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The $\mu \rightarrow e + \gamma$ Decay and the
Intermediate Charged Vector Boson Theory

by

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A B S T R A C T

The $\mu - e - \gamma$ vertex is calculated in the intermediate charged vector boson theory as a function of the square of the four momentum of the photon. Consistency arguments show that the boson anomalous magnetic moment should be taken equal to zero. The result is then specialized to a real photon. The experimental branching ratio

$$\rho = (\mu \rightarrow e + \gamma) / (\mu \rightarrow e + \gamma + \bar{\nu}) = (1.2 \pm 1.5) \times 10^{-6}$$

fixes the cut-off value at less than one fifth the intermediate boson mass. It is concluded that this theory does not reasonably account for the experimental data, no matter how massive the boson is assumed to be.

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1. INTRODUCTION

The universal V-A interaction among ordinary (non-strange) fermions and the non-observation of certain processes, e.g. $\mu^+ \rightarrow 2e^+ + e^-$ and $K^0 \rightarrow \mu^+ + \mu^-$, has led Feynman and Gell-Mann¹⁾ to postulate that a heavy charged vector boson serves as the intermediary connecting the two fermion lines. Such a structure of the four fermion interaction would lead, barring additional selection rules, to the decay of a free mu meson into an electron and a photon,

$$\mu \rightarrow e + \gamma,$$

at a rate proportional to the same power of the weak interaction coupling constant as the rate for ordinary beta decay. Several calculations of the branching ratio

$$\rho = \frac{w(\mu \rightarrow e + \gamma)}{w(\mu \rightarrow e + \gamma + \bar{\gamma})}$$

given by this theory have been reported²⁾⁻⁴⁾, and recently several experimental teams searched again for $\mu \rightarrow e + \gamma$ decay⁵⁾ with the result that

$$\rho_{\text{experimental}} = (1.2 \pm 1.5) \times 10^{-6}.$$

In trying to determine whether the theory is consistent with this small branching ratio, it should be borne in mind that the theory is not renormalizable, and in a strict sense is undefined.

However, the usual point vertex assumed for the four fermion interaction is itself non-renormalizable, and yet, in lowest order perturbation theory it is in excellent agreement with beta decay experiments. One may hope that in lowest order, the intermediate boson theory is also satisfactory. However, it is divergent in lowest order for the process $\mu \rightarrow e + \gamma$ decay.

The predicted branching ratio depends on the boson's assumed anomalous ⁶⁾ magnetic moment, $\lambda(e\hbar/2m_b c)$, where m_b is the boson mass, and on the ratio Λ of the cut-off to the boson mass,

$$\rho_{\text{theoretical}} = \rho(\lambda, \Lambda).$$

Our main contention is that λ should be taken equal to zero. There are two reasons for this. First, the boson is assumed not to have strong interactions. Any anomalous moment is thus due to the weak and electromagnetic couplings, and can arise only by higher order corrections to the process under consideration. Therefore, in any lowest order process there is no anomalous moment.

The other, perhaps not as compelling reason why λ should be set equal to zero, is not manifest if one calculates only the $\mu \rightarrow e + \gamma$ decay, but is evident from the more general case in which the photon is not real, as occurs for example in the closely related process in which a mu meson bound in the coulomb field of a nucleus converts into an electron ⁷⁾

$$\mu_{\text{bound}}^- + \text{Nucleus} \rightarrow e^- + \text{recoil Nucleus}.$$

In both processes the same vertex is involved, the only difference being that in one case the emitted photon is real, and in the other it is virtual. The difficulty with $\lambda \neq 0$ comes from the presence in the matrix element of a term of the form

$$\lambda(k^2 \not{A} - k \cdot A \not{k})C$$

where k is the four momentum carried off by the photon, A is the k^{th} component of the Fourier transform of the electromagnetic field, and C is a quadratically divergent constant. This is the only quadratically divergent term. For the special case of a real photon, the k -dependent factor is zero, so that if one specializes to this case immediately, as is natural if one is considering the $\mu \rightarrow e + \gamma$ decay, this term never appears and the result is apparently only logarithmically divergent ^{3) 4)}.

If, however, the matrix element is calculated for a momentum k even only slightly off the light cone, the result is quadratically divergent. This ambiguity in the order of divergence of the matrix element on the light cone seems to us to be physically inadmissible. In order to eliminate this ill-behaved term, we must take λ equal to zero ⁸⁾.

