Higgs-gauge boson interactions in the economical 3-3-1 model

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(Dated: March 16, 2006)

I. INTRODUCTION

Recent neutrino experimental results [1] establish the fact that neutrinos have masses and the standard model (SM) must be extended. Among the beyond-SM extensions, the models based on the SU(3)$_{C}$ $\otimes$ SU(3)$_{L}$ $\otimes$ U(1)$_{X}$ (3-3-1) gauge group have some intriguing features: Firstly, they can give partial explanation of the generation number problem. Secondly, the third quark generation has to be different from the first two, so this leads to the possible explanation of why top quark is uncharacteristically heavy.

There are two main versions of the 3-3-1 models. In one of them [2] the three known left-handed lepton components for each generation are associated to three SU(3)$_{L}$ triplets as ($\nu_{l}, l, l^{c}_{L}$)$_{L}$, where $l^{c}_{L}$ is related to the right-handed isospin singlet of the charged lepton $l$ in the SM. The scalar sector of this model is quite complicated (three triplets and one sextet). In the variant model [3] three SU(3)$_{L}$ lepton triplets are of the form ($\nu_{l}, l, \nu^{c}_{l}$)$_{L}$, where $\nu^{c}_{l}$ is related to the right-handed component of the neutrino field $\nu_{l}$ (a model with right-handed neutrinos). The scalar sector of this model requires three Higgs triplets, therefore, hereafter we call this version the 3-3-1 model with three Higgs triplets (331RH3HT). It is interesting to note that, in the 331RH3HT, two Higgs triplets have the same U(1)$_{X}$ charge with two neutral components at their top and bottom. Allowing these neutral components vacuum expectation values (VEVs), we can reduce number of Higgs triplets to be two. As a result, the dynamics symmetry breaking also affect lepton number. Hence it follows that the lepton number is also broken spontaneously at a high scale of energy. This kind of model was proposed in Ref.[4], its gauge boson mixing and currents have been in detail considered in Ref.[5].

Note that the mentioned model contains very important advantage, namely: There is no new parameter, but it contains very simple Higgs sector, hence the significant number of free parameters is reduced. To mark the minimal content of the Higgs sector, this version is going to be called the economical 3-3-1 model.

It is well known that the electroweak symmetry breaking in the SM is achieved via the Higgs mechanism. In the Glashow-Weinberg-Salam model there is a single complex Higgs doublet, where the Higgs boson $h$ is the physical neutral Higgs scalar which is the only remaining part of this doublet after spontaneous symmetry breaking (SSB). In the extended models there are additional charged and neutral scalar Higgs particles. The prospects for Higgs coupling measurements at the LHC have recently been analyzed in detail in Ref.[6]. The experimental detection of the $h$ will be great triumph of the SM of electroweak interactions and will mark new stage in high energy physics.
In extended Higgs models, which would be deduced in the low energy effective theory of new physics models, additional Higgs bosons like charged and CP-odd scalar bosons are predicted. Phenomenology of these extra scalar bosons strongly depends on the characteristics of each new physics model. By measuring their properties like masses, widths, production rates and decay branching ratios, the outline of physics beyond the electroweak scale can be experimentally determined.

The mass spectrum of the mentioned scalar sector has been presented in Ref. [4], and some couplings of the two neutral scalar fields with the charged $W$ and the neutral $Z$ gauge bosons in the SM were presented. From explicit expression for the $ZZh$ vertex, the authors concluded that two vacuum expectations responsible for the second step of SSB have to be in the same range: $u \sim v$, or theory needs one more the third scalar triplet. As we will show in the following, this conclusion is incorrect. That is why this work is needed.

The interesting feature compared with other 3-3-1 models is the Higgs physics. In the 3-3-1 models, the general Higgs sector is very complicated [7, 8] and this prevents the models’ predicability. The scalar sector of this model is a subject of the present study. As shown, by couplings of the scalar fields with the ordinary gauge bosons such as the photon, the $W$ and the neutral $Z$ gauge bosons, we are able to identify full content of the Higgs sector in the SM including the neutral $h$ and the Goldstone bosons eaten by their associated massive gauge ones. All interactions among Higgs-gauge bosons in the SM are recovered.

Production of the Higgs boson in the 331RH3HT at the CERN LHC has been considered in Ref. [9]. In scalar sector of the considered model, there exists the singly-charged boson $H_2^\pm$, which is a subject of intensive current studies (see, for example, Ref. [10, 11]). The trilinear coupling $ZW^\pm H^+$ which differs, at the tree level, from zero only in the models with Higgs triplets, plays a special role on study phenomenology of these exotic representations. We shall pay particular interest on this boson.

The paper is organized as follows. Sec II is devoted to a brief review of the model. The scalar fields and mass spectrum is presented in Sec III and their couplings with the ordinary gauge bosons are given in Sec IV. Production of the $H_2^\pm$ at the CERN LHC are calculated in Sec V. We outline our main results in the last section - Sec VI.

II. A REVIEW OF THE MODEL

The particle content in this model, which is anomaly free, is given as follows:

$$\psi_{aL} = (\nu_{aL}, l_{aL}, N_{aL})^T \sim (1, 3, -1/3), \quad l_{aR} \sim (1, 1, -1),$$

where $a = 1, 2, 3$ is a family index. Here the right-handed neutrino is denoted by $N_L \equiv (\nu_R)^c$.

$$Q_{1L} = (u_1, d_1, U)^T_L \sim (3, 1, 2/3), \quad Q_{aL} = (d_\alpha, -u_\alpha, D_\alpha)^T_L \sim (3^*, 0), \quad \alpha = 2, 3,$$

$$u_{aR} \sim (1, 2/3), \quad d_{aR} \sim (1, -1/3), \quad U_R \sim (1, 2/3), \quad D_{aR} \sim (1, -1/3).$$

Electric charges of the exotic quarks $U$ and $D_\alpha$ are the same as of the usual quarks, i.e. $q_U = 2/3$ and $q_{D_\alpha} = -1/3$.

The SU(3)$_L \otimes$ U(1)$_X$ gauge group is broken spontaneously via two steps. In the first step, it is embedded in that of the SM via a Higgs scalar triplet

$$\chi = (\chi_1^0, \chi_2^0, \chi_3^0)^T \sim (3, -1/3)$$

acquired with VEV given by

$$\langle \chi \rangle = \frac{1}{\sqrt{2}} (u, 0, \omega)^T.$$  

In the last step, to embed the gauge group of the SM in U(1)$_Q$, another Higgs scalar triplet

$$\phi = (\phi_1^+, \phi_2^0, \phi_3^+)^T \sim (3, 2/3)$$

is needed with the VEV as follows

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} (0, v, 0)^T.$$
The covariant derivative of a triplet is

\[ D_\mu = \partial_\mu - igT_\mu W_\mu - igX T_\mu X B_\mu \]

where the matrices in (7) are given by (3)

\[ \mathcal{P}_\mu^{\text{NC}} = \frac{g}{2} \begin{pmatrix} W_{3\mu} + \frac{1}{\sqrt{3}} W_{3\mu} + t \sqrt{\frac{2}{3}} X B_\mu & 0 & y_\mu \\ 0 & -W_{3\mu} + \frac{1}{\sqrt{3}} W_{3\mu} + t \sqrt{\frac{2}{3}} X B_\mu & 0 \\ y_\mu & 0 & -\frac{2}{\sqrt{3}} W_{3\mu} + t \sqrt{\frac{2}{3}} X B_\mu \end{pmatrix} \] (8)

and

\[ \mathcal{P}_\mu^{\text{CC}} = \frac{g}{\sqrt{2}} \begin{pmatrix} 0 & c_\theta W_\mu^+ + s_\theta Y_\mu^+ & X_\mu^0 \\ c_\theta W_\mu^+ + s_\theta Y_\mu^+ & 0 & c_\theta Y_\mu^+ - s_\theta W_\mu^+ \\ X_\mu^0 & c_\theta Y_\mu^+ - s_\theta W_\mu^+ & 0 \end{pmatrix} . \] (9)

Here \( t \equiv g_X / g = \frac{3\sqrt{2}w}{\sqrt{4c_W^2 - 1}} \), \( \tan \theta = \frac{\sqrt{2}}{2} \) and \( W^\pm, Y^\pm \) and \( X^0_\mu \) are the physical fields. The existence of \( y_\mu \) is a consequence of mixing among the real part \((X_\mu^0 + X_\mu^0)\) with \(W_{3\mu}, W_{8\mu}\) and \(B_\mu\); and its expression is determined from the mixing matrix \( U \) given in the Appendix of Ref. [3].

\[ y_\mu \equiv U_{42} Z_\mu + U_{43} Z_\mu^\prime + (U_{44} - 1) \frac{(X_\mu^0 + X_\mu^0)}{\sqrt{2}} \] (10)

with

\[ U_{42} = -t_\theta \left( c_\varphi \sqrt{1 - 4s_\theta^2 c_\varphi^2} - s_\varphi \sqrt{4c_W^2 - 1} \right), \]
\[ U_{43} = -t_\theta \left( s_\varphi \sqrt{1 - 4s_\theta^2 c_\varphi^2} + c_\varphi \sqrt{4c_W^2 - 1} \right), \]
\[ U_{44} = \sqrt{1 - 4s_\theta^2 c_\varphi^2}. \] (11)

We remind that \( \varphi \) is the \( Z - Z' \) mixing angle and \( \theta' \) is the similar angle of \( W_1, Z, Z' \) mixing defined by (3)

\[ t_{2\varphi} = \frac{\sqrt{(3 - 4s_\theta^2)(1 + 4t_{2\varphi}^2) \left\{ [c_2 W + (3 - 4s_\theta^2) t_{2\varphi}^2] u^2 - v^2 - (3 - 4s_\theta^2) t_{2\varphi}^2 \omega^2 \right\}}}{2s_\theta^4 W - 2s_\theta^4 W - 3 t_{2\varphi}^2 u^2 - [c_2 W + 2(3 - 4s_\theta^2) t_{2\varphi}^2] v^2 + [2c_\theta^2 + (3 - 4s_\theta^2) t_{2\varphi}^2] \omega^2}, \] (12)

\[ s_{\theta'} \equiv \frac{t_{2\theta}}{c_W \sqrt{1 + 4t_{2\theta}^2}}. \] (13)

After SSB the non-Hermitian physical gauge bosons \( W, X^0, Y^\pm \) gain masses given by

\[ M_{W}^2 \equiv \frac{g_0^2 v^2}{4}, \]
\[ M_{X}^2 \equiv \frac{g_0^2 (u^2 + v^2 + \omega^2)}, \]
\[ M_{\phi}^2 = M_{\phi}^2 - M_{\phi}^2. \] (14)

