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HIGH ENERGY SCATTERING OF HADRONS

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In these lectures we shall discuss some recent developments in the theory of elastic scattering of hadrons (strongly interacting particles) at high energy and small momentum transfers. By "high energy" we mean here the region $p_L \gtrsim 4 \text{ GeV}/c$, where p_L is the laboratory momentum of the incident particle; the highest value of p_L available with present accelerators is about $30 \text{ GeV}/c$ for protons. This region is characterized by the fact that cross-sections are found to be smooth, slowly varying, in certain cases even nearly constant, functions of p_L without any structure. By "small momentum transfers" we mean the region of the forward diffraction peak. At present, experimental high energy data are available for PP , PN , $\bar{P}P$, $\bar{P}N$, π^+P , K^+P and K^+N reactions, where P = proton, N = neutron. We shall deal with two recently proposed models which have been rather successful in predicting relations between the various meson-baryon and baryon-baryon cross-sections at high energies, namely the quark model ¹⁾⁻⁸⁾ (Ch. II), and the combination of Regge pole model and current algebra ^{9), 10)} (Ch. III). In Ch. I, we briefly mention a few general asymptotic properties of elastic scattering amplitudes, which are useful for the discussion in the last two Chapters.

I. - GENERALITIES

1 - Asymptotic expansion of the amplitude

Consider the elastic scattering process (A and B are hadrons ^{*)}),

$$A + B \rightarrow A + B, \quad (1)$$

together with the crossed reaction

$$\bar{A} + B \rightarrow \bar{A} + B, \quad (2)$$

^{*)} We always neglect spins.

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in the region of small momentum transfers ($0 \lesssim |t| \lesssim 1 \text{ (GeV/c)}^2$) and high energies ($s \gtrsim 8 \text{ GeV}^2$) where $t = -(\text{c.m. momentum transfer})^2$ and $s = -(\text{c.m. energy})^2$. In this region we write down the following asymptotic series expansions for the scattering amplitudes $T(s, t)$ and $\bar{T}(s, t)$ for reactions (1) and (2) respectively

$$T(s, t) \underset{s \rightarrow \infty}{\approx} \sum_{j=0,1,\dots} c_j(t) s_t^{\alpha_j(t)}, \quad (3)$$

$$\bar{T}(s, t) \underset{s \rightarrow \infty}{\approx} \sum_{j=0,1,\dots} \bar{c}_j(t) s_t^{\alpha_j(t)}, \quad (4)$$

in which $s_t = s + \frac{t}{2} - m_A^2 - m_B^2$. The coefficients c_j and $\bar{c}_j$ can, in general, be complex. The exponents α_j are taken real with the condition $\alpha_0 > \alpha_1 > \dots$, as suggested by the fact that no oscillations have been found experimentally in the variation of total cross-sections at high energy.

The existing high-energy data can be described in the form (3) and (4) with a small number of terms (two to four). Note that (3) and (4) correspond to dominance by poles in the angular momentum plane; if $\alpha(t)$ varies with t these poles are Regge poles, otherwise they are fixed poles. Logarithmic terms of the form $(\ell \ln s_t) \beta_j(t)$ in (3) and (4), which we have left out because present experiments are not accurate enough to decide on their presence or absence, would reflect the presence of fixed or moving cuts in the ℓ plane.

Using analyticity of T in s , crossing symmetry, and the assumption of slower than exponential growth of T in the upper-half s plane, a strong relation between the two expansions (3) and (4) can be derived ¹¹⁾⁻¹⁴⁾, namely

$$\bar{c}_j(t) = c_j^*(t) e^{-i\pi\alpha_j(t)}. \quad (5)$$

This important property holds only for the expansions (3) and (4) in which s_t , instead of s , is used as expansion parameter. An immediate consequence of (5) is that these expansions contain actually the same powers of s_t .

Consider the forward direction ($t = 0$). If the total cross-sections for scattering of hadrons tend to a finite non-zero limit at very high energies, i.e., using the optical theorem

$$\sigma_T = \frac{8\pi^2}{k\sqrt{s}} \operatorname{Im} T(s, 0) \quad , \quad (7)$$

(k = c.m. momentum, σ_T = total cross-section), if $\operatorname{Im} T(s, 0) \underset{s \rightarrow \infty}{\sim} \text{const.} \cdot s$, we have $\alpha_0(0) = 1$ and hence from (5)

$$\operatorname{Im} c_0(0) = \operatorname{Im} \bar{c}_0(0) \quad , \quad (8)$$

$$\operatorname{Re} c_0(0) = -\operatorname{Re} \bar{c}_0(0) \quad . \quad (9)$$

Relation (8) is the Pomeranchuk theorem $\sigma_T(\infty) = \bar{\sigma}_T(\infty)$ for total cross-sections¹⁵⁾. The present data^{16), 17)} extending up to

$p_L \sim 25$ GeV/c are compatible with it although even at 25 GeV/c there are still considerable differences $\bar{\sigma}_T - \sigma_T$. These have to be described by the second and higher terms in the expansions (3) and (4).

Relation (9) gives $X(s, 0) = -\bar{X}(s, 0)$ for $s \rightarrow \infty$, where $X(s, t) = \frac{\operatorname{Re} T(s, t)}{\operatorname{Im} T(s, t)}$. Since for $p_L \gtrsim 5$ GeV/c both $X(s, 0)$ and $\bar{X}(s, 0)$ are found to be negative (and of order -0.3 for PP and NP, -0.2 for $\pi^{\pm}P$) where measured¹⁸⁾, (9) suggests that $X(\infty, 0) = \bar{X}(\infty, 0) = 0$, which would mean that the measured values of X and $\bar{X}$ are due to the higher terms in (3) and (4). Note that for an amplitude which has positive exchange signature¹³⁾, i.e., which satisfies

$$\bar{T}(s, t) = \xi T(s, t) \quad , \quad (10)$$

with $\xi = +1$, the assumption $\alpha_0(0) = 1$ implies the asymptotic amplitude to be purely imaginary. This applies to the Regge pole model if the Pomeranchuk trajectory passes through one (Ch. III).

Suppose that in addition to the asymptotic term (the one with $j = 0$) only one term ($j = 1$) is needed to describe the data above ~ 5 GeV/c; we then have from (3), (4), (5) and (7) for an imaginary asymptotic amplitude, putting $\Delta = [\sigma_T(s) - \sigma_T(\infty)] k \sqrt{s}/8\pi^2$ and $\alpha_1(0) = \alpha_1$,

$$\Delta = -\cos(\alpha_1\pi) \bar{\Delta} - \sin(\alpha_1\pi) \text{Re} \bar{T}, \quad (11a)$$

$$\text{Re} T = \cos(\alpha_1\pi) \text{Re} \bar{T} - \sin(\alpha_1\pi) \bar{\Delta} \quad (11b)$$

The total cross-section curves suggest in the high-energy region $\alpha_1 \simeq \frac{1}{2}$. Since Δ and $\bar{\Delta}$ are found to be positive above 5 GeV/c for all hadrons, (11) predicts for this value of α_1 the real parts to be negative, in accordance with all existing measurements¹⁸⁾. Moreover, $\Delta < \bar{\Delta}$ implies $|\text{Re} \bar{T}| < |\text{Re} T|$ for all pairs of reactions (1) and (2). For instance, for PP and $\bar{P}P$ scattering ($P = \text{proton}$), we expect around 10 GeV/c and $t = 0$, $|\text{Re} \bar{T}| \ll |\text{Re} T|$, because here $\bar{\Delta} \gg \Delta$ (the bar refers to $\bar{P}P$) due to strong annihilation contributions in $\bar{P}P$ scattering¹⁴⁾. This agrees with recent experimental results¹⁹⁾.

We remark that the assumption $\alpha_0(0) = 1$ is not absolutely required, neither by theory nor by experiment. In fact, good fits to the existing cross-section data exist with $\alpha_0(0)$ slightly less than one (see Ch. III), implying vanishing asymptotic total cross-sections¹⁰⁾.