The problem is then to determine whether a value of β consistent with experiment can be obtained with a reasonable choice of the single remaining parameter Λ .

In Section 2, a sketch of the calculation is given, and in Section 3 are a brief resumé and our conclusions.

2. CALCULATION

It is convenient to calculate the matrix element for both $\mu \rightarrow e + \gamma$ decay and $\mu \rightarrow e$ conversion, together and to later specialize. Consider the three lowest order graphs for the $\mu \rightarrow e + \gamma$ decay, which are shown in Figure 1. The four momenta are indicated by arrows and labels. The momentum k carried out of the process by the photon is on the light cone, i.e. $k^2 = 0$. In the $\mu \rightarrow e$ conversion process the lowest order approximation for the matrix element consists of precisely the three graphs of Figure 1, with the sole difference that in place of a real photon being emitted, the external Coulomb field of the nucleus acts⁹⁾. In this case, of course, the momentum k carried off by the electromagnetic field, that is, the recoil four momentum of the nucleus, is not on the light cone; $k^2 \neq 0$. The "natural" units, $\hbar = c = 1$ are used. The boson-fermion vertex is taken to be

$$g \gamma_\sigma \frac{1}{2}(1 + i \gamma_5).$$

For the part of the boson-electromagnetic vertex that is linear in the electromagnetic field, we take

$$e(\Gamma_\mu)_{\rho\sigma} \equiv e \left\{ -(q_{1\mu} + q_{2\mu})\delta_{\rho\sigma} + q_{1\rho}\delta_{\mu\sigma} + q_{2\sigma}\delta_{\mu\rho} + \lambda [(q_{1\rho} - q_{2\rho})\delta_{\mu\sigma} - (q_{1\sigma} - q_{2\sigma})\delta_{\mu\rho}] \right\},$$

where a boson with four momentum q_1 and index σ emits or absorbs a photon with index μ and becomes a boson with four momentum q_2 and index ρ . The boson propagator is

$$\frac{-i}{(2\pi)^4} \frac{1}{q^2 - m_B^2} \left(\delta_{\mu\nu} - \frac{1}{m_B^2} q_\mu q_\nu \right).$$

The electromagnetic field operator is

$$A_\mu(x) = (2\pi)^{-3/2} \int d^4k A_\mu(k) e^{ik \cdot x},$$

and in the case that a real photon is involved is

$$A_\mu(x) = (2\pi)^{-3/2} \int d^3k \epsilon_\mu(\vec{k}) (2k_0)^{-\frac{1}{2}} (a_k e^{ik \cdot x} + a_k^\dagger e^{-ik \cdot x}),$$

where the polarization index is omitted from the polarization four vector $\epsilon_\mu(\vec{k})$. According to the usual rules¹⁰⁾, we then obtain for the graphs of Figure 1

$$A = K\bar{u}(2) \int \frac{d^4q_1 d^4q_2 d^4q_3}{q_3^2 (q_1^2 - m_B^2) (q_2^2 - m_B^2)} \delta^4(p_1 - q_1 - q_3) \delta^4(q_1 - k - q_2) \delta^4(q_2 + q_3 - p_2) x$$

$$\frac{1}{m_B} \gamma_\mu \not{q}_3 \gamma_\nu (m_B^2 \delta_{\mu\sigma} - q_{2\mu} q_{2\sigma}) (\not{T}_\tau)_\sigma \not{p} A_\tau (m_B^2 \delta_{\rho\nu} - q_{1\rho} q_{1\nu}) (1 + i\gamma_5) u(1)$$

$$B = -K\bar{u}(2) \int \frac{d^4q_1 d^4q_2 d^4q_3}{q_3^2 (q_1^2 - m_\mu^2) (q_2^2 - m_B^2)} \delta^4(p_1 - k - q_1) \delta^4(q_1 - q_2 - q_3) \delta^4(q_2 + q_3 - p_2) x$$

$$\gamma_\mu \not{q}_3 \gamma_\nu (m_B^2 \delta_{\mu\nu} - q_{2\mu} q_{2\nu}) (1 + i\gamma_5) (\not{q}_1 + m_\mu) \not{p} u(1)$$