The Yukawa interactions which induce masses for the fermions can be written in the most general form as

\[ \mathcal{L}_Y = (\mathcal{L}_Y^\chi + \mathcal{L}_Y^\phi) + \mathcal{L}_Y^{\text{mix}}, \] (16)

where

\[ (\mathcal{L}_Y^\chi + \mathcal{L}_Y^\phi) = h_{11} Q_{1L} \chi U_R + h_{1a} \bar{Q}_{aL} \chi^* D_R \]
\[ + h_{ab} \bar{Q}_{aL} \phi \bar{e}_b R + h_{ab} \bar{e}_{pmn} (\bar{Q}_{aL})^m (\phi)_n + h_{1a} \bar{Q}_{1L} \phi \bar{d}_{aR} + h_{a0} \bar{Q}_{aL} \phi^* u_{aR} + h.c., \]
\[ \mathcal{L}_Y^{\text{mix}} = h_{1a} \bar{Q}_{aL} u_{aR} + h_{a0} \bar{Q}_{aL} \chi^* d_{aR} + h_{1a} \bar{Q}_{aL} \phi D_{aR} + h_{a0} \bar{Q}_{aL} \phi^* U_R + h.c. \] (17)
The VEV \( \omega \) gives mass for the exotic quarks \( U \) and \( D_{\alpha} \), \( u \) gives mass for \( u_{\alpha}, d_{1} \) and all ordinary leptons. As mentioned above, the VEV \( \omega \) is responsible for the first step of symmetry breaking, while the second step is due to \( u \) and \( v \). Therefore the VEVs in this model have to be satisfied the constraints

\[
u, v < \omega.
\] (19)

The Yukawa couplings of Eq. (17) possess an extra global symmetry which is not broken by VEVs \( v, \omega \) but by \( u \). From these Yukawa couplings, one can find the following lepton symmetry \( L \) as in Table I (only the fields with nonzero \( L \) are listed, all other fields have vanishing \( L \)). Here, \( L \) is broken by \( u \) which is behind \( L(\chi_{1}^{0}) = 2 \) (see also [12]), i.e., \( u \) is a kind of the lepton-number violating parameter. It is interesting that the exotic quarks also carry the lepton number. Thus, this \( L \) obviously does not commute with gauge symmetry. One can construct a new conserved charge \( L \) through \( L \) by making the linear combination \( L = xT_{3} + yT_{8} + zX + LI \) where \( T_{3} \) and \( T_{8} \) are SU(3) \( _{L} \) generators. One finds the following solution \( x = 0, y = \frac{4}{\sqrt{3}}, z = 0 \), and

\[
L = \frac{4}{\sqrt{3}} T_{8} + LI.
\] (20)

Another useful conserved charge \( B \) which is not broken by \( u, v \) and \( \omega \) is usual baryon number \( B = BI \). The \( L \) and \( B \) charges for the fermion and Higgs multiplets are listed in Table II. Moreover, the Yukawa couplings of (18) conserve \( L \) and violate \( L \) with \( \pm 2 \) units which implies that these interactions are very small.

Taking into account of the famous experimental data \([13]\)

\[
R_{\mu\text{on}} \simeq \frac{M_{Y}}{M_{W}} \lessapprox 0.012.
\]

we get the constraint: \( R_{\mu\text{on}} \lessapprox \frac{M_{Y}}{M_{W}} \lessapprox 0.012 \). Therefore, it follows that \( M_{Y} > 230 \) GeV. However, the stronger bilepton mass bound has been derived from consideration of experimental limit on lepton-number violating charged lepton decays [14] of 440 GeV.

In the case of \( u \rightarrow 0 \), analyzing the \( Z \) decay width [13], the \( Z - Z' \) mixing angle is constrained by \(-0.0015 \leq \varphi \leq 0.001 \). From atomic parity violation in cesium, bounds for mass of the new exotic \( Z' \) and the \( Z - Z' \) mixing angles, again in the limit \( u \rightarrow 0 \), are given [13]

\[
-0.00156 \leq \varphi \leq 0.00105, \quad M_{Z'} \geq 2.1 \text{ TeV}
\] (22)

These values coincide with the bounds in the usual 331RH3HT [16].

From the \( W \) width, one gets an upper limit [3]:

\[
\sin \theta \leq 0.08.
\] (23)
III. HIGGS POTENTIAL

In this model, the most general Higgs potential has very simple form

\[ V(\chi, \phi) = \mu_1^2 \chi^1 \chi + \mu_2^2 \phi^1 \phi + \lambda_1 (\chi^1 \chi)^2 + \lambda_2 (\phi^1 \phi)^2 + \lambda_3 (\chi^1 \chi)(\phi^1 \phi) + \lambda_4 (\chi^1 \phi)(\phi^1 \chi). \]  

(24)

Note that there is no trilinear scalar coupling and this makes the Higgs potential much simpler than those in the 331RN3HT and closer to that of the SM. The analysis in Ref. \[1\] shows that after symmetry breaking, there are eight Goldstone bosons and four physical scalar fields. One of two physical neutral scalars is the SM Higgs boson.

Let us shift the Higgs fields into physical ones

\[ \chi = \left( \begin{array}{c} \chi_1^0 + \frac{u}{\sqrt{2}} \\ \chi_2^0 + \frac{\omega}{\sqrt{2}} \\ \chi_3^0 + \frac{v}{\sqrt{2}} \end{array} \right), \quad \phi = \left( \begin{array}{c} \phi_1^+ \\ \phi_2^0 + \frac{u}{\sqrt{2}} \\ \phi_3^+ \end{array} \right). \]  

(25)

The subscript \( P \) denotes physical fields as in the usual treatment. However, in the following, this subscript will be dropped. By substitution of (25) into (24), the potential becomes

\[ V(\chi, \phi) = \mu_1^2 \left[ \left( \chi_1^0 + \frac{u}{\sqrt{2}} \right) \left( \chi_1^0 + \frac{u}{\sqrt{2}} \right) + \chi_3^0 \chi_2^- + \left( \chi_3^0 + \frac{\omega}{\sqrt{2}} \right) \left( \chi_3^0 + \frac{\omega}{\sqrt{2}} \right) \right] + \mu_2^2 \left[ \phi_1^- \phi_1^+ + \left( \phi_2^0 + \frac{u}{\sqrt{2}} \right) \left( \phi_2^0 + \frac{u}{\sqrt{2}} \right) + \phi_3^- \phi_3^+ \right] + \lambda_1 \left[ \left( \chi_1^0 + \frac{u}{\sqrt{2}} \right) \left( \chi_1^0 + \frac{u}{\sqrt{2}} \right) + \chi_2^+ \chi_2^- + \left( \chi_3^0 + \frac{\omega}{\sqrt{2}} \right) \left( \chi_3^0 + \frac{\omega}{\sqrt{2}} \right) \right]^2 + \lambda_2 \left[ \phi_1^- \phi_1^+ + \left( \phi_2^0 + \frac{u}{\sqrt{2}} \right) \left( \phi_2^0 + \frac{u}{\sqrt{2}} \right) + \phi_3^- \phi_3^+ \right]^2 + \lambda_3 \left[ \left( \chi_1^0 + \frac{u}{\sqrt{2}} \right) \left( \chi_1^0 + \frac{u}{\sqrt{2}} \right) + \chi_2^+ \chi_2^- + \left( \chi_3^0 + \frac{\omega}{\sqrt{2}} \right) \left( \chi_3^0 + \frac{\omega}{\sqrt{2}} \right) \right] \times \left[ \phi_1^- \phi_1^+ + \left( \phi_2^0 + \frac{u}{\sqrt{2}} \right) \left( \phi_2^0 + \frac{u}{\sqrt{2}} \right) + \phi_3^- \phi_3^+ \right] + \lambda_4 \left[ \left( \chi_1^0 + \frac{u}{\sqrt{2}} \right) \phi_1^+ + \chi_3^0 \left( \phi_2^0 + \frac{u}{\sqrt{2}} \right) + \left( \chi_3^0 + \frac{\omega}{\sqrt{2}} \right) \phi_3^+ \right] \times \left[ \phi_1^- \left( \chi_1^0 + \frac{u}{\sqrt{2}} \right) + \left( \phi_2^0 + \frac{u}{\sqrt{2}} \right) \chi_2^- + \phi_3^\dagger \left( \chi_3^0 + \frac{\omega}{\sqrt{2}} \right) \right]. \]  

(26)

From the above expression, we get constraint equations at the tree level

\[ \mu_1^2 + \lambda_1 (u^2 + \omega^2) + \lambda_3 \frac{v^2}{2} = 0, \]  

(27)

\[ \mu_2^2 + \lambda_2 v^2 + \lambda_3 (u^2 + \omega^2) = 0, \]  

(28)

which imply that the Higgs vacuums are not SU(3)_L \( \otimes \) U(1)_X singlets. As a result, the gauge symmetry is broken spontaneously. The nonzero values of \( \chi \) and \( \phi \) at the minimum value of \( V(\chi, \phi) \) can be easily obtained by

\[ \chi^+ \chi = \frac{u^2 + \omega^2}{2} = \frac{\lambda_3 \mu_3^2 - 2\lambda_2 \mu_1^2}{4\lambda_1 \lambda_2 - \lambda_3^2}, \]  

(29)

\[ \phi^+ \phi = \frac{v^2}{2} = \frac{\lambda_3 \mu_3^2 - 2\lambda_1 \mu_2^2}{4\lambda_1 \lambda_2 - \lambda_3^2}. \]

It is worth noting that any other choice of \( u, \omega \) for the vacuum value of \( \chi \) satisfying (24) gives the same physics because it is related to (24) by an SU(3)_L \( \otimes \) U(1)_X transformation. Thus, in general, we assume that \( u \neq 0 \).
Since \( u \) is a parameter of lepton-number violation, therefore the terms linear in \( u \) violate the latter. Applying the constraint equations \( \text{(24)} \) and \( \text{(25)} \) we get the minimum value, mass terms, lepton-number conserving and violating interactions:

\[
V(\chi, \phi) = V_{\text{min}} + V_{\text{mass}}^N + V_{\text{mass}}^C + V_{\text{LNC}} + V_{\text{LNV}},
\]

where

\[
V_{\text{min}} = -\frac{\lambda_2}{4} v^4 - \frac{1}{4} (u^2 + \omega^2) [\lambda_1 (u^2 + \omega^2) + \lambda_3 v^2],
\]

\[
V_{\text{mass}}^N = \lambda_1 (u S_1 + \omega S_2)^2 + \lambda_2 v^2 S_2^2 + \lambda_3 v (u S_1 + \omega S_3) S_2,
\]

\[
V_{\text{mass}}^C = \frac{\lambda_4}{2} (u \phi_1^+ + v \chi_2^+ + \omega \phi_3^+)(u \phi_1^- + v \chi_2^- + \omega \phi_3^-),
\]

\[
V_{\text{LNC}} = \lambda_1 (\chi^+ \chi)^2 + \lambda_2 (\phi^+ \phi)^2 + \lambda_3 (\chi^+ \chi)(\phi^+ \phi) + \lambda_4 (\chi^+ \phi)(\phi^+ \chi)
+ 2 \lambda_1 \omega S_3 (\chi^+ \chi) + 2 \lambda_2 v S_2 (\phi^+ \phi) + \lambda_3 v S_2 (\chi^+ \chi) + \lambda_3 \omega S_3 (\phi^+ \phi)
+ \frac{\lambda_4}{\sqrt{2}} (v \chi_2^+ + \omega \phi_3^+)(\chi^+ \phi) + \frac{\lambda_4}{\sqrt{2}} (v \chi_2^- + \omega \phi_3^-)(\phi^+ \chi),
\]

\[
V_{\text{LNV}} = 2 \lambda_1 u S_1 (\chi^+ \chi) + \lambda_3 u S_1 (\phi^+ \phi) + \frac{\lambda_4}{\sqrt{2}} u [\phi_1^- (\chi^+ \phi) + \phi_3^+ (\phi^+ \chi)].
\]

In the above equations, we have dropped the subscript \( P \) and used \( \chi = (\chi_1^0, \chi_2^-, \chi_3^0)^T, \phi = (\phi_1^+, \phi_2^0, \phi_3^+)^T \). Moreover, we have expanded the neutral Higgs fields as

\[
\chi_1^0 = \frac{S_1 + i A_1}{\sqrt{2}}, \quad \chi_3^0 = \frac{S_3 + i A_3}{\sqrt{2}}, \quad \phi_2^0 = \frac{S_2 + i A_2}{\sqrt{2}}.
\]

In the literature, the real parts \( (S_i, i = 1, 2, 3) \) are also called CP-even scalar and the imaginary part \( (A_i, i = 1, 2, 3) \) – CP-odd scalar. In this paper, for short, we call them scalar and pseudoscalar field, respectively. As expected, the lepton-number violating part \( V_{\text{LNC}} \) is linear in \( u \) and trilinear in scalar fields.

