions in the $(\square_a, \overline{\square}_b)$ of $U(N_a) \times U(N_b)$. The number of chiral fermions in the 8d theory in the representation $(\square_a, \overline{\square}_b)$ of the final group is given by $m_a m_b$ times the index, namely

$$I_{ab} = m_a m_b \int_{\mathbf{T^2}} (F_a - F_b) = n_a m_b - m_a n_b \tag{9.15}$$

The result (9.10) is a simple generalization for the case of compactification on three two-tori.

An important property about these chiral fields is that they are localized at points in the internal space. From the string theory viewpoint this follows because boundary conditions for open strings with endpoints on D-branes with different magnetic fields require the absense of center of mass zero mode in the worldsheet mode expansion. From the low energy effective theory viewpoint, this follows because such strings behave as charged particles in a magnetic field. From elementary quantum mechanics, such particles feel a harmonic oscillator potential and are localized in the internal space. Excited states in the harmonic oscillator system (Landau levels) correspond to stringy oscillator (gonions [9] in T-dual picture).

Notice that the field theory argument to obtain the spectrum is valid only in the large volume limit. However, the chirality of the resulting multiplets protects the result, which can therefore be extended to arbitrarily small volumes. This kind of argument will be quite useful in the more involved situation with closed string field strength fluxes, where we do not have a stringy derivation of the results.

It is a simple exercise to verify that the above formula remains valid in situations where the open strings under consideration end on lower-dimensional D-branes. The result from directly quantizing open strings in these configurations is exactly reproduced by formally replacing the entires (n, m) associated to the transverse directions to the brane by the value $(0, 1)$. This should be interpreted as 'zero wrapping, delta function magnetic field', which is a possible description for a localized D-brane (a laymans version of the skyscraper (or delta-function) sheaf).

The relation of magnetised D-brane models to intersecting D-brane models is clear, by performing three T-dualities along say the vertical directions. This relation facilitates the computations of diverse results in the magnetised D-brane picture by translating them from the more geometric and intuitive intersecting D-brane picture. This relation will actually permeate the discussion in this lecture. It is useful nevertheless to rederive several results directly from the magnetised D-brane picture. For instance, the discussion of anomaly cancellation, as we do in the following.

Cancellation of anomalies

Following [29, 30], the gauge anomaly induced by each localized chiral fermion is cancelled by an anomaly inflow mechanism associated to the branes. Namely, the violation of charge induced by the anomaly is compensated by a charge inflow from the bulk of the intersecting branes. This explanation is sufficient in situations where the

branes are infinitely extended. In the compact context, however, within a single brane the charge 'inflowing' into an intersection must be compensated by charge 'outflowing' from other intersections. Consistency of anomaly inflow in a compact manifold imposes global constraints on the configuration.

From the point of view of the compactified four-dimensional effective field theory, which does not resolve the localization of the different chiral fermions, these global constraints correspond to cancellation of triangle gauge anomalies in the usual sense. In fact, the cancellation of cubic non-abelian anomalies for the gauge factor $SU(N_a)$ is

$$\sum_{b=1}^{K} I_{ab}\, N_b = 0 \tag{9.16}$$

Thus tadpole cancellation conditions imply the cancellation of cubic non-abelian anomalies. Namely, string theory consistency conditions imply consistency of the low-energy effective theory.

Mixed $U(1)$ anomaly cancellation

Mixed $U(1)$ anomalies are proportional to $A_{ab} = N_a I_{ab}$, and cancel by a Green-Schwarz mechanism, in analogy with intersecting brane models. We describe it directly in the picture of D9-branes with magnetic fluxes. The couplings on the world-volume of D9-branes to bulk RR fields are of the form (wedge products implied)

$$\begin{array}{ccccc} \int_{D9_a} C_0\, F_a^5 & ; & \int_{D9_a} C_2\, F_a^4 & ; & \int_{D9_a} C_4\, F_a^3 \\ \int_{D9_a} C_6\, F_a^2 & ; & \int_{D9_a} C_8\, F_a & ; & \int_{D9_a} C_{10} \end{array} \tag{9.17}$$

In order to obtain the four-dimensional version of these couplings, we define

$$\begin{array}{rclcrcl} C_2^I & = & \int_{(\mathbf{T^2})_\mathbf{I}} C_4 & ; & C_0^I & = & \int_{(\mathbf{T^2})_\mathbf{I}} C_2 \\ B_2^I & = & \int_{(\mathbf{T^2})_\mathbf{J}\times(\mathbf{T^2})_\mathbf{K}} C_6 & ; & B_0^I & = & \int_{(\mathbf{T^2})_\mathbf{J}\times(\mathbf{T^2})_\mathbf{K}} C_4 \\ B_2 & = & \int_{(\mathbf{T^2})_\mathbf{1}\times(\mathbf{T^2})_\mathbf{2}\times(\mathbf{T^2})_\mathbf{2}} C_8 & ; & B_0 & = & \int_{(\mathbf{T^2})_\mathbf{1}\times(\mathbf{T^2})_\mathbf{2}\times(\mathbf{T^2})_\mathbf{3}} C_6 \end{array}$$

where $I \neq J \neq K \neq I$ in second row. The fields C_2 and C_6, and also C_0 and C_8 are Hodge duals, while C_4 is self-dual. In four dimensions, the duality relations are

$$\begin{array}{rclcrcl} dC_0 & = & *dB_2 & ; & dB_0^I & = & *dC_2^I \\ dC_0^I & = & -*dB_2^I & ; & dB_0 & = & -*dC_2 \end{array}$$

In the dimensional reduction, one should take into account that integration of F_a along the I^{th} two-torus yields a factor m_a^I. Also, integrating the pullback of the RR forms on the (multiply wrapped) D9$_a$-brane over the I^{th} two-torus yields a factor n_a^I.

We obtain the couplings

$$\begin{array}{ccc} N_a\, n_a^1\, n_a^2\, n_a^3 \int_{M_4} C_2 \wedge F_a & ; & m_b^1\, m_b^2\, m_b^3 \int_{M_4} B_0 \wedge F_b \wedge F_b \\ N_a\, m_a^I\, n_a^J\, n_a^K \int_{M_4} C_2^I \wedge F_a & ; & m_b^J\, m_b^K\, n_b^I \int_{M_4} B_0^I \wedge F_b \wedge F_b \\ N_a\, m_a^J\, m_a^K\, n_a^I \int_{M_4} B_2^I \wedge F_a & ; & m_b^I\, n_b^J\, n_b^K \int_{M_4} C_0^I \wedge F_b \wedge F_b \\ N_a\, m_a^1\, m_a^2\, m_a^3 \int_{M_4} B_2 \wedge F_a & ; & n_b^1\, n_b^2\, n_b^3 \int_{M_4} C_0 \wedge F_b \wedge F_b \end{array}$$

As usual, the N_a prefactors arise from $U(1)_a$ normalization.

The GS amplitude where $U(1)_a$ couples to one untwisted field which propagates and couples to two $SU(N_b)$ gauge bosons is proportional to

$$\begin{aligned} -N_a\ \ n_a^1 n_a^2 n_a^3 m_b^1 m_b^2 m_b^3 + N_a \textstyle\sum_I m_a^I n_a^J n_a^K m_b^J m_b^K m_b^I - N_a \textstyle\sum_I m_a^I m_a^J n_a^K m_b^K n_b^I n_b^J + \\ N_a\, m_a^1 m_a^2 n_a^3 n_b^1 n_b^2 n_b^3 \ = N_a \textstyle\prod_I (m_a^I n_b^I - n_a^I m_b^I) = N_a I_{ab} \end{aligned} \qquad (9.18)$$

as required to cancel the residual mixed $U(1)$ anomaly.

Similarly to our discussion for intersecting brane models, the linear combinations of $U(1)$ gauge bosons with non-trivial $B \wedge F$ couplings become massive and disappear from the low energy dynamics.

9.3 Magnetised D-branes in toroidal orientifolds

We are interested in adding orientifold planes into this picture, since they are required to obtain supersymmetric fluxes. Consider type IIB on $\mathbf{T^6}$ (with zero NSNS B-field) modded out by ΩR, with $R : x_m \to -x_m$. This introduces 64 O3-planes, which we take to be all $O3^-$. It also requires the D9-brane configuration to be $\mathbf{Z_2}$ invariant. Namely, for the N_a D9_a-brane with topological numbers (m_a^i, n_a^i) we need to introduce their N_a ΩR images D$9_{a'}$ with numbers $(-m_a^i, n_a^i)$.

The RR tadpole cancellation conditions read

$$\sum_a N_a[\mathbf{Q}_a] + \sum_a N_a[\mathbf{Q}_{a'}] \, - 32\,[\mathbf{Q}_{O3}] \, = \, 0 \qquad (9.19)$$

with $[\mathbf{Q}_{O3}] = [\mathbf{0}]_1 \times [\mathbf{0}]_2 \times [\mathbf{0}]_3$. More explicitly

$$\begin{aligned} &\textstyle\sum_a N_a m_a^1 m_a^2 n_a^3 = 0 \quad \text{and permutations of } 1,2,3 \\ &\textstyle\sum_a N_a n_a^1 n_a^2 n_a^3 = 16 \end{aligned} \qquad (9.20)$$

Namely, cancellation of induced D7- and D3-brane charge. Notice that there is no net D9- or D5-brane charge, in agreement with the fact that the orientifold projection eliminates the corresponding RR fields

There is also an additional discrete constraint, which we would like to point out. It follows from a careful analysis of K-theory D-brane charge in the presence of orientifold planes. Following [31], the charge of D5-branes wrapped on some $\mathbf{T^2}$ in the presence of O3-planes is classified by a real K-theory group which is $\mathbf{Z_2}$. This statement is T-dual to the fact that D7-brane charge is $\mathbf{Z_2}$ valued in type I theory. Following [32] RR tadpole cancellation requires cancellation of the K-theory D-brane charge. Hence the total induced D5-brane charge on the D9_a-branes (without images) must be even in the above configurations. This amounts to the condition

$$\sum_a N_a m_a^1 n_a^2 n_a^3 = \text{even and permutations of } 1,2,3 \tag{9.21}$$

The condition is non-trivial, and models satisfying RR tadpole conditions in homology, but violating RR tadpole conditions in K-theory can be constructed [33]. Such models are inconsistent, as can be made manifest by introducing a D7-brane probe, on which world-volume the inconsistency manifests as a global gauge anomaly [32]. The condition is however happily satisfied by models in the literature, and also in our examples below.

The rules to obtain the spectrum are similar to the above ones, with the additional requirement of imposing the ΩR projections. This requires a precise knowledge of the ΩR action of the different zero mode sectors (in field theory language, on the harmonic oscillator groundstates for chiral fermions). The analysis is simplest in terms of the T-dual description, where it amounts to the geometric action of the orientifold on the intersection points of the D-branes. The result, which is in any case derivable in our magnetised brane picture, can be taken from [8].

The aa sector is mapped to the $a'a'$ sector, hence suffers no projection [6]. We obtain a 4d $U(N_a)$ gauge group, and superpartners with respect to the $\mathcal{N}=4$ supersymmetry unbroken by the brane.

The $ab+ba$ sector is mapped to the $b'a'+a'b'$ sector, hence does not suffer a projection. We obtain I_{ab} 4d chiral fermions in the representation $(\square_a, \overline{\square}_b)$. Plus additional scalars which are massless in the susy case, and tachyonic or massive otherwise.

The $ab'+b'a$ sector is mapped to the $ba'+a'b$. It leads to $I_{ab'}$ 4d chiral fermions in the representation $(\square_a, \square_b)$ (plus additional scalars).

The $aa'+a'a$ sector is invariant under ΩR, so suffers a projection. The result is $n_{\square\!\square}$

[6] We do not consider branes for which $a=a'$ here; they will be taken care of explicitly in the examples below.

and $n_{\Box\Box}$ 4d chiral fermions in the $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_a$, $\Box\Box_a$ representations, resp, with

$$\begin{aligned} n_{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}} &= \frac{1}{2}(I_{aa'} + 8I_{a,O3}) = -4\, m_a^1 m_a^2 m_a^3\, (n_a^1 n_a^2 n_a^3 + 1) \\ n_{\Box\Box} &= \frac{1}{2}(I_{aa'} - 8I_{a,O3}) = -4\, m_a^1 m_a^2 m_a^3\, (n_a^1 n_a^2 n_a^3 - 1) \end{aligned} \tag{9.22}$$

where $I_{a,O3} = [\mathbf{Q}_a] \cdot [\mathbf{Q}_{O3}]$.

9.4 Magnetised D-branes in the $\mathbf{T^6}/(\mathbf{Z_2} \times \mathbf{Z_2})$ orbifold

Finally, we will be interested in models with orbifold and orientifold actions. In particular, consider type IIB on the orbifold $\mathbf{T^6}/(\mathbf{Z_2} \times \mathbf{Z_2})$, modded out by ΩR. The model contains 64 O3-planes (with $-1/2$ units of D3-brane charge), and 4 $O7_i$-planes (with -8 units of $D7_i$-brane charge), transverse to the i^{th} two-torus. Their total charges are given by -32 times the classes

$$\begin{aligned} [\mathbf{Q}_{O3}] &= [\mathbf{0}_1] \times [\mathbf{0}_2] \times [\mathbf{0}_3] \quad ; \qquad [\mathbf{Q}_{O7_1}] = -[\mathbf{0}_1] \times [(\mathbf{T}^2)_2] \times [(\mathbf{T}^2)_3] \\ [\mathbf{Q}_{O7_2}] &= -[(\mathbf{T}^2)_1] \times [\mathbf{0}_2] \times [(\mathbf{T}^2)_3] \quad ; \qquad [\mathbf{Q}_{O7_3}] = -[(\mathbf{T}^2)_1] \times [(\mathbf{T}^2)_2] \times [\mathbf{0}_3] \end{aligned} \tag{9.23}$$

where the signs are related to the specific signs in the definition of the $\mathbf{Z_2} \times \mathbf{Z_2}$ action. We define $[\mathbf{Q}_{Op}] = [\mathbf{Q}_{O3}] + [\mathbf{Q}_{O7_1}] + [\mathbf{Q}_{O7_2}] + [\mathbf{Q}_{O7_3}]$. The RR charge is cancelled using magnetised D9-branes and their orientifold images (the orbifold projection maps each stack of D9-branes to itself), which carry just induced $D7_i$- and D3-brane charges. The RR tadpole conditions read

$$\sum_a N_a[\mathbf{Q}_a] + \sum_a N_a[\mathbf{Q}_{a'}] - 32\,[\mathbf{Q}_{Op}] = 0 \tag{9.24}$$

The models with magnetised D9-branes in this orientifold are T-dual to those in [19], whose main features are easily translated. The spectrum can be computed using the above techniques, taking care of the additional orbifold projections on the spectrum, or equivalently translated from [19]. The result is shown in table 5, where $I_{a,Op} = [Q_a] \cdot [Q_{Op}]$.