$$C = -K \bar{u}(2) \int \frac{d^4 q_1 d^4 q_2 d^4 q_3}{q_3^2 (q_1^2 - m_B^2) (q_2^2 - m_e^2)} \delta^4(p_1 - q_1 - q_3) \delta^4(q_1 + q_3 - q_2) \delta^4(q_2 - k - p_2) \times$$

$$\not{A}(\not{A}_2 + m_e) \gamma_\mu q_3 \gamma_\nu (m_B^2 \delta_{\mu\nu} - q_{1\mu} q_{1\nu}) (1 + i\gamma_5) u(1)$$

where

$$K = \frac{e}{2(2\pi)^{9/2}} \left(\frac{G}{m_B} \right)^2 \sqrt{\frac{m_\mu m_e}{E(1)E(2)}},$$

and $u(1)$ and $\bar{u}(2)$ are the mu meson and electron spinors. Each of these three terms is in fact quadratically divergent, and the result depends on which of the integrations are carried out with the aid of the δ functions. The most reasonable prescription is to use the δ functions to eliminate those integration variables that describe the propagation of charged particles. This gives a gauge invariant result which corresponds to the answer that would be given by the insertion of a cut-off on the propagator of the neutral particle. Also, no translations of the origins of the integration variables are allowed, because of the degree of the divergence. With this prescription we obtain

$$A = K \delta^4(p_1 - p_2 - k) \bar{u}(2) \int \frac{d^4 q}{q^2 D_1 D_2} \frac{1}{m_B^2} (m_B^2 \gamma_\sigma - \not{A}_2 q_{2\sigma}) \not{A} (m_B^2 \gamma_\rho - \not{A}_1 q_{1\rho}) \times$$

$$(T_\tau)_{\sigma\rho} A_\tau (1 + i\gamma_5) u(1)$$

$$B = K \delta^4(p_1 - p_2 - k) \bar{u}(2) \int \frac{d^4 q}{q^2 (m_e^2 - m_\mu^2) D_2} (2m_B^2 \not{A} + \not{A}_2 \not{A}_2) (\not{A} + 2p_2 \cdot A) x$$

$$(1 + i\gamma_5) u(1)$$

$$C = K \delta^4(p_1 - p_2 - k) \bar{u}(2) \int \frac{d^4 q}{q^2 D_1 (m_\mu^2 - m_c^2)} (-\not{A} + 2p_1 \cdot A) (2m_B^2 \not{A} + \not{A}_1 \not{A}_1) x$$

$$(1 + i\gamma_5) u(1)$$

where the subscript 3 has been omitted from the remaining integration variable, $D_1 = q_1^2 - m_B^2$, $D_2 = q_2^2 - m_B^2$, $q_1 = p_1 - q$ and $q_2 = p_2 - q$.

Now A, B, and C each have the factor

$$K \delta^4(p_1 - p_2 - k) \bar{u}(2)$$

at the left and the factor

$$(1 + i\gamma_5) u(1)$$

at the right. Let the operator (in the spinor space) A' be defined by $A = K \delta^4(p_1 - p_2 - k) \bar{u}(2) A' (1 + i\gamma_5) u(1)$, and similarly

for B' and C' . The sum $A' + B' + C'$ may be put into manifestly gauge invariant form by introducing a gauge invariant vector. It is convenient to define, in addition to the vector $k = p_1 - p_2$, the vector $p = \frac{1}{2}(p_1 + p_2)$, and to note that $2k \cdot p = p_1^2 - p_2^2 = m_\mu^2 - m_e^2 \neq 0$. The most general gauge invariant vector that is linear in the four vector potential A and that can be constructed with the vectors k , p , and A , is easily seen to be

$$\alpha Q + \beta k \cdot Q k + \delta k \cdot Q p ,$$

where

$$Q = A - \frac{p \cdot A}{k \cdot p} k ,$$

and α , β , δ are arbitrary invariant functions of k^2 . Actually we need only the gauge invariant vector Q and the gauge invariant scalar $k \cdot Q$. Note that $p \cdot Q = 0$. In A' replace A_τ by $Q_\tau + (p \cdot A / k \cdot p) k_\tau$; in B' replace $\cancel{A} + 2p_2 \cdot A$ by $\cancel{A} - k \cdot Q + 2p \cdot A$; and in C' replace $-\cancel{A} + 2p_1 \cdot A$ by $-\cancel{A} + k \cdot Q + 2p \cdot A$. All terms linear in Q are manifestly gauge invariant. The sum of the other terms of $A' + B' + C'$ vanishes identically algebraically, that is, without doing the integrations. The remaining terms, which are gauge invariant, are