In the pseudoscalar sector, all fields are Goldstone bosons: \( G_1 = A_1, G_2 = A_2 \) and \( G_3 = A_3 \) (cf. Eq. \text{(31)}). The scalar fields \( S_1, S_2 \) and \( S_3 \) gain masses via \text{(31)}), thus we get one Goldstone boson \( G_4 \) and two neutral physical fields-the SM \( H^0 \) and the new \( H_1^0 \) with masses

\[
m_{H^0}^2 = \lambda_2 v^2 + \lambda_1 (u^2 + \omega^2) - \sqrt{[\lambda_2 v^2 - \lambda_1 (u^2 + \omega^2)]^2 + \lambda_3^2 v^2 (u^2 + \omega^2)} \approx \frac{4 \lambda_1 \lambda_2 - \lambda_3^2}{2 \lambda_1} v^2,
\]

\[
M_{H_1^0}^2 = \lambda_2 v^2 + \lambda_1 (u^2 + \omega^2) + \sqrt{[\lambda_2 v^2 - \lambda_1 (u^2 + \omega^2)]^2 + \lambda_3^2 v^2 (u^2 + \omega^2)} \approx 2 \lambda_1 \omega^2.
\]

In terms of scalars, the Goldstone and Higgs fields are given by

\[
G_4 = \frac{1}{\sqrt{1 + t_\theta^2}} (S_1 - t_\theta S_3),
\]

\[
H^0 = c_\zeta S_2 - \frac{s_\zeta}{\sqrt{1 + t_\theta^2}} (t_\theta S_1 + S_3),
\]

\[
H_1^0 = s_\zeta S_2 + \frac{c_\zeta}{\sqrt{1 + t_\theta^2}} (t_\theta S_1 + S_3),
\]

where

\[
t_{2\zeta} = \frac{\lambda_3 M_W M_X}{\lambda_1 M_X^2 - \lambda_2 M_W^2}.
\]

From Eq. \text{(37)}, it follows that mass of the new Higgs boson \( M_{H^0} \) is related to mass of the bilepton gauge \( X^0 \) (or \( Y^\pm \) via the law of Pythagoras) through

\[
M_{H_1^0}^2 = \frac{8 \lambda_1}{g^2} M_X^2 \left[ 1 + \mathcal{O} \left( \frac{M_X^2}{M_X^2} \right) \right]
= \frac{2 \lambda_1 s_W^2}{\pi \alpha} M_X^2 \left[ 1 + \mathcal{O} \left( \frac{M_X^2}{M_X^2} \right) \right] \approx 18.8 \lambda_1 M_X^2.
\]
Here, we have used $\alpha = \frac{1}{2\pi}$ and $s_W^2 = 0.231$.
In the charged Higgs sector, the mass terms for $(\phi_1, \chi_2, \phi_3)$ is given by\cite{[62]}, thus there are two Goldstone bosons and one physical scalar field:

$$H^+_2 \equiv \frac{1}{\sqrt{u^2 + v^2 + \omega^2}}(u\phi_1^+ + v\chi_2^+ + \omega\phi_3^+)$$

(43)

with mass

$$M^2_{H^+_2} = \frac{\lambda_4}{2}(u^2 + v^2 + \omega^2) = 2\lambda_4 M^2_Y g^2$$

$$= \frac{s_W^2 \lambda_4}{2\pi \alpha} M^2_Y \approx 4.7\lambda_4 M^2_Y .$$

(44)

Two remaining Goldstone bosons are

$$G^+_5 = \frac{1}{\sqrt{1 + t_6^2}}(\phi_1^+ - t_6 \phi_3^+),$$

(45)

$$G^+_6 = \frac{1}{\sqrt{(1 + t_6^2)(u^2 + v^2 + \omega^2)}} [v(t_6 \phi_1^+ + \phi_3^+) - \omega(1 + t_6^2)\chi_2^+] .$$

(46)

Thus, all pseudoscalars are eigenstates and massless (Goldstone). Other physical fields are related to the scalars in the weak basis by the linear transformations:

$$\begin{pmatrix} H^0 \cr H^0_1 \cr G_4 \cr H^+_2 \cr G^+_5 \cr G^+_6 \end{pmatrix} = \begin{pmatrix} -s_\zeta s_\theta & c_\zeta & -s_\zeta c_\theta & (s_1) \\
 c_\zeta s_\theta & c_\zeta & c_\zeta c_\theta & (s_2) \\
 c_\theta & 0 & -s_\theta & (s_3) \\
 -s_\theta c_\phi & v c_\phi & -v c_\phi & (\phi_1^+) \\
 c_\theta & v c_\theta & -v c_\theta & (\phi_2^+) \\
 c_\theta & v c_\theta & -v c_\theta & (\phi_3^+) \end{pmatrix} .$$

(47)

(48)

From\cite{[60] and [61]}, we come to the previous result in Ref.\cite{[63]}

$$\lambda_1 > 0 , \hspace{1em} \lambda_2 > 0 , \hspace{1em} 4\lambda_1 \lambda_2 > \lambda_3^2 .$$

(49)

Eq.\cite{[43]} shows that mass of the massive charged Higgs boson $H^+_2$ is proportional to those of the charged bilepton $Y$ through a coefficient of Higgs self-interaction $\lambda_4 > 0$. Analogously, this happens for the SM Higgs boson $H^0$ ($M_{H^0} \sim M_Y$) and the new $H^0_1$ ($M_{H^0_1} \sim M_X$). Combining\cite{[43]} with the constraint equations\cite{[27], [28]} we get a consequence: $\lambda_3$ is negative ($\lambda_3 < 0$).

To finish this section, let us comment on our physical Higgs bosons. In the effective approximation $w \gg v, u$, from Eqs\cite{[47] and [48]} it follows that

$$H^0 \sim S_2 , \hspace{1em} H^0_1 \sim S_3 , \hspace{1em} G_4 \sim S_1 , \hspace{1em} H^+_2 \sim \phi^+_3 , \hspace{1em} G^+_5 \sim \phi^+_1 , \hspace{1em} G^+_6 \sim \chi^+_2 .$$

(50)

This means that, in the effective approximation, the charged boson $H^+_2$ is a scalar bilepton (with lepton number $L = 2$), while the neutral scalar bosons $H^0$ and $H^0_1$ do not carry lepton number (with $L = 0$).

IV. HIGGS - SM GAUGE BOSON COUPLINGS

There are a total of 9 gauge bosons in the SU(3)$_L \otimes$ U(1)$_X$ group and 8 of them are massive. As shown in the previous section, we have got just 8 massless Goldstone bosons - the justified number for the model. One of the neutral scalars is identified with the SM Higgs boson, therefore its couplings to ordinary gauge bosons such as the
photon, the Z and the $W^\pm$ bosons have to have, in the effective limit, usual known forms. To search Higgs bosons at future high energy colliders, one needs their couplings with ordinary particles, specially with the gauge bosons in the SM.

The interactions among the gauge bosons and the Higgs bosons arise in part from

$$\sum_{Y=\chi, \phi} (D_\mu Y)^+ (D^\mu Y).$$

In the following the summation over $Y$ is default and only the terms giving interested couplings are explicitly displayed.

First, we consider the relevant couplings of the SM $W$ boson with the Higgs and Goldstone bosons. The trilinear couplings of the pair $W^+W^-$ with the neutral scalars are given by

$$\sum_{Y} (P_{\mu}^\text{NC} (\chi))^+ (P^{\text{CC} \mu} (\chi)) + (P_{\mu}^\text{NC} (\phi))^+ (P^{\text{CC} \mu} (\phi)) + \text{h.c.} = \frac{g^2 v}{2} W^+ W^- S_2. \quad (51)$$

Because of $S_2$ is a combination of only $H$ and $H_1^0$, therefore, there are two couplings which are given in Table IV. Couplings of the single $W$ with two Higgs bosons exist in

$$i \left( Y + P_{\mu}^\text{CC} \partial^\mu Y - \partial^\mu Y + P_{\mu}^\text{CC} Y \right) = \frac{ig}{\sqrt{2}} W^- \left[ Y_2^\pm (c_0 \partial^\mu Y_1 - s_0 \partial^\mu Y_3) - \partial^\mu Y_2^\pm (c_0 Y_1 - s_0 Y_3) \right] + \text{h.c.}$$

$$= \frac{ig}{\sqrt{2}} W^- \left[ Y_2^\pm (c_0 \partial^\mu \lambda_1^0 - s_0 \partial^\mu \lambda_3^0) - \partial^\mu Y_2^\pm (c_0 \lambda_1^0 - s_0 \lambda_3^0) \right]$$

$$+ \phi_2^0 (c_0 \partial^\mu \phi_1^+ - s_0 \partial^\mu \phi_3^+) - \partial^\mu \phi_2^0 (c_0 \phi_1^+ - s_0 \phi_3^+) \right] + \text{h.c.} \quad (52)$$

The resulting couplings of the single $W$ boson with two scalar fields are listed in Table V where we have used a notation $A \partial_\mu B = A(\partial_\mu B) - (\partial_\mu A) B$. Vanishing couplings are

$$\mathcal{V}(W^- H_2^0 H_0^0) = \mathcal{V}(W^- H_2^0 H_1^0) = \mathcal{V}(W^- H^0 G_6^+) = 0.$$ 

Quartic couplings of $W^+W^-$ with two scalar fields arise in part from

$$\sum_{Y} (P_{\mu}^\text{CC} (\chi))^+ (P^{\text{CC} \mu} (\chi)) = \frac{g^2 v}{2} W^+ W^- \left[ \lambda_2^\pm \lambda_2^\mp + c_0^2 \lambda_1^0 \lambda_1^0 + s_0^2 \lambda_3^0 \lambda_3^0 \right]$$

$$+ c_0 s_0 (\lambda_1^0 \lambda_3^0 + \lambda_3^0 \lambda_1^0) + \phi_2^0 \phi_2^0$$

$$+ c_0^2 \phi_1^+ \phi_1^+ + s_0^2 \phi_3^+ \phi_3^+ \right] + \text{h.c.} \quad (53)$$