The discrete conditions arising from cancellation of K-theory torsion charges was carried out in [25] following ideas in [32]. Here we simply quote the result

$$\begin{aligned} \textstyle\sum_\alpha N_\alpha m_\alpha^1 m_\alpha^2 m_\alpha^3 &\in 4\mathbf{Z}, \\ \textstyle\sum_\alpha N_\alpha n_\alpha^1 n_\alpha^2 m_\alpha^3 &\in 4\mathbf{Z}, \\ \textstyle\sum_\alpha N_\alpha n_\alpha^1 m_\alpha^2 n_\alpha^3 &\in 4\mathbf{Z}, \\ \textstyle\sum_\alpha N_\alpha m_\alpha^1 n_\alpha^2 n_\alpha^3 &\in 4\mathbf{Z}. \end{aligned} \tag{9.25}$$

Sector	**Representation**
aa	$U(N_a/2)$ vector multiplet 3 Adj. chiral multiplets
$ab+ba$	I_{ab} $(\square_a, \overline{\square}_b)$ fermions
$ab'+b'a$	$I_{ab'}$ $(\square_a, \square_b)$ fermions
$aa'+a'a$	$\frac{1}{2}(I_{aa'} - 4I_{a,Op})$ $\square\!\square$ fermions $\frac{1}{2}(I_{aa'} + 4I_{a,Op})$ $\begin{array}{c}\square\\[-4pt]\square\end{array}$ fermions

Table 5: General chiral spectrum on generic magnetised D9$_a$-branes in the ΩR orientifold of $\mathbf{T^6}/(\mathbf{Z_2} \times \mathbf{Z_2})$. The models may contain additional non-chiral pieces which we ignore here. In supersymmetric situations, scalars combine with the fermions given above to form chiral supermultiplets.

Notice that these are actually $\mathbf{Z_2}$ charge constraints, since N_α are already even integers. In K-theory language, we are imposing the global cancellation of $\mathbf{Z_2}$ RR charges, carried by fractional $D5_i - \overline{D5}_i$ and $D9 - \overline{D9}$ pairs.

Second, we would like to construct models free of NSNS tadpoles, that is, such that the tensions of the objects in the configuration do also cancel. In a magnetised D-brane configuration with vanishing RR tadpoles, this can be achieved by requiring that every set of D-branes preserves the same $\mathcal{N} = 1$ supersymmetry unbroken by the orientifold. This usually implies a condition on the Kähler parameters, which in the present context reads[7]

$$\sum_i \tan^{-1}\left(\frac{m_a^i A_i}{n_a^i}\right) = 0, \tag{9.26}$$

where A_i is the area of $(\mathbf{T}^2)_i$ in α' units. A small deviation from this condition can be understood as a non-vanishing FI-term in the $D = 4$ effective theory [19, 15].

10 MSSM-like models

The most practical way to deal with the model building applications of magnetised D-brane models is to translate them from the similar discussion for intersecting D-branes (as is done in the literature). Hence, here we simply recover the models in lecture 1.

[7] This formula is actually only valid for the case $n_a^i \geq 0$. See below for some other important cases.

Again, the idea is to embed in a globally consistent way a local structure of D-branes leading to a MSSM-like structure. Here we now interpret the integers (n, m) as wrapping numbers and magnetic monopole quanta of the corresponding D-branes. Again, our general arguments on the computation of the spectrum guarantee that any model containing such subsector will reproduce a gauge theory with MSSM like chiral spectrum (plus additional exotics, whose structure depends on the detailed set of additonal branes in the model).

To provide one example, we simply present a generalization of the example studied in the previous lecture, shown in table 10 [25]. It is easy to check that these magnetic numbers satisfy the tadpole conditions, by simply imposing

$$g^2 + N_f = 14. \tag{10.1}$$

Notice that this give us an upper bound for the number of generations, namely $g \leq 3$.

The gauge group of this model is

$$SU(3) \times SU(2) \times SU(2) \times U(1)_{B-L} \times [U(1)' \times USp(8N_f)]\,, \tag{10.2}$$

where $U(1)' = [U(1)_a + U(1)_d] - 2g\,[U(1)_{h_1} - U(1)_{h_2}]$ is the only Abelian factor that, besides $U(1)_{B-L}$, survives the generalised Green-Schwarz mechanism. The $USp(8N_f)$ gauge group will only remain as such when all the D3-branes are placed on top of an orientifold singularity. Eventually, by moving them away it can be Higgsed down to $U(1)^{2N_f}$. Of course, the new D-brane sectors will also imply new chiral matter, some of it charged under the Left-Right MSSM gauge group. We will explain below how to deal with these chiral exotics.

Clearly the most interesting solution is $n = 0$, $g = 3$, $N_f = 5$, on which we center in what follows. Indeed, this corresponds to the example studied in previous lecture.

In agreement with its T-dual, in order to satisfy the $N = 1$ susy conditions, we need

$$\begin{gathered} A_2 = A_3 \\ \tan^{-1}(A_1/2) + \tan^{-1}(A_2/3) + \tan^{-1}(A_3/4) = \pi \end{gathered} \tag{10.3}$$

These fix the Kähler parameters A_i in terms of the overall volume $A_1A_2A_3$. This 'fixing' of moduli should not be thought of as a dynamical stabilisation process. As explained in [19], changes in the Kahler moduli lead to Fayet-Illiopoulos terms on the D-branes. They force some of the charged scalars in ab sector to acquire a vev and break gauge symmetry, but preserving $N = 1$ supersymmetry. This corresponds to recombining some of the branes in teh model into bound states. Hence, the condition

N_α	(n_α^1, m_α^1)	(n_α^2, m_α^2)	(n_α^3, m_α^3)
$N_a = 6$	$(1,0)$	$(g,1)$	$(g,-1)$
$N_b = 2$	$(0,1)$	$(1,0)$	$(0,-1)$
$N_c = 2$	$(0,1)$	$(0,-1)$	$(1,0)$
$N_d = 2$	$(1,0)$	$(g,1)$	$(g,-1)$
$N_{h_1} = 2$	$(-2,1)$	$(-3,1)$	$(-4,1)$
$N_{h_2} = 2$	$(-2,1)$	$(-4,1)$	$(-3,1)$
$8N_f$	$(1,0)$	$(1,0)$	$(1,0)$

Table 6: D-brane magnetic numbers giving rise to an $\mathcal{N} = 1$ MSSM-like model.

(10.3) correspond simply to imposing that the model is supersymmetric with branes as they stand in table (10).

The resulting low-energy spectrum and its discussion is completely identical to those in lecture 1, hence we skip these details here.

11 D-branes at singularities

As mentioned in the introduction, D-branes at orbifold singularities provide another very tractable class of B-type brane model. We describe some of its main features in this section. As mentioned the orbifold configuration does not really correspond to a large volume regime (since there are collapsed cycles, which make α' corrections important). Hence, the system should be studied by directly quantizing open strings in the orbifold configuration. This is easily done by applying the techniques developed in [34]. It is however important to point out that certain topological and protected quantities (like the chiral spectrum and the world-volume superpotential) can be computed in the large volume limit and reliably extrapolated to the orbifold configuration, in the spirit of [35]. For $\mathbf{C^3}/\mathbf{Z_3}$ this analysis has been carried out in [36], where the identification of the appropriate large volume bundles for the involved B-type branes was carried out. We skip this interesting discussion, and work directly at the orbifold point.

For concreteness, let us center of a stack of n D3-branes sitting at the Origin of a $\mathbf{C^3}/\mathbf{Z_N}$ orbifold singularity. These models were first Considered in [34]. The $\mathbf{Z_N}$ generator θ acts on the three complex coordinates of $\mathbf{C^3}$ as follows

$$(z_1, z_2, z_3) \to (e^{2\pi I\, a_1/N} z_1, e^{2\pi i\, a_2/N} z_2, e^{2\pi i\, a_3/N} z_3) \tag{11.1}$$

where the $a_i \in \mathbf{Z}$ in order to have and order N action [8]. We will center on orbifolds that preserve some supersymmetry, hence their holonomy must be in $SU(3)$ and thus we require $a_1 \pm a_2 \pm a_3 = 0$ mod N, for some choice of signs.

The closed string spectrum in the configuration can be easily obtained. However, this sector is uncharged under the gauge group on the D-brane world-volume, so we skip its discussion.

Concerning the open string sector, the main observation is that there are no twisted sectors. This follows because the definition of twisted sectors in closed strings made use of the periodicity in the worldsheet direction σ, and this is not allowed in open strings. Hence, the spectrum of open strings on a set of D3-branes at a $\mathbf{C^3}/\mathbf{Z_N}$ orbifold singularity is simply obtained by considering the open string spectrum on D3-branes in flat space $\mathbf{C^3}$, and keeping the $\mathbf{Z_N}$-invariant ones. Each open string state on D3-branes in flat space is given by a set of oscillators acting on the vacuum, and an $n \times n$ Chan-Paton matrix λ encoding the $U(n)$ gauge degrees of freedom. The action of θ on one such open string state is determined by the action on the corresponding set of oscillators and the action on the Chan-Paton matrix. For concreteness,let us center on massless states. The eigenvalues of the different sets of oscillators for these states are

Sector	State	θ eigenvalue
NS	$(0,0,0,\pm)$	1
	$(\underline{+,0,0},0)$	$e^{2\pi i\, a_i/N}$
	$(\underline{-,0,0},0)$	$e^{-2\pi i\, a_i/N}$
R	$\pm\frac{1}{2}(+,+,+,-)$	1
	$\frac{1}{2}(\underline{-,+,+},+)$	$e^{2\pi i\, a_i/N}$
	$\frac{1}{2}(\underline{+,-,-},-)$	$e^{-2\pi i\, a_i/N}$

The eigenvalues can be described as $e^{2\pi i\, r\cdot v}$, where r is The $SO(8)$ weight and $v = (a_1, a_2, a_3, 0)/N$. The above action can easily be understood by decomponsing the $SO(8)$ representation with respect to the $SU(3)$ subgroup in which the $\mathbf{Z_N}$ is embedded. In fact we have $8_V = 3+\overline{3}+1+1$, and $8_C = 3+\overline{3}+1+1$, and noticing that (11.1) defines the action on the representation 3. Notice that the fact that bosons and fermions have the same eigenvalues reflects the fact that the orbifold preserves $\mathcal{N}=1$ supersymmetry on the D-brane world-volume theory. In fact we see that the different states group into a vector multiplet V, with eigenvalue 1, and three chiral multiplets, Φ_i with eigenvalue $e^{2\pi i\, a_i/N}$.

[8] One also needs $N\sum_i a_i =$ even (so that the quotient is a spin manifold, i.e. allows spinors to be defined).

On the other hand, the action of θ on the Chan-Paton degrees of freedom corresponds to a $U(n)$ gauge transformation. This is defined by a unitary order N matrix $\gamma_{\theta,3}$, which without loss of generality we can diagonalize and write in the general form

$$\gamma_{\theta,3} = \mathrm{diag}\,(1_{n_0}, e^{2\pi i/N} 1_{n_1}, \ldots, e^{2\pi i(N-1)/N} 1_{n_{N-1}}) \tag{11.2}$$

with $\sum_{a=0}^{N-1} n_a = n$. The action on the Chan-Paton wavefunction (which transforms in the adjoint representation) is

$$\lambda \to \gamma_{\theta,3} \lambda \gamma_{\theta,3}^{-1} \tag{11.3}$$

We now have to keep states invariant under the combined action of θ on the oscillator and Chan-Paton piece. For states in the $\mathcal{N}=1$ vector multiplet, the action on the oscillators is trivial, hence the surviving states correspond to Chan-Paton matrices satisfying the condition

$$\lambda = \gamma_{\theta,3} \lambda \gamma_{\theta,3}^{-1} \tag{11.4}$$

The surviving states correspond to a block diagonal matrix. The gauge group is easily seen to be

$$U(n_0) \times \ldots \times U(n_{N-1}) \tag{11.5}$$

For the i^{th} chiral multiplet Φ_i, the oscillator part picks up a factor of $e^{2\pi i\, a_i/N}$. So surviving states have Chan-Paton wavefunction must satisfy

$$\lambda = e^{2\pi i\, a_i/N} \gamma_{\theta,3} \lambda \gamma_{\theta,3}^{-1} \tag{11.6}$$

The surviving multiplets correspond to matrices with entries in a diagonal shifted by a_i blocks. It is easy to see that the surviving multiplets transform in the representation

$$\sum_{i=1}^{3} \sum_{a=0}^{N-1} (\Box_a, \overline{\Box}_{a+a_i}) \tag{11.7}$$

We clearly see that in general the spectrum is chiral, so we have achieved the construction of D-brane configurations with non-abelian gauge symmetries and charged chiral fermions. Moreover, we see that in general the different fermions have different quantum numbers. The only way to obtain a replication of the fermion spectrum (i.e. a structure of families, like in the Standard Model), we need some of the a_i to be equal (modulo N). The most interesting example is obtained for the $\mathbf{C^3/Z_3}$ singularity, with $v = (1, 1, -2)/3$. The spectrum on the D3-brane world-volume is given by

$$\begin{array}{ll} \mathcal{N}=1 \text{ Vect.Mult.} & U(n_0) \times U(n_1) \times U(n_2) \\ \mathcal{N}=1 \text{ Ch.Mult.} & 3\,[\,(n_0, \overline{n}_1, 1) + (1, n_1, \overline{n}_2) + (\overline{n}_0, n_1, 1) \end{array} \tag{11.8}$$

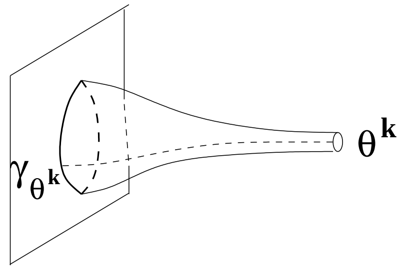

Figure 24: D3-branes at singularities are charged under RR forms in the θ^k twisted sector, via a disk diagram. Worldsheet degrees of freedom suffer the action of θ^k as they go around the cut, shown as a dashed line. The amplitude is proportional to $\mathrm{tr}\,\gamma_{\theta^k}$.

we see there is a triplication of the chiral fermion spectrum. Hence in this setup the number of families is given by the number of complex planes with equal eigenvalue.

We would like to point out that, as usual in models with open strings, there exist some consistency conditions, known as cancellation of RR tadpoles. Namely, there exist disk diagrams, see figure 24, which lead to the coupling of D-branes at singularities to RR fields in the θ^k twisted sector. When the θ^k twist has the origin as the only fixed point, the corresponding RR fields do not propagate over any dimension transverse to the D-brane. This implies that they have compact support, and Gauss law will impose the corresponding charges must vanish, namely that the corresponding disk diagrams cancel. The coefficient of the disk diagram is easy to obtain: from the figure, we see that any worldsheet degree of freedom must suffer the action of θ^k as it goes around the closed string insertion. In particular it means that the Chan-Paton degrees of freedom suffer the action of $\gamma^k_{\theta^k,3=(\gamma_{\theta,3}}$ as they go around the boundary. Hence the disk amplitude is proportional to $\mathrm{tr}\,\gamma_{\theta^k,3}$, and the RR tadpole condition reads

$$\mathrm{Tr}\,\gamma_{\theta^k,3} = 0 \quad ,\text{for } ka_i \neq 0 \bmod N \tag{11.9}$$

For instance, for the above $\mathbf{Z_3}$ model these constraint require $n_0 = n_1 = n_2$ In general, the above constrains ensure that the 4d chiral gauge field theory on the volume of the D3-branes is free of anomalies.

