On the limit cycle instability in magnetized accretion discs

Andrea Merloni & Sergei Nayakshin

1 Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Strasse 1, D-85741, Garching, Germany
2 Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH, UK

ABSTRACT
Observational evidence accumulated over the past decade indicates that accretion discs in X-ray binaries are viscously stable unless they accrete very close to the Eddington limit. This is at odds with the most basic standard accretion disc theory, but could be explained by either having the discs to be much cooler whereby they are not radiation pressure dominated, or by a more sophisticated viscosity law. Here we argue that the latter is taking place in practice, on the basis of a stability analysis that assumes that the magneto-rotational-instability (MRI) responsible for generating the turbulent stresses inside the discs is also the source for a magnetically dominated corona. We show that observations of stable discs in the high/soft states of black hole binaries, on the one hand, and of the strongly variable microquasar GRS 1915+105 on the other, can all be explained if the magnetic turbulent stresses inside the disc scale proportionally to the geometric mean of gas and total pressure with a constant of proportionality (viscosity parameter) having a value of a few times $10^{-2}$. Implications for bright AGN are also briefly discussed.

Key words: accretion, accretion discs; black hole physics; X-rays: binaries

1 INTRODUCTION

According to the widely adopted standard solutions (Shakura & Sunyaev 1973), luminous accretion discs close to the Eddington rate should be radiation pressure dominated and therefore unstable to perturbations of both mass flow (Lightman & Eardley 1974) and heating rate (Pringle, Rees & Pacholcyz 1973) for the commonly adopted assumption that viscous stresses within the disc are proportional to the total (gas+radiation) pressure ($\alpha$-viscosity prescription). Taking into account the stabilizing effect of radial advection near the Eddington rate, often modeled with the “slim disc” solutions (Abramowicz et al. 1988), a limit cycle-type of behaviour should be expected, which has been confirmed by numerical simulations of time-dependent discs (Taam & Lin 1984; Honma, Matsumoto & Kato 1991; Szuszkiewicz & Miller 1997).

These instabilities may also operate in accretion discs of supermassive black holes. It is now believed that the main phase of growth of these black holes must have occurred in short-lived episodes of near-Eddington accretion (Yu & Tremaine 2002; Merloni 2004; Hopkins et al. 2006), most likely associated with bright quasar phases. The lifetimes and duty-cycles of such luminous objects, crucial to understand their cosmological evolution, may be influenced by the radiation-pressure instabilities (or lack thereof) of accretion discs.

Galactic black holes in binary systems provide an important laboratory to study these instabilities on humanly observable time scales. Among all of them, only one – the famous microquasar GRS 1915+105 – displays a variability which can be explained by some kind of viscous instability (Belloni et al. 1997; Nayakshin, Rappaport & Melia 2000; Janiuk, Czerny & Siemiginowska 2000; for a recent review and a more comprehensive list of references on this enigmatic source, see Fender & Belloni 2004). On the other hand, the vast majority of transient galactic black holes are observed to be stable in a disc-dominated state (the so-called high/soft state, or thermal dominant state, see McClintock & Remillard 2006) at luminosities which are a few tenths of the Eddington one, which is hard to reconcile with the original viscosity prescription of stresses proportional to total pressure.

Two physically plausible ways to make luminous accretion disks stable are well known. First of all, the discs may be cooling much faster than the standard solution assumes due to an additional rapid energy transfer from the disc mid-plane into a corona, a jet or a wind, so that the radiation pressure simply never dominates in the disk (e.g., Svensson & Zdziarski 1994). Secondly, the anomalous viscosity of accretion discs, now understood to be due to MRI, may not scale with the total disc pressure (e.g., Lightman & Eardley 1974). Numerical MHD simulations of turbulent accretion flows are the most promising tools for differentiating between these possibilities from first principles (see, in particular, Sano et al. 2004 and references therein). However, due to immense numerical challenges, one will have to wait...
2 VISCOSITY LAW