With the help of (A1) and (A2), we get the interested couplings of $W^+W^-$ with two scalars which are listed in Table VI. Our calculation give following vanishing couplings

$$\mathcal{V}(W^+ W^- H_2^2 G_5^-) = \mathcal{V}(W^+ W^- G_5^+ G_6^-)$$

$$= \mathcal{V}(W^+ W^- H^0 G_6^+) = \mathcal{V}(W^+ W^- H_1^0 G_4^+) = 0. \quad (54)$$

Now we turn on the couplings of neutral gauge bosons with Higgs bosons. In this case, the interested couplings exist in

$$i \left( Y + P_{\mu}^\text{NC} \partial^\mu Y - \partial^\mu Y + P_{\mu}^\text{NC} Y \right) = -\frac{ig}{2} \left( W_{\mu}^3 \left( \partial_\mu \lambda_1^0 \lambda_1^0 - \partial_\mu \lambda_2^0 \lambda_2^0 + \partial_\mu \phi_1^+ \phi_1^+ - \partial_\mu \phi_2^0 \phi_2^0 \right) \right)$$

$$+ W_{\mu}^8 \left( \partial_\mu \lambda_1^0 \lambda_1^0 + \partial_\mu \lambda_2^0 \lambda_2^0 + \partial_\mu \phi_1^+ \phi_1^+ + \partial_\mu \phi_2^0 \phi_2^0 - 2 \partial_\mu \lambda_3^0 \lambda_3^0 - 2 \partial_\mu \phi_3^+ \phi_3^+ \right)$$

$$+ i \frac{1}{\sqrt{3}} B^\mu \left[ - \frac{1}{3} (\partial_\mu \lambda_1^0 \lambda_1^0 + \partial_\mu \lambda_2^0 \lambda_2^0 + \partial_\mu \lambda_3^0 \lambda_3^0) + \frac{2}{3} (\partial_\mu \phi_1^+ \phi_1^+ + \partial_\mu \phi_2^0 \phi_2^0$$

$$+ \partial_\mu \phi_3^+ \phi_3^+) \right] + \text{h.c.} \right) \quad (55)$$
It can be checked that, as expected, the photon $A_\mu$ does not interact with neutral Higgs bosons. Other vanishing couplings are

$$\mathcal{V}(A H_2^+ G^-_3) = \mathcal{V}(A H_2^+ G^-_6) = \mathcal{V}(A G_2^+ G^-_3) = 0 \quad (56)$$

and

$$\mathcal{V}(A A H^0) = \mathcal{V}(A A H^0_1) = \mathcal{V}(A A G_4) = 0, \\
\mathcal{V}(A Z H^0) = \mathcal{V}(A Z H^0_1) = \mathcal{V}(A Z G_4) = 0, \\
\mathcal{V}(A Z' H^0) = \mathcal{V}(A Z' H^0_1) = \mathcal{V}(A Z' G_4) = 0.$$

The nonzero electromagnetic couplings are listed in Table VII. It should be noticed that the electromagnetic interaction is diagonal, i.e. the non-zero couplings, in this model, always have a form

$$i e q_H A^\mu H^+ \partial_\mu H.$$

For the $Z$ bosons, the following observation is useful

$$W_3^\mu = U_{12} Z^\mu + \cdots, \quad W_8^\mu = U_{22} Z^\mu + \cdots, \\
B^\mu = U_{32} Z^\mu + \cdots, \quad y^\mu = U_{42} Z^\mu + \cdots \quad (58)$$

Here

$$U_{12} = c_\varphi c_{\theta_1} c_W, \quad U_{22} = \frac{c_\varphi (s_W^2 - 3 c_W^2 s_\varphi^2) - s_\varphi \sqrt{(1 - 4 s_W^2 c_W^2)} (4 c_W^2 - 1)}{\sqrt{3} c_W c_\varphi}, \\
U_{32} = -\frac{t_W (c_\varphi \sqrt{4 c_W^2 - 1} + s_\varphi \sqrt{1 - 4 s_W^2 c_W^2})}{\sqrt{3} c_\varphi} \quad (60)$$

are elements in the mixing matrix of the neutral gauge bosons given in the Appendix of Ref. [5]. From (55) and (58), it follows that the trilinear couplings of the single $Z$ with charged Higgs bosons exist in part from the Lagrangian terms

$$-\frac{ig}{2} Z^\mu \left[ \left( U_{12} - \frac{U_{22}}{\sqrt{3}} + \frac{t}{3} \sqrt{\frac{2}{3}} U_{32} \right) \partial_\mu \chi_2^- \chi_2^+ + \left( U_{12} + \frac{U_{22}}{\sqrt{3}} + \frac{2t}{3} \sqrt{\frac{2}{3}} U_{32} \right) \partial_\mu \phi_1^- \phi_1^+ \\
+ \left( -\frac{2}{\sqrt{3}} U_{22} + \frac{2t}{3} \sqrt{\frac{2}{3}} U_{32} \right) \partial_\mu \phi_3^- \phi_3^+ + U_{42} \left( \partial_\mu \phi_1^- \phi_3^+ + \partial_\mu \phi_3^- \phi_1^+ \right) \right] + \text{h.c.} \quad (61)$$

From (61) we get trilinear couplings of the $Z$ with the charged Higgs bosons which are listed in Table VIII. The limit sign (→) in the Tables is the effective one.

In the effective limit, the $ZG_5 G_5$ vertex gets an exact expression as in the SM. Hence $G_5$ can be identified with the charged Goldstone boson in the SM ($G_{W_2}$).

Now we search couplings of the single $Z_\mu$ boson with neutral scalar fields. With the help of the following equations

$$
\chi_1^0 \partial_\mu \chi_1^0, \quad \chi_3^0 \partial_\mu \chi_3^0, \quad \phi_2^0 \partial_\mu \phi_2^0 = i G_2 \partial_\mu S_2, \\
\partial_\mu \chi_1^0 \chi_3^0 + \partial_\mu \chi_3^0 \chi_1^0 = \frac{1}{2} \left[ \partial_\mu S_1 S_3 + \partial_\mu S_3 S_1 + \partial_\mu G_1 G_3 + \partial_\mu G_3 G_1 + i G_3 \partial_\mu S_1 + i G_1 \partial_\mu S_3 \right], \quad (62)
$$

the necessary parts of Lagrangian are

$$\frac{g}{2} \left[ \left( U_{12} + \frac{U_{22}}{\sqrt{3}} - \frac{t}{3} \sqrt{\frac{2}{3}} U_{32} \right) G_1 \partial_\mu S_1 + U_{42} G_1 \partial_\mu S_3 \\
+ \left( -\frac{2}{\sqrt{3}} U_{22} - \frac{t}{3} \sqrt{\frac{2}{3}} U_{32} \right) G_3 \partial_\mu S_3 + U_{42} G_3 \partial_\mu S_1 + \left( -U_{12} + \frac{U_{22}}{\sqrt{3}} + \frac{2t}{3} \sqrt{\frac{2}{3}} U_{32} \right) G_2 \partial_\mu S_2 \right]. \quad (63)$$
The resulting couplings are listed in Table IX. From Table IX, we conclude that $G_2$ should be identified to $G_Z$ in the SM. For the $Z'$ boson, the following remark is again helpful

\[ W_3^\mu = U_{13} Z' \mu + \cdots, \quad W_8^\mu = U_{23} Z' \mu + \cdots, \]
\[ B^\mu = U_{33} Z' \mu + \cdots, \quad y^\mu = U_{43} Z' \mu + \cdots, \]  

(64)

where

\[ U_{13} = s_\varphi c_\theta c_W, \quad U_{23} = s_\varphi (s_W^2 - 3 c_\varphi c_W s_\theta^2) + c_\varphi \sqrt{(1 - 4 s_\varphi^2 c_W^2)(4 c_W^2 - 1)}, \]
\[ U_{33} = t_W (s_\varphi \sqrt{4 s_W^2 - 1} - c_\varphi \sqrt{1 - 4 s_\varphi^2 c_W^2}). \]  

(65)

(66)

Thus, with the replacement $Z \rightarrow Z'$ one just replaces column 2 by 3, for example, trilinear coupling constants of the $Z'_\mu$ with two neutral Higgs bosons are given in Table IX.

Next, we search couplings of two neutral gauge bosons with scalar fields which arise in part from

\[ Y^+ P^\mu_{\nu} P_{\nu} Y = \frac{g^2}{4} \left[ Y^+_1 (A^{\mu\nu}_1 A_{11\mu} + y_\mu y^\nu) + Y^+_3 (A_{11\mu} y^\nu + A_{33\mu} y^\nu) \right] Y_1 + (A^{\mu\nu}_{22} A_{22\mu}) Y_2 Y_2 + \left[ Y^+_1 (A_{11\mu} y^\nu + A_{33\mu} y^\nu) + Y^+_3 (A_{11\mu} A_{33\mu} + y_\mu y^\nu) \right] Y_3, \]
\[ = \frac{g^2}{4} \left\{ \left[ \lambda_1^0 (A^{\mu\nu}_1 A_{11\mu} + y_\mu y^\nu) + \lambda_3^0 (A_{11\mu} y^\nu + A_{33\mu} y^\nu) \right] \chi^0_1 + (A^{\mu\nu}_{22} A_{22\mu}) \chi^0_2 \chi^0_2 \right. \]
\[ + \left[ \lambda_1^0 (A^{\mu\nu}_1 A_{11\mu} + y_\mu y^\nu) + \lambda_3^0 (A_{11\mu} A_{33\mu} + y_\mu y^\nu) \right] \chi_3^0 + \left( A^{\mu\nu}_{22} A^{\mu\nu}_{22} \right) \phi^0_2 \phi^0_2 \]
\[ + \left[ \lambda_1^0 (A_{11\mu} y^\nu + A_{33\mu} y^\nu) + \phi_3^0 \left( A_{11\mu} A_{33\mu} + y_\mu y^\nu \right) \right] \phi^0_3 \right\}. \]  

(67)

(68)

Here $A^\mu_{ii} (i = 1, 2, 3)$ is a diagonal element in the matrix $P^\mu_{\nu} P_{\nu}$ which is dependent on the $U(1)_X$ charge:

\[ A^{\mu\nu}_1 = W_3^\mu + \frac{W_8^\mu}{\sqrt{3}} - \frac{t}{3} \sqrt{\frac{2}{3}} B^\mu, \]
\[ A^{\mu\nu}_{11} = W_3^\mu + \frac{W_8^\mu}{\sqrt{3}} + \frac{t}{3} \sqrt{\frac{2}{3}} B^\mu, \]
\[ A^{\mu\nu}_{22} = - W_3^\mu + \frac{W_8^\mu}{\sqrt{3}} - \frac{t}{3} \sqrt{\frac{2}{3}} B^\mu, \]
\[ A^{\mu\nu}_{33} = -2 \frac{W_8^\mu}{\sqrt{3}} - \frac{t}{3} \sqrt{\frac{2}{3}} B^\mu, \]
\[ A^{\mu\nu}_{33} = -2 \frac{W_8^\mu}{\sqrt{3}} + \frac{2t}{3} \sqrt{\frac{2}{3}} B^\mu. \]  

(69)

(70)

Quartic couplings of two $Z$ with neutral scalar fields are given by

\[ \frac{g^2}{4} \left\{ \left[ \lambda_1^0 (A^{\mu\nu}_1 A_{11\mu} + y_\mu y^\nu) + \lambda_3^0 (A_{11\mu} y^\nu + A_{33\mu} y^\nu) \right] \chi_1^0 + \left( A^{\mu\nu}_{22} A^{\mu\nu}_{22} \right) \phi^0_2 \phi^0_2 \right\} \]
\[ + \left[ \lambda_1^0 (A_{11\mu} y^\nu + A_{33\mu} y^\nu) + \phi_3^0 \left( A_{11\mu} A_{33\mu} + y_\mu y^\nu \right) \right] \phi^0_3 \right\}. \]
In this case, the couplings are listed in Table XI.