At non-forward directions ($t \neq 0$, but small) we have at once from (5) in the limit of infinite energies for each pair of processes (1) and (2)

$$|T| = |\bar{T}|, \text{ or } \frac{d\sigma_{el}}{dt} = \frac{d\bar{\sigma}_{el}}{dt} \quad (13)$$

This is the generalized Pomeranchuk theorem²⁰⁾. Since all measured differential cross-sections for $0 \leq |t| \lesssim 1(\text{GeV}/c)^2$ can be parametrized in the form

$$\frac{d\sigma_{el}}{dt} = \left(\frac{d\sigma_{el}}{dt} \right)_{t=0} e^{at + bt^2}, \quad (14)$$

with the slope a always in the neighbourhood of $10(\text{GeV}/c)^{-2}$ and $b^2/a \ll 1$, (13) implies

$$a = \bar{a}, \quad b = \bar{b}, \quad \text{for } s \rightarrow \infty. \quad (15)$$

The existing data above $\sim 5 \text{ GeV}/c$ are again compatible with this result¹⁶⁾. For instance, for $\pi^\pm P$ scattering, (15) is well satisfied with constant a and b . The slopes $\bar{a}$ and a for $\bar{P}P$ and PP show a variation with energy, more pronounced for $\bar{a}$ than for a . The latter seems to increase with increasing energy (shrinking of the diffraction peak), the former decreases (expansion of the diffraction peak), both tending to a common limit of about $10(\text{GeV}/c)^{-2}$, which is about the same value as obtained from $\pi^\pm P$ scattering²¹⁾. Around $10 \text{ GeV}/c$, however, a and $\bar{a}$ are still very different [$a \simeq 9(\text{GeV}/c)^{-2}$, $\bar{a} \simeq 12.5(\text{GeV}/c)^{-2}$]. This difference and the expansion of the $\bar{P}P$ diffraction peak are connected with the presence and variation with energy of $\bar{P}P$ annihilation contributions²²⁾. Similar results are known for K nucleon scattering, where again the slopes a and $\bar{a}$ tend to common limits. From this we may conclude that the leading term in the expansions (3) and (4) has exponent $\alpha_0(t) = \alpha_0(0)$ for $|t| \lesssim 1(\text{GeV}/c)^{-2}$, with $\alpha_0(0) = 1$, or very close to one.

2 - Unitarity condition

We write the unitarity condition in the form

$$\frac{16\pi^2}{k\sqrt{s}} \operatorname{Im} T(s, t) = \sum_n \langle AB | \tau^\dagger | n \rangle \langle n | \tau | AB \rangle, \quad (16)$$

where the sum extends over all elastic and inelastic final states and where τ is the properly normalized scattering operator. Conservation of energy and momentum is assumed to be taken into account. For $t = 0$, (16) becomes

$$2\sigma_T(AB) = \sum_n \langle AB | \tau^\dagger | n \rangle \langle n | \tau | AB \rangle \quad (17)$$

Consider the reaction

$$A + B \rightarrow \text{inelastic final state } n,$$

in the c.m. system. The jet structure of high energy inelastic collisions (small transverse momenta in c.m.) suggests that the particles in the final state may be roughly divided into two groups A' and B' , one containing the particles which go forward in the c.m. system and one containing the particles going backward. In general A' and B' are many-particle systems. In analogy with two-body reactions, we can now, according to the possible exchanges in the t channel in reactions of the type $A + B \rightarrow A' + B'$, roughly subdivide the contributions to the sum in (17) (the same, of course, holds for (16)), i.e., to $\sigma_T(AB)$, into three classes :

- a) annihilation or baryon exchange contributions $\sigma_A(AB)$;
- b) charge, isospin and strangeness exchange contributions $\sigma_E(AB)$;
- c) Pomanchuk contributions $\sigma_P(AB)$.

Class a) contributes only when A and B have opposite baryon number. Class c) refers to those contributions for which vanishing internal quantum numbers are exchanged in the t channel of $A + B \rightarrow A' + B'$. One expects that at very high energies only the latter contributions survive, i.e.,

$$\frac{\sigma_A}{\sigma_P} \rightarrow 0, \quad \frac{\sigma_E}{\sigma_P} \rightarrow 0, \quad \text{for } s \rightarrow \infty, \quad (18)$$

and that at finite energies the measured differences $\sigma_T(\bar{A}B) - \sigma_T(AB)$ are mainly determined by contributions of classes a) and b).

As a consequence of the above subdivision we may write approximately *)

$$\sigma_T(AB) = \sigma_P(AB) + \sigma_A(AB) + \sigma_E(AB). \quad (19)$$

The contribution from elastic final states in (17), as being mainly diffractive, is supposed to be included in σ_P^P . From (19) simple inequalities between total cross-sections follow ⁵⁾. For instance,

$$\sigma_T(PN) > \sigma_T(PP),$$

which one obtains by noting that $\sigma_P(PP) \simeq \sigma_P(PN)$, $\sigma_A(PP) = \sigma_A(PN) = 0$ and $\sigma_E(PN) > \sigma_E(PP)$, the last inequality arising from the fact that charge can be exchanged in more ways between N and P than between P and P. Similarly, because of strong annihilation contributions in $\bar{P}N$, one has

$$\sigma_T(\bar{P}N) > \sigma_T(PP).$$

*) We neglect the fact that in general the three types of contributions affect each other due to absorption corrections. For instance, the presence of σ_A affects $\sigma_{e\ell}$ and hence σ_P .

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Similar inequalities can be obtained for meson-baryon scattering. Although these considerations are very crude, all inequalities between measured high-energy cross-sections derived on the basis of (19) turn out to agree with the facts ⁵⁾.

II. - QUARK MODEL FOR HIGH-ENERGY SCATTERING

1 - The additivity assumption

A very successful approach to high-energy scattering of hadrons has recently ¹⁾⁻⁸⁾ been developed on the basis of the quark model ²³⁾. In the latter the hadrons are assumed to be composed of $SU(3)$ triplets, called quarks, which have fractional charge and baryon number. We shall denote the triplet of quarks by p, n, λ , which have the following quantum numbers

$$\begin{aligned} p : \quad B = \frac{1}{3}, \quad Q = \frac{2}{3}e, \quad S = 0, \quad I = \frac{1}{2} \\ n : \quad B = \frac{1}{3}, \quad Q = -\frac{1}{3}e, \quad S = 0, \quad I = \frac{1}{2} \\ \lambda : \quad B = \frac{1}{3}, \quad Q = -\frac{1}{3}e, \quad S = -1, \quad I = 0. \end{aligned} \quad (20)$$

The corresponding triplet of antiquarks is denoted by $\bar{p}, \bar{n}, \bar{\lambda}$. In the quark model the baryons and decuplet members have quark composition $(q q' q'')$, the pseudoscalar and vector mesons $(q \bar{q}')$. For instance, $P = (ppn)$, $\pi^+ = (p\bar{n})$, $K^+ = (p\bar{\lambda})$, etc. In the following we shall always forget about the spin variables; more precisely, we shall consider only spin averaged cross-sections ²⁴⁾.

The basic assumption one makes in the application of the quark model to high-energy scattering is the so-called additivity assumption, in which the amplitude for a two-body reaction $A + B \rightarrow A' + B'$ is written as a sum of corresponding two-body quark scattering amplitudes, the quarks inside hadrons being considered as quasi-free particles ^{*)}.

*) As to the nature of the quarks we are completely ignorant; it may be that quarks exist as truly free particles although they have never been detected as yet; it may also be that they can only exist as a kind of quasi-particles which cannot be separated from the hadrons, like phonons in a crystal; it may also be ...

Restricting ourselves in these lectures to elastic scattering amplitudes ($A' = A$, $B' = B$) we can put this assumption into mathematical form in the following way ^{3), 5)}

$$T'_{AB}(s, t) = \sum_{ij} f_i^A(t) f_j^B(t) T'_{ij}(s, t), \quad (21)$$

where i and j label the quarks and antiquarks contained in A and B respectively. The function f_i^A is the form factor for the i -th quark (or antiquark) in A ; with a similar definition for f_j^B . The primes in (21) indicate that the amplitudes are non-covariant amplitudes, i.e., matrix elements of the S matrix. This means that, in contrast to the amplitudes T and $\bar{T}$ defined in Section I-1, their normalization is such that at high energy s independence of the amplitudes implies s independence of cross-sections. It is expected that (21) is applicable only at high energy ($s \gtrsim 10 \text{ GeV}^2$) and small momentum transfers [$|t| \lesssim 1(\text{GeV}/c)^2$]. Its main applications up to now have been in the forward direction where the imaginary part of the left-hand side becomes proportional to the total cross-section $\sigma_T(AB)$. It should be mentioned that SU_3 need not be an exact symmetry of the S matrix; the only point at which it intervenes in the quark model is in the construction of the bound state wave functions.

The energy dependence of T'_{ij} in (21) deserves a comment. The value of $T'_{ij}(s, t)$ for given s and t and fixed quark labels i and j will, in general, not be the same for a nucleon-nucleon and a meson-nucleon collision, the reason being that the c.m. energy of the $q_i + q_j$ collision will be different for the two cases and unequal to the energy of the $A + B$ collision. If the meson and baryon have the same laboratory momentum p_L , the effective momenta of the constituting quarks should be on the average $\frac{1}{2}p_L$ and $\frac{1}{3}p_L$ respectively ^{*)}.