Clearly the above model is not realistic. However, more involved models of this kind, with additional branes (like D7-branes, also passing through thesingularity), can lead to models much closer to the Standard Model, see [37], also [38].

Following the general arguments in section 3.1, the strategy to obtain a field theory with standard model gauge group from the $\mathbf{Z_3}$ singularity is to choose a D3-brane

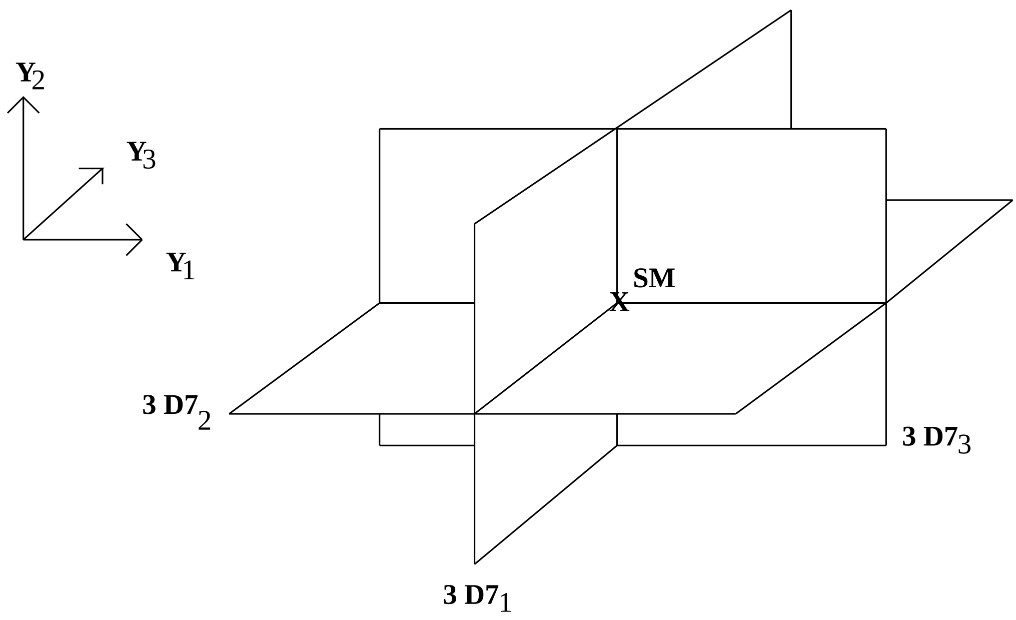

Figure 25: A non-compact Type IIB $\mathbf{Z_3}$ orbifold singularity yielding SM spectrum. Six D3 branes sit on top of a $\mathbf{Z_3}$ singularity at the origin. Tadpoles are canceled by the presence of intersecting D7-branes with their worldvolumes transverse to different complex planes.

Chan-Paton embedding

$$\gamma_{\theta,3} \quad = \quad \mathrm{diag}\,(\mathrm{I}_3, \alpha \mathrm{I}_2, \alpha^2 \mathrm{I}_1) \tag{11.10}$$

The simplest way to satisfy the tadpole conditions is to introduce only one set of D7-branes, e.g. D7$_3$-branes, with Chan-Paton embedding $u_0^3 = 0$, $u_0^1 = 3$, $u_0^2 = 6$. The gauge group on the D3-branes is $U(3) \times U(2) \times U(1)$, whereas in the D7$_3$-branes is $U(3) \times U(6)$ on each. Note that, before compactification, the latter behave as global symmetries in the worldvolume of the D3-branes. The D7$_3$-branes group can be further broken by global effects, since the corresponding branes are extended along some internal dimensions.

An alternative procedure to obtain a smaller group on the D7-branes is to use all three kinds of D7-branes, as depicted in Figure 25. For instance, a very symmetrical choice consistent with (11.10) is $u_0^r = 0$, $u_1^r = 1$, $u_1^r = 2$, for $r = 1, 2, 3$. Each kind of D7-brane then carries a $U(1) \times U(2)$ group.

The spectrum for this latter model is given in table 7 (for later convenience we have also included states in the $7_r 7_r$ sectors; their computation is analogous to the computation of the 33 sector). In the last column we give the charges under the

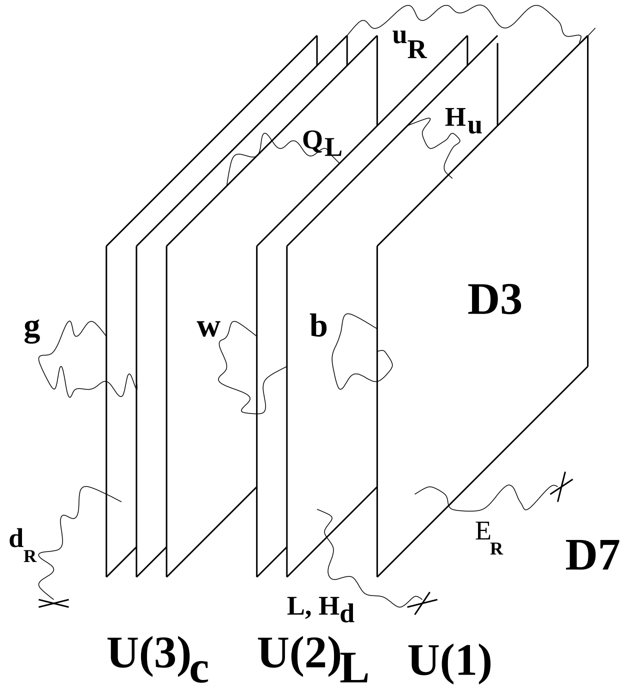

Figure 26: D-brane configuration of a SM $\mathbf{Z_3}$ orbifold model. Six D3-branes (with worldvolume spanning Minkowski space) are located on a $\mathbf{Z_3}$ singularity and the symmetry is broken to $U(3) \times U(2) \times U(1)$. For the sake of visualization the D3-branes are depicted at different locations, even though they are in fact on top of each other. Open strings starting and ending on the same sets of D3-branes give rise to gauge bosons; those starting in one set and ending on different sets originate the left-handed quarks, right-handed U-quarks and one set of Higgs fields. Leptons, and right-handed D-quarks correspond to open strings starting on some D3-branes and ending on the D7-branes (with world-volume filling the whole figure).

anomaly-free combination

$$Y = -\left(\frac{1}{3}\, Q_3 \; + \; \frac{1}{2}\, Q_2 \; + \; Q_1 \right) \tag{11.11}$$

As promised, it gives the correct hypercharge assignments for standard model fields. A pictorial representation of this type of models is given in Figure 26.

Matter fields	Q_3	Q_2	Q_1	$Q_{u_1^r}$	$Q_{u_2^r}$	Y
33 sector						
$3(3,2)$	1	-1	0	0	0	1/6
$3(\bar{3},1)$	-1	0	1	0	0	-2/3
$3(1,2)$	0	1	-1	0	0	1/2
$\mathbf{37}_r$ sector						
$(3,1)$	1	0	0	-1	0	-1/3
$(\bar{3},1;2')$	-1	0	0	0	1	1/3
$(1,2;2')$	0	1	0	0	-1	-1/2
$(1,1;1')$	0	0	-1	1	0	1
$\mathbf{7}_r\mathbf{7}_r$ sector						
$3(1;2)'$	0	0	0	1	-1	0

Table 7: Spectrum of $SU(3)\times SU(2)\times U(1)$ model. We present the quantum numbers under the $U(1)^9$ groups. The first three $U(1)$'s come from the D3-brane sector. The next two come from the D7_r-brane sectors, written as a single column with the understanding that e.g. fields in the $\mathbf{37}_r$ sector are charged under the $U(1)$ in the $\mathbf{7_r 7_r}$ sector.

We find it remarkable that such a simple configuration produces a spectrum so close to that of the standard model. In particular, we find encouraging the elegant appearance of hypercharge within this framework, as the only linear combination of $U(1)$ generators which is naturally free of anomalies in systems of D3-branes at orbifold singularities.

The model constructed above, once embedded in a global context, may provide the simplest semirealistic string compactifications ever built. Indeed, in Section 4 we will provide explicit compact examples of this kind. Let us once again emphasize that, however, many properties of the resulting theory will be independent of the particular global structure used to achieve the compactification, and can be studied in the non-compact version presented above, as we do in Section 5.

One may wonder about the mirror version of this construction, which should be in terms of intersecting D6-branes in the mirror geometry. This has been worked out in [39, 13], to which we refer the reader for details.

12 Final remarks

We have described a new class of type IIB compactification leading to interesting chiral physics in four dimensions. They moreover have a beautiful relation to intersecting brane models via mirror symmetry. Notice that, despite the equivalence in string theory of both kinds of constructions, very often one side is far simpler than the other, and allows for more efficient discussion of the physics. For instance, in toroidal models the discussion in terms of intersecting branes can be considered more intuitive and pedagogial (and that is why they went first in these lectures). However, in general Calabi-Yaus it is far simpler to construct holomorphic stable bundles than to construct special lagrangian submanifolds, hence the discussion of model building in terms of type IIB theory is more practical.

Another important point is that the equivalence of Calabi-Yau compactifications with A- and B- branes does not hold (in this form) in the presence of fluxes, to be introduced in the coming lecture. Hence, it is extremely useful to have a well-developed intuition about each of these two pictures independently in order to address further developments.

Lecture 3. Flux compactifications and D-brane systems

13 Introduction

Models in the previous lecture center essentially on the construction of D-brane configurations leading to interesting guage sectors from the corresponding open strings. The closed string sector is however also relavant for low energy dynamics, and leads to an additional set of open questions. The main such question corresponds to the existence of moduli, massless fields with flat potential and hence undetermined vevs. Stabilization of these moduli (namely, providing them with a potential which fixes their vevs and gives them masses) is an important theoretical question, of immediate relevance to phenomenological model building. A recent proposal to address the issue, namely the study of compactification with non-trivial backgrounds for the field strengths of different 10d p-form fields has been put forth. The proposal is vary natural, since it involves considering the most general backgrounds available for compactification (there is no reason, beyond seemingly simplicity, to have stuck to pure metric backgrounds in the CY era). In addition, the introduction of such fluxes leads naturally to a nice and tractable way to break supersymmetry (another one of the important open questions).

Much work is being devoted to understanding flux compactifications, and in fact several speakers in this conference have dealt with the topic. We refer the reader to those reference for further details. In this lecture we would like to describe different issues and constructions that arise when combining flux compactifications with D-brane model building. For simplicity an concreteness we center on the particular setup of type IIB CY compactifications with 3-form fluxes. Similar analysis should (and could in principle) be carried out for other setups.

14 Flux compactifications

Compactifications of type II theories (or orientifolds thereof) with NSNS and RR field strength fluxes have been considered, among others, in e.g. [41, 42, 43, 44, 45]. In this section we review properties of type IIB compactifications with 3-form fluxes. The reason to center on type IIB theory is that the framework and results is more established, while for type IIA compactifications strong activity on setting the basic arena is in progress [46].

14.1 Consistency conditions and moduli stabilization

Type IIB compactification on a Calabi-Yau threefold X_6 with non-trivial NSNS and RR 3-form field strength backgrounds H_3, F_3 have been extensively studied. In particular the analysis in [43] provided in a very general setup the consistency conditions such fluxes should satisfy. They must obey the Bianchi identities

$$dF_3 = 0 \quad dH_3 = 0 \tag{14.1}$$

and they should be properly quantized, namely for any 3-cycle $\Sigma \subset \mathbf{X}_6$

$$\frac{1}{(2\pi)^2\alpha'}\int_\Sigma F_3 \in \mathbf{Z} \quad ; \quad \frac{1}{(2\pi)^2\alpha'}\int_\Sigma H_3 \in \mathbf{Z} \tag{14.2}$$

The fluxes hence define integer 3-cohomology classes in $H^3(X_6, \mathbf{Z})$.

A subtlety in flux quantization in toroidal orientifolds was noticed in [44]. Namely, if flux integrals along some 3-cycle are integer but odd, consistency requires the corresponding 3-cycle to pass through an odd number of exotic O3-planes (studied in [47]). For simplicity we restrict to the case where all flux integrals are even integers.

An important observation in [43] is that, in order to avoid previous no-go theorems about the existence of configurations of fluxes satisfying the equations of motion, it is crucial to include orientifold 3-planes in the compactification, so we consider type IIB orientifolds with these objects. The simplest way to understand the need of these objects, is to notice the type IIB supergravity Chern-Simons coupling

$$\int_{M_4\times X_6} H_3 \wedge F_3 \wedge C_4 \tag{14.3}$$

where C_4 is the IIB self-dual 4-form gauge potential. This coupling implies that upon compactification the flux background contributes to a tadpole for C_4, with positive coefficient $N_{\rm flux}$ [9] (in D3-brane charge units). Moreover, fluxes contribute positively to the energy of the configuration, due to the 2-form kinetic terms. The only way to cancel these tadpoles is to introduce objects with negative RR C_4-charge and negative tension, to cancel both the RR tadpole and also to compensate the vacuum energy of the configuration. Having O3-planes in the configuration, it is natural to consider the possibility of adding N_{Q_3} explicit D3-branes as well. The RR tadpole cancellation constraint hence reads

$$N_{Q_3} + N_{\rm flux} + Q_{O3} = 0 \tag{14.4}$$

[9]This is so if we require the flux to preserve the same supersymmetry as the O3-planes. This is implicit in the literature, since fluxes leading to negative RR 4-form charge would lead to uncancelled NSNS tadpoles, and hence to non-Poincare invariant 4d theories.

we normalize charge such that a D3-brane in covering space has charge +1. With this convention an O3-plane has charge $-1/2$, and

$$N_{\rm flux} \;=\; \frac{1}{(4\pi^2\alpha')^2}\int_{X_6} H_3\wedge F_3 \;=\; \frac{1}{(4\pi^2\alpha')^2}\,\frac{i}{2\phi_I}\int_{X_6} G_3\wedge \overline{G}_3 \tag{14.5}$$

where ϕ_I is the imaginary part of the IIB complex coupling $\phi = a + i/g_s$, and

$$G_3 = F_3 - \phi H_3 \tag{14.6}$$

Finally, in order to satisfy the equations of motion, the flux combination G_3 must be imaginary self-dual with respect to the Hodge operation defined in terms of the Calabi-Yau metric in X_6

$$*_6\, G_3 \;=\; i\, G_3 \tag{14.7}$$

This can be regarded as the minimization of the scalar potential following form the flux-induced superpotential [48]

$$W \;=\; \int_{X_6} G_3\wedge\Omega \tag{14.8}$$

Given these conditions, the analysis in [43] guarantees the existence of a consistent supergravity solution for the different relevant fields in the configuration, metric, and 4-form, which have the form of a warped compactification (similar to a black 3-brane solution, since the same fields are sourced). [10]

In particular the metric and 5-form background take the form

$$\begin{aligned} ds^2 \;&=\; Z(y^m)^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu \;+\; Z(y^m)^{1/2} g^{CY}_{mn}dy^m dy^n \\ F_5 \;&=\; d\xi_4 + *d\xi_4 \qquad \xi_4 \;=\; \frac{1}{4\kappa Z}\, dx^0dx^1dx^2dx^3 \end{aligned} \tag{14.9}$$

where g^{CY} is the $SU(3)$ holonomy metric of the Calabi-Yau, and the warp factor is given by the solution to the Laplace equation

$$-\nabla^2 Z(y^m) \;=\; (2\pi)^4\alpha'^2 g_s\rho_3(y^m) \;+\; \frac{\kappa^2}{12}G_{mnp}(G^{mnp})^* \tag{14.10}$$

[10]Interestingly, the above configurations lead, at the classical supergravity level, to 4d Poincare invariant solutions (i.e. vanishing cosmological constant) and flat potential for the overal Kahler parameter, even in the absence of supersymmetry [43], although α' corrections [49] in general spoil this property. Even with supersymmetry, spacetime non-perturbative effects [63] may generate supersymmetry breaking effects and generate a cosmological constant. We will have nothing to say about this familiar problem.

where ρ_3 is the density of localized sources (D3-branes and O3-planes), and the laplacian and raising of indices are carried out with the CY metric. Notice that the integrability condition of this equation is the expression (14.4).