Trilinear couplings of the pair $ZZ$ with one scalar field are obtained via the following terms:

$$
\frac{g^2}{4} \left[ v S_2 A_2^\phi A_2^\phi + u S_1 A_1^\phi A_1^\phi + \omega S_3 A_3^\phi A_3^\phi 
+ (u S_1 + \omega S_3) y_\mu y^\mu - (\omega S_1 + u S_3) y_\mu A_2^\phi A_2^\phi \right].
$$

(71)

The obtained couplings are given in Table XI.

Because of (63), for the $ZZ'$ couplings with scalar fields, the above manipulation is good enough. For example, Table XI is replaced by Table XIII.

Now we turn on the interested coupling $ZW^±H_2^±$ arisen in part from

$$
Y^+p^{NC}p^{CC}_\mu Y + \text{h.c.} = \frac{g^2}{2\sqrt{2}} \left\{ W_\mu^- A_2^\mu Y^* \left( c_\theta y_1 - s_\theta y_3 \right) + W_\mu^+ \left[ (c_\theta A_1^\mu - s_\theta y_2^\mu) Y_1^* + (c_\theta y_3 - s_\theta A_1^\mu) Y_3^* \right] \right\} + \text{h.c.}
$$

(72)

For our Higgs triplets, one gets

$$
\frac{g^2}{2\sqrt{2}} \left\{ W_\mu^- A_2^\mu \left( c_\theta (\chi^-_2 + A_2^\phi \phi_2^0) - s_\theta \phi_2^+ \right) + W_\mu^+ \left[ (c_\theta A_1^\mu - s_\theta y_2^\mu) \chi_1^+ + (c_\theta y_3 - s_\theta A_1^\mu) \chi_3^+ \right] \right\} + \text{h.c.}
$$

(73)

From Eq. (72), the trilinear couplings of the $W$ boson with one scalar and one neutral gauge bosons exist in a part

$$
\frac{g^2}{4} W_\mu^+ \left\{ v \phi^- \left[ c_\theta \left( \frac{2}{\sqrt{3}} W_2^\mu + \frac{4t}{3} \sqrt{\frac{2}{3}} B^\mu \right) - s_\theta y_2^\mu \right] + v \phi_3 \left[ c_\theta y_3 - s_\theta \left( -W_3^\mu - \frac{4t}{3} \sqrt{\frac{2}{3}} B^\mu \right) \right] + \omega \chi_2^\pm \left( s_\theta (W_3^\mu + \sqrt{3} W_2^\mu) + \frac{c_\theta y}{\sqrt{3}} y^\mu \right) \right\} + \text{h.c.}
$$

(74)

From the above equation, we get necessary nonzero couplings, which are listed in Table XIV. Vanishing couplings are

$$
\mathcal{V}(AW^+H_2^±) = \mathcal{V}(AW^+G_0^-) = 0.
$$

(75)

Eq. (75) is consistent with an evaluation in Ref. 11, where authors neglected the diagrams with the $\gamma W^±H^±$ vertex.

From (41), it follows that, to get couplings of the bilepton gauge boson $Y^+$ with $ZH^+_2$, one just makes in (74) the replacement: $c_\theta \rightarrow -s_\theta, \ s_\theta \rightarrow c_\theta$.

Finally, we can identify the scalar fields in the considered model with that in the SM as follows:

$$
H \leftrightarrow h, \ G_5^+ \leftrightarrow G_W^+, \ G_2 \leftrightarrow G_Z.
$$

(76)

In the effective limit $\omega \gg v, u$ our Higgs can be represented as

$$
\chi = \left( \begin{array}{c} \frac{1}{\sqrt{2}} u + G_{X^0} \\ \frac{1}{\sqrt{2}} (\omega + H_{Z^0}^0 + iG_Z) \end{array} \right), \ \phi = \left( \begin{array}{c} G_{W^+} \\ \frac{1}{\sqrt{2}} (v + h + iG_Z) \end{array} \right)
$$

(77)

where $G_3 \sim G_{Z^0}, \ G_{G_0} \sim G_{Y^-}$ and

$$
G_4 + i G_1 \sim \sqrt{2} G_{X^0}
$$

(78)

Note that identification in (78) is possible due to the fact that both scalar and pseudoscalar parts of $\chi_1^0$ are massless. In addition, the pseudoscalar part is decouple from others. While its scalar part mixes with the same ones as in the gauge boson sector (for details, see [3]).

We emphasize again, in the effective approximation, all Higgs-gauge boson couplings in the SM are recovered (see Table XVI). In contradiction with the previous analysis in Ref. 4, the condition $u \sim v$ or introduction of the third triplet are not necessary.
V. PRODUCTION OF $H_{2}^\pm$ VIA WZ FUSION AT LHC

The possibility to detect the neutral Higgs boson in the minimal version at $e^+e^-$ colliders was considered in [18] and production of the SM-like neutral Higgs boson at the CERN LHC was considered in Ref. [9]. This section is devoted to production of the charged $H_{2}^\pm$ at the CERN LHC.

Let us firstly discuss on the mass of this Higgs boson. Eq. (44) gives us a connection between its mass and those of the singly-charged bilepton $Y$. Through the coefficient of Higgs self-coupling $\lambda_4$, note that in the considered model, the nonzero majoron couplings of $G_{X\alpha}$ with the leptons exist only in the loop-levels. To keep the smallness of these couplings, the mass $M_{H_{2}^\pm}$ can be taken in the electroweak scale with $\lambda_4 \sim 0.01$ [19]. From (44), taking the lower limit for $M_Y$ to be 1 TeV, the mass of $H_{2}^\pm$ is in range of 200 GeV.

Taking into account that, in the effective approximation, $H_{2}^-$ is the bilepton, we get the dominant decay channels as follows

$$H_{2}^- \rightarrow \nu_\ell, \; \bar{u}d_\alpha, \; D_\alpha \bar{u}_a, \; ZW^-, \; Z'W^-, \; XW^-, \; ZY^-.$$  

(79)

Assuming that masses of the exotic quarks $(U, D_\alpha)$ are larger than $M_{H_{2}^\pm}$, we come to the fact that, the hadron modes are absent in decay of the charged Higgs boson. Due to that the Yukawa couplings of $H_{2}^i \bar{l}^\pm \nu$ are very small, the main decay modes of the $H_{2}^\pm$ are in the second line of (79). Note that the charged Higgs bosons in doublet models such as two-Higgs doublet model or MSSM, has both hadronic and leptonic modes [10]. This is a specific feature of the model under consideration.

Because of the exotic $X, Y, Z'$ gauge bosons are heavy, the coupling of a singly-charged Higgs boson $(H_{2}^\pm)$ with the weak gauge bosons, $H_{2}^\pm W^\mp Z$, may be main. Here, it is of particular importance for the electroweak symmetry breaking. Its magnitude is directly related to the structure of the extended Higgs sector under global symmetries [20]. This coupling can appear at the tree level in models with scalar triplets, while it is induced at the loop level in multi scalar doublet models. The coupling, in our model, differs from zero at the triple level due to the fact that the $H_{2}^\pm$ belongs to a triplet.

Thus, for the charged Higgs boson $H_{2}^\pm$, it is important to study the couplings given by the interaction Lagrangian

$$\mathcal{L}_{int} = f_{ZW} H_{2}^\pm W_\mu^\mp Z^\mu,$$  

(80)

where $f_{ZW}$, at tree level, is given in Table XIV. The same as in [11], the dominant rate is due to the diagram connected with the $W$ and $Z$ bosons. Putting necessary matrix elements in Table XIV, we get

$$f_{ZW} = -\frac{g^2 v \omega s_{2\theta}}{4\sqrt{\omega^2 + c_{2\theta}^2}} \frac{c_{\phi} - s_{\phi} \sqrt{(4c_{W}^2 - 1)(1 + 4t_{2\theta}^2)}}{\sqrt{(1 + 4t_{2\theta}^2)c_{W}^2 + (4c_{W}^2 - 1)t_{2\theta}^2}}.$$  

Thus, the form factor, at the tree-level, is obtained by

$$F = \frac{f_{ZW}}{gM_W} = -\frac{\omega s_{2\theta}}{2\sqrt{(\omega^2 + c_{2\theta}^2)(1 + 4t_{2\theta}^2)}} \frac{c_{\phi} - s_{\phi} \sqrt{(4c_{W}^2 - 1)(1 + 4t_{2\theta}^2)}}{c_{W}^2 + (4c_{W}^2 - 1)t_{2\theta}^2}.$$  

(81)

The decay width of $H_{2}^\pm \rightarrow W_i^\pm Z_i$, where $i = L, \; T$ represent respectively the longitudinal and transverse polarizations, is given by [11]

$$\Gamma(H_{2}^\pm \rightarrow W_i^\pm Z_i) = M_{H_{2}^\pm} \frac{\lambda^{1/2}(1, w, z)\lambda_{1/2}(1, w, z)}{16\pi} |M_{i1}|^2,$$  

(82)

where $\lambda(1, w, z) = (1 - w - z)^2 - 4wz$, $w = M_{W}^2/M_{H_{2}^\pm}^2$ and $z = M_{Z}^2/M_{H_{2}^\pm}^2$. The longitudinal and transverse contributions are given in terms of $F$ by

$$|M_{LL}|^2 = \frac{g^2}{4z}(1 - w - z)^2 |F|^2,$$  

(83)

$$|M_{TT}|^2 = 2g^2 w|F|^2.$$  

(84)

