The Jeans mass and the origin of the knee in the IMF

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ABSTRACT

We use numerical simulations of the fragmentation of a 1000 M⊙ molecular cloud and the formation of a stellar cluster to study how the initial conditions for star formation affect the resulting initial mass function (IMF). In particular, we are interested in the relation between the thermal Jeans mass in a cloud and the knee of the initial mass function, i.e. the mass separating the region with a flat IMF slope from that typified by a steeper, Salpeter-like, slope. In three isothermal simulations with \(M_{\text{Jeans}} = 1M_\odot\), \(M_{\text{Jeans}} = 2M_\odot\) and \(M_{\text{Jeans}} = 5M_\odot\), the number of stars formed, at comparable dynamical times, scales roughly with the number of initial Jeans masses in the cloud. The mean stellar mass also increases (though less than linearly) with the initial Jeans mass in the cloud. It is found that the IMF in each case displays a prominent knee, located roughly at the mass scale of the initial Jeans mass. Thus clouds with higher initial Jeans masses produce IMFs which are shallow to higher masses. This implies that a universal IMF requires a physical mechanism that sets the Jeans mass to be near 1 M⊙. Simulations including a barotropic equation of state as suggested by Larson, with cooling at low densities followed by gentle heating at higher densities, are able to produce realistic IMFs with the knee located at \(\approx 1M_\odot\), even with an initial \(M_{\text{Jeans}} = 5M_\odot\). We therefore suggest that the observed universality of the IMF in the local Universe does not require any fine tuning of the initial conditions in star forming clouds but is instead imprinted by details of the cooling physics of the collapsing gas.

Key words: stars: formation – stars: luminosity function, mass function – open clusters and associations: general.

1 INTRODUCTION

One of the goals of studies of star formation is to understand the origin of the initial mass function (IMF). Since Salpeter’s initial study (Salpeter 1955), we have known that a decreasing number of stars form with increasing mass. The form of this mass function,

\[
dN \propto m^{-\gamma}dm
\]

with \(\gamma = 2.35\), has proven to be remarkably robust for intermediate and high-mass stars (Kroupa 2002). More recently, we have seen that the distribution of lower-mass stars is less steep (Miller & Scalo 1979; Scalo 1986; Kroupa et al. 1993), \(\gamma = 1.5\), eventually becoming nearly flat for brown dwarfs (Allen et al. 2005; Kroupa 2002). This knee in the slope of the mass distribution suggests that there exists a change in the physical process which determines the stellar masses (Elmegreen 2004) for low and high-masses. The location of this knee at \(\approx 1M_\odot\) is found to be fairly consistent across the Galactic stellar populations (Kroupa 2002; Munch et al. 2002; Chabrier 2003) and thus provides a useful reference point for models of star formation and the initial mass function.

There have been many theories developed to explain the origin of the initial mass function. These theories are based upon physical processes such as fragmentation (Zinnecker 1984; Larson 1985; Klessen, Burkert & Bate 1998; Klessen & Burkert 2001; Klessen 2001), turbulence (Padoan & Nordlund 2002), coagulation (Silk & Takahashi 1979; Murray & Lin 1996), accretion (Zinnecker 1982; Bonnell et al. 1997; Klessen & Burkert 2000; Bonnell et al. 2001b; Basu & Jones 2004; Bate & Bonnell 2005), stellar mergers (Bonnell, Bate & Zinnecker 1998; Bonnell & Bate 2002) and feedback (Silk 1995), or a combination of multiple processes (Adams & Fatuzzo 1996). In addition to being able to reproduce the
slope of the IMF, any theory needs to be able to explain the characteristic stellar mass. This characteristic mass occurs due to the break in the IMF and it is therefore crucial to understand why there is a knee in the IMF. It is also crucial that these theories establish secondary indicators, such as the initial mass segregation in stellar clusters (Bonnell & Davies 1998; Bonnell et al. 2001a) in order to assess their relevance.

Numerical simulations of the collapse and fragmentation of turbulent molecular clouds are now capable of following the formation of sufficient numbers of stars to resolve the IMF from the low-mass brown dwarfs (Bate, Bonnell & Bromm 2003) to high-mass O stars (Bonnell, Bate & Vine 2003). These simulations produce nearly flat IMFs ($\gamma \approx 1$) for low-mass objects (Bate et al. 2003; Bate & Bonnell 2005), rolling over to $\gamma = 1.5$ near $m \lesssim 1M_\odot$ and then to $\gamma \approx 2.5$ for higher-mass stars (Bonnell et al. 2001b; Bonnell et al. 2003).