*) This makes sense only if the effective mass m_q of bound quarks is of order $m_q \simeq \frac{1}{3} m_B \simeq \frac{1}{2} m_M$, where m_B and m_M are average values of the baryon and meson masses respectively.

This implies that in checking relations, obtained from (21) by eliminating quark amplitudes, which involve both meson-baryon and baryon-baryon cross-sections, these cross-sections should be taken at laboratory momenta in the ratio 2 to 3 respectively ^{3), 5)}.

Before applying the additivity assumption to specific processes we have to make an important restriction concerning its applicability to antibaryon-baryon and baryon-baryon reactions ⁸⁾. Consider the imaginary parts of the elastic amplitudes together with the unitarity condition (16). If in (16) $A = \bar{B}'$ = antibaryon and B = baryon, at finite energies an appreciable contribution [called class a) in Chapter I] to the sum over n comes from annihilation channels $\bar{B}' + B \rightarrow$ mesons (Fig. 1). Since such a process involves baryon, or triple quark, exchange, it can clearly not be described by the additivity formula. In other words, Fig. 2, which describes (16) under the additivity assumption cannot be a good approximation to the annihilation process of Fig. 1. So $\bar{B}'B$ annihilation contributions have to be excluded from the additivity assumption for the imaginary part of the $\bar{B}'B$ amplitude ^{*)}; for these contributions an assumption of multiplicative, rather than additive, type to express them in terms of quarks seems more natural (see the end of this Chapter). The exclusion of annihilation processes from the additivity assumption for $\text{Im } T_{\bar{B}'B}$ may in general affect also $\text{Im } T_{B'B}$, as well as $\text{Re } T_{\bar{B}'B}$ and $\text{Re } T_{B'B}$, because these amplitudes are related by the crossing relations (5). Take for example the case of PP and $\bar{P}P$ scattering in the forward direction. If only two terms are necessary to describe the high-energy data we can apply equations (11) to this case (the bar refers to $\bar{P}P$). Present high energy data suggest the value $\alpha_1 \simeq \frac{1}{2}$, so that (11) becomes

$$\text{Re } T = -\bar{\Delta}, \quad \text{Re } \bar{T} = -\Delta.$$

*) A similar restriction applies to meson-baryon backward scattering. Its contribution to the sum in (16) for small angle meson-baryon scattering is, however, very small.

Since $\bar{\Delta} \simeq \sigma_A$, where σ_A is the cross-section for annihilation (see below), the exclusion of annihilation from the additivity assumption implies in this case the same for the main part of $\text{Re } T$. However, Δ , and therefore $\text{Im } T$, and $\text{Re } \bar{T}$ are not affected. This is fortunate since we wish to define additivity for the complete non-annihilation part of the sum in (16), i.e., in the case of PP scattering, for the imaginary part of the complete amplitude. Note that for $\alpha_1 \simeq 0$ we would have an inconsistency. In that case (11) becomes

$$\text{Re } T = \text{Re } \bar{T}, \quad \Delta = -\bar{\Delta},$$

so that, assuming additivity to hold for the complete imaginary amplitude $\text{Im } T$, the same would be true for $\text{Im } \bar{T}$, including annihilation.

The part to be excluded in the additivity scheme from the sum in (17) for $\sigma_T(\bar{B}'B)$ is just the cross-section for annihilation $\sigma_A(\bar{B}'B)$,

$$2\sigma_A(\bar{B}'B) = \sum_{n'} \langle \bar{B}'B | \tau^\dagger | n' \rangle \langle n' | \tau | \bar{B}'B \rangle,$$

which is an experimental quantity ($|n'\rangle$ contains only mesons). However, only one high-energy annihilation cross-section has been measured so far, namely $\sigma_A(\bar{P}P) = 22.5 \pm 2 \text{ mb}$ at $p_L = 5.7 \text{ GeV/c}$ ²⁵⁾. At this momentum one has $\sigma_T(\bar{P}P) = 62.5 \pm 1 \text{ mb}$ and $\sigma_T(PP) = 41.0 \pm 1 \text{ mb}$ so that

$$\sigma_A(\bar{P}P) = [\sigma_T(\bar{P}P) - \sigma_T(PP)] \pm (1 \div 2) \text{ mb.} \quad (22)$$

In order to be able to compare our theoretical results with experiment we shall assume that (22) continues to hold for $p_L \gtrsim 6 \text{ GeV/c}$ and that the part of the total cross-section $\sigma_T(\bar{B}'B)$ which is additive in the quark amplitudes is $\sigma_T(\bar{B}'B) - \sigma_A(\bar{B}'B)$. As long as accurate data on

annihilation cross-sections are lacking, a more rigorous procedure which takes care of small corrections due to the fact that annihilation and non-annihilation contributions are not completely independent²² does not make much sense.

The diagrams of Fig. 2 describe for $B\bar{B}'$ reactions the two types of contributions that approximate in the additivity scheme the non-annihilation part of (16). Similar diagrams can be drawn for baryon-baryon and meson-baryon scattering. The right-hand half of Fig. 2a contains antiquark-quark annihilation, i.e., single quark exchange between hadrons A and B in $\langle AB|T|n\rangle$. Such inelastic reactions, however, have never been found to occur experimentally. We therefore expect that only the diagrams of the type depicted in Fig. 2b and not those of Fig. 2a can provide a good approximation to the non-annihilation part of the sum in (16). Hence, we also exclude antiquark-quark annihilation contributions of the type given by Fig. 2a from the additivity scheme [see also remarks following (28)]. Note that if quarks do not exist as separate particles but only in bound combinations²⁵⁾, it would be a priori clear that a process as shown in Fig. 2a cannot exist.

We now use the additivity assumption, expressed by (21), limited to non-annihilation contributions as discussed above, to derive certain relations between hadron cross-sections. We first consider the forward direction ($t = 0$), for which $f_i^A = f_j^B = 1$. We obtain from (21) the following relations

$$\tilde{S}^{\pm}(PP) = 5 \tilde{S}^{\pm}(pp) + 4 \tilde{S}^{\pm}(np) \quad , \quad (23a)$$

$$\tilde{S}^{\pm}(PN) = 4 \tilde{S}^{\pm}(pp) + 5 \tilde{S}^{\pm}(np) \quad , \quad (23b)$$

$$S^+(\pi^+P) = 3 \tilde{S}^+(pp) + 3 \tilde{S}^+(np) \quad , \quad (23c)$$

$$S^-(\pi^+P) = \tilde{S}^-(pp) - \tilde{S}^-(np) \quad , \quad (23d)$$

$$S^{\pm}(K^+P) = 2 \tilde{S}^{\pm}(pp) + \tilde{S}^{\pm}(np) \pm 3 \tilde{S}^{\pm}(\lambda p), \quad (23e)$$

$$S^{\pm}(K^+N) = \tilde{S}^{\pm}(pp) + 2 \tilde{S}^{\pm}(np) \pm 3 \tilde{S}^{\pm}(\lambda p), \quad (23f)$$

with the definitions

$$\left. \begin{aligned} \tilde{S}^{\pm}(AB) &= S^{\pm}(AB) - \sigma_A(\bar{A}B) \\ S^{\pm}(AB) &= \sigma_T(\bar{A}B) \pm \sigma_T(AB) \end{aligned} \right\} \quad (24)$$