$$(A') = \int \frac{d^4 q}{q^2 D_1 D_2} \frac{1}{m_B^2} (m_B^2 \gamma_\sigma - \not{q}_2 \gamma_{2\sigma}) \not{A} (m_B^2 \gamma_\rho - \not{q}_1 \gamma_{1\rho}) \times$$

$$\left\{ -\delta_{\sigma\rho} (q_1 + q_2) \cdot Q + q_{1\sigma} Q_\rho + q_{2\rho} Q_\sigma + \lambda (k_\sigma Q_\rho - k_\rho Q_\sigma) \right\}$$

$$(B') = \int \frac{d^4 q}{q^2 D_2} \frac{1}{2k \cdot p} (2m_B^2 \cancel{q} + \cancel{q}_2 \cancel{q}_2) (-\cancel{q} + k \cdot Q)$$

$$(C') = \int \frac{d^4 q}{q^2 D_1} \frac{1}{2k \cdot p} (-\cancel{q} + k \cdot Q) (2m_B^2 \cancel{q} + \cancel{q}_1 \cancel{q}_1)$$

where (A') stands for the part of A' linear in Q , and likewise for (B') and (C') .

It may be noted that (A') appears to contain a quartically divergent, λ -independent part. This, however, is illusory because of the identity

$$q_{2\sigma} q_{1\rho} \left\{ -\delta_{\sigma\rho} (q_1 + q_2) \cdot Q + q_{1\sigma} Q_\rho + q_{2\rho} Q_\sigma \right\} = 0,$$

which causes it to vanish. Evaluation of (A') , (B') , and (C') is carried out by applying the identity

$$\int a^{-b} \mathcal{J}^{-1} = a^{-1} + a^{-1} b \int a^{-b} \mathcal{J}^{-1}$$

to the factors $D_1^{-1} = \int (q^2 - m_B^2) - (2p_1 \cdot q - p_1^2) \mathcal{J}^{-1}$ and $D_2^{-1} = \int (q^2 - m_B^2) - (2p_2 \cdot q - p_2^2) \mathcal{J}^{-1}$. Explicitly,

$$D_1^{-1} = D^{-1} + P_1 D^{-2} + P_1^2 D^{-3} + P_1^3 D^{-4} + P_1^4 D^{-4} D_1^{-1},$$

and likewise for D_2^{-1} , where D stands for the spherically symmetric expression $q^2 - m_B^2$, $P_1 = 2p_1 \cdot q - p_1^2$, and $P_2 = 2p_2 \cdot q - p_2^2$.

This identity permits the angular dependence to be removed from the integrand term by term, except for the last term. It also has the advantage that each term has at least one power of q less than the preceding, and is thus more convergent. The infinite terms are evaluated exactly, that is, formally. In the finite terms, we keep only those that have the lowest power of (Fermion mass/ m_B)², where the Fermion mass may be either m_μ or m_e . This is the only approximation made in evaluating the matrix element. We then obtain (it is convenient to write the integrals in dimensionless form, so that henceforth q means q/m_B),

$$\begin{aligned}
A' + B' + C' &= \frac{\lambda}{4} (k^2 \not{q} - k \cdot Q \not{Q}) \int d^4 q (q^2 - 1)^{-1} \\
&+ \left\{ \begin{aligned} &\not{q} (1-\lambda) \frac{1}{2} (\not{k} \not{p} - \not{p} \not{k}) + \left(\frac{1}{6} - \frac{5}{4} \lambda \right) k^2 \not{q} + \not{q} \left(\frac{1}{3} + \frac{5}{4} \lambda \right) \not{k} + (1-\lambda) \not{p} \not{k} \cdot Q \\ &+ \frac{\lambda}{4m_B^2} \left(\not{q} \frac{1}{2} (\not{k} \not{p} - \not{p} \not{k}) - \frac{1}{12} k^2 - p^2 \not{q} k^2 \not{q} + \not{q} \left(\frac{1}{12} k^2 + p^2 \right) \not{k} \right. \\ &\quad \left. + (k^2 - 2k \cdot p) \not{p} \not{q} \right) \end{aligned} \right\} \int d^4 q (q^2 - 1)^{-2} \\
&+ \left\{ \not{q} (1+2\lambda) \frac{1}{2} (\not{k} \not{p} - \not{p} \not{k}) + \frac{1}{6} k^2 \not{q} + \not{q} \frac{1}{6} \not{k} + (1+2\lambda) \not{p} \not{q} \cdot Q \right\} \int d^4 q (q^2 - 1)^{-3} \\
&+ \left\{ \not{q} (3) \frac{1}{2} (\not{k} \not{p} - \not{p} \not{k}) + 2k^2 \not{q} + \not{q} (-2 \not{k} + 3 \not{p}) \not{q} \cdot Q \right\} \int d^4 q (q^2 - 1)^{-4}
\end{aligned}$$