We remark that the above condition (14.7) should not be regarded as an additional constraint on the fluxes. Rather, for a set of fluxes in a fixed topological sector (i.e. in a fixed cohomology class), eq. (14.7) is a condition on the scalar moduli which determine the internal metric. The scalar potential is minimized at points in moduli space where (14.7) is satisfied, while fluxes induce a positive scalar potential at other points. Hence introduction of fluxes leads to a natural mechanism to stabilize moduli. Explicit expressions will be discussed later on, for the moment let us state the result in [43, 44] that generically all complex structure moduli and most Kahler moduli are stabilized by this mechanism.

14.2 Supersymmetry

The conditions for a configurations with 3-form fluxes to preserve some supersymmetry have been studied in [50], and applied in explicit constructions in [44]. Let us review these results.

The 10d $\mathcal{N}=2$ type IIB real supersymmetry transformation parameters ϵ_L, ϵ_R, can be gathered into a complex one $\epsilon = \epsilon_L + i\epsilon_R$. It is chiral in 10d, satisfying $\Gamma_{10d}\epsilon = -\epsilon$, with $\Gamma_{10d} = \gamma^0 \dots \gamma^9$. Compactification on $\mathbf{X_6}$ splits this spinor with respect to $SO(6) \times SO(4)$ e.g. as

$$\epsilon_L = \xi \otimes u + \xi^* \otimes u^* \tag{14.11}$$

where ξ is a 6d chiral spinor $\Gamma_{6d}\xi = -\xi$, and u is a 4d chiral spinor $\Gamma_{4d}u = u$. For $\mathbf{X_6}$ of generic $SU(3)$ holonomy only one component of ξ is covariantly constant and provides susy transformations in 4d.

On the other hand, the presence of the O3-planes and D3-branes in the background preserves only those ϵ satisfying

$$\epsilon_R = -\gamma^4 \dots \gamma^9 \epsilon_L \tag{14.12}$$

Such spinors are of the form $\epsilon = 2\xi \otimes u$.

The conditions for a flux to preserve a supersymmetry associated to a particular spinor component of ξ are [50]

$$G\xi = 0 \quad ; \quad G\xi^* = 0 \quad ; \quad G\gamma^m \xi^* = 0 \tag{14.13}$$

where $G = \frac{1}{6} G_{mnl} \gamma^{[m} \gamma^n \gamma^{l]}$.

To understand this a bit better, let us introduce complex coordinates $z^i, \overline{z}^i$, where the gamma matrix algebra reads

$$\{\gamma^i, \gamma^j\} = \{\gamma^{\overline{i}}, \gamma^{\overline{j}}\} = 0 \quad ; \quad \{\gamma^i, \gamma^{\overline{j}}\} = \delta^{i\overline{j}} \tag{14.14}$$

Introducing the highest weight state ξ_0 satisfying $\gamma^{\overline{i}} \xi_0 = 0$, the spinor representation is

State	$SO(6)$ weight	State	$SO(6)$ weight
ξ_0	$\frac{1}{2}(+++)$	$\gamma^i \xi_0$	$\frac{1}{2}(\underline{-++})$
$\gamma^1 \gamma^2 \gamma^3 \xi_0$	$\frac{1}{2}(---)$	$\gamma^i \gamma^j \xi_0$	$\frac{1}{2}(\underline{--+})$

The O3-planes preserve ξ_0 and $\gamma^{ij} \xi_0$. Of these, a general Calabi-Yau (on which z_i are complex coordinates) preserves only ξ_0, since it is $SU(3)$ invariant.

The conditions that a given flux preserves ξ_0, can be described geometrically [50, 44] as

a) G_3 is of type $(2, 1)$ in the corresponding complex structure

b) $G_3 \wedge J = 0$ where J is the Kahler form

For explicit discussion of these conditions see below. Notice that a G_3 flux which is not $(2, 1)$ in a complex structure, may still be supersymmetric if it preserves other spinor ξ_0' (although it does not preserve ξ_0). In such case, G_3 would be of type $(2, 1)$ in a different complex structure where ξ_0' is the spinor annihilated by the new $\gamma^{\overline{i}'}$.

Since the techniques to find consistent (possibly supersymmetric) fluxes at particular values of the stabilized moduli (and vice versa) have been discussed in the literature, we will not dwelve into their discussion.

In a generic Calabi-Yau compactification, where the holonomy is $SU(3)$ but not a subgroup of $SU(2)$), there is a unique component of the spinor which is covariantly constant with respect to the spin connection. In this kind of situations, there is a preferred complex structure, that in which that spinor is annihilated by $\gamma^{\overline{i}}$. We will be interested in fluxes that preserve that spinor (so we call it ξ_0) and then G_3 should be $(2, 1)$ in that complex structure.

In toroidal compactifications, the holonomy preserves several spinors, so we can play with different complex structures. In toroidal orbifolds, the orbifold projections in general project out some spinor components, leaving others invariant (just one if the orbifold group is in $SU(3)$ but not in $SU(2)$). In such case some complex structure is preferred, in analogy with the Calabi-Yau case.

14.3 Fluxes in the $\mathbf{T^6}/(\mathbf{Z_2} \times \mathbf{Z_2})$

In the following discussion, we center on compactifications on orientifolds of $\mathbf{T^6}/(\mathbf{Z_2} \times \mathbf{Z_2})$. Hence, we would like to briefly describe the specific set of fluxes we are to introduce on this manifold.

For simplicity of the geometry, we will work in the orbifold limit. This implies that there are collapsed 3-cycles at the orbifold singularities [11]. Since they are not in the large volume regime, we do not introduce fluxes along these 3-cycles and simply consider untwisted fluxes. These are flux choices which exist in the parent toroidal theory and are invariant under the $\mathbf{Z_2} \times \mathbf{Z_2}$ symmetry. This requires that the 3-forms have one leg along each of the $\mathbf{T^2}$'s in the $\mathbf{T^6} = \mathbf{T^2} \times \mathbf{T^2} \times \mathbf{T^2}$. As mentioned before, we will center on ISD G_3 fluxes, which lead to 4d Minkowski backgrounds. This implies that G_3 should have just $(2,1)$ and $(0,3)$ components. Hence in complex notation, G_3 is a linear combination of the forms

$$d\overline{z}_1 d\overline{z}_2 d\overline{z}_3 \quad , \quad dz_1 dz_2 d\overline{z}_3 \quad , \quad dz_1 d\overline{z}_2 dz_3 \quad , \quad d\overline{z}_1 dz_2 dz_3 \tag{14.15}$$

Of course, a choice of G_3 is not a consistent choice for arbitrary choices of dilaton and complex structure moduli. We have yet to impose the quantization conditions of F_3 and H_3. In fact, the condition that flux quantization imposes on moduli is simply a rephrasing of the statement that the flux potential stabilizes some of the moduli.

About quantization, we consider fluxes F_3, H_3 quantized to even integers on $\mathbf{T^6}/(\mathbf{Z_2} \times \mathbf{Z_2})$, to avoid exotic O3-planes. Since we will work on the coverin space, namely the parent $\mathbf{T^6}$, this will require quantizing our toroidal fluxes to multiples of 8. This implies that the contribution of the fluxes to the 4-form tadpole should be a multiple of 64, namely $N_{\text{flux}} = 64n$.

15 Model building with fluxes and D-branes

Since we are interested in using flux compactification of type IIB theory, we need to use the model building techniques in lecture 2. Consequently there are two classes of simple models that may be constructed, namely using magnetised D-branes and using D-branes at singularities. In this section we propose some illustrative examples of both kinds of construction.

[11] Recall we are working with the choice of discrete torsion that leads to hodge numbers $(h_{11}, h_{2,1}) = (3, 51)$, namely 3-cycles in twisted sectors.

Notice that the requirement that the point in moduli space at which moduli stabilize should be in the regime of the classical supergravity approximation is not automatic. If this is not the case, the analysis of the stabilization mechanism is note really justified, and the examples constructed should be regarded as toy models of the kind of computations to be carried out in more flexible setups, allowing for the required fine-tuning.

15.1 A model with magnetised D-branes

In this section we construct some $\mathcal{N} = 1$ and $\mathcal{N} = 0$ models with ISD 3-form fluxes and configurations of magnetised D-branes. Given the material presented in previous sections our strategy should be clear by now. We consider Type IIB string theory, compactified on $\mathbf{T^6}/(\mathbf{Z_2} \times \mathbf{Z_2})$. In order to be a consistent string theory construction, we should satisfy the RR tadpole conditions, including the contribution form th 3-form flux. In addition, in order to have a compactification to 4d Minkowski space (without tadpoles for NSNS fields) we consider D-brane configurations satisfying the 'angle conditions' (so that their tensions are canceled by the orientifold planes) and 3-form fluxes which are ISD.

In building models with 3-form fluxes, the quantization conditions imply that the latter have a large contribution of the latter to the RR 4-form tadpole. Indeed, the D3-brane RR tadpole condition reads

$$\sum_\alpha N_\alpha n_\alpha^1 n_\alpha^2 n_\alpha^3 + n \cdot 32 = 16, \tag{15.1}$$

This seems to suggest it is not possible to build consistent models without introducing supersymmetry breaking antibranes [51, 52].

However, it was realized in [25] that magnetised D9-branes may carry either anti-D3-brane or anti-D7$_i$-brane charge, while still preserving the $\mathcal{N} = 1$ supersymmetry of the orientifold background. The T-dual version of this striking fact was already pointed out in [19], where some explicit models with this property were constructed. And this is in fact the case for the example presented in lecture 2. Hence we can exploit that model for model building with fluxes.

Recall the the magnetised D-brane model 7.1 in lecture 2. Since the model is already familiar to us, we skip the details of its spectrum and phenomenology, and proceed with the construction. We would like simply to emphasize that, although the open string sector in the presence of fluxes is not exactly solvable, the massless chiral sector is topological, and hence protected, and can be computed by the same index

computation in lecture 2. Hence, the chiral matter content is exactly as in previous sections. On the other hand, non-chiral sectors can in general acquire mass terms induced by the fluxes (see later) and disappear from the massless spectrum.

In order for the D-branes to have their tension canceled by the O-planes, we need to require the angle conditions

$$\begin{gathered} A_2 = A_3 \\ \tan^{-1}(A_1/2) + \tan^{-1}(A_2/3) + \tan^{-1}(A_3/4) = \pi \end{gathered} \tag{15.2}$$

When introducing fluxes, the first RR 4-form tadpole condition gets modified. Since flux quantisation conditions imply that $N_{\text{flux}} = n \cdot 64$, $n \in \mathbf{N}$, this translates into

$$g^2 + N_f + 4n = 14. \tag{15.3}$$

which has several solutions. Among them we find:

- $n = 0, \quad g = 3, \quad N_f = 5$
- $n = 1, \quad g = 3, \quad N_f = 1$
- $n = 2, \quad g = 2, \quad N_f = 2$
- $n = 3, \quad g = 1, \quad N_f = 1$

Let us discuss the above solutions in some detail. The first corresponds to the 3 generation model without 3-form flux studied in lecture 2. The last of these solutions is also very interesting. The quantum of flux $N_{\text{flux}} = 3 \cdot 64 = 192$ can be achieved by considering the 3-form flux

$$G_3 \; = \; \frac{8}{\sqrt{3}} \, e^{-\frac{\pi i}{6}} \, (d\overline{z}_1 dz_2 dz_3 + dz_1 d\overline{z}_2 dz_3 + dz_1 dz_2 d\overline{z}_3) \tag{15.4}$$

which is well quantised at the particular value $\tau_1 = \tau_2 = \tau_3 = \tau = e^{2\pi i/3}$ for the untwisted complex structure moduli and the dilaton. These are indeed the values where those fields get fixed after the scalar potential generated by G_3 is minimised [44, 52]. Notice that the flux (15.4) is a combination of $(2,1)$ 3-forms, and hence the closed string background as a whole preserves $\mathcal{N} = 1$ supersymmetry [50]. We thus find that it is actually possible to find chiral $\mathcal{N} = 1$ string theory vacua involving 3-form fluxes and magnetised D-branes. Since this explicit model has only one SM family, it should be regarded as a toy model illustrating that it is possible to get chirality, supersymmetry in explicit flux compactifications. Model building in more general CY manifolds allows for enough freedom to obtain more realistic gauge sectors.

If we relax the requirement that fluxes preserve supersymmetry, more possibilities open up. Indeed the second solution to (15.3) involves a MSSM-like spectrum with 3 generations and $N_{\rm flux} = 64$. The latter can be achieved by

$$G_3 \;=\; 2\,(d\overline{z}_1 dz_2 dz_3 + dz_1 d\overline{z}_2 dz_3 + dz_1 dz_2 d\overline{z}_3 + d\overline{z}_1 d\overline{z}_2 d\overline{z}_3). \tag{15.5}$$

which contains a $(0,3)$ and hence breaks supersymmetry (although with vanishing cosmological constant at leading order). A particular value of the moduli where such flux is well quantised is given by $\tau_1 = \tau_2 = \tau_3 = \tau = i$. Notice that this give us $g_s = 1$ and the string perturbation theory may seem no longer reliable. It turns out that the scalar potential derived from the flux (15.5) has several flat directions. In particular, it vanishes when one imposes the complex structure moduli and the dilaton to be pure imaginary

$$\begin{aligned} &\tau_i = i t_i, \qquad t_i \in \mathbf{R} \\ &\tau = i/g_s, \end{aligned} \tag{15.6}$$

and to satisfy the constraint

$$g_s t_1 t_2 t_3 = 1. \tag{15.7}$$

So, in principle, by varying the parameters t_i one can consider the string coupling to be very close to zero. Of course, α' corrections may modify this picture and lift the flat directions left by (15.6), (15.7), dynamically fixing g_s. The evaluation of such effects is beyond the scope of the present lectures.

As discussed in previous lectures, one can improve the model by considering suitable D-brane bound states. We will not discuss the issue further, and simply mention if for future convenience.