The nature and extent of the posited limit cycle instabilities at high accretion rates depend critically on the poorly understood prescription for the viscous torques: it is well known that assuming viscous stresses scale proportionally to the gas pressure results in accretion discs which are stable throughout, even at the highest accretion rates (Lightman and Eardley 1974; Stella and Rosner, 1984). In fact, our ignorance of the physical mechanisms giving rise to the disc viscosity, and in particular, its exact scaling, has led many authors to consider the outcome of radiation pressure dominated discs, in particular of its exact scaling, has led many authors to consider the outcome of radiation pressure dominated discs. Thus assuming viscous stresses scale proportionally to the gas pressure results in accretion discs which are stable throughout, even at the highest accretion rates (Lightman and Eardley 1974; Stella and Rosner, 1984) and therefore cannot be directly associated with the observed limit cycle instabilities. One should always keep in mind, however, that this is by no means the only way in which energy can be removed non-radiatively from the optically thick disc.

Let us consider now the general case of the viscosity prescription (1), with $0 < \mu < 2$. We can calculate analytically the value of the disc parameters at which the instability sets in by studying the stability properties of the stationary solutions. In the one (vertical) zone limit, the equation for hydrostatic equilibrium in the vertical direction is

$$P_{\text{tot}} = \frac{GM\Sigma H}{2R^3},$$

while the angular momentum conservation equation reads:

$$P_{\text{gas}} \rho (1 - \mu/2) = \frac{3\Omega K M J(R)}{8\pi \alpha_0 H},$$

where $\mu = 2(1 + \xi)$, $\xi = P_{\text{mag}}/P_{\text{gas}}$ is the ratio of the radiation to the gas pressure. Note that in this approach $f$ is an implicit function of radius, through the radial dependence of the pressures.

Recent progress in numerical studies of the disc-corona coupling has been made by simulating a gas-pressure dominated local patch of an accretion disc (with vertical gravity included) in which heating by dissipation of the MHD turbulence is balanced by radiative cooling (Hirose, Krolik & Stone 2006; see also Miller & Stone 2000). In broad accordance with eq. (3), it was found that the fraction of power released outside the disc main body was less than about 10% for a measured stress parameter of $\alpha_0 \approx 0.02$. However, due to the increased magnetic pressure support in the upper disc layers, most of the Poynting flux emerging from the disc main body is dissipated below the photosphere, and therefore cannot be directly associated with the observed hot, optically thin X-ray emitting plasma. Obviously, global radiative simulations are needed to assess the role of long-wavelength Parker instability modes, and the scaling with the radiation pressure predicted by eq. (3) for the generation of genuinely hot coronae from disc magnetic fields.

3 STABILITY ANALYSIS

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2 Magneto-Rotationally Unstable discs and the generation of the corona

Numerical studies of the last decade have shed new light on the nature of viscosity in accretion discs, by elucidating the crucial role of MHD turbulence for their enhanced transport properties. Since magneto-rotational instability (MRI; see Balbus & Hawley 1998, and reference therein) is the primary driver of the angular momentum transfer in the discs, the turbulent magnetic stresses scale with magnetic pressure, and therefore $t_{\phi} \propto P_{\text{mag}}$, where $P_{\text{mag}} = |B_{\text{disc}}|^2/8\pi$ is the magnetic pressure inside the disc.

One of the main open issues in the physics of black hole accretion discs is the relationship between the disc MRI-driven turbulent viscosity and the generation of the hot coronae that are usually postulated in order to explain the observed X-ray emission (Liang & Price 1977; Galeev, Rosner & Vaiana 1979; Blackman & Field 2000; Kuncic & Bicknell 2004). Phenomenological models usually assume that at each radius, a fraction $f$ of the internally generated power is transferred vertically outside the disc, and powers a magnetically dominated corona (Haardt & Maraschi 1991; Svensson & Zdziarski 1994). As customary (see e.g. Svensson & Zdziarski 1994; Merloni 2003), we assume that in MRI-turbulent discs such a fraction $f$ of the binding energy is transported from large to small depths by some form of collective mean electromagnetic action (Poynting flux).