For the case of $M_{H_{2}^\pm} \gg M_{Z}$, we have $|M_{TT}|^2/|M_{LL}|^2 \sim 8M_{W}^2M_{Z}^2/M_{H_{2}^\pm}^4$ which implies that the decay into a longitudinally polarized weak boson pair dominates that into a transversely polarized one. The form factor $F$ and the mixing
angle \( t_\varphi \) are presented in Table III where we have used: \( s_{W}^2 = 0.2312 \), \( v = 246 \text{ GeV} \), \( \omega = 3 \text{ TeV} \) (or \( M_Y = 1 \text{TeV} \)) as the typical values to get five cases corresponding with the \( s_\vartheta \) values under the constraint \[23\] which was given in \(\text{Eq.}\). Next, let us study the impact of the \( H_+^\pm W_+^\mp \) vertex on the production cross section of \( pp \rightarrow W_+^\pm Z^\ast X \rightarrow H_+^\pm X \) which is a pure electroweak process with high \( p_T \) jets going into the forward and backward directions from the decay of the produced scalar boson without color flow in the central region. The hadronic cross section for \( pp \rightarrow H_+^\pm X \) via \( W_+^\pm Z^\ast \) fusion is expressed in the effective vector boson approximation \[21\] by

\[
\sigma_{\text{eff}}(s, M_{H_+^\pm}^2) \simeq \frac{16\pi^2}{\lambda(1,w,z)M_{H_+^\pm}^2} \sum_{\lambda=\tau,L} \Gamma(H_+^\pm \rightarrow W_+^\pm Z_\lambda) \tau \left. \frac{d\mathcal{L}}{d\tau} \right|_{pp/W_+^\pm Z_\lambda},
\]

where \( \tau = M_{H_+^\pm}^2/s \), and

\[
\left. \frac{d\mathcal{L}}{d\tau} \right|_{pp/W_+^\pm Z_\lambda} = \sum_{ij} \int_{\tau}^{1} \frac{d\tau'}{\tau'} \int_{\tau'}^{1} \frac{dx}{x} f_i(x) f_j(x') \left. \frac{d\mathcal{L}}{d\xi} \right|_{q_i,q_j/W_+^\pm Z_\lambda},
\]

with \( \tau' = \hat{s}/s \) and \( \xi = \tau/\tau' \). Here \( f_i(x) \) is the parton structure function for the \( i \)-th quark, and

\[
\left. \frac{d\mathcal{L}}{d\xi} \right|_{q_i,q_j/W_+^\pm Z_\lambda} = \frac{c}{16\pi^4 \xi} \ln \left( \frac{\hat{s}}{M_W^2} \right) \ln \left( \frac{\hat{s}}{M_Z^2} \right) \left[ (2 + \xi)^2 \ln(1/\xi) - 2(1 - \xi)(3 + \xi) \right],
\]

\[
\left. \frac{d\mathcal{L}}{d\xi} \right|_{q_i,q_j/W_+^\pm Z_\lambda} = \frac{c}{16\pi^4 \xi} \left[ (1 + \xi) \ln(1/\xi) + 2(\xi - 1) \right],
\]

where \( c = \frac{s_{W}^4}{64\pi^4 w_{W}^2} \left[ g_1 V(q_j) + g_2 A(q_j) \right] \) with \( g_1 V(q_j), g_2 A(q_j) \) for quark \( q_j \) are given in Table I of Ref. \[23\]. Using CTEQ6L \[23\], in Fig. 1 we have plotted \( \sigma_{\text{eff}}(s, M_{H_+^\pm}^2) \) at \( \sqrt{s} = 14 \text{ TeV} \), as a function of the Higgs boson mass corresponding five cases in Table III.

Assuming discovery limit of 25 events corresponding to the horizontal line, and taking the integrated luminosity of 300 \( fb^{-1} \) \[23\], from the figure, we come to conclusion that, for \( s_\vartheta = 0.08 \) (the line on top), the charged Higgs boson \( H_+^\pm \) with mass larger than \( 1700 \text{ GeV} \), cannot be seen at the LHC. These limiting masses are denoted by \( M_{H_+^\pm}^{\text{max}} \) and listed in Table III. If the mass of the above mentioned Higgs boson is in range of \( 200 \text{ GeV} \) and \( s_\vartheta = 0.08 \), the cross section can exceed 260 \( fb \): i.e., \( 78000 \) of \( H_+^\pm \) can be produced at the integrated LHC luminosity of 300 \( fb^{-1} \). This production rate is about ten times larger than those in Ref. \[11\]. The cross-sections decrease rapidly as mass of the Higgs boson increases from 200 GeV to 400 GeV.

<table>
<thead>
<tr>
<th>( s_\vartheta )</th>
<th>0.08</th>
<th>0.05</th>
<th>0.02</th>
<th>0.009</th>
<th>0.005</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_\varphi )</td>
<td>-0.0320698</td>
<td>-0.0156778</td>
<td>-0.00598729</td>
<td>-0.00449063</td>
<td>-0.00422721</td>
</tr>
<tr>
<td>( F )</td>
<td>-0.087481</td>
<td>-0.0561693</td>
<td>-0.022803</td>
<td>-0.0102847</td>
<td>-0.00571598</td>
</tr>
<tr>
<td>( M_{H_+^\pm}^{\text{max}}[^{\text{GeV}}] )</td>
<td>1700</td>
<td>1300</td>
<td>700</td>
<td>420</td>
<td>320</td>
</tr>
</tbody>
</table>

### Table III: Values of \( F, t_\varphi \) and \( M_{H_+^\pm}^{\text{max}} \) for given \( s_\vartheta \).
FIG. 1: Hadronic cross section of $W^\pm Z$ fusion process as a function of the charged Higgs boson mass for five cases of $\sin \theta$. Horizontal line is discovery limit (25 events).

bilepton gauge bosons $G_{\chi^0}$ is decouple, while its CP-even counterpart has the mixing by the same way in the gauge boson sector. Despite the mixing among the photon with the non-Hermitian neutral bilepton $X^0$ as well as with the $Z$ and the $Z'$ gauge bosons, the electromagnetic couplings remain unchanged.

It is worth mentioning that, masses of all physical Higgs bosons are related to that of gauge bosons through the coefficients of Higgs self-interactions. All gauge-scalar bosons couplings in the standard model are recovered. The coupling of the photon with the Higgs bosons are diagonal.

It should be mentioned that in Ref.[4], to get nonzero coupling $ZZh$ at the tree level, the authors suggested the following solution: (i) $u \sim v$ or (ii) by introducing the third Higgs scalar with VEV $(\sim v)$. This problem does not happen in our consideration.

After all we focused attention to the singly-charged Higgs boson $H^{\pm}$ with mass proportional to the bilepton mass $M_Y$ through the coefficient $\lambda_4$. Mass of the $H^{\pm}$ is estimated and is in a range of 200 GeV. This boson, in difference with those arisen in the Higgs doublet models, does not have the hadronic and leptonic decay modes. The trilinear coupling $ZW^\pm H^{\pm}$ which differs, at the tree level, while the similar coupling of the photon $\gamma W^\pm H^{\pm}$ as expected, vanishes. In the model under consideration, the charged Higgs boson $H^{\pm}_2$ with mass larger than 1700 GeV, cannot be seen at the LHC. If the mass of the above mentioned Higgs boson is in range of 200 GeV, however, the cross section can exceed 260 fb: i.e., 78000 of $H_2^{\pm}$ can be produced at the LHC for the luminosity of 300 $fb^{-1}$. By measuring this process we can obtain useful information to determine the structure of the Higgs sector.

Detailed analysis of discovery potential of all these Higgs bosons will be presented elsewhere.

APPENDIX A: MIXING MATRICES OF SCALAR SECTOR

For the sake of convenience in practical calculations, we give here some mixing matrices

1. Neutral scalar bosons

$$
\begin{pmatrix}
S_1 \\
S_2 \\
S_3
\end{pmatrix}
= \begin{pmatrix}
-s_\zeta s_\theta & c_\zeta s_\theta & c_\theta \\
c_\zeta & s_\zeta & 0 \\
-s_\zeta c_\theta & c_\zeta c_\theta & -s_\theta
\end{pmatrix}
\begin{pmatrix}
H \\
H_4^0 \\
G_4
\end{pmatrix},
$$

(A1)
2. Singly-charged scalar bosons

\[
\left( \phi_+^1, \phi_+^2, \phi_+^3, \chi_2^+ \right) = \frac{1}{\sqrt{\omega^2 + c_0^2 v^2}} \begin{pmatrix}
\omega s_\theta & c_\theta \sqrt{\omega^2 + c_0^2 v^2} & \frac{\omega s_\theta}{2} & H_2^+
\omega c_\theta & 0 & -\omega & G_5^+
\omega c_\theta & -s_\theta \sqrt{\omega^2 + c_0^2 v^2} & vc_\theta & G_6^+
\end{pmatrix}.
\]  

\[\text{(A2)}\]

**APPENDIX B: HIGGS-GAUGE BOSON COUPLINGS**

**TABLE IV:** Trilinear coupling constants of $W^+ W^- H$ with neutral Higgs bosons.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^+ W^- H$</td>
<td>$\frac{g}{2} \sqrt{\omega^2 + c_0^2 v^2}$</td>
</tr>
<tr>
<td>$W^+ W^- H_0$</td>
<td>$\frac{g}{2} \sqrt{\omega^2 + c_0^2 v^2}$</td>
</tr>
</tbody>
</table>

**TABLE V:** Trilinear coupling constants of $W^-$ with two Higgs bosons.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^\mu H_2^\pm \partial_\mu G_4$</td>
<td>$\frac{igv_\mu s_\theta}{2\sqrt{\omega^2 + c_0^2 v^2}}$</td>
</tr>
<tr>
<td>$W^\mu - G_5^\pm \partial_\mu H$</td>
<td>$-\frac{g}{2} \sqrt{\omega^2 + c_0^2 v^2}$</td>
</tr>
<tr>
<td>$W^\mu - G_5^\pm \partial_\mu G_4$</td>
<td>$\frac{igv_\mu c_\theta}{2\sqrt{\omega^2 + c_0^2 v^2}}$</td>
</tr>
<tr>
<td>$W^\mu H_2^\pm \partial_\mu G_2$</td>
<td>$-\frac{g}{2} \sqrt{\omega^2 + c_0^2 v^2}$</td>
</tr>
<tr>
<td>$W^\mu H_2^\pm \partial_\mu G_3$</td>
<td>$\frac{igv_\mu s_\theta}{2\sqrt{\omega^2 + c_0^2 v^2}}$</td>
</tr>
</tbody>
</table>
TABLE VI: Nonzero quartic coupling constants of $W^+W^-$ with Higgs bosons.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Coupling</th>
<th>Vertex</th>
<th>Coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^+W^- H_1^+ H_2^-$</td>
<td>$\frac{g^2\omega_v^2}{2(\omega^2+\nu^2+\theta^2)}$</td>
<td>$W^+W^- G_1^0 G_1^0$</td>
<td>$\frac{\omega_v^2}{2}$</td>
</tr>
<tr>
<td>$W^+W^- G_3^+ G_5^-$</td>
<td>$\frac{\omega_v^2}{2}$</td>
<td>$W^+W^- G_3^0 G_4^0$</td>
<td>$\frac{\omega_v^2}{2}$</td>
</tr>
<tr>
<td>$W^+W^- G_6^+ G_6^-$</td>
<td>$\frac{g^2\omega_v^2}{2(\omega^2+\nu^2+\theta^2)}$</td>
<td>$W^+W^- G_6^0 G_4^0$</td>
<td>$\frac{g^2}{2}$</td>
</tr>
<tr>
<td>$W^+W^- H_1^+ H_1^-$</td>
<td>$\frac{g^2\omega_v^2}{2(\omega^2+\nu^2+\theta^2)}$</td>
<td>$W^+W^- H_1^0 H_1^0$</td>
<td>$\frac{g^2\omega_v^2}{4}$</td>
</tr>
<tr>
<td>$W^+W^- HH$</td>
<td>$\frac{g^2\omega_v^2}{2}$</td>
<td>$W^+W^- G_1^0 G_3^0$</td>
<td>$-\frac{g^2\omega_v^2}{4}$</td>
</tr>
<tr>
<td>$W^+W^- H_1^0 H_0^0$</td>
<td>$\frac{g^2\omega_v^2}{2}$</td>
<td>$W^+W^- G_2^0 G_2^0$</td>
<td>$\frac{g^2}{4}$</td>
</tr>
</tbody>
</table>