15.2 A model with D-branes at singularities and fluxes

In this section we would like to discuss the construction of orientifold of type IIB on $\mathbf{T^6}/\mathbf{Z_3}$ by the orientifold action ΩR with $R : z_i \to -z_i$, and NSNS and RR 3-form field strength fluxes. This is a T-dual version of the model in [53]. As discussed in the main text, the models turn out be non-supersymmetric in the closed string sector since the orbifold and the $\mathbf{Z_3}$ invariant fluxes necessarily preserve different supersymmetries.

A direct way of constructing would be to introduce fluxes in the $\mathbf{T^6}/\mathbf{Z_3}$ orientifold, which will necessarily be non-supersymmetric, but should at least be imaginary self-dual with respect to the metric of the 6d internal space.

We prefer instead to take an indirect route, which takes more advantage of our discussions in the main text, and carry out the construction as follows. We start with

type IIB on $\mathbf{T^6}/\mathbf{Z_3}$ modded out by ΩR, and introduce a 3-form flux of the form

$$G_3 \;=\; 2\, dz_1 dz_2 d\overline{z}_3 \tag{15.8}$$

This flux stabilizes moduli at a factorized product of three two-tori (i.e. off-diagonal Kahler parameters are frozen to zero) with complex structure parameters $\tau_i = e^{2\pi i/3}$, and stabilizes the dilaton at $\phi = e^{2\pi i/3}$. It is also properly quantized over 3-cycles in the torus, with integrated fluxes giving even numbers. Notice that this flux preserves some of the supersymmetries of the underlying $\mathbf{T}^6/(\Omega R)$ geometry. Concretely it preserves the spinor ξ_0 (satisfying $\gamma^{\bar{i}}\xi_0 = 0$ in the above complex coordinates), as well as the spinors $\gamma^1\gamma^3\xi_0$ and $\gamma^2\gamma^3\xi_0$ (hence preserves 4d $\mathcal{N} = 3$ supersymmetry). Being supersymmetric the flux is automatically imaginary self-dual with respect to the underlying metric. For the above flux we have $N_{\rm flux} = 12$, hence cancellation of RR tadpoles requires the introduction of 20 D3-branes.

We now mod out the configuration by the $\mathbf{Z_3}$ orbifold generated by

$$\theta : (z_1, z_2, z_3) \longrightarrow (e^{2\pi i/3} z_1, e^{2\pi i/3} z_2, e^{4\pi i/3} z_3) \tag{15.9}$$

namely $z_i \longrightarrow e^{2\pi v_i} z_i$ with $v = 1/3(1,1,2)$.

For this quotient to be possible it is crucial that the flux (15.8) is invariant under the action of θ, so it corresponds to a possible flux in $\mathbf{T^6}/\mathbf{Z_3}$ (fulfills condition i) in section 14.3). It is also important to notice that the $\mathbf{Z_3}$ quotient of $\mathbf{T^6}/(\mathbf{\Omega R})$ does not contain closed 3-cycles which are not closed in $\mathbf{T^6}/\mathbf{Z_3}$. hence proper quantization is not spoilt (point ii in section 14.3). Also, the collapsed cycles at $\mathbf{Z_3}$ singularities are 2- and 4-cycles, hence do not impose additional quantization constraints (point iii in 14.3). Finally, the flux is imaginary self-dual in the metric of the orbifold space (at the orbifold point in moduli space), since the metric is inherited from that of $\mathbf{T^6}$ (point iv in 14.3). Finally, the spinor preserved by the orbifold projection is $\gamma^3\xi^0$, which is not any of the supersymmetries preserved by the flux. Hence the final model is not supersymmetric.

In this model, supersymmetry broken in the closed string sector by the interplay between the orbifold projection and the flux. It is possible that this kind of breaking of supersymmetry has some particularly nice features, since the interactions between untwisted modes is sensitive to supersymmetry breaking only via effects involving twisted modes. It would be interesting to analyze the impact of this property on the violations of the no-scale structure of the low energy supergravity effective theory for these models (i.e. the degree of protection against α' or g_s corrections).

To define the model completely, we need to specify the configuration of the 20 D3-branes, which are still required to cancel the untwisted RR tadpole. In the ΩR orientifold of $\mathbf{T^6}$ there is one point, the origin $(0,0,0)$, fixed under ΩR and θ. At this point, cancellation of RR twisted tadpoles requires the presence of D3-branes, with a Chan-Paton matrix satisfying

$$\mathrm{Tr}\ \gamma_{\theta,3} = -4 \tag{15.10}$$

In addition, there are other 26 points fixed under θ (and gathered in 13 pairs under ΩR), where there is no twisted RR tadpole. If D3-branes are present, they should have traceless Chan-Paton matrix. Finally there are 63 points fixed under ΩR (gathered in 21 trios under $\mathbf{Z_3}$) at which we may locate any number (even or odd) of D3-branes. A simple solution would be to locate the 20 D3-branes at the origin, with

$$\gamma_{\theta,3} = \mathrm{diag}\,(\mathrm{I}_4, e^{2\pi i/3}\mathrm{I}_8, e^{4\pi i/3}\mathrm{I}_8) \tag{15.11}$$

leading to an $\mathcal{N}=1$ supersymmetric sector (to leading approximation, since interactions with the closed sector would transmit supersymmetry breaking), with spectrum

$$\begin{array}{rl} \mathcal{N}=1 \text{ vect.mult.} & SO(4)\times U(8) \\ \mathcal{N}=1 \text{ ch.mult.} & 3\,[\,(\Box,\Box)+(1,\overline{\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}})\,] \end{array} \tag{15.12}$$

A more interesting possibility, which we adapt from [54], is to locate 11 D3-branes at the origin, with

$$\gamma_{\theta,3} = \mathrm{diag}\,(\mathrm{I}_1, e^{2\pi i/3}\mathrm{I}_5, e^{4\pi i/3}\mathrm{I}_5) \tag{15.13}$$

This leads to a gauge sector with

$$\begin{array}{rl} \mathcal{N}=1 \text{ vect.mult.} & U(5) \\ \mathcal{N}=1 \text{ ch.mult.} & 3\,(\,5+\overline{10}\,) \end{array} \tag{15.14}$$

One should now be careful in locating the additional D3-branes. Introducing an odd number of D3-branes on top of the O3-plane at the origin implies that it is an $\widetilde{O3}^-$-plane in notation of [47], i.e. there exists a $\mathbf{Z_2}$ B_{RR} background on an $\mathbf{RP}_2$ around the O3-plane. In order to be consistent with the fact that our flux has even integral over the different 3-cycles implies that there should exist other $\widetilde{O3}^-$-planes in the configuration. The conditions in [44] state that for any 3-plane over which the integrated flux of H_3 is even (any 3-plane in our case), the number of $\widetilde{O3}^-$-planes must be even. The

configuration in [54] turns out to satisfy the corresponding consistency conditions (the underlying reason being that it is consistent for zero fluxes, which is a particular case of even quanta on 3-cycles).

To adapt this configuration, denote A, B or C the coordinate of an O3-plane in a complex plane, according to whether $z_i = 1/2$, $z_i = e^{2\pi i/3}/2$ or $z_i = (1 + e^{2\pi i/3})/2$. The remaining 9 D3-branes in the model are located on top of O3-planes at the points

$$\begin{array}{ccc} (A,A,A) & (B,B,C) & (C,C,B) \\ (A,0,0) & (B,0,0) & (C,0,0) \\ (0,A,0) & (0,B,0) & (0,C,0) \end{array} \tag{15.15}$$

This set is invariant under exchange of fixed points by $\mathbf{Z_3}$, and introduces the right number of $\widetilde{O3}^-$-planes at the right places. The additional D3-branes do not lead to additional gauge symmetries.

Thus the final model contains a 3-family $SU(5)$ GUT gauge sector (although without adjoint chiral multiplets to break it down to the Standard Model), as the only gauge sector of the theory. In addition, its closed string sector is non-supersymmetric. It would be interesting to estimate the impact of the supersymmetry breaking on the gauge sector [55]. It would also be interesting to construct other models based on the $\mathbf{Z_3}$ orbifold, or other orbifold models. We leave these interesting question for future work.

16 Effects of fluxes on D-branes

In the above constructions we have combined D-branes and fluxes seemingly without second thoughts. However, there is a lot of physics in the interplay between both ingredients, which in fact has some impact on the way D-branes can be consistently embedded in a flux background, and on the important issue of stabilization of the open string moduli.

16.1 The Freed-Witten topological constraint

The first important issue is a new topological constraint on the allowed D-brane configurations. It is known that the K-theory group that specifies the allowed D-brane charges in a string background is modified in the presence of NSNS 3-form fluxes (and becomes the so-called twisted K-theory). The physical interpretation for this phenomenon is

the following [56, 57]. Any Dp-brane has a world-volume coupling $\int \tilde{A}_{p-2} \wedge H_3$, where $\tilde{A}_{p-2}$ is the dual potential to the world-volume gauge field A_1 [12]. This implies that the world-volume gauge field has a modified Bianchi identity $dF_2 = H_3$. This is inconsistent for any D-brane wrapped on a cycle such that the pullback of H_3 on the world-volume is cohomologically non-trivial. This is the physical realization of the fact that the corresponding D-brane charge does not exist in the corresponding twisted K-theory (although it exists in cohomology). There are ways to modify the configurations to render them consistent [57], by adding D-branes ending on the troublesome D-branes, but we will not consider them any further.

For model building the above constraint has non-trivial effects, but not many. Recall that our models are essentially built by using D3-branes, sitting at points in the internal space, and D7-branes wrapped on holomorphic 4-cycles. The pullback of the H_3 background on the D3-branes is obviously trivial, so there is no problem. On the other hand, generically, 4-cycles on Calabi-Yau manifolds do not contain non-trivial 1-cycles, hence (by Poincare duality) they do not support non-trivial 3-forms and the pullback of H_3 is automatically trivial. Of course, in specific Calabi-Yau manifold, and for specific 4-cycles (like a $\mathbf{T^4}$ on $\mathbf{T^6}$), the constrain may be non-trivial and restrict the set of possible D-branes.

In fact, in our above examples for $\mathbf{T^6}/(\mathbf{Z_2} \times \mathbf{Z_2})$ we had an additional kind of brane, namely the magnetised D9-branes, see lecture 2. Clearly, the pullback of H_3 on such objects is non-trivial, and the corresponding D-branes, as they stand, are inconsistent. One possible way out is to add D-branes 'ending' on the D9-branes, and acting as compensating magnetics sources (see appendix in [52]). A second possibility is to recombine the D9-branes, and their images, to obtain a bound state with no D9-brane charge, for which there is no inconsistency (since the induced H_3 on the bound state volume is now trivial). This is the simplest possibility, which for the model of section 2 in lecture 2 amounts to the recombination of branes h_1, h_2 and their images (as we discussed there, albeit the motivation was phenomenological).

16.2 Dynamical effects of fluxes on D-branes

- dynamical effects. Eg on D7s as is manifest from F-theory. argue against F-theory. Comment on D-brane action and interpretation: soft terms. Give results (no deriva-

[12] The above interaction may be obtained as follows. Start with a D3-brane, which contains a coupling $F_2 \wedge B_2 \simeq A_1 \wedge H_3$. By S-duality we obtain the promised term for a D3-brane. Further use of T-duality provides the promised term for other Dp-branes.

tion) with physical interpretation. Connection with effective lagrangian.

A particularly interesting class of models is obtained by considering a supersymmetric set of D-branes embedded in flux compactifications. Our purpose in this talk is to compute the effect of fluxes on such D-brane sectors. Concretely, the fluxes induce diverse new terms in the action for the D-brane world-volume theory, which correspond to supersymmetry breaking soft terms induced by non-supersymmetric flux components, and superpotential terms induced by supersymmetric flux components. For simplicity we refer to all of these as soft terms (since even superpotential terms can be regarded as softly breaking the extended supersymmetry on D-branes in (locally) flat space).

17 Supersymmetry breaking soft terms

The computation of these soft terms has been carried out in [55, 58, 59] for D3-branes, and in [60] (see also [61]) for D3/D7-brane systems. There are essentially two different approaches, that lead to equivalent results:

• Since D-branes are sensitive only to local background around them, we can consider D-brane world-volume action coupled to the local supergravity background, and expand perturbatively on the fluxes.

• We can describe effect of fluxes in the 4d effective lagrangian, which contains gauge sectors corresponding to the D-branes. The computation of the effects of fluxes on D-branes

Let us briefly sketch the computations in the two approaches in turn.

17.1 The local analysis

Let us briefly discuss the first approach, in a qualitative fashion, directing the reader to the references for more details. We start with the simpler configuration of a stack of n D3-brane in flat space. The world-volume gauge theory is a 4d $N=4$ $U(n)$ gauge theory. In $N=1$ terms, it contains a vector multiplet $V=(A_\mu,\lambda)$, and three adjoint chiral multiplets $\Phi_i=(\Phi_i,\Psi_i)$. In the latter, the scalars are Goldstone bosons of the spontaneously broken translational symmetries in the transverse directions. Hence vevs for these scalars correspond to the transverse coordinates of the D3-brane.

This also implies that the DBI+CS action for world-volume fields describes the dynamics of the D3-brane. Conversely, determining the dynamics of the D3-brane in the flux background will determine the action for the world-volume fields, including the flux-induced soft terms.

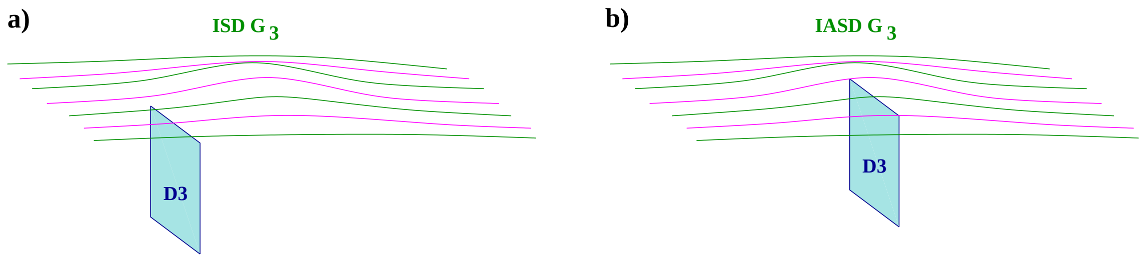

Figure 27: D3-branes in the presence of ISD or IASD fluxes.

One may work in a sort of expansion in the flux density. At lowest order we have the D3-brane in flat space without fluxes. The D3-brane world-volume action is $N = 4$ super-Yang-Mills (SYM). Equivalently, $N = 1$ SYM coupled to three adjoint chiral multiplets with a superpotential $W = \mathrm{tr}\,\Phi_1[\Phi_2\Phi_3]$.

Introducing the effect of fluxes induces soft terms for this theory. These effects depend strongly on the ISD/IASD properties of the flux density G_3, as follows, see figure 27.

• An ISD flux has positive tension and RR 4-form charge. A D3-brane in its background experiences effects from gravitational attraction and Coulomb-like repulsion. The cancellation implies that there are no soft terms generated in this situation.