In (24) A and B may refer to hadrons as well as to quarks. In all equations (23)-(24) the upper or the lower signs have to be taken simultaneously. For meson-baryon scattering $\tilde{S}^{\pm}$ and $S^{\pm}$ are of course identical. In writing down (23) we have used isospin and charge conjugation invariance, and the optical theorem (7). Equations (23) imply four relations between the quantities $\tilde{S}^{\pm}$, namely ⁸⁾

$$\tilde{S}^+(PP) + \tilde{S}^+(PN) = 3 S^+(\pi^+P), \quad (25a)$$

$$\tilde{S}^+(PP) - \tilde{S}^+(PN) = S^+(K^+P) - S^+(K^+N), \quad (25b)$$

$$S^-(K^+P) - S^-(K^+N) = S^-(\pi^+P), \quad (25c)$$

$$\tilde{S}^-(PP) - \tilde{S}^-(PN) = S^-(\pi^+P). \quad (25d)$$

Relation (25c) is the "good" Johnson-Treiman relation ²⁾, which is known to be in excellent agreement with the facts. As to relations (25a,b,d) a detailed comparison with experiment has to await better data on nucleon-antinucleon annihilation at high energy. This is in particular true for (25b,d) which involve cross-section differences

which are of the order of the error in (22) so that a significant test is not possible at present ; these relations are however fully compatible with the data. To compare relations (25a) with the data we make use of (22), which can be written as $\tilde{S}^-(PP) \simeq 0$. Using this in (23a,b,c) we get $\tilde{S}^-(PN) \simeq -S^-(\pi^+P)$, so that $\tilde{S}^+(PP) \simeq 2\sigma_T(PP)$ and $\tilde{S}^+(PN) \simeq 2\sigma_T(PN) - S^-(\pi^+P)$. Remembering that we have to compare nucleon-nucleon and meson-nucleon cross-sections at laboratory momenta in the ratio $\sim \frac{3}{2}$, we find for the left and right-hand sides of (25a) respectively

$$157 \pm 3 \text{ mb}, \quad 157.8 \pm 0.6 \text{ mb} \quad \text{at} \quad \begin{array}{l} p_L^\pi = 8 \text{ GeV/c} \\ p_L^P = 12 \text{ GeV/c} \end{array},$$

$$154 \pm 3 \text{ mb}, \quad 150.0 \pm 0.6 \text{ mb} \quad \text{at} \quad \begin{array}{l} p_L^\pi = 12 \text{ GeV/c} \\ p_L^P = 18 \text{ GeV/c} \end{array},$$

where we have used the data of Ref. 17). Note that the agreement would have been bad if annihilation contributions would have been included on the left-hand side of (25a), increasing the latter by 15-20 %. Many other relations between cross-sections follow from the additivity assumption (plus only isospin and C invariance), which, however, involve one or more unmeasured cross-sections 2),3). We will not write these down here.

Inserting the data on the left of (23) we can find numbers for the quark "cross-sections". At 10 GeV/c we get, taking $\tilde{S}^-(PP) = 0$ and using the data of Ref. 17),

$$\begin{aligned} \tilde{S}^-(pp) &\simeq 0.9 \text{ mb}, \quad \tilde{S}^-(np) \simeq -1.1 \text{ mb}, \quad \tilde{S}^-(\lambda p) \simeq -1.5 \text{ mb}, \\ \tilde{S}^+(pp) &\simeq 9.5 \text{ mb}, \quad \tilde{S}^+(np) \simeq 7.7 \text{ mb}, \quad \tilde{S}^+(\lambda p) \simeq 4.4 \text{ mb}. \end{aligned} \quad (26)$$

From this we find for the non-annihilation parts of the quark "cross-sections", $\sigma(qq')$,

$$\begin{aligned}\sigma(\bar{p}p) &\simeq 5.2 \text{ mb}, \quad \sigma(\bar{p}n) \simeq 3.3 \text{ mb}, \quad \sigma(\bar{\lambda}p) \simeq 1.5 \text{ mb}, \\ \sigma(pp) &\simeq 4.3 \text{ mb}, \quad \sigma(pn) \simeq 4.4 \text{ mb}, \quad \sigma(\lambda p) \simeq 3 \text{ mb}.\end{aligned}\quad (27)$$

At this stage we have to make a remark about the signs of $\tilde{S}^-(qq')$. It is reasonable to assume that the considerations of Section I-2 can also be applied to quark scattering. Remembering that by definition the $\tilde{S}^-(qq')$ do not involve quark-antiquark annihilation contributions, so that they can be only non-zero because there is a difference in the number of ways (integral) charge and/or hypercharge can be exchanged in $q + q'$ and $\bar{q} + q'$ collisions, one predicts, using (20),

$$\tilde{S}^-(pp) > 0, \quad \tilde{S}^-(np) < 0, \quad \tilde{S}^-(\lambda p) < 0. \quad (28)$$

For instance, the first inequality arises because charge can be exchanged in more ways between p and $\bar{p}$ than between p and p . We see that these predictions are satisfied by (26); it can be verified, assuming (22), that this is true for the whole momentum range $6 \leq p_L \leq 18 \text{ GeV/c}$. This is consistent with the fact that quark-antiquark annihilation contributions (Fig. 2a) are not included on the right of (23), in the sense that, if one would have found from (23) $\tilde{S}^-(\lambda p)$ or $\tilde{S}^-(np)$ positive this would have been an indication that in (23) the quantities $S^+(qq')$ which include $q\bar{q}$ annihilation, rather than $\tilde{S}^+(qq')$, should have occurred. Of course, the results given by (26) do not by themselves necessarily require the absence of $q\bar{q}'$ annihilation on the right-hand side of (23), but indicate that annihilation could certainly not dominate, at least for np and λp ; one should always have $\sigma_A(\bar{n}p) < |\tilde{S}^-(np)|$ and $\sigma_A(\bar{\lambda}p) < |\tilde{S}^-(\lambda p)|$.

From (23) we can obtain further relations between measured cross-sections if we make some extra assumptions concerning the $\tilde{S}^{\pm}(qq')$. In the first place, assuming in (23) the condition

$$\tilde{S}^+(pp) + \tilde{S}^+(pn) = 4 \tilde{S}^+(\lambda p) \quad , \quad (29)$$

we obtain 8)

$$3 S^+(\pi^+P) = 2 [S^+(K^+P) + S^+(K^+N)] \quad . \quad (30)$$

This relation is remarkably successful in the whole range $6 \lesssim p_L \lesssim 18$ GeV/c for instance, at 10 GeV/c we have left and right 154 mb and 156 mb respectively. We thus have the empirical fact that the quark cross-sections satisfy the condition (29); its dynamical meaning, however, is unclear. It is interesting to write relations (25a) and (30) in slightly different form by introducing the average cross-sections $\tilde{S}_P, S_\pi, S_K, \tilde{S}_p, \tilde{S}_\lambda$, defined as

$$\begin{aligned} \tilde{S}_P &= \frac{1}{4} [\tilde{S}^+(PP) + \tilde{S}^+(PN)] \\ S_\pi &= \frac{1}{2} S^+(\pi^+P) \\ S_K &= \frac{1}{4} [S^+(K^+P) + S^+(K^+N)] \\ \tilde{S}_p &= \frac{1}{4} [\tilde{S}^+(pp) + \tilde{S}^+(pn)] \\ \tilde{S}_\lambda &= \frac{1}{2} \tilde{S}^+(\lambda p) \quad . \end{aligned} \quad (31)$$

We can then write (25a), (29) and (30) as

$$\tilde{S}_P : S_\pi : S_K = 6 : 4 : 3 \quad , \quad (32)$$

$$\tilde{S}_p : \tilde{S}_\lambda = 2 : 1 \quad , \quad (33)$$

which are excellently verified from ~ 6 GeV/c up to the highest present accelerator energies. Extrapolating to $s \rightarrow \infty$ and using Pomernanchuk's theorem, (32) becomes

$$\sigma_T^\infty(PP) : \sigma_T^\infty(\pi P) : \sigma_T^\infty(KP) = 6 : 4 : 3. \quad (34)$$

Next, $SU(3)$ invariance, i.e.,

$$\tilde{S}^\pm(\lambda p) = \tilde{S}^\pm(np), \quad (35)$$

gives two further relations, namely

$$\frac{1}{2} S^-(K^+P) = S^-(\pi^+P) \quad (36)$$

$$\tilde{S}^+(PP) = 2 S^+(\pi^+P) - \frac{1}{2} S^+(K^+P). \quad (37)$$

Comparing (35) with (26) we see that $SU(3)$ is badly broken for the quark cross-sections, reflecting the well-known fact that the Johnson-Treiman relation (36) is not as good as (25c) ^{*)}. Relation (37) agrees very well with the facts, the reason being that it is much less sensitive to symmetry breaking than (36). Again the agreement would have been much worse if annihilation contributions were included on the left-hand side.

^{*)} Strictly speaking $SU(3)$ is not needed to obtain (36); what is required is the condition $\tilde{S}^-(\lambda p) = \tilde{S}^-(np)$ which is not necessarily a reflection of $SU(3)$ symmetry ⁷⁾.