We can now estimate the vertical Poynting flux, $F_p$, in the simplest way, assuming that $F_p \approx v_D P_{\text{mag}}$, where $v_D$ is the upward drift velocity of a magnetic flux tube within the disc. In Merloni (2003) it was argued that $v_D$ should be of the order of the Alfvén speed $v_A$. This translates into the following expression for the fraction of power dissipated in the corona, uniquely relating this quantity to the magnetic disc viscosity parameter $\alpha_0$ (Merloni 2003; Hirose, Krolik & Stone 2006):

$$f \approx v_A/v_c = \sqrt{2\alpha_0 \beta \mu^2},$$

where $\beta = 1/(1 + \xi)$, is the ratio of gas to total pressure, $\xi = P_{\text{mag}}/P_{\text{gas}}$ is the ratio of the radiation to the gas pressure. Note that in this approach $f$ is an implicit function of radius, through the radial dependence of the pressures.

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where the function $J(R) = (1 - \sqrt{R_{\text{in}}/R})$, with $R_{\text{in}} = 3R_S$, describes the Newtonian approximation of the net torque at the inner boundary condition for a disc around a Schwarzschild black hole. Finally, the energy balance equation is given by

$$F_{\text{rad}} \simeq \frac{c P_{\text{rad}}}{\tau_{\text{rad}}} = \frac{3}{2} \sigma T^4 \Omega H = \frac{3 T^6 R_B J(R)(1 - f)}{8 \pi},$$

where $F_{\text{rad}}$ is the vertical radiation flux and we have $f \propto \beta^{1/4}$, from eq. (3).

By differentiating logarithmically the above expressions, together with the equation of state (2), with respect to $M$, we obtain four equations in terms of the logarithmic derivatives of $P_{\text{tot}}$, $\rho$, $T$ and $H$, that can be solved for $d \log \rho/d \log M$ and $d \log H/d \log M$ as functions of $\beta$ and $f$. The lower turning point of the $M(\Sigma)$ curve, which indicates the local instability condition is then found by setting

$$\frac{d \log \Sigma}{d \log M} = \frac{d \log \rho}{d \log M} + \frac{d \log H}{d \log M} = 0,$$

which in turn gives:

$$\beta_{\text{crit}} = \frac{\left( \frac{P_{\text{gas}}}{P_{\text{tot}}} \right)}{\frac{1}{f} - f_{\text{crit}}} = \frac{7 \mu (2 - 3 f) - 16 (1 - f)}{7 \mu (2 - 3 f) - 16 (1 - 1)}.$$

The instability sets in at a transition radius which can be found by solving the following algebraic equation:

$$\frac{R_{\text{tr}}}{J(R_{\text{tr}})^{16/21}} \simeq 350 R_S \left( \frac{\beta_{\text{crit}}}{1 - \beta_{\text{crit}}} \right)^{20/21} \times \left( \alpha_0 m \right)^{2/21} m^{16/21} (1 - f)^{6/7}$$

where we have defined $m \equiv M_{\text{BH}}/M_\odot$.

In the limit of $f = 0$, we obtain the known result (see Szuksziewicz 1990) that, for the instability condition to be satisfied, the gas pressure needs to be smaller than 0.4 times the total pressure if $\mu = 0$, and just 1/13 times if $\mu = 1$. Thus, the larger is $\mu$, the more difficult is to excite the instability. By using eq. (3) to relate $f$ and $\beta$, we can then explore the whole space spanned by the parameters $\alpha_0$ and $\mu$. In Figure 1, where we plot as contours the surface of critical gas to total pressure ratio $\beta_{\text{crit}}$, we can see the whole space spanned by the parameters $\alpha_0$ and $\mu$.