TABLE VII: Trilinear electromagnetic coupling constants of $A^\mu$ with two Higgs bosons.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>$A^\mu H_1^+ \partial^\mu H_2^+$</th>
<th>$A^\mu G_1^0 \partial^\mu G_1^0$</th>
<th>$A^\mu G_6^0 \partial^\mu G_6^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coupling</td>
<td>$ie$</td>
<td>$ie$</td>
<td>$ie$</td>
</tr>
</tbody>
</table>

TABLE VIII: Trilinear coupling constants of $Z^\mu$ with two charged Higgs bosons.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z^\mu H_2^+ \bar{\partial}_\mu H_2^+$</td>
<td>$\frac{i g}{2(v^2 s^2 + c^2)} \left{ (v^2 s^2 + \omega^2 c^2) U_{12} + \frac{\omega^2 (1 - 3 c^2)}{2} - \frac{v^2 c^2}{2} \sqrt{\frac{3}{2}} U_{32} + \omega^2 s_{2h} U_{42} \right} \rightarrow -g s_W t_W$</td>
</tr>
<tr>
<td>$Z^\mu G_5^+ \bar{\partial}_\mu G_5^+$</td>
<td>$\frac{i g}{2(v^2 s^2 + c^2)} \left{ c^2 U_{12} + (1 - 3 s^2) U_{22} + \frac{\omega^2}{2} \sqrt{\frac{3}{2}} U_{32} - s_{2h} U_{42} \right} \rightarrow \frac{i g}{2 c_W} (1 - 2 s^2_W)$</td>
</tr>
<tr>
<td>$Z^\mu G_6^+ \bar{\partial}_\mu G_6^+$</td>
<td>$\frac{i g}{2(v^2 s^2 + c^2)} \left{ (\omega^2 + v^2 s^2 c^2) U_{12} + \frac{\omega^2 (1 - 3 c^2)}{2} - \frac{v^2 c^2}{2} \sqrt{\frac{3}{2}} (\omega^2 + 2 v^2 c^2) U_{32} + 2 v^2 s_{2h} U_{42} \right} \rightarrow \frac{i g}{2 c_W} (1 - 2 s^2_W)$</td>
</tr>
<tr>
<td>$Z^\mu H_2^+ \bar{\partial}_\mu G_5^+$</td>
<td>$\frac{i g \omega}{4 \sqrt{\omega^2 + c^2}} (s_{2h} U_{12} + \sqrt{3} s_{2h} U_{22} + 2 c_{2h} U_{42}) \rightarrow 0$</td>
</tr>
<tr>
<td>$Z^\mu H_2^+ \bar{\partial}_\mu G_6^+$</td>
<td>$\frac{i g \omega}{2 \sqrt{\omega^2 + c^2}} \left{ -c^2 U_{12} + (2 - 3 c^2) U_{22} + \frac{\omega}{2} \sqrt{\frac{3}{2}} U_{32} + s_{2h} U_{42} \right} \rightarrow 0$</td>
</tr>
<tr>
<td>$Z^\mu G_5^+ \bar{\partial}_\mu G_6^+$</td>
<td>$\frac{i g \omega}{4 \sqrt{\omega^2 + c^2}} (s_{2h} U_{12} + \sqrt{3} s_{2h} U_{22} + 2 c_{2h} U_{42}) \rightarrow 0$</td>
</tr>
</tbody>
</table>
TABLE IX: Trilinear coupling constants of $Z_\mu$ with two neutral Higgs bosons.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z^\mu G_1 \partial_{\mu} H$</td>
<td>$-\frac{g_2 c}{2} \left[ \left( U_{12} + \frac{U_{22}}{\sqrt{3}} - \frac{t_3}{\sqrt{2}} U_{32} \right) s_\theta + U_{42} c_\theta \right] \longrightarrow 0$</td>
</tr>
<tr>
<td>$Z^\mu G_2 \partial_{\mu} H$</td>
<td>$\frac{g_2 c}{2} \left( -U_{12} + \frac{U_{22}}{\sqrt{3}} + \frac{2t_3}{\sqrt{2}} U_{32} \right) c_\zeta \longrightarrow -\frac{g}{2c_W}$</td>
</tr>
<tr>
<td>$Z^\mu G_3 \partial_{\mu} H$</td>
<td>$\frac{g_2 c}{2} \left[ \left( \frac{2}{\sqrt{3}} U_{22} + \frac{t_3}{\sqrt{2}} U_{32} \right) c_\theta - U_{42} s_\theta \right] \longrightarrow 0$</td>
</tr>
<tr>
<td>$Z^\mu G_1 \partial_{\mu} H_1$</td>
<td>$\frac{g_2 c}{2} \left[ \left( U_{12} + \frac{U_{22}}{\sqrt{3}} - \frac{t_3}{\sqrt{2}} U_{32} \right) s_\theta + U_{42} c_\theta \right] \longrightarrow 0$</td>
</tr>
<tr>
<td>$Z^\mu G_2 \partial_{\mu} H_1$</td>
<td>$\frac{g_2 c}{2} \left( -U_{12} + \frac{U_{22}}{\sqrt{3}} + \frac{2t_3}{\sqrt{2}} U_{32} \right) s_\zeta \longrightarrow 0$</td>
</tr>
<tr>
<td>$Z^\mu G_3 \partial_{\mu} H_1$</td>
<td>$-\frac{g_2 c}{2} \left[ \left( \frac{2}{\sqrt{3}} U_{22} + \frac{t_3}{\sqrt{2}} U_{32} \right) c_\theta - U_{42} s_\theta \right] \longrightarrow 0$</td>
</tr>
<tr>
<td>$Z^\mu G_1 \partial_{\mu} G_4$</td>
<td>$\frac{g_2 c}{2} \left[ \left( U_{12} + \frac{U_{22}}{\sqrt{3}} - \frac{t_3}{\sqrt{2}} U_{32} \right) c_\theta - U_{12} s_\theta \right] \longrightarrow \frac{g}{2c_W}$</td>
</tr>
<tr>
<td>$Z^\mu G_2 \partial_{\mu} G_4$</td>
<td>$0$</td>
</tr>
<tr>
<td>$Z^\mu G_3 \partial_{\mu} G_4$</td>
<td>$\frac{g_2 c}{2} \left[ \left( \frac{2}{\sqrt{3}} U_{22} + \frac{t_3}{\sqrt{2}} U_{32} \right) s_\theta + U_{42} c_\theta \right] \longrightarrow 0$</td>
</tr>
</tbody>
</table>
TABLE X: Trilinear coupling constants of $Z'_\mu$ with two neutral Higgs bosons.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z'^\mu G_1 \partial_\mu H$</td>
<td>$-\frac{gs}{2} \left[ \left( U_{13} + \frac{U_{31}}{\sqrt{3}} - \frac{i}{3} \sqrt{\frac{2}{3}} U_{33} \right) s_\theta + U_{43} c_\theta \right] \rightarrow 0$</td>
</tr>
<tr>
<td>$Z'^\mu G_2 \partial_\mu H$</td>
<td>$\frac{g}{2} \left( -U_{13} + \frac{U_{23}}{\sqrt{3}} + \frac{2i}{3} \sqrt{\frac{2}{3}} U_{33} \right) c_\zeta \rightarrow \frac{g}{2iW \sqrt{4c^2_W - 1}}$</td>
</tr>
<tr>
<td>$Z'^\mu G_3 \partial_\mu H$</td>
<td>$\frac{gs}{2} \left[ \left( \frac{2}{\sqrt{3}} U_{23} + \frac{1}{3} \sqrt{\frac{2}{3}} U_{33} \right) c_\theta - U_{43} s_\theta \right] \rightarrow 0$</td>
</tr>
<tr>
<td>$Z'^\mu G_1 \partial_\mu H_1^0$</td>
<td>$\frac{gs}{2} \left[ \left( U_{13} + \frac{U_{31}}{\sqrt{3}} - \frac{i}{3} \sqrt{\frac{2}{3}} U_{33} \right) s_\theta + U_{43} c_\theta \right] \rightarrow 0$</td>
</tr>
<tr>
<td>$Z'^\mu G_2 \partial_\mu H_1^0$</td>
<td>$\frac{g}{2} \left( -U_{13} + \frac{U_{23}}{\sqrt{3}} + \frac{2i}{3} \sqrt{\frac{2}{3}} U_{33} \right) s_\zeta \rightarrow 0$</td>
</tr>
<tr>
<td>$Z'^\mu G_3 \partial_\mu H_1^0$</td>
<td>$-\frac{gs}{2} \left[ \left( \frac{2}{\sqrt{3}} U_{23} + \frac{1}{3} \sqrt{\frac{2}{3}} U_{33} \right) c_\theta - U_{43} s_\theta \right] \rightarrow -\frac{gs_\theta}{2iW \sqrt{4c^2_W - 1}}$</td>
</tr>
<tr>
<td>$Z'^\mu G_1 \partial_\mu G_4$</td>
<td>$\frac{g}{2} \left[ \left( U_{13} + \frac{U_{31}}{\sqrt{3}} - \frac{i}{3} \sqrt{\frac{2}{3}} U_{33} \right) s_\theta + U_{43} c_\theta \right] \rightarrow \frac{gs_\theta}{2iW \sqrt{4c^2_W - 1}}$</td>
</tr>
<tr>
<td>$Z'^\mu G_2 \partial_\mu G_4$</td>
<td>$0$</td>
</tr>
<tr>
<td>$Z'^\mu G_3 \partial_\mu G_4$</td>
<td>$\frac{g}{2} \left[ \left( \frac{2}{\sqrt{3}} U_{23} + \frac{1}{3} \sqrt{\frac{2}{3}} U_{33} \right) s_\theta + U_{43} c_\theta \right] \rightarrow 0$</td>
</tr>
</tbody>
</table>
TABLE XI: Quartic coupling constants of ZZ with two scalar bosons.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZZG₁G₁</td>
<td>$\frac{g^2}{2} \left( U_{12} + \frac{U_{24}}{\sqrt{3}} - \frac{1}{\sqrt{3}} \sqrt{\frac{2}{3}} U_{32} \right)^2 + U_{42}^2 \rightarrow \frac{g^2}{2c_{1/4}}$</td>
</tr>
<tr>
<td>ZZG₂G₂</td>
<td>$\frac{g^2}{2} \left( -U_{12} + \frac{U_{24}}{\sqrt{3}} + \frac{2}{\sqrt{3}} \sqrt{\frac{2}{3}} U_{32} \right)^2 \rightarrow \frac{g^2}{2c_{1/4}}$</td>
</tr>
<tr>
<td>ZZG₃G₃</td>
<td>$\frac{g^2}{2} \left( \frac{2}{\sqrt{3}} U_{22} + \frac{1}{\sqrt{3}} \sqrt{\frac{2}{3}} U_{32} \right)^2 + U_{42}^2 \rightarrow 0$</td>
</tr>
<tr>
<td>ZZG₄G₄</td>
<td>$\frac{g^2}{2} \left[ \frac{2}{\sqrt{3}} U_{22} + \frac{1}{\sqrt{3}} \sqrt{\frac{2}{3}} U_{32} \right)^2 + U_{42}^2 \rightarrow 0$</td>
</tr>
<tr>
<td>ZZH₁H₁</td>
<td>$\frac{g^2}{2} \left[ \frac{2}{\sqrt{3}} U_{22} + \frac{1}{\sqrt{3}} \sqrt{\frac{2}{3}} U_{32} \right)^2 + U_{42}^2 + s_{20} U_{42} \left( U_{12} - \frac{U_{24}}{\sqrt{3}} - \frac{2}{\sqrt{3}} \sqrt{\frac{2}{3}} U_{32} \right) \rightarrow \frac{g^2}{2c_{1/4}}$</td>
</tr>
<tr>
<td>ZZH₂H₂</td>
<td>$\frac{g^2}{2} \left[ \frac{2}{\sqrt{3}} U_{22} + \frac{1}{\sqrt{3}} \sqrt{\frac{2}{3}} U_{32} \right)^2 + U_{42}^2 + s_{20} U_{42} \left( U_{12} - \frac{U_{24}}{\sqrt{3}} - \frac{2}{\sqrt{3}} \sqrt{\frac{2}{3}} U_{32} \right) \rightarrow 0$</td>
</tr>
<tr>
<td>ZZG₃G₄</td>
<td>$\frac{g^2}{2} \left[ \frac{2}{\sqrt{3}} U_{22} + \frac{1}{\sqrt{3}} \sqrt{\frac{2}{3}} U_{32} \right)^2 + U_{42}^2 + s_{20} U_{42} \left( U_{12} - \frac{U_{24}}{\sqrt{3}} - \frac{2}{\sqrt{3}} \sqrt{\frac{2}{3}} U_{32} \right) \rightarrow \frac{g^2}{2c_{1/4}}$</td>
</tr>
<tr>
<td>ZZH₁H₁</td>
<td>$\frac{g^2}{4} \left[ s_{20} U_{42} + s_{20} U_{42} \left( U_{12} + \frac{U_{24}}{\sqrt{3}} - \frac{2}{\sqrt{3}} \sqrt{\frac{2}{3}} U_{32} \right) \rightarrow 0$</td>
</tr>
<tr>
<td>ZZH₂G₄</td>
<td>$\frac{g^2}{4} \left( U_{12} - \frac{U_{24}}{\sqrt{3}} - \frac{2}{\sqrt{3}} \sqrt{\frac{2}{3}} U_{32} \right) \rightarrow 0$</td>
</tr>
<tr>
<td>ZZH₂G₄</td>
<td>$\frac{g^2}{4} \left( U_{12} - \frac{U_{24}}{\sqrt{3}} - \frac{2}{\sqrt{3}} \sqrt{\frac{2}{3}} U_{32} \right) \rightarrow 0$</td>
</tr>
</tbody>
</table>
### TABLE XII: Trilinear coupling constants of $ZZ$ with one scalar bosons.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ZZH$</td>
<td>[ \frac{g^2}{2} \left[ v_{C} \left( U_{12} - \frac{V_{22}}{\sqrt{3}} - \frac{\sqrt{2}}{\sqrt{3}} U_{32} \right)^2 - u_{S} \left( U_{12} + \frac{V_{22}}{\sqrt{3}} - \frac{1}{\sqrt{2}} U_{23} \right)^2 - \omega_{S} c_{b} \left( U_{12} + \frac{V_{22}}{\sqrt{3}} + \frac{1}{\sqrt{2}} U_{23} \right)^2 \right] \rightarrow \frac{g^2}{2} ]</td>
</tr>
<tr>
<td>$ZZH'$</td>
<td>[ \frac{g^2}{2} \left[ v_{C} \left( U_{12} - \frac{V_{22}}{\sqrt{3}} - \frac{2}{3} \sqrt{2} U_{32} \right)^2 + u_{C} s_{b} \left( U_{12} + \frac{V_{22}}{\sqrt{3}} - \frac{1}{3} \sqrt{2} U_{23} \right)^2 + \omega_{C} c_{b} \left( U_{12} + \frac{V_{22}}{\sqrt{3}} + \frac{1}{3} \sqrt{2} U_{23} \right)^2 \right] \rightarrow 0 ]</td>
</tr>
<tr>
<td>$ZZG_{4}$</td>
<td>[ \frac{g^2}{2} \left[ s_{b} \left( U_{12} + \sqrt{3} U_{22} \right) + \frac{\sqrt{2}}{\sqrt{3}} U_{12} \right] \left[ U_{12} - \frac{V_{22}}{\sqrt{3}} - \frac{2}{3} \sqrt{2} U_{32} \right] \rightarrow 0 ]</td>
</tr>
</tbody>
</table>