If one sets k^2 and $k \cdot Q$ equal to zero, one obtains for the real photon matrix element

$$A+B+C = K \delta^4(p_1 - p_2 - k) \bar{u}(2) \left[\frac{\lambda}{4} \times \text{zero} \times \int d^4 q (q^2 - 1)^{-1} \right. \\ \left. + \frac{1}{2} (\not{p}_1 - \not{p}_2) \not{k} \{ -i\pi^2 N \} \right] (1 + i\gamma_5) u(1)$$

where

$$\{ -i\pi^2 N \} = (1 - \lambda) \int d^4 q (q^2 - 1)^{-2} + (1 + 2\lambda) \int d^4 q (q^2 - 1)^{-3} + 3 \int d^4 q (q^2 - 1)^{-4} .$$

The first of the two terms in the square bracket is indeterminate, as it consists of zero multiplied by a quadratically divergent integral. For a virtual photon the factor zero is replaced by the finite expression $k^2 \not{A} - k \cdot A \not{k}$, in which case this term is without question quadratically divergent¹¹⁾. It is the presence of this term which causes the previously mentioned ambiguity on the light cone if λ is different than zero. The second term in the square bracket is well behaved as concerns the neighbourhood of the light cone because λ is ^{taken} different than 1¹²⁾.

We will continue to use the symbol N for the expression given above, although henceforth it will mean this expression evaluated with $\lambda = 0$. With this understanding, the matrix element

is

$$(\Lambda+B+C)_{\mu \rightarrow e+\gamma} = \frac{e}{2(2\pi)^{9/2}} \left(\frac{G}{m_B} \right)^2 \sqrt{\frac{m_\mu m_e}{E(1)E(2)}} \delta^4(p_1 - p_2 - k) \bar{u}(2)^{\frac{1}{2}} (\not{p} - \not{k}) x$$

$$\not{\epsilon}(1+i\gamma_5)u(1) \left\{ -i\pi^2 N \right\} .$$

Since we want the $\mu \rightarrow e + \gamma$ decay rate in the mu meson's rest system, $E(1) = m_\mu$. Also $p \cdot \epsilon = p_1 \cdot \epsilon = 0$ because p_1 has only a time component and ϵ has only space components. If we use the fact that $m_\mu \gg m_e$, then $\bar{u}(2)^{\frac{1}{2}} (\not{p} - \not{k}) \approx \frac{1}{2} m_\mu^2 \bar{u}(2)$. The S-matrix element is then

$$S_{\mu \rightarrow e+\gamma} = \frac{e}{4(2\pi)^{9/2}} \left(\frac{G}{m_B} \right)^2 \sqrt{\frac{m_e}{E(2)}} m_\mu^2 \delta^4(p_1 - p_2 - k) \bar{u}(2) \frac{1}{\sqrt{2k_0}} x$$

$$\not{\epsilon}(1+i\gamma_5)u(1) \left\{ -i\pi^2 N \right\} ,$$

which gives the transition rate

$$\omega(\mu \rightarrow e + \gamma) = 2^{-14} \pi^{-5} e^2 (G/m_B)^4 m_\mu^5 N^2 .$$