Finally, we consider briefly reactions involving deuterons. Treating the deuteron (denoted by D) as a single isoscalar object and using additivity and isospin invariance one obtains the relation, analogous to (25a)

$$\tilde{S}^+(PD) = \frac{3}{2} S^+(\pi^+D), \quad (37a)$$

where the tilde on the left-hand side indicates that contributions due to annihilation of P with one of the nucleons composing the deuteron, should be subtracted from $\sigma_T(PD)$. Because of lack of data for these contributions we cannot test (37a) directly. However, using the Glauber formula

$$\sigma_T(AD) = \sigma_T(AP) + \sigma_T(AN) - \frac{\langle r^{-2} \rangle}{4\pi} \sigma_T(AP) \sigma_T(AN),$$

where r measures the separation of nucleons in the deuteron, and noting that for the non-annihilation parts the Glauber correction terms are to first approximation the same for $A = P$ and $A = \bar{P}$, we have

$$\tilde{S}^-(PD) \approx \tilde{S}^-(PP) + \tilde{S}^-(PN) \approx -S^-(\pi^+P),$$

where we have used (22) and (23). Therefore $\tilde{S}^+(PD) \approx 2\sigma_T(PD) - \tilde{S}^-(\pi^+P)$. With this result we find, using the data of Ref. ¹⁷⁾, at $p_L^\pi = 10, 12, 14$ GeV/c and $p_L^P = \frac{3}{2}p_L^\pi$, $\tilde{S}^+(PD)/S^+(\pi^+D) = 1.48, 1.50, 1.51$, respectively, to be compared with the value 1.50 predicted by (37a).

We may conclude that the quark model, with additivity assumption for the non-annihilation parts of the elastic amplitudes, is in excellent agreement with the data as far as high-energy total cross sections are concerned. The model has also been successfully applied

to a number of inelastic two-body reactions ²⁷⁾⁻²⁹⁾, although in most of these cases complicating features like spins and form factors, together with inaccuracy of the data, make it difficult to test additivity proper without recourse to further assumptions ^{*)}. Why this simple model works so well is a mystery ; a priori dynamical justifications for it are lacking. However, as will be seen below, by considering non-forward directions one may obtain in certain extreme cases a posteriori justifications for the additivity hypothesis.

We shall discuss briefly the implications of (21) for non-forward directions ³⁾⁻⁵⁾ [$t \neq 0$, but limited to small values, $|t| \lesssim 1(\text{GeV}/c)^2$] in the limit $s \rightarrow \infty$. For more details see Ref. 5). We shall assume that in this limit the quark-quark elastic scattering amplitude becomes purely imaginary (diffractive scattering) and has an energy-independent cross-section (see Ch. I). Furthermore we assume that the generalized Pomerenchuk theorem (13) is valid for quarks and that only contributions of type c) (Section I.2) contribute at $s = \infty$. Restricting ourselves to pion-nucleon and nucleon-nucleon scattering, i.e., non-strange quarks, we can then write asymptotically [see remark following (21)]

$$T'_{AB}(s, t) \xrightarrow{s \rightarrow \infty} i g(t) \left[\sum_i f_i^A(t) \right] \left[\sum_j f_j^B(t) \right], \quad (38)$$

*) For inelastic reactions of the type $P\bar{P} \rightarrow B'\bar{B}$, where B and B' are baryons or baryon resonances, there is a considerable (in some cases perhaps even dominating) contribution from annihilation channels in the corresponding sum in (16), which should be excluded from additivity. This contribution is, in contrast to the case of $B'\bar{B} \rightarrow B'\bar{B}$ where it is $\sigma_A(B'\bar{B})$, not a measurable quantity, and the same is true for the non-annihilation part. This implies that the additivity model does not make much sense for these particular inelastic reactions, neither for the corresponding crossed reactions (see p. 11)

with

$$T'_{ij}(s, t) \xrightarrow{s \rightarrow \infty} i g_{ij}(t) \quad (39)$$

(g_{ij} real)

$$g_{\bar{p}p} = g_{\bar{n}n} = g_{pp} = g_{nn} = g_{pn} = g_{np} = g_{\bar{p}n} = g_{p\bar{n}} = g. \quad (40)$$

In (40) we have used isospin and charge conjugation invariance. Equation (38) implies immediately the factorization theorem

$$T'_{pp}(s, t) T'_{\pi\pi}(s, t) = [T'_{\pi p}(s, t)]^2, \quad (41)$$

derived previously in the Regge pole model under the assumption of one-pole dominance³⁰⁾. If SU(3) were exact, (40) could be extended to include also the strange quarks and we would have a factorization property for all hadrons.

From (38) we can obtain in two extreme cases an understanding of why additivity can be a good approximation³⁾⁻⁵⁾. Case I is obtained when the entire t dependence of $\lim_{s \rightarrow \infty} T'_{AB}(s, t)$ in the region of the diffraction peak comes from the quark-quark amplitude $g(t)$, i.e., when in that region the form factors f are approximately constant and equal to one. In that case (38) becomes

$$T'_{AB}(s, t) \xrightarrow{s \rightarrow \infty} i g(t) n_A n_B, \quad (42)$$

where n_A and n_B are the number of quarks composing A and B . This equation implies that all non-strange hadrons have asymptotic diffraction curves of the same shape, in accordance with the data (see Ch. I). Moreover, this common value of a [see (14)] for the hadrons is identical to that for quark-quark scattering. Since the size of a particle is

related to the slope of its diffraction curve, this means that in this extreme case quarks have roughly the same size as hadrons. At the same time, since we have

$$\sigma_T^\infty(qq) = \frac{1}{9} \sigma_T^\infty(PP) , \quad (43)$$

the quarks have very weak absorption, i.e., are very transparent. It is this high transparency which in the present picture may give an a posteriori justification for the additivity assumption.

Case II is the opposite of Case I. Now the t dependence of $\lim_{s \rightarrow \infty} T_{AB}^i(s, t)$ in the diffraction peak is assumed to be entirely determined by the form factors in (38), i.e., $g(t)$ is constant in t . In this case the quarks have spatial dimensions which are small compared to those of the corresponding hadrons. This small size of the quarks makes multiple scattering effects rare, so that also in this extreme picture we can understand why additivity works. A remarkable relation ^{4),5)} is obtained by noting that the form factors $f_i^{A,B}(t)$ should, in this limiting case of small quarks composing large hadrons, be proportional to the electromagnetic form factors $G_E^{A,B}(t)$ for hadrons A and B. Using this in (38) with $g(t) \equiv g(0)$ for small t we obtain, for the case of proton-proton scattering,

$$\frac{d\sigma_{ee}(PP)}{dt} = \left(\frac{d\sigma_{el}(PP)}{dt} \right)_{t=0} \left[G_E^P(t) \right]^4 , \quad (44)$$

which as shown by Fig. 4 is in excellent agreement with experiment for the range $|t| \lesssim 1(\text{GeV}/c)^2$ *). For πP scattering the corresponding relation cannot be tested because data on $G_E^\pi(t)$ are not available at present. On the other hand, from the experimental fact that the πP

*) Using completely different arguments, Wu and Yang ³¹⁾ derived (44) for the case of large t , where agreement with experiment seems to be less good.

and PP diffraction curves seem to have similar shapes at very high energy we may conclude from relations of type (44) that the same is true for G_E^π and G_E^P . It should be mentioned that relation (44) can also be made plausible in Case I^{4),5)}.

2 - Quarks and high-energy annihilation

In the foregoing we have excluded the annihilation part $\sigma_A(\bar{B}'B)$ of antibaryon-baryon cross-sections from the additivity assumption. The question arises whether for annihilation at high energy one can assume some other simple mechanism through which it is possible to express $\bar{B}'B$ annihilation in terms of $\bar{q}q$ annihilation, and in so doing to relate the various antibaryon-baryon annihilation cross-sections to each other. The simplest assumption one can propose is the factorization assumption⁸⁾, in which one writes the $\bar{B}'B$ annihilation amplitude as a product of three $\bar{q}'q$ annihilation amplitudes, momentum and energy being approximately conserved in each of the latter. It amounts to approximating Fig. 1 by Fig. 3. The assumption is compatible with the experimental fact that for $\bar{P}P$ annihilation in flight the dominant contributions to $\sigma_A(\bar{P}P)$ come from states with six or more mesons (resonances counted as single mesons) in the final state²⁵⁾.

The factorization assumption for $\bar{B}'B$ annihilation can be expressed in mathematical form as follows

$$\sigma_A(\bar{B}'B) = \sum_{\{j_1, j_2, j_3\}} \prod_{i=1,2,3} Z(j_i, i), \quad (45)$$

where j_1, j_2, j_3 run over the permutations of 1, 2, 3 and $Z(j_i, i)$ describes the contribution of all annihilations of antiquark j_i in $\bar{B}'$ with quark i in B (see Fig. 3). Each Z , which has dimension of $(\text{mb})^{1/3}$, is the product of an antiquark-quark annihilation cross-section and a factor of dimension $(\text{length})^{-4/3}$ determined by the wave function of the baryon as three-quark bound state. This factor is assumed to be the same for all baryons. Its explicit form is not needed for our purpose.