1. In the following, for the sake of consistency with our Newtonian inner boundary condition, we will assume $\epsilon = \epsilon_0 \equiv 1/12$. However, it should be kept in mind that for a black hole of arbitrary spin, the true critical value of $L/L_{\text{Edd}}$ should be rescaled by a factor $\epsilon/\epsilon_0$ with respect to what shown in Fig. 2.

**Figure 1.** The critical value of the gas to total pressure ratio, $\beta_{\text{crit}}$, below which the disc is unstable as a function of the viscosity parameter $\alpha_0$ and index $\mu$. Calculations are for the case of no large scale magnetic field torques, and for a black hole mass $M_{\text{BH}} = 10M_\odot$. The solid lines mark the values $\beta_{\text{crit}} = 0.1, 0.2, 0.4, 0.8$. The horizontal dashed line mark the $\mu = 1$ case discussed in section 2.

**3.1 Observational constraints**

It is an observational fact that only the microquasar GRS 1915+105, the most luminous transient black hole binary in our galaxy, displays strong variability on timescales compatible with the limit cycle behaviour expected from thermal instabilities (see Belloni et al. 1997; Nayakshin et al. 2000; Janiuk et al. 2000, 2002). Many other systems are known to be stable up to luminosities which are a few tens of percent of the Eddington luminosity. This is clearly impossible for standard viscosity prescriptions ($\mu = 0$), unless the viscosity parameter $\alpha_0$ (and thus the fraction of power released into the corona) is high enough. But, as figures 1 and 2 show, there is only a very limited region of the parameter space for which this is possible, and within such a region, the fraction of power released outside the optically thick disc is always large, in contradiction with the observed spectral properties of high/soft state black hole binaries.

On the other hand, if $\mu \approx 1$, as argued, for example in Merloni & Fabian (2002) or Merloni (2003), we can have accretion discs which are stable at more than half of the Eddington luminosity, and this holds over a very large range in possible viscosity parameters $\alpha_0$. This can indeed explain the observed lack of unstable, disc-dominated galactic black hole binaries up to luminosities of about a half of the Eddington one (Gierliński & Done 2004), while at the same time allowing for some instability, taking in place only in those systems which are constantly accreting very close to (or above)
the Eddington rate, as supposedly is GRS 1915+105 (Done, Wardziński & Gierliński 2004).

We can extend this result to the case of supermassive black holes (SMBH). It is well known that for any value of $\mu$ and $\alpha_0$ the critical accretion rate scales with mass as $m_{\text{crit}} \propto M^{-1/8}$. Then for a $10^9$ solar masses black hole the limit cycle instability should set in at a luminosity, in units of Eddington, which is just one tenth of that of a binary black hole, both viscosity law and black hole spin being the same. If indeed most of the bright, high redshift Quasars that dominate the growth history of SMBH have $L/L_{\text{Edd}} > 0.1$, as recently proposed (Vestergaard 2004; McLure & Dunlop 2004), then they should be undergoing limit cycle instability.

4 TIME DEPENDENT EVOLUTION

In order to test the above conclusions based on the stability analysis of stationary solutions, we have carried out a set of time dependent simulations for different values of the parameters $\alpha_0$ and $m$. From the equation of conservation of mass and angular momentum, we can write (Pringle, 1981; Livio & Pringle 1992)

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} \left[ R^{1/2} \frac{\partial}{\partial R} \left( \nu \Sigma R^{1/2} \right) \right].$$

(10)