The same theory gives for the normal decay mode of the mu meson the rate

$$\omega(\mu \rightarrow e + \nu + \bar{\nu}) = \frac{1}{3} 2^{-9} \pi^{-3} (G/m_B)^4 m_\mu^5 ,$$

and therefore the predicted branching ratio is

$$\rho = \frac{3\alpha}{8\pi} N^2 = 0.87 \times 10^{-3} N^2 .$$

Because N is itself logarithmically divergent, a numerical value for ρ can only be obtained by introducing a cut-off. The Feynman cut-off, $-\Lambda^2(q^2 - \Lambda^2)^{-1}$, is sufficiently strong to give finite results. The above expression for N is thus replaced by (recall that $\lambda = 0$)

$$\begin{aligned} i(\pi/\Lambda)^2 N = & \int d^4 q (q^2 - 1)^{-2} (q^2 - \Lambda^2)^{-1} + \int d^4 q (q^2 - 1)^{-3} (q^2 - \Lambda^2)^{-1} \\ & + 3 \int d^4 q (q^2 - 1)^{-4} (q^2 - \Lambda^2)^{-1} . \end{aligned}$$

Evaluation of the integrals by the standard technique of Feynman gives

$$N = \frac{-\Lambda^2}{2(1-\Lambda^2)^4} \left\{ 2\Lambda^2 (\Lambda^4 - \Lambda^2 + 3) \ln \Lambda^2 + (1 - \Lambda^2)(2\Lambda^4 + \Lambda^2 + 3) \right\} .$$

The values of ρ thus obtained, in the range $0.1 \leq \Lambda \leq 0.9$, are shown in Figure 2, where the experimental upper limit on ρ , which we take to be 2.7×10^{-6} , is indicated by the dotted horizontal line. From this one sees that the cut-off would have to be at least 5 times smaller than m_B .

3. SUMMARY AND CONCLUSIONS

Because the intermediate charged vector boson theory with electromagnetic interaction ^{is non-renormalizable,} and because the Fermi point interaction theory which it is supposed to replace is itself non-renormalizable, we have adopted the viewpoint that only lowest order perturbation is to be considered. The boson is assumed to have no strong interactions, and therefore, in lowest order in the weak and electromagnetic interactions it has no anomalous moment, i.e. $\lambda = 0$. In addition we have imposed the criterion that the order of divergence of the perturbation theory matrix element for a physical process involving a photon must be a continuous function of the photon four momentum. Fulfillment of this condition also requires that in lowest order $\lambda = 0$ ¹³⁾. With $\lambda = 0$, and with the Feynman cut-off function, it then follows that the cut-off must be less than $m_B/5$ in order that $\rho_{\text{theoretical}}$ be consistent with the upper limit on $\rho_{\text{experimental}}$.

We reject the theory because $\Lambda \leq \frac{1}{5}$ seems to us to be unreasonable. There is the practical objection that the answer is highly cut-off dependent, because of the steepness of the logarithmic function between 0 and 1. A change in Λ from 0.1 to 0.2 produces a 13-fold change in the branching ratio. In this

region the logarithmic term is far more Λ -dependent than would be a quadratically divergent term.

There is also a theoretical objection. Of course nothing in the formalism of field theory excludes the possibility of a cut-off much smaller than the rest mass of the associated virtual quantum. However, since a cut-off is inserted ad hoc in order to damp out the otherwise infinite contribution of high virtual four momenta, and represents therefore a "domain of ignorance" whose size is measured by the inverse of the cut-off value, it seems unreasonable to attribute meaning to a "theory" for which the domain of ignorance far exceeds the size of the quantum involved, in this case the boson. In other words, the need for a cut-off at small virtual momentum transfer means that the prediction of the vector boson theory for the fermion-neutrino-boson vertex is incorrect even for small virtual momenta. The boson propagator is badly mutilated for almost all momenta. Since only the ratio of the cut-off to m_B is involved in the branching ratio, the difficulty of a large domain of ignorance cannot be overcome by taking m_B very large.

The original reason for introducing an intermediate charged vector boson is that it gives in a natural way the V-A interaction and the charge exchange character of the four fermion vertex. It should be borne in mind that the $\mu \rightarrow e + \gamma$ decay can be forbidden even if the Λ interaction is mediated by an intermediate charged vector boson. For example, if there are two kinds of neutrinos, ⁽⁴⁾ Λ those coupled to the electron-boson vertex, and those coupled to the mu-meson-boson vertex, then the $\mu \rightarrow e + \gamma$ decay is forbidden. It may also be that the lowest order perturbation result is not correct in such a non-renormalizable theory. Our conclusion is solely that in lowest order, the form of the intermediate boson theory that has been used is not consistent with experiment unless it is so badly mutilated by the cut-off function that it is in fact no longer the theory with which one starts. We therefore think that it is still necessary to search experimentally for evidence of a structure in the four fermion interaction.