Using isospin and charge conjugation invariance and writing

$$Z_1 = Z(\bar{p}p), \quad Z_2 = Z(\bar{p}n), \quad Z_3 = Z(\bar{\lambda}p), \quad (46)$$

one has the following expressions for the annihilation cross-sections for reactions with protons as targets

$$\begin{aligned} \sigma_A(\bar{p}p) &= 2 Z_1^3 + 4 Z_1 Z_2^2, \\ \sigma_A(\bar{n}p) &= 2 Z_2^3 + 4 Z_1^2 Z_2, \\ \sigma_A(\bar{\Lambda}p) &= \sigma_A(\bar{\Sigma}^0 p) = 2 Z_1 Z_2 Z_3 + 2 Z_1^2 Z_3 + 2 Z_2^2 Z_3, \\ \sigma_A(\bar{\Sigma}^+ p) &= 2 Z_1^2 Z_3 + 4 Z_1 Z_2 Z_3, \\ \sigma_A(\bar{\Sigma}^- p) &= 2 Z_2^2 Z_3 + 4 Z_1 Z_2 Z_3, \\ \sigma_A(\bar{\Xi}^- p) &= 2 Z_1 Z_3^2 + 4 Z_2 Z_3^2, \\ \sigma_A(\bar{\Xi}^0 p) &= 2 Z_2 Z_3^2 + 4 Z_1 Z_3^2, \\ \sigma_A(\bar{\Omega}^- p) &= 6 Z_3^3. \end{aligned}$$

(47)

At present, experimental information at high-energy exists only for the first cross-section of the list. This means that we cannot yet compare the factorization assumption with experiment.

It will be clear that neither the additivity assumption as used here nor the factorization assumption by themselves can produce relations for hadron cross-sections that connect annihilation and non-annihilation effects. Relations of that type can only be obtained if one makes such a connection "ad hoc" on the level of the quarks. Examples of such relations are the Freund relations ^{6), 7), 32)}

$$\begin{aligned} S^-(PP) &= 5 S^-(\pi^+P) \\ S^-(PN) &= 4 S^-(\pi^+P) \end{aligned} \quad , \quad (48)$$

and the Levinson-Wall-Lipkin relations ³³⁾

$$\begin{aligned} S^-(PP) &= 2 S^-(K^+P) + S^-(K^+N) \\ S^-(PN) &= S^-(K^+P) + 2 S^-(K^+N) \end{aligned} \quad , \quad (49)$$

which essentially express nucleon-antinucleon annihilation in terms of meson-nucleon scattering, $S^-(PP)$ and $S^-(PN)$ being dominated by annihilation contributions. It is easy to see, using (22), (23) and (47), that (48) and (49) imply the following relations between quark annihilation and non-annihilation cross-sections :

from (48)

$$Z_1^3 = Z_2^3 = -\frac{3}{2} \tilde{S}^-(np) \quad , \quad (50)$$

and from (49)

$$Z_1^3 = Z_2^3 = -\frac{3}{2} \tilde{S}^-(\lambda p) \quad . \quad (51)$$

Remember that both right-hand sides are positive [see (28)]. The dynamical meaning of conditions (50) and (51) is unclear ; they should be regarded as expressing within the model the empirical facts that (48)

is rather well and (49) very well satisfied by the data ^{*)}. As such their status is analogous to, for instance, (29).

The equality $Z_1 = Z_2$ implied by (50) or (51), used in (47), leads to

$$\frac{\sigma_A(0)}{\sigma_A(1)} = \frac{\sigma_A(1)}{\sigma_A(2)} = \frac{\sigma_A(2)}{\sigma_A(3)} \quad (52)$$

where $\sigma_A(S) = \sigma_A(\bar{B}P)$ for strangeness S of $\bar{B}$, σ_A being independent of the charge and isospin of $\bar{B}$.

*) In the original form of the quark model in which annihilation was supposed to be included in the additivity assumption, (48) and (49) were obtained by putting $S^-(np) = 0$ and $S^-(\lambda p) = 0$ respectively ^{6),7),33)}. By writing them as $\sigma_A(\bar{n}p) = -\tilde{S}^-(np)$ and $\sigma_A(\bar{\lambda}p) = -\tilde{S}^-(\lambda p)$ it is clear that, analogously to (50) and (51), the nature of these conditions is to express for quark scattering annihilation contributions in terms of non-annihilation contributions.

III. - REGGE POLE MODEL AND CURRENT ALGEBRA

In this last part we shall discuss a model, proposed recently by Cabibbo, Horwitz and Ne'eman (CHN) ⁹⁾, which essentially consists of the Regge pole model endowed with an algebraic structure for the residue functions, or more precisely for the vertex strength functions. The original motivation for this model was to obtain an algebraic foundation for the successful high energy results produced by the quark model (Ch. II). Up to now the model has been formulated only for elastic scattering in the forward direction with absence of spins.

The model is based on the following assumptions :

- 1) - The Regge pole model for elastic scattering, i.e., the amplitude $T_{AB}(s,t)$ for reaction (1) at high energy can be written in the form (3) with

$$C_j^{AB}(t) = -\beta_j^{AB}(t) \xi_j^{\pm}(t) \frac{\Gamma(\alpha_j(t) + \frac{3}{2})}{\Gamma(\alpha_j(t) + 1)} \quad (53)$$

In here $\beta_j^{AB}(t)$ is the residue function associated with the Regge pole $\alpha_j(t)$. Using the factorization theorem ³⁰⁾ we can express this function as a product of two vertex strength functions, as follows

$$\beta_j^{AB}(t) = \gamma_j^A(t) \gamma_j^B(t) \quad (54)$$

$\xi_j^{\pm}(t)$ is the signature factor, given by

$$\xi_j^{\pm}(t) = \frac{1 \pm e^{-i\pi\alpha_j(t)}}{\sin \pi\alpha_j(t)} \quad (55)$$

where the plus (minus) sign has to be taken if the physical states on the Regge trajectory $\alpha_j(t)$ have even (odd) angular momentum.

2) - Two unitary nonets of Regge trajectories dominate high-energy scattering, one, $s_j(j=0, \dots, 8)$, of even signature having the 2^+ mesons (A_2, K^{*}, f_0, f'_0) as physical particles and one, $v_j(j=0, \dots, 8)$, of odd signature corresponding to the nonet of 1^- mesons ($\rho, K^*, \omega, \varphi$). The Pomeron trajectory, describing the asymptotic behaviour of the amplitude, is assumed to be included in the even signature nonet.

3) - The functions $\gamma_j^{A,B}$ occurring in (54) are for $t = 0$ given by the matrix elements of operators S_j and V_j belonging to a $U(12)$ algebra,

$$\delta^3(\underline{p}_A - \underline{p}'_A) \gamma_{s_j}^A(0) = \langle A | S_j | A \rangle \quad \left. \vphantom{\delta^3(\underline{p}_A - \underline{p}'_A)} \right\} j=0, \dots, 8 \quad (56)$$

$$\delta^3(\underline{p}_A - \underline{p}'_A) \gamma_{v_j}^A(0) = \langle A | V_j | A \rangle \quad \left. \vphantom{\delta^3(\underline{p}_A - \underline{p}'_A)} \right\} j=0, \dots, 8 \quad (57)$$

with

$$S_j = \int D(\beta \lambda_j, \underline{x}, 0) d^3x, \quad (58)$$

$$V_j = \int D(\lambda_j, \underline{x}, 0) d^3x, \quad (59)$$

the integrals being carried out in the rest frame of the incident particle. The D 's are current densities, which in a quark representation have the form

$$D(M_j, \underline{x}, 0) = \frac{1}{2} q^\dagger M_j q, \quad (60)$$

where q and $q^\dagger$ are the quark fields. The densities associated with S_j and V_j are of scalar and vector type respectively. The V_j 's have the same algebraic properties as the unitary spin generators.

Together with the S_j they form a $U(3) \times U(3)$ algebra isomorphic to the $[U(3) \times U(3)]_\beta$ contained in the $[U(6) \times U(6)]_\beta$ of Dashen and Gell-Mann³⁴⁾. They satisfy

$$\begin{aligned} [S_i, S_j] &= i f_{ijk} V_k, \\ [V_i, S_j] &= i f_{ijk} S_k \end{aligned} \quad (61)$$

with the f_{ijk} (just as the unitary matrices λ_i) defined in Ref. 35). These commutation relations fix the relative scales of γ_{S_j} and γ_{V_j} . For elastic scattering, we need only $j = 0, 3, 8$ in (56) and (57), $j = 0$ referring to the unitary singlet, $j = 3, 8$ to the centre elements of the octet. Hence each elastic amplitude $T_{AB}(s, t)$ is, in general, a sum of six terms, three arising from the even signature nonet, coupled to the quantities S_j , in which $j = 0$ corresponds to the Pomernanchuk trajectory, and three from the odd signature nonet, coupled to the quantities V_j .