The time-dependent energy equation needed to close the set must take into account large radial gradients of temperature, and therefore includes a number of additional terms with respect to the standard heating and cooling terms of the stationary solution. The form of the energy equation that we use follows the formalism of Nayakshin et al. (2000):

$$P_{\text{rad}} H \left( \frac{4-3\beta}{\Gamma_3} - 1 \right) \left( \frac{\partial \ln \Sigma}{\partial \tau} + v_R \frac{\partial \ln \Sigma}{\partial R} \frac{H}{\partial \tau} \right) + \left( \frac{\partial \ln \Sigma}{\partial \tau} + v_R \frac{\partial \ln \Sigma}{\partial R} \frac{H}{\partial \tau} \right) = F^+ - F^- - \frac{2}{R} \frac{\partial (RF_H H)}{\partial R} + J, \quad (11)$$

where $\gamma$ is the ratio of specific heats $(\gamma = 5/3)$ and $\Gamma_3$ is given in Abramowicz et al. (1995). Here, the radial velocity $v_R$ induced by viscous stresses is given by (Eq. 5.7 of Frank et al. 1992):

$$v_R = -\frac{3}{\Sigma R^{1/2}} \frac{\partial}{\partial R} \left[ (\nu \Sigma R^{1/2}) \right].$$

(12)

The terms on the left hand side of Equation (11) represent the full time derivative (e.g., $\partial/\partial \tau + v_R \partial/\partial R$) of the gas entropy, while the terms on the right are the viscous heating, the energy flux in the vertical direction, the diffusion of radiation in the radial direction, and the viscous diffusion of thermal energy. Following Cannizzo (1993; and references therein), we take $J = 2c_p \nu (\Sigma/R) [\partial \ln \Sigma/R/\partial R] \partial R/\partial R$ to be the radial energy flux carried by viscous thermal diffusion, where $c_p$ is the specific heat at constant pressure. $F^+$ is the accretion disk heating rate per unit area, and is given by

$$F^+ = (9/4) \nu \Sigma R^{1/2} (1 - f).$$

(13)

The radiation flux in the radial direction is

$$F_R = -2c P_{\text{rad}} \frac{H}{\tau_T} \frac{\partial \ln T}{\partial R},$$

(14)

where $\tau_T$ is the optical depth of the disk, $\tau_T \equiv \kappa \Sigma / 2$, and $\kappa$ is the radiative opacity (assumed here to be dominated by electron scattering opacity). Finally, the radiative cooling rate in the vertical direction is given by

$$F^- = c P_{\text{rad}} / \tau_T.$$  

(15)

The results of our time-dependent simulations for the case $\mu = 1$ and $M_{\text{BH}} = 14M_\odot$ (as appropriate for GRS 1915+105) are shown in figure 3 as a set of lightcurves plotted over a time typically of the order of the viscous time at the outer domain boundary. Shown separately are the total disc (thin red solid lines) and coronal (blue dashed lines) emissions, together with their sum (thick black solid lines), representing the total dissipated energy in the disc–corona system. These results broadly confirm the analysis presented in section 3, in that we observe stable discs at $\dot{m} > 0.4$ and 0.2 for $\alpha_0 = 0.03$ and 0.1, respectively (see figure 2).

We find in general a relationship between the amplitude of the instability and its duty-cycle (i.e. the ratio of the “burst duration” to the period of the oscillation), whereas small duty cycles are associated with large amplitude variability. The amplitude itself is much smaller than in standard $\alpha$-discs, and grows with the ratio $\dot{m}/m_{\text{crit}}$. For accretion rates just above the critical values, the large cycles imply a “refilling” time of the inner disc faster than the viscous time at the transition radius. In fact, we found that the viscosity prescription, $\mu \approx 1$, reproduces many features of the phenomenological one introduced by Nayakshin et al. (2000).
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Figure 3. Set of lightcurves for magnetized discs with the viscosity law corresponding to $\mu = 1$, calculated for different accretion rates ($\dot{m} = 0.2, 0.4, 0.5$) and viscosity parameters ($\alpha_0 = 0.03, 0.1, 0.2$). Each panel shows one lightcurve over a timescale approximately equal to one tenth of the viscous time at the outer boundary of our discs ($r = 200$). Thick black lines represent the total (disc plus corona) emission, thin red lines the optically thick disc emission, and thin blue dashed ones the coronal (X-ray) emission. The black hole mass is fixed to $10 M_\odot$ in order to explain the time variability of GRS 1915+105 (a task which is beyond the aim of this work).