In order for fragmentation to occur, significant structure is needed in the molecular clouds. Turbulence can generate significant structure due to shocks driven into the gas (e.g. Larson 1981; Padoan et al. 2001). A detailed analysis of the pre-star formation evolution shows that the turbulent-driven structure is generally unbound and needs to grow through coagulation and accretion onto the clumps before gravitational collapse occurs (Clark & Bonnell 2005). Significantly, the mass at which the clumps become gravitationally bound is that of the thermal Jeans mass in the unperturbed cloud. The Jeans mass is given by

$$M_{\text{Jeans}} = \left(\frac{5 R_g T}{2 G \rho}\right)^{3/2} \left(\frac{4}{\pi^7}\right)^{-1/2},$$

where $\rho$ is the gas density, $T$ the temperature, $R_g$ is the gas constant, $G$ is the gravitational constant and $\mu$ is the mean molecular weight of the gas.

Larson (1985, 2005) has suggested that a change in the gas cooling with densities may provide an important determinant of the Jeans mass at the point of fragmentation. This change occurs as the cooling is dominated at lower densities by atomic and molecular line emission whereas at higher densities the gas is coupled to the dust and it is dust cooling that dominates. The resulting equation of state is suggested to involve cooling at lower densities followed by a gentle heating at higher densities. Jappsen et al. (2005) have used such an equation of state in numerical simulations and shown that it can set the mass scale for star formation.

It is therefore likely that the thermal Jeans mass plays an important role in determining the characteristic stellar mass (Larson 2005). Fragmentation, and ejections, most probably play a key role in determining the low-mass end of the IMF (Larson 1985; Bate et al. 2003; Bate & Bonnell 2005) whereas competitive accretion or other coagulation process dominates the formation of intermediate and massive stars (Larson 1982; Zinnecker 1982; Bonnell et al. 2001a; Bonnell & Bate 2002; Bonnell, Vine & Bate 2004; Bonnell & Bate 2005). In this paper we use numerical simulations to investigate the direct relation between the thermal Jeans mass and the knee of the IMF.

<table>
<thead>
<tr>
<th>Sim</th>
<th>$M_J$</th>
<th>EoS</th>
<th>Radius</th>
<th>$N_{\text{stars}}$</th>
<th>$M_{\text{tot}}$</th>
<th>$M_{\text{med}}$</th>
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<td>A</td>
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<td>330</td>
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<td>231</td>
<td>316</td>
<td>0.58</td>
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</tr>
<tr>
<td>C</td>
<td>5</td>
<td>Iso</td>
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<td>91</td>
<td>195</td>
<td>0.63</td>
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</tr>
<tr>
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<td>5</td>
<td>Lar</td>
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<td>334</td>
<td>333</td>
<td>0.47</td>
<td>1.00</td>
</tr>
<tr>
<td>E</td>
<td>5</td>
<td>Iso</td>
<td>0.9</td>
<td>119</td>
<td>253</td>
<td>0.60</td>
<td>2.13</td>
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</tbody>
</table>

### Table 1. Status of simulations at $t = 1.5t_{\text{ff}}$. The radii of the clouds is in pc and the masses are in $M_\odot$.

### 2 CALCULATIONS

We use Smoothed Particle Hydrodynamics (SPH) (Monaghan 1992, Benz et al. 1990) simulations to follow the fragmentation of 1000 $M_\odot$ molecular clouds and the formation of sufficient numbers of stars to resolve the form of the IMF. The goal is to investigate the dependency of the knee of the IMF on the Jeans mass in the cloud. We perform three isothermal simulations varying the initial size of the cloud while maintaining the temperature at 10 K. The gas is assumed to remain isothermal in these simulations. The cloud sizes are set (see Table 1) such that the initial Jeans mass of the cloud is either $1M_\odot$, $2M_\odot$, or $5M_\odot$. We also report on one non-isothermal simulation where we use a barotropic equation of state to mimic the transition from line to dust cooling invoked by Larson (2005). This equation of state has an initial cooling of the form

$$T = T_c \left(\frac{\rho}{\rho_c}\right)^{-0.25}, \quad \rho < \rho_c$$

followed by a gentle heating

$$T = T_c \left(\frac{\rho}{\rho_c}\right)^{0.1}, \quad \rho > \rho_c$$

once the gas is well coupled to the dust. The critical temperature and density was chosen to be $T_c = 4.9K$ at $\rho_c = 3 \times 10^{-18}$ g cm$^{-3}$. This simulation started from a density of $\rho = 2.2 \times 10^{-20}$ g cm$^{-3}$ and a temperature of 16 K, and thus a Jeans mass of $5M_\odot$. In order to facilitate comparison, we have also performed an isothermal simulation from the same initial conditions.