On the basis of the above assumptions the following expressions for the total cross-sections are obtained

$$\begin{aligned} S^\pm(PP) &= 12 t_0^\pm + 6 t_8^\pm + 2 t_3^\pm, \\ S^\pm(PN) &= 12 t_0^\pm + 6 t_8^\pm - 2 t_3^\pm, \\ S^+(\pi^+P) &= 8 t_0^+ + 4 t_8^+, \\ S^-(\pi^+P) &= 4 t_3^-, \\ S^+(K^+P) &= 8 t_0^+ - 2 t_8^+ + 2 t_3^+, \\ S^-(K^+P) &= 6 t_8^- + 2 t_3^-, \\ S^+(K^+N) &= 8 t_0^+ - 2 t_8^+ - 2 t_3^+, \\ S^-(K^+N) &= 6 t_8^- - 2 t_3^-. \end{aligned} \quad (62)$$

In here the quantities $t_j^\pm$ are given by

$$t_j^\pm = \frac{\Gamma(\alpha_j^\pm + \frac{3}{2})}{\Gamma(\alpha_j^\pm + 1)} \left(\frac{\nu}{\nu_j^\pm} \right)^{\alpha_j^\pm(0)}, \quad (63)$$

with $\nu = s_0 = s - m_A^2 - m_B^2$ and where we have explicitly taken out scale factors $\nu_j^\pm$. The latter are unknown parameters. The superscript $\pm$ refers to the signature. In calculating the matrix elements in (56) and (57) the $[U(6) \times U(6)]_F$ assignments of $(56,1)^+$ and $(6,6^*)^-$ for baryons and mesons are used, which in the quark model correspond to the usual quark composition of the hadrons. The normalization $\text{tr } \lambda_1^2 = 2$ is used. The signs in (62) are determined by the signature factors (55) and by the requirement that at the physical poles the exchange of 2^+ and 1^- particles gives rise to forces between equal particles which are respectively attractive and repulsive. The non-trivial assumption is then made that the extrapolation to $t = 0$ is not accompanied by changes in sign. If such a change in sign occurs between $t = 0$ and the physical pole the corresponding $t_j^\pm$ in (62) has to be replaced by $-t_j^\pm$ (see remark 4 below).

No F/D and octet-singlet mixing is as yet taken into account (see below); (62) corresponds to pure F coupling for the vector trajectories, while the coupling of the scalar trajectories to baryons and mesons is pure F and D respectively.

From (62) the following four relations between cross-sections are obtained after eliminating $t_j^\pm$:

$$S^+(PP) + S^+(PN) = 3 S^+(\pi^+P) \quad , \quad (64a)$$

$$S^+(PP) - S^+(PN) = S^+(K^+P) - S^+(K^+N) \quad , \quad (64b)$$

$$S^-(K^+P) - S^-(K^+N) = S^-(\pi^+P) \quad , \quad (64c)$$

$$S^-(PP) - S^-(PN) = S^-(\pi^+P) \quad . \quad (64d)$$

These relations have to be compared with the quark model results (25). They only differ in the fact that, in contrast to (25), they include annihilation contributions. As mentioned in Ch. II, this affects mainly (64a), which around 10 GeV/c has a discrepancy with experiment of the order of $\sim 20\%$. Of course, asymptotically (25a) and (64a) become identical and predict $\sigma_T(\bar{P}P)/\sigma_T(\pi P) = \frac{5}{2}$. As to relations (64b and d), due to the large experimental uncertainties in $\sigma_T(\bar{P}N)$ similar remarks apply to them as made with respect to (25b and d). We come back to relations (64) at the end of this Chapter.

With the signs in (62) fixed by the above-mentioned assumption a number of inequalities follows; examples are

$$\begin{aligned}
 S^-(AP) &> 0, \quad \text{for } A = K^+, \pi^+, P, \\
 S^+(AP) &> S^+(AN), \quad \text{for } A = K^+, P, \\
 S^-(PP) &> S^-(PN), \\
 \sigma_T(\bar{P}P) &> \sigma_T(PN), \quad \sigma_T(\bar{P}P) > \sigma_T(\bar{P}N), \quad (65)
 \end{aligned}$$

which are all in accordance with the data above 6 GeV/c.

Other relations are obtained from (62) by assuming equalities among the t_j^+ . For instance, U(3) symmetry for the vector contributions, i.e., $t_3^- = t_8^- = t_0^-$, is equivalent to the Freund relations (48); the equality $t_0^- = t_8^-$ implies the Levinson-Wall-Lipkin relations (49). Exchange degeneracy³⁶⁾ for the $I = 1$ exchanges, i.e., $t_3^- = t_3^+$, gives

$$\sigma_T(K^+P) = \sigma_T(K^+N),$$

which seems to agree very well with the facts ^{17) *}). SU(3) for the vector and scalar trajectories gives respectively the Johnson-Treiman relation (36), and

$$S^+(PP) = 2 S^+(\pi^+P) - \frac{1}{2} S^+(K^+P). \quad (66)$$

The latter relation is the counterpart of (37) ; it is, however, less well satisfied by the data.

The model also predicts another interesting property ¹⁰⁾. A good fit to the entire data on total cross-sections can only be obtained within the model if one assumes all σ_T to decrease at very high energy as

$$\sigma_T \sim \nu^{-(0.075 \pm 0.008)} \quad (67)$$

corresponding to $\alpha_0^+(0) = 0.925 \pm 0.008$, where $\alpha_0^+(0)$ is the intercept of the Pomanchuk trajectory. This can best be seen by considering the average cross-sections $S_P = \frac{1}{4} [S^+(PP) + S^+(PN)]$ and S_π and S_K , defined in (31), which in terms of t_j^+ become

$$S_P = 6 t_0^+ + 3 t_8^+ \quad (68a)$$

$$S_\pi = 4 t_0^+ + 2 t_8^+ \quad (68b)$$

$$S_K = 4 t_0^+ - t_8^+ \quad (68c)$$

with

$$t_j^+(\nu) = t_j^+(1) \nu^{\alpha_j^+(0) - 1} \quad (69)$$

*) In the quark model this equality is implied by the equality of $\sigma(pp)$ and $\sigma(np)$ [see (27) and (74) below].

If one assumes $\alpha_0^+(0) = 1$, i.e., $t_0^+ = \text{const.} \neq 0$, Eqs. (68) predict that S_P and S_π approach their asymptotic limits $6t_0^+$ and $4t_0^+$ from above, but S_K approaches its limit $4t_0^+$ from below. This runs counter to the experimental fact that for $p_L \gtrsim 5 \text{ GeV/c}$ all hadron total cross-sections are known to decrease with increasing energy. Hence with $\alpha_0^+(0) = 1$ Eqs. (68) cannot fit the data. In fact, since from (68b and c) one has

$$\frac{1}{12} (S_\pi + 2 S_K) = t_0^+ \quad , \quad (70)$$

a straight line fit to the experimental values for the left-hand side on a logarithmic plot (Fig. 5) gives $\alpha_0^+(0) \simeq 0.93$. To fit the entire data, including the nucleon-nucleon data, it is necessary to allow for F/D admixture at the scalar trajectory nucleon vertex together with some octet-singlet mixing. The reason is that Eqs. (68) predict $S_P/S_\pi = \frac{3}{2}$, i.e., relation (64a), which, as mentioned earlier, is in disagreement with experiment (remember that S_P includes annihilation). The modified equations become

$$\begin{aligned} S_P &= (\alpha\sqrt{6} + \beta\lambda\sqrt{3})^2 t_0^+ + (\beta\sqrt{6} - \alpha\lambda\sqrt{3})^2 t_8^+ , \\ S_\pi &= 2(\alpha\sqrt{2} + \beta)(\alpha\sqrt{2} + \beta\lambda) t_0^+ + 2(\beta\sqrt{2} - \alpha)(\beta\sqrt{2} - \alpha\lambda) t_8^+ , \\ S_K &= (2\alpha\sqrt{2} - \beta)(\alpha\sqrt{2} + \beta\lambda) t_0^+ + 2(\beta\sqrt{2} + \alpha)(\beta\sqrt{2} - \alpha\lambda) t_8^+ , \end{aligned} \quad (71)$$

with $\lambda = F - \frac{1}{3} D$, $\alpha = \cos \varphi$, $\beta = \sin \varphi$, φ being the (t_0^+, t_8^+) mixing angle. An excellent fit to the data of Ref. ¹⁷⁾ is obtained with the following parameters

$$\begin{aligned} t_0^+(1) &= 7.43 \pm 0.56 , \quad t_8^+(1) = 2.08 \pm 0.26 \\ \alpha_0^+(0) &= 0.925 \pm 0.008 , \quad \alpha_8^+(0) = 0.76 \pm 0.04 \\ \tan \varphi &= -0.066 \pm 0.048 , \quad \lambda = 2.44 \pm 0.09 . \end{aligned} \quad (72)$$

This result remains essentially unchanged if we put $\varphi = 0$. In that case we have from (71) in the limit $\nu \rightarrow \infty$, $S_A \rightarrow 0$ as $\nu^{-0.075}$ ($A = P, \pi, K$) and

$$\frac{S_P}{S_\pi} \rightarrow \frac{3}{2}, \quad \frac{S_K}{S_\pi} \rightarrow 1. \quad (73)$$

With the small mixing angle given by (72) the latter values change by a few per-cent. Compare (73) with (34) and remember that the latter result is obtained from an extrapolation of the empirical relation (30), i.e., the quark model does not necessarily contradict (73) (see also remark 3 below). It should be remarked that also with Eqs. (71) no acceptable fit to the data can be obtained with $\alpha_0^+(0) = 1$.