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R E F E R E N C E S

- 1) R.P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).
- 2) R.P. Feynman and M. Gell-Mann, 1958 Annual International Conference on High Energy Physics at CERN.
- 3) G. Feinberg, Phys. Rev. 110, 1482 (1958).
- 4) M.E. Ebel and F.J. Ernst, to be published. These authors find a larger branching ratio than that obtained by Feinberg. The present authors also find the larger value.
- 5) J. Ashkin, T. Fazzini, G. Fidecaro, N.H. Lipman, A.W. Morrison and H. Paul, Nuov. Cim. (1959), preceding paper.
- 6) By "normal" magnetic moment we mean a moment of magnitude $e\hbar/2m_B c$, (i.e. one boson magneton) which one obtains from the free Lagrangian,

$$\mathcal{L} = -\frac{1}{2} \sum_{\mu, \nu} \left(\partial \phi_\mu^* / \partial x_\nu - \partial \phi_\nu^* / \partial x_\mu \right) \left(\partial \phi_\mu / \partial x_\nu - \partial \phi_\nu / \partial x_\mu \right) - m_B^2 \sum_{\mu} \phi_\mu^* \phi_\mu$$
 by assuming the minimal electrodynamic interaction.
- 7) A general phenomenological treatment of this process has been given recently by S. Weinberg and G. Feinberg, Phys. Rev. Lett. 3, 111 (1959), and by N. Cabibbo and R. Gatto, to be published. Our results obtained with the intermediate charged vector boson theory will be reported in a subsequent paper.
- 8) A similar situation occurs in the closely related process, the scattering of vector bosons by a coulomb field. If $\lambda \neq 0$ one obtains a physically unreasonable cross-section which increases indefinitely as the square of the boson energy. See W. Pauli, Rev. Mod. Phys. 13, 203 (1941), Table I.

- 9) Actually the finite size of the nuclear charge distribution should be taken into account, as well as the incoherent process in which the nucleus is left in an excited state.
- 10) As for example in Schweber, Bethe, and de Hoffman, Mesons and Fields, Vol. I, published by Row, Peterson and Co., Evanston Illinois (1955).

- 11) The factor $k^2 \not{A} - k \cdot A \not{k}$ is the matrix element in momentum space, that corresponds to a contact interaction of the form

$$j_\nu^{\text{ext}}(x) \bar{\psi}^{(e)}(x) \gamma_\nu \psi^{(\mu)}(x),$$

where $j_\nu^{\text{ext}}(x)$ is the "external" source of the electromagnetic field. On the one hand the $\mu \rightarrow e + \gamma$ decay occurs in the absence of any external field and this term does not contribute. On the other hand, the $\mu \rightarrow e$ conversion requires an external source, e.g. the charge of the nucleus, and this term does not vanish.

- 12) If λ were equal to 1 there would be, in addition to the ambiguity with the quadratically divergent term, a similar ambiguity with the logarithmically divergent term.
- 13) If both these arguments are rejected, one can take instead the point of view that since a cut-off must in fact be inserted into the integrals, it may as well be inserted at the beginning of the calculation. In this case all terms are finite. The "quadratic" terms proportional to λ no longer cause an ambiguity in the neighbourhood of the light cone. Everything is "well defined" even with λ different than zero. However, one would still require on physical grounds that the transition amplitude be a smooth function of k^2 in the neighbourhood of the light cone. Therefore one would be obliged to use instead of the Feynman cut-off function, $-\Lambda^2(q^2 - \Lambda^2)^{-1}$, which was used by Feynberg and by Ebel and Ernst, a stronger function, such as $\Lambda^4(q^2 - \Lambda^2)^{-2}$, which would be needed to make the quadratically divergent term finite.
- 14) J. Schwinger, Ann. Phys. 2, 407 (1957).

Figure 1. The three lowest order graphs for the $\mu \rightarrow e + \gamma$ decay in the intermediate charged vector boson theory. They differ only in that the photon is emitted by the charged boson in A, by the mu meson in B, and by the electron in C.

Fig 2 The branching ratio, $f = w(\mu \rightarrow e + \gamma) / w(\mu \rightarrow e + \nu + \bar{\nu})$ as a function of the ratio of the cut-off value to the intermediate boson mass.