Star formation is modeled by the inclusion of sink-particles (Bate, Bonnell & Price 1995) that interact only through self-gravity and through gas accretion. Sink-particle creation occurs when dense clumps of gas have $\rho > 1.5 \times 10^{-15}$ g cm$^{-3}$, are self-gravitating, and are contained in a region such that the SPH smoothing lengths are smaller than the ‘sink radius’ of 200 AU. Gas particles are accreted if they fall within a sink-radius (200 AU) of a sink-particle and are bound to it. In the case of overlapping sink-radii, the gas particle is accreted by the sink-particle to which it is most bound. The gravitational forces between sink-particles are smoothed at 160 AU using the SPH kernel. The simulations contain $5 \times 10^5$ particles such that the minimum resolvable mass is 0.15$M_\odot$ (Bate & Burkert 1997; Bate et al. 2003). We therefore cannot resolve any star formation below this value although sink-particles commonly form with initial masses typically half this value, corresponding to the number of neighbours in one SPH smoothing length. These ‘stars’ quickly accrete up to the minimum resolvable mass. The code has variable smoothing lengths in time and in
The knee of the IMF

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Figure 1. The evolution of the cluster forming in simulation A with $M_{\text{Jeans}} = 1M_\odot$ is shown at $t = 0.9t_{\text{ff}}$, $t = 1.34t_{\text{ff}}$, and $t = 1.72t_{\text{ff}}$. The column density is plotted from a minimum value of 0.0075 to a maximum value of 150 g cm$^{-2}$.

Figure 2. The mean (solid lines) and maximum (dashed lines) stellar masses for the three isothermal simulations are plotted as a function of the free-fall time. The mean stellar masses are, from top to bottom, for initial Jeans masses of 5, 2 and 1 $M_\odot$. The mean stellar masses increase with the Jeans mass. The maximum stellar masses are basically indistinguishable and reflect solely the total mass being accreted into the forming stellar clusters.

3 ISOTHERMAL CLUSTER FORMATION AND STELLAR MASSES

The early evolution of the simulations is similar to that reported in Bonnell, Bate & Vine (2003). The initially uniform density is stirred up due to the chaotic motions present from the initial ‘turbulent’ velocity field. Shock compression leads to filamentary structures being formed very quickly, but these do not proceed to form stars until larger, bound structures are formed through self-gravity (e.g. Clark & Bonnell 2005). Once gravitationally bound clumps form along the filaments, they collapse to form stars. These stars fall towards their local potential minima and there form small clusters which grow by accreting stars and gas (e.g. Bonnell et al. 2003). This process is illustrated in Figure 1 for simulation A. Note that apart from a different random seed for the initial velocity field, the simulation is the same as reported in Bonnell et al. (2003).

The primary difference in the three isothermal simulations is the number of stars that form. Table 1 denotes the various parameters of the three simulations, the number of stars formed by $t = 1.5t_{\text{ff}}$, and the associated total mass, as well as the average and maximum stellar mass. Simulation A, with a Jeans mass of $M_{\text{Jeans}} = 1M_\odot$, forms over 400 stars by $t = 1.5t_{\text{ff}}$ (over 500 by $t = 1.72t_{\text{ff}}$) whereas simulation B, with $M_{\text{Jeans}} = 2M_\odot$, forms just over 200 and simulation C, with $M_{\text{Jeans}} = 5M_\odot$, forms under 100 stars. The lower density in the higher Jeans mass simulations means that the clumps need to be more massive to overcome thermal (and kinetic) support to become gravitationally bound and collapse. Thus fewer of the clumps form stars and these stars are also more massive.