• An IASD flux has positive tension and negative RR 4-form charge. Gravitational and Coulomb effects on D3-brane add up, and D3-branes are attracted to region of maximum IASD flux density. The non-trivial potential for the D3-brane position implies non-trivial soft terms.

Let us sketch the computation of the soft terms underlying the above qualitative picture. One starts with the D3-brane action, whose bosonic Dirac-Born-Infeld and Chern-Simons pieces are as follows [62]. The DBI piece reads

$$S_{DBI} = -\mu_3 \int d^4x\, \mathrm{Tr}\left(e^{-\phi}\sqrt{-\det\left(P\left[E_{\mu\nu} + E_{\mu m}(Q^{-1}-\delta)^{mn}\, E_{n\nu}\right] + \sigma\, F_{\mu\nu}\right)\, \det(Q)}\right) \qquad (17.1)$$

where $P[M]$ denotes the pullback of the 10d background M onto the D3-brane world-volume, and

$$\begin{aligned} E_{MN} &= G_{MN} - B_{MN} \\ Q^m{}_n &= \delta^m{}_n + i\sigma\,[\phi^m, \phi^p]\, E_{pn} \\ \sigma &= 2\pi\alpha' \end{aligned} \qquad (17.2)$$

The CS piece is given by

$$S_{CS} = \mu_3 \int \mathrm{Tr}\, \left(P \left[e^{i\sigma\, \mathbf{i}_\phi \mathbf{i}_\phi} \left(\sum_n C^{(n)} + \frac{1}{2} B_2 \wedge C_2 \right) e^{-B} \right] e^{\sigma\, F} \right) \quad . \tag{17.3}$$

where $i_\phi C_{(p)}$ denotes contraction of a leg of the p-form, transverse to the D3-brane, with the associated worldvolume scalar [62].

It is important to recall the interpretation of worldvolume scalars as coordinates in transverse space, via

$$x^m = 2\pi\alpha'\, \phi^m \quad . \tag{17.4}$$

The dependence of the above action on the D3-brane is thus transformed into a dependence on the world-volume scalars. By expanding the supergravity backgroun in powers of x^m around the D3-brane location, one obtains a set of higher-dimension operators in the world-volume scalars, deforming the original $N = 4$ super-Yang-Mills theory. A rather general ansatz for the supergravity background around the branes is

$$\begin{aligned}
ds^2 &= Z_1(x^m)^{-1/2} \eta_{\mu\nu}\, dx^\mu dx^\nu + Z_2(x^m)^{1/2}\, {ds^2}_{CY} \\
\tau &= \tau(x^m) \\
G_3 &= \frac{1}{3!} G_{lmn}(x^m)\, dx^l dx^m dx^n \\
\chi_4 &= \chi(x^m)\, dx^0 dx^1 dx^2 dx^3 \\
F_5 &= d\chi_4 + *_{10}\, d\chi_4
\end{aligned} \tag{17.5}$$

Since we are in principle interested only in the most relevant terms, up to dimension three in the world-volume action, it is enough to truncate the expansion of the supergravity background as follows

$$\begin{aligned}
Z_1^{-1/2} &= 1 + \frac{1}{2} K_{mn}\, x^m x^n + \ldots \\
Z_2^{1/2} &= 1 + \ldots \\
\tau &= \tau_0 + \frac{1}{2} \tau_{mn}\, x^m x^n \\
\chi_4 &= (\text{const.} + \frac{1}{2} \chi_{mn}\, x^m x^n + \ldots)\, dx^0 dx^1 dx^2 dx^3 \\
G_{lmn}(x^m) &= G_{lmn} + \ldots
\end{aligned} \tag{17.6}$$

Following [58], the quantitative results upon computation of the world-volume action in a general flux background are, for soft terms up to dimension three gives

$$\begin{aligned}
\text{Scalar masses } m: &\qquad \mathrm{Tr}\, m^2 = \tfrac{g_s}{6} \left(\sum |G^-_{lmn}|^2 - Re(G^-_{lmn} \overline{G}^+_{lmn}) \right) \\
\text{Scalar trilinears } A: &\qquad \tfrac{g_s}{3}\, G^-_{lmn} \phi^l \phi^m \phi^n \\
\text{Fermion masses } M: &\qquad (g_s^{1/2}/\sqrt{2})\, G^-_{lmn} \Psi \Gamma^{lmn} \Psi .
\end{aligned} \tag{17.7}$$

We see that they all vanish for pure ISD flux $G_3^- = 0$. A particular example of IASD flux induced soft terms is provided by a (3,0) G_3 flux

$$m^2 = \frac{g_s}{6}\,|G_{123}|^2 \quad , \quad M^a = \frac{g_s^{1/2}}{\sqrt{2}}\,G_{123} \quad , \quad A^{ijk} = -\epsilon^{ijk}\,g_s\,G_{123} \tag{17.8}$$

which correspond to the dilaton-dominated soft terms in susy phenomenology.

An important observation is that the conclusions are reversed if the gauge sector is localized on anti-D3-branes: Soft terms arise for ISD flux, and vanish for IASD flux. This may be relevant in some scenarios with anti-D3-brane [63].

One can perform a similar local analysis for D7-branes [60], which is nevertheless more involved since D7-branes wrap 4-cycles Σ_4 in X_6, and the 4d physics depends strongly on the 4-cycle geometry. In particular, the local symmetry of the configuration is only $SO(4) \times SO(2)$, far smaller that the $SO(6)$ of D3-brane systems. It is thus convenient to decomponse the ISD and IASD pieces of G_3 under $SO(4) \times SO(2)$, as follows

$$\begin{aligned} \text{ISD} \quad 10 &= (3,1)_+ + (1,3)_- + (2,2)_0 \\ \text{IASD} \quad \overline{10} &= (3,1)_- + (1,3)_+ + (2,2)_0 \end{aligned} \tag{17.9}$$

Let us denote by G and G' the ISD pieces in the $(3,1)_+$ and $(1,3)_-$ representations, and by $\tilde{G}$ and $\tilde{G}'$ the IASD ones in the $(3,1)_-$ and $(1,3)_+$.

A further subtlety for D7-branes is that in the presence of field strength fluxes there are non-trivial consistency conditions for brane wrapping, namely the cohomology class of the pullback of H_3 on the 4-cycle should be trivial. In certain examples, most notably the toroidal models to be considered below, this could lead to inconsistency of the configuration. The problems are avoided if the components in the $(2,2)_0$ are absent, as we consider henceforth.

Finally, after determining the 8d world-volume action including the flux-induced terms, one needs to perform a Kaluza-Klein reduction to 4d. This requires a detailed knowledge of the wrapped 4-cycle, hence our discussion below assumes a particularly simple case, the four-torus. Partial results for other case are described in [60].

For a D7-brane on $\mathbf{T^4} \times \mathbf{C}$, the 4d effective action, in the absence of fluxes is an $N = 4$ $U(n)$ gauge theory, with the one complex scalar Φ_3 associated to the D7-brane position in transverse complex plane, and two, denoted Φ_1, Φ_2 associated to Wilson line degrees of freedom. Skipping the detailed computation, the results for the soft terms are

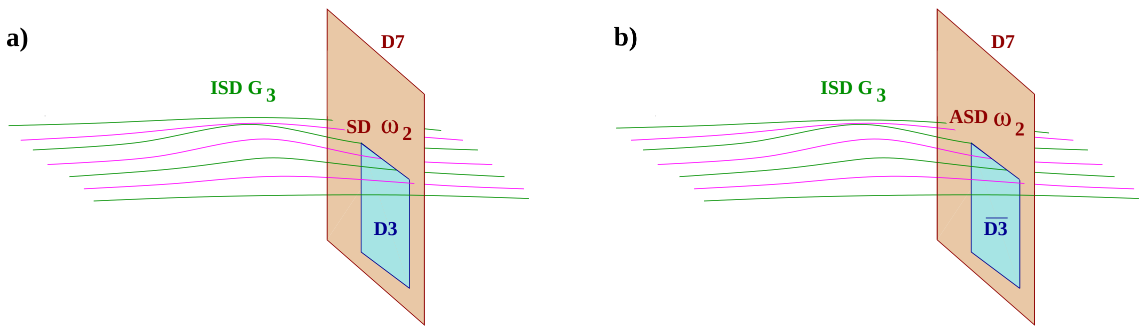

Figure 28: D7-branes in the presence of ISD flux which is SD or ASD when restricted to its world-volume.

Transverse position scalar masses

$$-\frac{g_s}{18}\,\{\,[\,(\tilde{G}^*)^2+(G'^*)^2\,]\,\Phi^3\Phi^3\;+\;\text{h.c.}\;+\;\frac{1}{2}\,(\,|\tilde{G}|^2+|G'|^2)\,\Phi^3\Phi^{\overline{3}}\,\} \tag{17.10}$$

Scalar trilinears

$$-\frac{g_s}{18}\,\{\,(\Phi^{\overline{1}},\Phi^{\overline{2}})(\tilde{G}\cdot\sigma)\begin{pmatrix}\Phi^1\\ \Phi^2\end{pmatrix}\;+\;(\Phi^{\overline{1}},\Phi^2)(G'\cdot\sigma)\begin{pmatrix}\Phi^1\\ \Phi^{\overline{2}}\end{pmatrix}\,\}\,\Phi^3\;+\;\text{h.c.} \tag{17.11}$$

Fermion masses

$$-\frac{g_s}{6\sqrt{2}}\,\{\,(\overline{\lambda},\overline{\Psi}^3)(G'\cdot\sigma)\begin{pmatrix}\lambda\\ \Psi^3\end{pmatrix}\;+\;(\Psi^1,\Psi^2)(\tilde{G}\cdot\sigma)\begin{pmatrix}\Psi^1\\ \Psi^2\end{pmatrix}\;+\;\text{h.c.}\,\} \tag{17.12}$$

The results, and in particular the fact tha only G' and $\tilde{G}$ appear in the soft terms, suggest a simple physical interpretation. Consider splitting $\mathbf{X_6}$ locally as $\Sigma_4\times R^2$, and the 3-form G_3 in $\mathbf{X_6}$ as 2-form ω_2 on Σ_4 and a 1-form on R^2. Roughly speaking, we write $G_3=\omega_2\wedge dz_3$, etc. Due to certain world-volume couplings on the D7-branes, a selfdual/anti-selfdual (SD/ASD) ω_2 induces a lower-dimensional D3/anti-D3 -brane on the world-volume. The appearance or not of soft terms can be understood as the existence or not of interaction between the induced branes and the flux background.

Hence, centering on ISD 3-form fluxes, we may split them in pieces corresponding to SD/ASD ω_2 on the D7-brane world-volume. In the former case, figure 28a, we have an induced D3-brane on the D7-brane world-volume. The latter has no interaction with the ISD flux. Hence the configuration leads to no soft terms. In the latter case, figure 28b, we have and induced anti-D3-brane on the D7-brane world-volume. The latter interacts non-trivially with the ISD flux. Hence there are non-trivial soft terms. (Similar conclusions follow for an IASD G_3 with ASD/SD ω_2)

Besides the quantitative results, the main novelty of this configuration, as compared with the D3-brane case, is that there are non-zero soft terms even with ISD fluxes. Since these fluxes can appear in global compact models, the latter are string compactification with supersymmetry breaking and non-trivial soft terms, solving classical supergravity equations of motion. It is important to emphasize that these are the first string models of this kind.

An additional interesting comment is that bundle moduli, namely wilson line moduli, or moduli associated to instanton backgrounds (for instance D3-D7 fields) do not get soft masses. This is ultimately related the origin of these scalars as modes of the 8d gauge bosons, for which mass terms are forbidden by 8d gauge invariance.

Once the results for D-branes in flat space are known, it is straightforward to perform an orbifold quotient and obtain the soft terms for semirealistic models like the one in figure 30 (see e.g. [64]. For applications to models of magnetised D-branes, see e.g. [65].

17.2 The 4d effective action approach

Let us conclude by mentioning that these results can be recovered from the second approach, namely from the 4d effective lagrangian. Consider the 4d effective action including the D3- and D7- gauge sectors, including only the dependence on the overall Kahler modulus. This contains the gauge kinetic functions

$$f_{D3} = -i\tau \quad ; \quad f_{D7} = \rho$$

the Kähler potential for the diverse geometric and brane moduli [66]

$$\begin{aligned} K \; = \; & -\log(S+S^*) - 3\log(T+T^*) \\ & + \; \frac{|\Phi_{77}^3|^2}{(S+S^*)} + \frac{1}{(T+T^*)}\left[\; (\sum_{a=1}^{3}|\Phi_{33}^a|^2) \; + \; (\sum_{b=1}^{2}|\Phi_{77}^b|^2) \; + \; (|\Phi_{37}|^2+|\Phi_{73}|^2)\;\right], \end{aligned} \tag{17.13}$$

and the superpotential (14.8).

In this language, flux components correspond to auxiliary fields of dilaton, Kähler and complex structure moduli chiral multiplets. The breaking of supersymmetry by fluxes correspond to an spontaneous breaking of supersymmetry in the closed string sector. For instance, the $(3,0)$ and $(0,3)$ flux components correspond to vevs for auxiliary fields of the dilaton and overall Kähler moduli

$$\begin{aligned} F_\tau \; & \simeq \; \textstyle\int {G_{(3)}}^* \wedge \Omega \\ F_\rho \; & \simeq \; \textstyle\int G_{(3)} \wedge \Omega \end{aligned}$$

Introducing these relations in the supergravity expressions [67, 68, 69], results in a set of soft terms in full agreement with the above sketched local analysis, see [60] for details.

18 Model building in warped throats

18.1 Throats and the hierarchy

Theories with strongly warped extra dimensions (warped throats) have revealed novel features compared with standard factorised compactifications [70, 71]. Such theories have been intensively applied to phenomenological model building beyond the Standard Model (SM) to address a variety of questions. The prototypical example of such applications is the RS1 construction [70], where a slice of AdS_5, with two boundaries, is regarded as a 5d compactification to 4d with an exponential warp factor in the extra dimension. Location of SM fields at the strongly warped end (infrared brane) leads to an exponential suppression of 4d scales as compared with the mildly warped end (ultraviolet brane), thus providing a new approach to the Planck/electroweak hierarchy. Inspired by the AdS/CFT correspondence in string theory, results in RS phenomenology have been interpreted in purely 4d terms by replacing the warped throat by a strongly interacting 4d conformal field theory. However, lack of a microscopic understanding of holography in the effective field theory approach prevents this picture to go beyond a qualitative rephrasing.

A microscopic construction of warped throats and their holographic description can be obtained in string theory. As we have described in section 14 warped extra dimensions appear in string theory in compactifications with non-trivial field strength fluxes, due to their backreaction on the underlying metric [42, 43]. In particular, warped throats with exponential warp factors appear when fluxes are (in intuitive terms) associated to 3-cycles localized in small regions of the compactification space.