We end with a few remarks.

1) - It is possible to escape the vanishing of total cross-sections by adding to the two nonets of trajectories introduced in assumption 2 above a second even signature singlet trajectory, which could then play the role of the Pomeron trajectory and have intercept going through one. Good fits to the data, based on such a picture, are known to exist ^{37), 38)}. However, such a possibility would mean an extension of the algebraic structure defined in assumption 3 and would therefore be outside the scope of the present model ^{*)}. Without altering the basic algebraic structure of the model one is led to (67). Needless to say,

*) An additional singlet, being outside the algebra of $[U(6) \times U(6)]_\beta$, would add to the right of (62) extra terms $\Gamma_N, \Gamma_\pi, \Gamma_K$, for $S^+(PN), S^+(\pi^+P), S^+(K^+N)$, respectively (N = nucleon), which would be arbitrary parameters. The model would therefore lose much of its predictive power; for instance the asymptotic prediction $\sigma(PN)/\sigma(\pi P) = \frac{3}{2}$ would not be obtained.

the above considerations do not prove the vanishing of cross-sections, no more than the considerations in Ref. 37), 38) prove their constancy, but show that such a possibility, fully compatible with the existing experimental situation including cosmic ray data, exists, and that it can be linked with an economical and appealing scheme.

2) - At very high energy the t_0^+ contributions will dominate each cross-section, so that in a $(\log \nu, \log \nu)$ plot the latter approach a straight line asymptotically. Certain cross-sections, such as $\sigma_T(K^+N)$ and $\sigma_T(K^+P)$, will approach this asymptote from below. Because of this, they may appear to be nearly constant throughout a large energy region, as is actually the case above 6 GeV/c for the just-mentioned cross-sections. Finally, from the fact that all elastic amplitudes behave asymptotically as

$$T(\nu, t) \underset{\nu \rightarrow \infty}{\sim} \xi^+(t) \nu^{\alpha_0^+(t)},$$

the value for $\alpha_0^+(0)$ given by (72) gives a non-vanishing ratio of real to imaginary part $X(\nu, 0)$ at $t = 0$. One finds for all hadrons $X(\nu, 0) \xrightarrow{\nu \rightarrow \infty} -0.12$. This value is compatible with existing experimental results 10, 18).

3) - It is easy to verify that by forming the following combinations of quark cross-sections

$$t_0^\pm = \frac{1}{4} [\tilde{S}^\pm(pp) + \tilde{S}^\pm(np) + \tilde{S}^\pm(\lambda p)] ,$$

$$t_8^\pm = \frac{1}{4} [\tilde{S}^\pm(pp) + \tilde{S}^\pm(np) - 2 \tilde{S}^\pm(\lambda p)] ,$$

$$t_3^\pm = \frac{1}{4} [\tilde{S}^\pm(pp) - \tilde{S}^\pm(np)] ,$$

(74)

and denoting them by t_j^+ as indicated above, one can formally put (23) into the form (62) and vice versa (see also Ref. ³⁹), replacing in the latter set of equations $S^\pm(NN')$ by $\tilde{S}^\pm(NN')$. In the CHN model, each t_j^+ is associated with a single Regge pole in the t channel of (1). In the quark model each combination in (74) may be associated with more than one Regge pole, or more general, may have more than one term in an expansion in powers of ν . Using (74) in (23) we can write in the quark model, analogous to (68),

$$\begin{aligned}\tilde{S}_P &= 6t_0^+ + 3t_8^+, \\ S_\pi &= 4t_0^+ + 2t_8^+, \\ S_K &= 4t_0^+ - t_8^+.\end{aligned}\tag{75}$$

In general the simplest way to parametrize t_0^+ and t_8^+ is as follows : $t_0^+ = x_1 + x_2 \nu^{x_3}$ and $t_8^+ = x_4 + x_5 \nu^{x_6}$. The case of vanishing cross-sections discussed above and in Ref. ¹⁰) corresponds to putting $x_1 = x_4 = 0$ and fitting the remaining parameters. The situation in which $S_\pi \neq S_K \neq 0$ for $s \rightarrow \infty$ (as considered in Refs. ^{37),38}) requires all $x_i (i=1, \dots, 6)$ to be non-zero.

4) - The last remark is of speculative nature. We have seen in the foregoing Chapter that in the quark model different mechanisms control the annihilation and non-annihilation parts of the total cross-sections. This rests on the fact that annihilation effects correspond to baryon exchange, while all non-annihilation processes are of different type, corresponding to exchange of vanishing baryon number. One might also

say that the latter processes are peripheral and the annihilation processes non-peripheral or central. This difference in nature of the two types of processes may require different descriptions for them, not only in the quark model, but also in other dynamical models⁸⁾. This may be true in particular for the Regge pole model⁴⁰⁾. In fact, from the observation that additivity of quark amplitudes corresponds to t channel exchanges of singlet and octet type, one might speculate about the possibility that the Regge pole model in which only singlet and octet trajectories are exchanged, like the CHN model, describes only the non-annihilation part of the total cross-sections. If this would be true, the relations obtained in the quark model and the CHN model would become identical [in (64) and (66) we have to replace $S^\pm(NN')$ by $\tilde{S}^\pm(NN')$, N = nucleon (see (24))]. Since these relations are in very good agreement with experiment we would no longer need the D/F admixture in the scalar trajectory-nucleon vertex [see (72)] ; indeed, this parameter was introduced precisely to account for the disagreement of (64a) and (66) with the facts. The conclusion (67) would obviously remain unshaken. The cross-section differences would present a difficulty. It is clear from (62) that in order to satisfy (22) one would need

$$6t_0^- + 3t_8^- + t_3^- \simeq 0. \quad (76)$$

Since from the $K^\pm N$ and $\pi^\pm p$ data one has $t_3^- > 0$ and $t_8^- > 0$, (76) would imply $t_0^- < 0$, i.e., the residue function of the ν_0 trajectory would have to change sign between $t = 0$ and the t value corresponding to the physical pole. This is perhaps not nice, but there is no compelling reason against such a sign change. In fact, there are for instance good reasons to believe that the residue function of the ρ trajectory changes sign at $t \simeq -0.2(\text{GeV}/c)^2$ ⁴¹⁾. It is easy to verify that t_0^- , t_8^- mixing would not alter this conclusion.

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FIGURE CAPTIONS

- Figure_1 Diagram representing antibaryon-baryon annihilation contributions to the sum in (16). The solid lines represent quarks and antiquarks, the wavy lines mesons.
- Figure_2 Diagrams describing, for antibaryon-baryon scattering, the sum in (16) under the additivity assumption for quark amplitudes, with (diagram a) and without (diagram b) quark-antiquark annihilation. Similar diagrams can be drawn for baryon-baryon and meson-baryon scattering.
- Figure_3 Diagram showing the factorization of antibaryon-baryon annihilation into three antiquark-quark annihilations.
- Figure_4 Demonstration of relation (44). The full lines represent $[\bar{d}\sigma_{e\ell}(PP)/dt]/[\bar{d}\sigma_{e\ell}(PP)/dt]_{t=0}$ at 12.8 GeV/c and the asymptotic curve obtained by extrapolating PP and $\bar{P}P$ diffraction curves to infinite energy. The dots give the 4th power of the nucleon electromagnetic form factor.
- Figure_5 Fit of the form $t_0^+(1) \gamma^{\alpha_0^+(0)-1}$ to the left-hand side of (70). The experimental points are taken from Ref. 17).

FIG.1

FIG.2

FIG.3

FIG.4

FIG.5