### TABLE XIII: Trilinear coupling constants of $ZZ'$ with one scalar bosons.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ZZ' H$</td>
<td>[ \frac{g^2}{2} \left[ v_{C} \left( U_{12} - \frac{V_{22}}{\sqrt{3}} - \frac{\sqrt{2}}{\sqrt{3}} U_{32} \right) \left( U_{13} - \frac{V_{22}}{\sqrt{3}} - \frac{\sqrt{2}}{\sqrt{3}} U_{33} \right) - u_{S} \left( U_{12} + \frac{V_{22}}{\sqrt{3}} - \frac{1}{\sqrt{2}} U_{23} \right) \left( U_{13} + \frac{V_{22}}{\sqrt{3}} - \frac{1}{\sqrt{2}} U_{23} \right) \right] \rightarrow \frac{g^2}{2} ]</td>
</tr>
<tr>
<td>$ZZ' H'$</td>
<td>[ \frac{g^2}{2} \left[ v_{C} \left( U_{12} - \frac{V_{22}}{\sqrt{3}} - \frac{2}{3} \sqrt{2} U_{32} \right) \left( U_{13} - \frac{V_{22}}{\sqrt{3}} - \frac{2}{3} \sqrt{2} U_{33} \right) + u_{C} s_{b} \left( U_{12} + \frac{V_{22}}{\sqrt{3}} - \frac{1}{3} \sqrt{2} U_{23} \right) \left( U_{13} + \frac{V_{22}}{\sqrt{3}} - \frac{1}{3} \sqrt{2} U_{23} \right) \right] \rightarrow 0 ]</td>
</tr>
<tr>
<td>$ZZ' G_{4}$</td>
<td>[ \frac{g^2}{2} \left[ \left( U_{12} + \frac{V_{22}}{\sqrt{3}} - \frac{1}{\sqrt{2}} U_{23} \right) \left( U_{13} + \frac{V_{22}}{\sqrt{3}} - \frac{1}{\sqrt{2}} U_{23} \right) - \left( \frac{V_{22}}{\sqrt{3}} U_{22} + \frac{1}{\sqrt{2}} U_{23} \right) \left( \frac{V_{22}}{\sqrt{3}} U_{23} + \frac{1}{\sqrt{2}} U_{23} \right) \right] \rightarrow 0 ]</td>
</tr>
</tbody>
</table>
TABLE XIV: Trilinear coupling constants of neutral gauge bosons with $W^+$ and the charged scalar boson.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AW^+G_5^-$</td>
<td>$\frac{s^2}{2}vSW$</td>
</tr>
<tr>
<td>$ZW^+H_3^-$</td>
<td>$\frac{s^2 v \omega}{2 \sqrt{v^2 + s^2 v \omega}} [s_\theta c_\theta (U_{12} + \sqrt{3} U_{22}) + c_\theta U_{42}]$</td>
</tr>
<tr>
<td>$Z'W^+H_3^-$</td>
<td>$\frac{s^2 v \omega}{2 \sqrt{v^2 + s^2 v \omega}} [s_\theta c_\theta (U_{13} + \sqrt{3} U_{23}) + c_\theta U_{43}] \to 0$</td>
</tr>
<tr>
<td>$ZW^+G_6^-$</td>
<td>$\frac{s^2 L_2}{4} [-s_\theta^2 U_{12} + (2 - 3s_\theta^2) \frac{U_{22}}{v} + \frac{4t_3}{3} \sqrt{3} U_{32} - s_\theta U_{42}] \to -\frac{s^2}{2}vSW tW$</td>
</tr>
<tr>
<td>$ZW^+G_6^-$</td>
<td>$\frac{s^2 (s_\theta^2, \frac{s_\theta^2}{2} \to ^2)}{8c_\theta \sqrt{v^2 + s^2 v \omega}} [s_\theta (U_{12} + \sqrt{3} U_{22}) + 2c_\theta U_{42}] \to 0$</td>
</tr>
</tbody>
</table>

TABLE XV: The SM coupling constants in the effective limit.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$WWhh$</td>
<td>$\frac{g^2}{4}$</td>
</tr>
<tr>
<td>$WWh$</td>
<td>$\frac{g^2}{2}v$</td>
</tr>
<tr>
<td>$WGWh$</td>
<td>$\frac{g}{2}$</td>
</tr>
<tr>
<td>$WGhG_Z$</td>
<td>$\frac{g}{2}$</td>
</tr>
<tr>
<td>$ZZhh$</td>
<td>$\frac{g^2}{2c_\theta v}$</td>
</tr>
<tr>
<td>$AWGw$</td>
<td>$\frac{g^2}{2vSW}$</td>
</tr>
<tr>
<td>$ZGzh$</td>
<td>$-\frac{g}{2c_\theta v}$</td>
</tr>
<tr>
<td>$WGhW$</td>
<td>$\frac{-g}{2c_\theta v}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{W}G_{W}A$</td>
<td>$ie$</td>
</tr>
<tr>
<td>$WWG_{2}G_{Z}$</td>
<td>$\frac{g^2}{2}$</td>
</tr>
<tr>
<td>$WWG_{W}G_{W}$</td>
<td>$\frac{g^2}{2}$</td>
</tr>
<tr>
<td>$ZZG_{2}G_{Z}$</td>
<td>$\frac{g^2}{2c_\theta v}$</td>
</tr>
<tr>
<td>$ZWG_{W}$</td>
<td>$-\frac{g^2}{2c_\theta vSW tW}$</td>
</tr>
<tr>
<td>$ZG_{W}G_{W}$</td>
<td>$\frac{g^2}{2c_\theta v} (1 - 2s_\theta^2)$</td>
</tr>
<tr>
<td>$AG_{W}G_{W}$</td>
<td>$ie$</td>
</tr>
</tbody>
</table>