Another general property of the simulated lightcurves is that, thanks to the nature of the magnetic viscosity law adopted, the fraction $f$ of power dissipated outside the optically thick disc is higher when gas pressure dominates, i.e., when the inner disc is denser and cooler, thus making coronal X-ray emission more prominent during the low luminosity parts of the instability cycles.

Figure 4 shows a set of simulated lightcurves with $\mu = 1$, $\alpha_0 = 0.03$, for different accretion rates ($\dot{m} = 0.05, 0.1, 0.2$) and black hole masses ($M_{\text{BH}} = 10^6, 10^7, 10^8 M_\odot$). Each panel shows one lightcurve over a timescale approximately equal to half of the viscous time at the outer boundary of our discs ($r = 200$). Linestyles are as in Fig. 3.

5 DISCUSSION

The stability analysis we have presented here can be used to explain the observed properties of accreting stellar mass black holes in different states. First of all, a major constraint can be derived from the absence of any limit-cycle instability in disc-dominated spectra. Those can be effectively selected in two complementary ways: by detail modeling, choosing those spectra in which the power-law component contribute to less than a fixed amount (say, 10%); and by studying the luminosity-temperature relation of the disc component, selecting those spectra for which $L_{\text{disc}} \propto T^4$ as expected from an optically thick disc extending down to the innermost stable orbit (Kubota & Makishima 2004). Modulo the distance uncertainty, these studies unambiguously demonstrated that accretion discs can be viscously stable up to at least $L_{\text{disc}}/L_{\text{Edd}} \gtrsim 0.3$ (Gierliński & Done 2004).

From Figure 2, then, we conclude that only two disjoint regions of parameter space are consistent with the observed phenomenology of black hole binaries. The first corresponds to $\alpha_0 \gtrsim 0.3$ and, within our theoretical framework, it corresponds to accretion discs sandwiched by powerful, stabilizing coronae as postulated by Svensson & Zdziarski (1994). The second acceptable region of the parameter space is roughly delimited by $\mu \gtrsim 0.8$ and $\alpha_0 \lesssim 0.15$ and would indicate that the stability properties of high/soft state black holes are dictated by a modified viscosity law.
The former constraint, however, implying a large fraction of power transported vertically by magnetic fields, seems inconsistent with the spectral properties of the observed black holes in the high/soft state (but see however Hirose et al. 2006 or Blaes et al. 2006). Moreover, for the only source that exhibits limit-cycle instabilities, the microquasar GRS 1915+105, one can associate the observed variability time-scales with the disc viscous time (Belloni et al. 1997; Nayakshin et al. 2000; Janiuk et al. 2000). These models consistently suggest that \( \alpha_0 \sim 1 \), with a constant of proportionality \( \alpha_0 \) of about a few percent. This in turn implies quite low values of \( f \) and consequent small contribution from the power-law component to the observed X-ray spectra.

5.1 The low/hard state

The scenario we have outlined above applies only to the high/soft state of black hole binaries. It seems unlikely that the soft-to-hard transition can be solely due to a decrease of the external accretion rate and a consequent increase of the gas pressure-dominated part of the disc, accompanied by an increase of the strength of the corona (Merloni & Fabian 2002). In fact, low values of \( \alpha_0 \) in the high/soft state also imply low values of \( f \approx \sqrt{2\alpha_0} \) in the low state, if the disc physics is not dramatically altered by the \( \dot{m} \) variation. Thus, there must be an additional physical mechanism, not included in the simple treatment of the coupled magnetized disc+corona system presented here, which is responsible for the observed state change. Obvious candidates are: a radial transition to an inner optically thin, radiatively inefficient flow, due to some kind of disc evaporation (Meyer & Meyer-Hofmeister 1994; Dullemond & Turolla 1998; Spruit & Deufel 2002), or a global rearrangement of a large scale poloidal magnetic field, so that the accretion energy can be dissipated almost entirely into the bulk flow of a relativistic jet (Livio, Pringle & King 2003).