The prototypical example is provided by the Klebanov-Strassler throat [72]. It is based on the local geometry of the deformed conifold, that is the cotangent bundle over $\mathbf{S^3}$. This can be written as

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = \epsilon \tag{18.1}$$

The finite size $\mathbf{S^3}$ is manifest by taking $x_i/\epsilon \in \mathbf{R}$. One turns on M units of RR 3-form flux on the $\mathbf{S^3}$ at its tip, and a suitable density of NSNS 3-form flux on its dual non-compact 3-cycle. If the latter is considered compact by setting a cutoff distance

(or by embedding in a global compactification), one denotes by K the total NSNS flux. For $K \gg g_s M$, where g_s is the IIB string coupling, fluxes backreact on the geometry and create a warped throat, whose geometry is approximately $AdS_5 \times T^{1,1}$ (with the cosmological constant of AdS_5 and the 5-form over $T^{1,1}$ slowly varying along the radial direction). The roles of the IR and UV branes in RS1 are played by the $\mathbf{S^3}$ of the deformed conifold and the cutoff/compactification, respectively. The relative warp factors between both endpoints is $e^{-\frac{2\pi K}{3 g_s M}}$ [73, 72, 43]. See figure 29

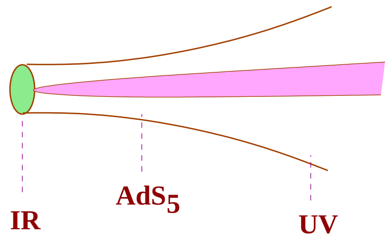

Figure 29: Schematic representation of the KS throat.

The holographic dual description is given by the (almost conformal) $\mathcal{N} = 1$ supersymmetric 4d gauge theory on $N = KM$ D3-branes at the singular conifold in the presence of M fractional branes. The gauge theory [74] has a gauge group $SU(N) \times SU(N + M)$, chiral multiplets A_i, B_i, $i = 1, 2$ in the representations $(\square, \overline{\square})$, $(\overline{\square}, \square)$ respectively, and a superpotential $W = \epsilon^{ij} \epsilon^{kl} \mathrm{tr}\, A_i B_k A_j B_l$, which is marginal for the strongly interacting conformal theory. Along the renormalization group (RG) flow to the infrared, the theory suffers a cascade of Seiberg dualities [75], in which the effective N decreases in steps of M. Eventually, at an infrared scale achieved after $K = N/M$ duality steps, the theory confines and running stops. This corresponds to the IR ending of the dual throat, with the infrared confinement scale related to the $\mathbf{S^3}$ size. The warp factor is associated to the ratio of UV and IR scales $e^{-\frac{2\pi K}{3 g_s M}}$ generated by the RG flow.

As emphasised in [73, 43], the situation is highly reminiscent of [70], with a region looking like an slice of AdS$_5$ space, ending at the infrared (by the finite size $\mathbf{S^3}$) and at the ultraviolet (by the remaining piece of the compact Calabi-Yau). In fact, this was advocated in [43], as a string theory realisation of the Randall-Sundrum proposal. In order to make this more precise, and to do real model building, we need to construct throats which have rich enough geometry at their tip to allow for chiral configurations of D-branes. This is the topic of this lecture.

18.2 Throats with singularities

Unfortunately, the Klebanov-Strassler throat is too simple to generate chiral physics in the infrared. On the gravity side, the geometry is smooth after the deformation, while in the field theory side, the light degrees of freedom are simply the glueballs of the confining theory. Hence the throat does not allow embedding the SM degrees of freedom at the IR end.

The simplest possibility is to consider throats which contain a singularity at their bottom (e.g. a $\mathbf{C^3}/\mathbf{Z_3}$ orbifold singularity), at which we would locate a stack of D3-branes (plus possibly some D7-branes). This is because D-branes at singularities are very local configurations, as compared with magnetised branes, or branes wrapped on finite-size holomorphic cycles; it would nevertheless be interesting to construct throats allowing for model building using the latter.

The most naive idea, namely to construct a strongly warped throat, similar to the KS model, at the bottom of which we have an orbifold singularity (preferably $\mathbf{C^3}/\mathbf{Z_3}$), would be to construct the quotient of the deformed conifold by a $\mathbf{Z_3}$ action with isolated fixed points. Unfortunately, the deformed conifold does not admit such symmetries. For instance, we can change variables and write (18.1) as

$$xy - uv = \epsilon \tag{18.2}$$

There is a $\mathbf{Z_3}$ symmetry $(x,u) \to e^{2\pi i/3}(x,u)$ and $(y,v) \to e^{-2\pi i/3}(y,v)$, which unfortunately is freely acting. Other possible $\mathbf{Z_3}$ symmetries, like $x \to e^{2\pi i/3}x$, $y \to e^{-2\pi i/3}y$, leaving u, v invariant, have a whole complex curve (defined by $uv = \epsilon$) of fixed points, so that locally the singularity is $\mathbf{C}\times\mathbf{C^2}/\mathbf{Z_3}$. This singularity leads to $\mathcal{N} = 2$ world-volume theories on $\overline{\mathrm{D3}}$-brane probes, which are non-chiral and thus not interesting.

Hence, it is not possible to consider a deformed conifold invariant under a $\mathbf{Z_3}$ symmetry of the required kind. An alternative is to construct geometries which also admit a complex deformation to a smooth space, but do have a $\mathbf{Z_3}$ symmetry. This was carried out in [76] for a double elliptic fibration geometry, and in [77] using the suspended pinch point (SPP) geometry, which is far simpler. In the remainder of this section we overview the latter models.

New progress in understanding warped throats for other geometries generalising the conifold, as well as their interpretation in terms of duality cascades [78, 79, 80] and infrared confinement [80], provides the techniques to implement these ideas. Skipping a more detailed exploration, it suffices for our purposes to consider one particular example. Consider the suspended pinch point (SPP) singularity, which can be described

as the hypersurface in $\mathbf{C^4}$ given by the equation

$$xy - zw^2 = 0 \tag{18.3}$$

This geometry admits a complex deformation [80, 81] to the smooth geometry

$$xy - zw^2 = \epsilon w \tag{18.4}$$

which contains a finite size $\mathbf{S^3}$.

The deformed geometry is moreover invariant under a $\mathbf{Z_3}$ symmetry acting as

$$x \to \alpha x \quad , \quad y \to \alpha y \quad , \quad z \to \alpha z \quad , \quad w \to \alpha^2 w \tag{18.5}$$

with $\alpha = e^{2\pi i/3}$. Notice that the $\mathbf{Z_3}$ action leaves invariant the holomorphic 3-form $\Omega = \frac{dxdydz}{zw}$ of the SPP, guaranteeing that the quotient is a new CY singularity. The $\mathbf{Z_3}$ action moreover has the origin as the unique fixed point, which hence descends to a $\mathbf{C^3}/\mathbf{Z_3}$ orbifold singularity in the quotient.

On general grounds it is expected that turning on M units of RR flux on the finite-size 3-cycle (as well as a suitable NSNS flux on its dual (non-compact) 3-cycle) leads to a warped throat. At its bottom, the throat is cut off by the finite-size 3-cycles, and it contains a $\mathbf{C^3}/\mathbf{Z_3}$ singularity. A chiral gauge sector at this bottom is obtained by introducing a small set of D3-branes at this singularity. The latter are considered as probes, and do not modify the structure of the throat substantially.

The gauge theory on these branes thus leads to a chiral gauge theory

$$\begin{array}{rc} \text{Vect.mult} & U(N) \times U(N) \times U(N) \\ \text{Ch.mult} & 3 \times [\,(N,\overline{N},1) + (1,N,\overline{N}) + (\overline{N},1,N)\,] \end{array} \tag{18.6}$$

where two of the $U(1)$ factors are anomalous and become massive. Also, the ranks of the three gauge factors are equal due to tadpole/anomaly cancellation. Following [37] (see lecture 2), we achieve non-equal ranks by introducing additional D7-branes in the configuration. Hence the above theory can be considered a toy model for the more realistic SM-like configurations to come.

The explicit metric for the deformed SPP has only recently become available, as a particular case of a cone over the $L^{a,b,c,}$ family Sasaki-Einstein metrics [82]. Hence, it is difficult to be more precise about the detailed structure of the throat. Nevertheless from the geometric viewpoint, the size of the 3-cycle is stabilized by fluxes [42, 43]. A precise determination of this size in terms of the underlying parameters (the fluxes) would require determining the dependence of the periods of the holomorphic 3-form

with the complex structure, around the point in moduli space where the size vanishes, as done for the deformed conifold in [43]. Even without this information, it is reasonable to expect the parametric dependence of the size, and the warp factor at the bottom of the throat, to be similar to the conifold case. Namely, the ratio of the warp factors at the tip, as compared with that at a radial distance r is $\simeq e^{K/Mg_s}$, where $N \simeq KM$ is the RR 5-form flux (and 5d cosmological constant) over the 5d horizon at the radial distance r.

It is easy to introduce additional D7-branes, by simply wrapping on them on a non-compact holomorphic 4-cycle. As discussed in [83], the D7-branes may be treated in the probe approximation as long as their number is much smaller than the fluxes. A concrete possibility is to consider a set of D7-branes, spanning the 4-cycle $w = 0$ in the parent SPP geometry, which passes through the D3-branes located at $x = y = z = w = 0$.

Hence locally at the singularity, we locally have a system of D3- and D7-brane probes. By choosing a suitable Chan-Paton action of $\mathbf{Z_3}$, for instance

$$\begin{aligned} \gamma_{\theta,3} &= \operatorname{diag}(\mathrm{I}_3, \alpha \mathrm{I}_2, \alpha^2 \mathrm{I}_1) \\ \gamma_{\theta,7} &= \operatorname{diag}(\alpha \mathrm{I}_3, \alpha^2 \mathrm{I}_6) \end{aligned} \tag{18.7}$$

consistent with cancellation of RR tadpoles, we are led to a realization of the semirealistic models studied in lecture 2.

Since the model is locally $\mathbf{C^3}/\mathbf{Z_3}$, local properties of the configuration (like the 33 and 37 spectrum) can be analysed ignoring the fluxes and the global geometry. The local analysis thus follows [37]. As discussed in [37], a single linear combination of the D3-brane $U(1)$s remains light, and plays the role of hypercharge. The group on the D7-branes leads to 8d gauge dynamics, which should be considered as a global symmetry from the D3-brane viewpoint. Upon embedding the throat in a global compactification, the D7-brane gauge dynamics becomes 4d, but the unbroken 4d gauge group and matter content depend on the compactification boundary conditions and are model-dependent. They will not be discussed in our local approach. The spectrum of the model corresponds to the quiver diagram 30, already described in lecture 2.

Hence, we have succeeded in constructing a throat with a semirealistic chiral D-brane sector at its tip. Moreover, as we sketch below, the throat admits a tractable holographic dual. The above construction realizes a configuration of D-branes leading to an interesting chiral gauge sector localized at the IR end of a warped throat. The construction is supersymmetric, although one may attempt to do non-supersymmetric

model building.

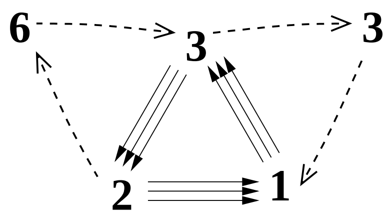

Figure 30: Quiver for the gauge theory at the infrared of the cascading throat. It is a 3-family SM-like theory.

18.3 Holographic field theory dual

The SPP throat has a known holographic description, in terms of a quiver field theory with fractional branes. The throat corresponds to an RG flow along which the quiver theory suffers a cascade of Seiberg dualities, similar to the conifold case [80]. At the infrared, this theory suffers partial confinement and the generation of a dynamical scale. This corresponds to the presence of a non-trivial 3-cycle at the tip of the dual throat. As discussed in [77], these phenomena survive the orbifolding by the $\mathbf{Z_3}$ symmetry, hence the quotient field theory describes the holographic dual of the throat ending on the singularity. One can give a quite precise characterization of the duality cascades of the dual field theory, and the appearance of the SM degrees of freedom as infrared light degrees of freedom after confinement. These issues are somewhat beyond the scope of the present lecture, and we refer the reader to the literature for more details.

19 Final comments

We have discussed some of the main issues involved in model building with branes and fluxes. Clearly many other further questions (both phenomenological and theoretical) have not been properly addressed in these lectures. For instance, the different approaches to the computation of the effects of non-supersymmetric fluxes on supersymmetric systems of D-branes (i.e. soft breaking terms), and their phenomenological properties. Or, on the formal side, the discussion of more general flux compactification possibilities, and the inclusion of additional effects (like non-perturbative contributions to the superpotential), the possibility of constructing deSitter vacua, etc... We nevertherless hope to have conveyed the feeling that interesting new things are happening, and hopefully will continue to do so in the search of better string models our our world.

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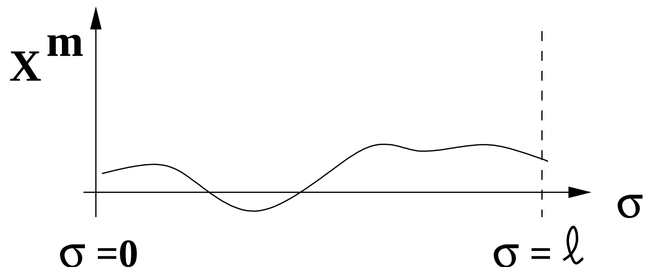

Figure 31: A string configuration is specified (in the light-cone gauge) by the position $X^i(\sigma)$ in transverse space for the point at the coordinate value σ along the string.

A Spectrum of open strings

A.1 Single D-brane in flat 10d space

In this appendix we describe a simplified calculation of the spectrum of open strings for a configuration of a single type II Dp-brane in flat 10d space.

In string theory the physical degrees of freedom for the string oscillation (in the light-cone gauge) are described by a set of functions $X^m(\sigma, t)$, which, at each (world-sheet) time t, define the graph of the string oscillation in the $i^{\rm th}$ transverse dimension, with $m = 2, \ldots, 9$. The coordinate σ parametrizes the length of the string, and runs from 0 to $\ell = 4\pi\alpha' p^+$, where $\alpha' = M_s^{-2}$ is the inverse of the string tension, and p^+ is the light cone momentum. This is shown in figure 31. For each such transverse direction, denoted generically $X(\sigma, t)$, one can perform a general mode expansion

$$X(\sigma,t) = x + \frac{p}{p^+}t + i\sqrt{\frac{\alpha'}{2}}\sum_\nu \frac{\alpha_\nu}{\nu}\,\exp[-\pi i\,\nu\,(\sigma+t)/\ell] + i\sqrt{\frac{\alpha'}{2}}\sum_{\tilde\nu}\frac{\tilde\alpha^i_{\tilde\nu}}{\tilde\nu}\,\exp[-\pi i\,\tilde\nu\,(\sigma-t)/\ell]$$

We need to impose that the open string endpoints can move freely along the coordinates spanned by the D-brane world-volume, denoted X^μ, with $\mu = 2, \ldots, p$, but are fixed at the D-brane location in the transverse coordinates, denoted X^i, with $i = p+1, \ldots, 9$. This is implemented by the boundary conditions (of Neumann and Dirichlet type)

$$\begin{array}{llll} \mu = 2,\ldots,p & \partial_\sigma X^\mu(\sigma,t) = 0 & \text{at } \sigma = 0,\ell & NN \\ i = p+1,\ldots,9 & \partial_t X^i(\sigma,t) = 0 & \text{at } \sigma = 0,\ell & DD \end{array} \qquad (A.1)$$

Imposing these constraints, the expansions become

$$X^\mu(\sigma,t) \;=\; x^\mu + \frac{p^\mu}{p^+}t + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha^\mu_n}{n}\cos[\pi n\sigma/\ell]\exp[-\pi i n t/\ell]$$

$$X^i(\sigma,t) \quad = \quad x^i + \sqrt{2\alpha'} \sum_{n\neq 0} \frac{\alpha_n^i}{n} \sin[\pi n\sigma/\ell] \exp[-\pi int/\ell] \qquad \text{(A.2)}$$

The parameters x^μ are arbitrary, while x^i must correspond to the coordinates of the D-brane in the corresponding dimension. Hence the string centre of mass is localized on the D-brane world-volume, as announced.