In all three simulations, the dissipation of the supporting kinetic energies in shocks allows for a larger scale gravitational collapse. This increases the gas density and thus lowers the Jeans mass relative to its initial value. Thus stars that form later can have smaller initial masses. Gas accretion onto all the stars increases their masses with those in the centre of clusters accreting more and therefore attaining the highest masses (Bonnell et al. 2001a; Bonnell et al. 2004). This results in an increasing dispersion in stellar masses with time and populates the resulting mass spectrum. Figure 2 plots the evolution of the average and maximum stellar mass as a function of free-fall time for the three simulations. The average stellar mass remains nearly constant for each simulation, and scales (though not linearly) with the initial Jeans mass. In contrast, the maximum stellar mass increases with time but does not depend on the initial Jeans mass. This is because the mass of these stars is from gas accretion and not from the fragmentation process (Bonnell et al. 2001b;
Bonelli et al. 2004). The total mass accreted by a star depends on the gas inflowing into the cluster's potential well and not on the Jeans mass.

### 3.1 Initial mass functions

The resulting initial mass functions (IMFs) for the three isothermal simulations are shown in figure 3. The IMF for simulation A with $M_{\text{Jeans}} = 1 \, M_\odot$ closely resembles that produced by simulations using similar initial conditions (Bonelli et al. 2003) as well as the observed IMF of young stellar clusters. The shallow slope at low-masses extends to 0.5 to 1 $M_\odot$ before developing into a steeper Salpeter-type slope for higher masses. In contrast, the IMFs for simulations B and C show marked departures in that the shallow part of the mass spectrum extends to significantly higher masses. Simulation B, with $M_{\text{Jeans}} = 2 \, M_\odot$ has a shallow IMF extending to $\approx 2$ or $3 \, M_\odot$ with a steeper Salpeter-like slope at higher masses. Simulation C, with $M_{\text{Jeans}} = 5 \, M_\odot$ has the shallow part of the IMF extending to masses of $\approx 5$ to $10 \, M_\odot$, with only a slight indication of a steeper slope at higher masses. It is clear from these IMFs that the knee or break-point between the shallow and steep sections of the IMF increases with the Jeans mass. To a fairly good approximation, the knee is located at the Jeans mass of the initial conditions.

The overall form of the IMF from simulation C resembles that found in the case of a low-mass cloud with a Jeans mass of $M_{\text{Jeans}} = 1 \, M_\odot$ and resolving down to the opacity limit (Bate et al. 2003). Both IMFs have a shallow (flat in log mass) IMF which ends near the Jeans mass of the cloud. Similarly, when the Jeans mass in the lower-mass cloud is reduced, this break point also moves to lower masses (Bate et al. 2000b) as seen in Simulation B. This lends credence to our conclusion that it is the Jeans mass which sets at which mass the IMF becomes steeper.

The physical interpretation behind this result is that clumps become bound near the initial Jeans mass of the unperturbed cloud (Clark & Bonnell 2005; Bonnell et al. 2004). These clumps can subfragment as they are at least partially kinetically supported, such that the thermal energy comprises only a fraction of the virial support in the clump. Higher-mass stars require substantial accretion from beyond the initial clump (Bonelli et al. 2004). The steep Salpeter-like slope appears once competitive accretion in a stellar dominated regime occurs (Bonelli et al. 2001b), and this can only happen once the stellar masses are above the masses of the initial clumps, and hence the Jeans mass.

### 4 NON-ISOTHERMAL CLUSTER FORMATION

If the resulting IMF depends on the initial Jeans mass, as these experiments suggest, then a universal IMF would appear to require fine tuning of the cloud properties such that the initial Jeans mass is always the same, with a value around a solar mass. This is an uncomfortable conclusion, since, for a cloud of fixed mass, the Jeans mass varies with both the temperature and cloud radius to a high power (1.5). It would be far more reassuring if we could instead invoke some physical process that yields a Jeans mass of around a solar mass from a variety of initial cloud conditions.

One straightforward way that this can occur is if the thermal physics results in an equation of state with structure in it such that the temperature reaches a minimum at a certain density. Based on the expected thermal physics of molecular clouds, Larson (2005) has recently suggested that the equation of state of the form $P = k \rho^\eta$ ($T \propto \rho^{n-1}$) should include some deviation from isothermality such that the gas cools ($\eta < 1$) with compression at low densities while at higher densities it heats ($\eta > 1$) slowly with increasing density. While the gas is cooling, the Jeans mass actually decreases under compression at a greater relative rate than does the local dynamical time whereas once the heating starts, the reverse occurs. The free-fall time denotes how quickly an object can reconfigure itself due to self-gravity.