5.2 Variability in AGN

Since the radiation-pressure dominated regime sets in at smaller dimensionless accretion rates for higher black hole masses, large scale variability due to radiation-pressure driven viscous instabilities must be more widespread in AGN than in stellar mass accreting black holes. However, observing such variability in practice is only feasible for lower mass super-massive black holes. Indeed, the viscous timescale, \( t_{\text{visc}} = (1/\Omega)(1/\alpha^{3/2})(R/H)^2 \), calculated at the transition radius (see eq. 9) is approximately given by

\[
t_{\text{visc}}(R_{\text{tr}}) \approx 400 \alpha_0^{-2/3} \dot{m}^{1/3} m^{-2/3} \beta_{\text{crit}}^{10/3 - \mu/2} (1 - \beta_{\text{crit}})^{10/3} \text{ s. (16)}
\]

This ranges from a few tens to a few hundreds of years if \( m=10^7 \), consistent with estimates of switch on/off times for some “changing look” AGN (Guainazzi et al. 2005), to \( 10^4 - 10^5 \) years for a 10\(^6\) solar masses black hole. This, interestingly, coincides with the proposed intermittency time of radio galaxies, based on radio sources number counts and on the properties of radio galaxies size distribution (Reynolds & Begelman 1997).

Another interesting aspect of the predicted variability for AGN concerns their feedback on galaxy formation. Indeed, in our model, black holes accreting at a time-averaged sub-Eddington rate may spend a fraction of their time accreting at above-Eddington accretion rate. If AGN feedback (for example in the form of powerful relativistic jets) is significant when \( L \sim L_{\text{Edd}} \), as suggested by the phenomenology of microquasars (see Fender, Belloni & Gallo 2004), then such a source would produce feedback while on the “hot” branch of the S-curve, whereas a completely stable source at the same time-averaged accretion rate would not.

6 CONCLUSIONS

In this paper we discussed the implications of the observed long term stability of the vast majority of black hole X-ray binaries in the so-called high/soft state (or thermal dominant state). This fact, together with the properties of the lightcurves of GRS 1915+105, the most luminous galactic black hole and the only such system to display limit-cycle-type of instabilities, can be used to put constraints on the nature of the viscosity law in accretion discs.

We performed a stability analysis for accretion flows with a flexible viscosity prescription under the hypothesis that the Magneto-Rotational-Instability responsible for generating the turbulent stresses inside the disc is also the source for a magnetically dominated corona. By varying the scaling index of the viscosity law, \( \mu \), and its overall normalization, \( \alpha_0 \), we have identified those regions of the parameter space for which limit-cycle instability can develop as a function of the accretion rate. From this analysis a coherent picture emerges for black hole binaries in the high/soft state in which MRI-driven turbulence generates stresses which scale with the disc pressure as the geometric mean of gas and total pressure (Merloni 2003), i.e. \( \mu \approx 1 \), with a constant of proportionality \( \alpha_0 \) of about a few percent. This in turn implies quite low values of \( f \) and consequent small contribution from the power-law component to the observed X-ray spectra. Provided that the viscosity law itself does not change dramatically at lower accretion rates, a consequence of this result is that the transition to the low/hard state must be caused by additional physics, such as the evaporation of, or a global large scale magnetic field re-arrangement in the inner portions of the disc.

The scaling with black hole mass of the accretion equations is such that limit cycle instabilities should play a role in all bright AGN and Quasars. The typical timescale for these oscillations, however, grows with black hole mass faster than linearly. Only smaller mass black holes (less than a few times \( 10^7 \) solar masses) may have limit cycle instability timescales of just a few years. Even considering the uncertainties in the absolute values of the predicted critical luminosity that come from uncertainties in the black hole mass and spin, it would be interesting to see for any evidence of the predicted behaviour by looking at distribution of observed disk lumi-
nosities in large samples of AGN with available estimates of the central black hole mass.

REFERENCES


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