The oscillation modes α_n^μ, α_n^i satisfy the commutation relations

$$[\alpha_n^\mu, \alpha_{-m}^\nu] = n\delta_{n,m}\delta^{\mu\nu} \quad ; \quad [\alpha_n^i, \alpha_{-m}^j] = n\delta_{n,m}\delta^{ij} \qquad \text{(A.3)}$$

corresponding to one infinite set of decoupled harmonic oscillators, for each spacetime dimension.

Recall that type II superstrings also have fermionic oscillation degrees of freedom. They can be similarly described in terms of an infinite set of decoupled fermionic harmonic oscillators, for each spacetime dimension. Hence we have an additional set of operators, $\Psi_{n+\nu}^\mu$, Ψ_{n+r}^i, with $r = \frac{1}{2}, 0$ for fermions with NS or R boundary (or periodicity) conditions, obeying the anticommutation relations

$$\{\Psi_{n+r}^\mu, \alpha_{-(m+r)}^\nu\} = \delta_{n,m}\delta^{\mu\nu} \quad ; \quad \{\Psi_{n+r}^i, \Psi_{-(m+r)}^j\} = \delta_{n,m}\delta^{ij} \qquad \text{(A.4)}$$

To construct the Hilbert space of string oscillation states, one first defines a vacuum state given by the product of groundstates of the infinite (bosonic and fermionic) harmonic oscillators, namely annihilated by all positive modding oscillators. Next one builds physical states by applying raising operators, corresponding to negative modding oscillators. In string theory, each string oscillation quantum state corresponds to a particle in spacetime. Its spacetime mass is given by

$$\alpha' M^2 = N_B + N_F - \frac{1}{2} r(1-r) \qquad \text{(A.5)}$$

where N_B and N_F are the bosonic and fermionic oscillator number operators

$$N_B = \sum_{n=1}^{\infty} \sum_{\mu,i} \alpha_{-n}\alpha_n \quad ; \quad N_F = \sum_{n=0}^{\infty} \sum_{\mu,i} (n+r)\Psi_{-(n+r)}\Psi_{n+r}. \qquad \text{(A.6)}$$

We will be interested in the lightest (in fact, massless, states).

In the NS sector (where world-sheet fermions satisfy NS periodicity conditions) there are no fermion zero modes and the vacuum is non-degenerate. The lightest states, along with their mass and their interpretation as particles is the $(p+1)$-dimensional D-brane world-volume, are

State	$\alpha' M^2$	GSO proj.	$(p+1)$-dim.field
$\|0\rangle$	$-\frac{1}{2}$	Out	-
$\psi^{\mu}_{-\frac{1}{2}}\|0\rangle$	0	OK	A_μ
$\psi^{i}_{-\frac{1}{2}}\|0\rangle$	0	OK	ϕ^i

We have also indicated the effect of the GSO projection (required from open-closed duality and the GSO projection for closed strings) on these fields. Notice that the tachyonic mode in the first line is projected out and removed from the physical spectrum, while the massless modes survive.

In the R sector, there are fermion zero modes Ψ_0^μ, Ψ_0^i. Since their application has zero cost in energy, the vacuum state is degenerate. Denoting zero modes collectively by Ψ_0^k, they satisfy the algebra $\{\Psi_0^k, \Psi_0^l\} = \delta^{kl}$. This implies that the degenerated vacuum states form a representation of this Clifford algebra, namely they transform as spinors of its $SO(8)$ invariance group. Hence the groundstates can be labeled by the two $SO(8)$ spinor representations of opposite chiralities, denoted $\mathbf{8}_{\mathrm{S}}$, $\mathbf{8}_{\mathrm{C}}$. The corresponding particles in the $(p+1)$-dimensional world-volume transform in the representations of the Lorentz group obtained by decomposing the representations of $SO(8)$ under $SO(p-1)$ (the subgroup of $SO(p+1)$ manifest in light-cone gauge). This corresponds to a set of $(p+1)$-dimensional fermions, whose detailed structure is dimension-dependent, but straightforward to determine in each specific case. The set of light states, along with their masses, behaviour under GSO, and $(p+1)$-dimensional interpretation, are

State	$\alpha' M^2$	GSO proj.	$(p+1)$-dim.field
$\mathbf{8}_{\mathrm{S}}$	0	Out	-
$\mathbf{8}_{\mathrm{C}}$	0	OK	λ_α

The final result is that the set of massless particles on the Dp-brane world-volume is given by a $U(1)$ gauge boson, $9-p$ real scalars and some fermions. The scalars (resp. fermions) can be regarded as Goldstone bosons (resp. Goldstinos) of the translational symmetries (resp. supersymmetries) of the vacuum broken by the presence of the D-brane. The open string sector fills out a $U(1)$ vector multiplet with respect to the 16 supersymmetries unbroken by the D-brane.

As described in the main text, when n parallel D-branes overlap, the world-volume gauge theory is enhanced to an $U(n)$ gauge group, and matter fields transform in the adjoint representation.

A.2 Open string spectrum for intersecting D6-branes

In this section we carry out the computation of the spectrum of open strings in the configuration of two stacks of intersecting D6-branes [29]. In particular, we explicitly show the appearance of 4d chiral fermions from the sector of open strings stretching between the different D6-brane stacks. The key point in getting chiral fermions is that the non-trivial angles between the branes removes fermion zero modes in the R sector, and leads to a smaller Clifford algebra.

As discussed above, the spectrum of states for open strings stretched between branes in the same stack is exactly as in section 2.2. It yields an $U(n_a)$ vector multiplet in the 7d world-volume of the a^{th} D6-brane stack.

We thus center in the computation of the spectrum of states for open strings stretched between two stacks a, b. The open string boundary conditions for the coordinates along M_4 are of the NN kind, and lead to the oscillators α_n^μ, Ψ_{n+r}^μ. For the directions where the branes form non-trivial angles, for instance in the 45 2-plane, we have boundary conditions

$$\begin{aligned} \partial_\sigma X^4|_{\sigma=0} &= 0 \\ \partial_t X^5|_{\sigma=0} &= 0 \\ \left(\cos\pi\theta^1\,\partial_\sigma X^4 + \sin\pi\theta^1\,\partial_\sigma X^5\right)|_{\sigma=\ell} &= 0 \\ \left(-\sin\pi\theta^1\,\partial_t X^4 + \cos\pi\theta^1\,\partial_t X^5\right)|_{\sigma=\ell} &= 0 \end{aligned} \tag{A.7}$$

where $\pi(\theta_1)_{ab}$ is the angle from the a^{th} to the b^{th} D6-brane, written $\pi\theta_1$ for short. One has similar expression for the coordinates associated to the remaining two-planes.

It is convenient to define complex coordinates $Z^i = X^{2i+2} + iX^{2i+3}$, $i = 1, 2, 3$. The boundary conditions for ab open strings thus read

$$\begin{aligned} \partial_\sigma(\operatorname{Re} Z^i)|_{\sigma=0} &= 0 \qquad ; \quad \partial_t(\operatorname{Im} Z^i)|_{\sigma=0} = 0 \\ \partial_\sigma[\operatorname{Re}(e^{i\theta_i} Z^i)]|_{\sigma=0} &= \ell \qquad ; \quad \partial_t[\operatorname{Im}(e^{i\theta_i} Z^i)]|_{\sigma=\ell} = 0 \end{aligned} \tag{A.8}$$

These boundary conditions shift the oscillator moddings by an amount $\pm\theta^i$. The oscillator operators (which are now associated to complex coordinates) are $\alpha^i_{n+\theta_i}$, $\alpha^{\bar{i}}_{n-\theta_i}$, $\Psi^i_{n+r+\theta_i}$, $\Psi^{\bar{i}}_{n+r-\theta_i}$. It is important to point out that the centre of mass degrees of freedom are frozen in these directions, so that the open strings are localized at the intersection between the D6-branes.

All these operators satisfy decoupled harmonic oscillator (anti)commutation relations. As before, the Hilbert space of string oscillation modes is obtained by first

constructing a vacuum (annihilated by all positive modding oscillators) and then applying creation operators to it (corresponding to negative modding oscillators). Each oscillation state corresponds to a particle that propagates on the 4d intersection of the D6-brane world-volumes. Its spacetime mass is given by

$$\alpha' M^2 = N_B + N_F + E_0 \tag{A.9}$$

where N_B and N_F are the oscillator numbers, and $E_0 = -\frac{1}{2}(1 + \sum_i \theta_i)$ in the NS sector and $E_0 = 0$ in the R sector (in the normal ordering here and in what follows, we have assumed that $0 \leq \theta_i \leq 1$).

In the NS sector, the groundstate is non-degenerate. The lightest states surviving the GSO projection are (for the above range of θ_i)

State	$\alpha' M^2$	4d field
$\Psi^1_{-\frac{1}{2}+\theta_1}\lvert 0\rangle$	$\frac{1}{2}(-\theta_1 + \theta_2 + \theta_3)$	Scalar
$\Psi^2_{-\frac{1}{2}+\theta_2}\lvert 0\rangle$	$\frac{1}{2}(\theta_1 - \theta_2 + \theta_3)$	Scalar
$\Psi^3_{-\frac{1}{2}+\theta_3}\lvert 0\rangle$	$\frac{1}{2}(\theta_1 + \theta_2 - \theta_3)$	Scalar
$\Psi^1_{-\frac{1}{2}+\theta_1}\Psi^2_{-\frac{1}{2}+\theta_2}\Psi^3_{-\frac{1}{2}+\theta_3}\lvert 0\rangle$	$1 - \frac{1}{2}(-\theta_1 - \theta_2 - \theta_3)$	Scalar

These scalars are complexified by similar scalars in the ba open string sector. Hence we obtain complex scalars, with masses given above, in the bi-fundamental representation $(n_a, \overline{n}_b)$ of the $U(n_a) \times U(n_b)$ gauge factor.

In the R sector, there are two fermion zero modes associated to the M_4 directions, hence the vacuum is degenerate. They satisfy a Clifford algebra, so the vacuum fills out two opposite-chirality spinor representations of $SO(2)$. Denoting them by $\pm\frac{1}{2}$, where the label corresponds to the 4d chirality, the $+\frac{1}{2}$ state is projected out by the GSO projection, while the $-\frac{1}{2}$ state survives. Taking into account a similar state surviving in the ba sector, in total we have a 4d left-handed chiral fermion in the bi-fundamental representation $(n_a, \overline{n}_b)$. The chirality of the fermion is determined by the orientation of the intersection.

B Boundary conditions for open strings ending on D-branes with magnetic fields

In this section we describe the quantization of open strings stretching between D-branes with different constant $U(1)$ magnetic fields on their world-volume. We also make

manifest the connection via T-duality with D-branes at angles. Some early references on this kind of system are [40].

Consider the world-sheet action for an open string stretching between two coincident D-branes, labeled a and b, carrying constant world-volume $U(1)$ magnetic fields F_a, F_b in a 2-plane. For simplicity, we consider the dynamics only in the 2-plane, whose coordinates we denote X^4, X^5. Sketchily, in the conformal gauge we have

$$S = \frac{1}{4\pi\alpha'} d^2\xi \int_\Sigma \partial_a X^m \partial_a X^m + \frac{1}{2\pi\alpha'} \left[\int dt\, (A_a)_m \partial_t X^m \mid_{\sigma=0} - \int dt\, (A_b)_m \partial_t X^m \mid_{\sigma=\ell} \right] \quad (B.1)$$

For constant magnetic fields, we may use $A_m = \frac{1}{2} F_{mn} X^n$ In order to find the appropriate boundary conditions, we require that upon variation of this action, the boundary terms drop. The variation, keeping carefully the boundary terms from integration by parts, is given by

$$\begin{aligned} \delta S &= \frac{1}{2\pi\alpha'} \int_\Sigma d^2\xi\, \partial_a X^m \partial_a \delta X^m + \\ &+ \frac{1}{2\pi\alpha'} \left[\int dt\, (F_a)_{mn} \delta X^n \partial_t X^m \mid_{\sigma=0} - \int dt\, (F_b)_{mn} \delta X^n \partial_t X^m \mid_{\sigma=\ell} \right] = \\ &= \frac{1}{2\pi\alpha'} \int_\Sigma d^2\xi\, \delta X^m \partial_a \partial_a \delta X^m - \frac{1}{2\pi\alpha'} \left[\int dt\, \left(\partial_\sigma X^m \delta X^m + (F_a)_{mn} \delta X^m \partial_t X^n \right) \mid_{\sigma=0} + \right. \\ &\left. - \int dt\, \left(\partial_\sigma X^m \delta X^m + (F_b)_{mn} \delta X^m \partial_t X^n \right) \mid_{\sigma=\ell} \right] \end{aligned} \quad (B.2)$$

Since the variations δX^m are arbitrary, the boundary conditions are

$$\partial_\sigma X^m + F_{mn}\, \partial_t X^n = 0 \quad \text{at}\, \sigma = 0, \ell \quad (B.3)$$

For simplicity let us center in the particular case of $F_a = 0$, and denote $(F_b)_{45} = \tan\theta$, we have

$$\begin{aligned} \sigma = 0 \qquad & \partial_\sigma X^4 = 0 \quad , \quad \partial_\sigma X^5 = 0 \\ \sigma = \ell \qquad & \cos\theta \partial_\sigma X^4 + \sin\theta \partial_t X^5 = 0 \\ & -\sin\theta \partial_t X^4 + \cos\theta \partial_\sigma X^5 = 0 \end{aligned} \quad (B.4)$$

Now recall that T-duality interchanges Neumann and Dirichlet boundary conditions (and hence $\partial_\sigma X$ and $\partial_t X$). Using T-duality along X^5, these boundary conditions are related to the boundary conditions for open strings stretching between two D-branes, labelled a, b, at angles 0 and θ with respect to the X^4 axis. In general, any D-brane with magnetic field F in a 2-plane is related to a D-brane at angle $\theta = \tan^{-1} F$ in the T-dual.

This mapping facilitates the computation of the open string spectrum, by relating it to a known answer (we leave the direct computation using the boundary conditions

(B.3) as an exercise). An open string stretched between two D-branes with magnetic fields F_a, F_b leads to the same spectrum as an open string stretching between two D-branes with relative angle $\theta = \tan^{-1} F_b - \tan^{-1} F_a$. The generalization of these ideas to several factorized 2-planes is straightforward.

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