For an isothermal gas, the free-fall time and Jeans mass evolve in unison such that while the Jeans mass decreases, gravity induces a central higher density region which contains exactly one Jeans mass. But, with cooling, a collapsing object can develop multiple Jeans masses before it can reconfigure itself into a more stable, condensed configuration.
Separate sub-entities can therefore develop, become gravitationally unstable and collapse to form lower-mass objects. With heating under compression, the relative decrease in the Jeans mass is slowed such that a collapsing clump can reconfigure itself to maintain one single, albeit more centrally condensed, object. It then maintains the same object mass. The Jeans mass effectively gets hung up at the point where the equation of state changes and this sets a mass scale for star formation.

In order to test this possibility, we performed one simulation with a barotropic equation of state including both a cooling term at low densities and a heating term at higher densities. Starting with an initial Jeans mass of $M_{\text{Jeans}} = 5 M_\odot$, which in the isothermal runs gave a flat IMF, we followed the collapse and fragmentation that forms a stellar cluster. This simulation formed 334 stars within $t = 1.55 t_\text{ff}$, with a median and mean stellar mass of 0.47 and 1.0 $M_\odot$, respectively. The maximum stellar mass is 20 $M_\odot$. These values are very similar to the isothermal $M_{\text{Jeans}} = 1 M_\odot$ simulation even though the initial Jeans mass was 5 times greater.

The corresponding IMF is shown in Figure 4 along with that from the corresponding isothermal simulation. The two simulations started from the exact same initial conditions with a Jeans mass of $M_{\text{Jeans}} = 5 M_\odot$ but produce remarkably different IMFs. The isothermal simulation produces a flat IMF that extends from low masses up to $\approx 5 M_\odot$ as found above. In contrast, the barotropic equation of state produces an IMF that has a broad peak that extends up to $\approx 1 M_\odot$ and a Salpeter-like slope at higher masses. This observationally realistic IMF is similar to that produced when the initial Jeans mass is $M_{\text{Jeans}} = 1 M_\odot$. This is due to the cooling which decreases the Jeans mass with gas compression. Thus, we can confirm that the physically motivated barotropic equation of state suggested by Larson (2005) allows for more general initial conditions to produce realistic IMFs.

A further point worth noting here is that this relaxation of the initial conditions also allows for more flexibility in the resulting stellar densities in the newly formed clusters without changing the location of the knee of the IMF. The median stellar densities in simulation D reported here are a factor of ten smaller than those found in Bonnell et al. (2003). This is most likely due to the larger initial Jeans radius in this simulation as the gas density is lower ($R_J \propto \rho^{-0.5}$). The Jeans radius sets the scale for the fragmentation such that more widely spaced fragments result in a less dense stellar cluster.

5 CONCLUSIONS

Numerical simulations of the isothermal fragmentation of turbulent molecular clouds, and the subsequent competitive accretion in the forming stellar clusters, are able to reproduce the observed stellar IMF only if the initial Jeans mass is $\approx 1 M_\odot$. We find that the knee of the IMF, is approximately given by the Jeans mass. Thus, simulations with $M_{\text{Jeans}} = 5 M_\odot$ result in a shallow low-mass IMF that extends to $\gtrsim 5 M_\odot$ in disagreement with observations of the field and cluster star IMF.

The main implication of this result is that in order to produce a universal IMF from a variety of cloud initial conditions, the thermal physics in molecular clouds must result in a Jeans mass of order $1 M_\odot$ when star formation is initiated. A barotropic equation of state, involving a cooling term at low densities followed by a gentle heating term once dust cooling dominates, is able to produce a realistic IMF even when the initial Jeans mass is high ($M_{\text{Jeans}} = 5 M_\odot$).

We can conclude therefore that the small departures from isothermal physics invoked by Larson (2005) is sufficient to reproduce the characteristic stellar mass and thus realistic IMFs independent of the exact initial conditions used (see also Jappsen et al.2005). Variations in cooling physics, at different epochs and in different metallicity regimes, would however result in a shift of the location of the knee, implying a top heavy IMF in regions of Population III star formation (Clarke & Bromm 2003; Bromm & Larson 2004).
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