CM-P00057573

Atchire
Ref. TH. 795

RADIATIVE CORRECTIONS TO LEPTONIC DECAYS
AND COMPOSITE MODELS FOR STRONG INTERACTIONS

N. Cabibbo
CERN - Geneva
and
Istituto di Fisica dell'Università, Roma

L. Maiani
Istituto Superiore di Sanità, Roma
and
Istituto di Fisica dell'Università, Firenze

G. Preparata
Istituto di Fisica dell'Università, Firenze
and
Istituto di Fisica dell'Università, Roma

ABSTRACT

On the basis of the requirement of finite radiative corrections for leptonic decays, we propose the abandonment of the quark model for strong interactions, in favour of models based on integrally charged triplets. We discuss a model of this kind - the SUB model - based on a proposal of Han and Nambu in which most of the results of the non-relativistic quark model hold.

20 June 1967
TH. 795

Radiative Corrections to Leptonic Decays and Composite Models for Strong Interactions

N. CABIBBO

CERN - Geneva, Switzerland and Istituto di Fisica dell'Università, Roma

L. MAIANI

Istituto Superiore di Sanità, Roma and Istituto di Fisica dell'Università, Firenze

G. PREPARATA

Istituto di Fisica dell'Università, Firenze and Istituto di Fisica dell'Università, Roma

We have shown in a previous paper⁽¹⁾ that the order- α radiative corrections to the $\pi^- \beta$ -decay can be finite, provided that a certain sum rule holds. This sum rule insures that the universal divergence of the corrections to the vector part of the β -decay established by Bjorken⁽²⁾ and Abers et al.⁽³⁾ is cancelled by an axial vector term⁽⁴⁾. This term depends on a tensor $T_{\lambda\mu}^A$:

$$T_{\lambda\mu}^A = \int d^4x e^{iKx} \langle \pi^0 | T (j_{\lambda}^{e.m.}(x) A_{\mu}^+(0)) | \pi^- \rangle + S_{\lambda\mu}^A =$$

$$= \epsilon_{\lambda\mu\nu\rho} K_{\nu} (\pi^- + \pi^0)_{\rho} \Phi(K^2, (K\pi)) \quad (1)$$

and in order for the radiative corrections to be finite, the high K^2 behaviour of Φ has to be⁽⁵⁾

$$\Phi \sim - \frac{\sqrt{2}}{K^2} \quad (2)$$

Following an argument given in ref. (2), for high timelike K_μ ,

$$T_{\lambda\mu}^A \sim \frac{i}{K_0} \int d^3x \langle \pi^0 | \left[j_\lambda^{\text{e.m.}}(\mathbf{x}) A_\mu^+(0) \right]_{x_0=0} | \pi^- \rangle \quad (3)$$

where only the space components of the currents (with $\lambda \neq \mu$) are relevant, because of eq. (1). In the quark model with a V-A weak interaction,

$$\left[j_i^{\text{e.m.}}(\mathbf{x}), A_j^+(0) \right]_{x_0=0} = \frac{1}{3} \delta^{(3)}(\vec{x}) \epsilon_{ijk} V_k^+(\mathbf{x}) \quad (4)$$

so that

$$\Phi \sim + \frac{\sqrt{2}}{3K^2}$$

thus preventing any cancellation of the divergent terms.

This result coincides with what one would expect if the hadrons are considered as bound states of pointlike quarks, with a V-A interaction. In fact eq. (3) simply states that in the high frequency limit the expectation value of a T-product of two currents, being proportional to an equal time commutator, is additive with respect to the elementary components of the hadrons⁽⁶⁾.

This suggests that the convergence of the radiative corrections is automatically insured if one builds a theory of hadrons based on elementary constituents such that the radiative corrections to their

β -decays are finite, when they are treated as point-like particles. This can be achieved only if the following two conditions are satisfied⁽⁷⁾:

- a) all the elementary constituents (which we assume to be spin 1/2 particles) have charge 0 or ± 1 ;
- b) the transition current in β -decay is of the form:

$$\gamma_\mu (1 - \gamma_5) \quad \text{for} \quad (Q = 0) \rightarrow (Q = + 1)$$

and $\gamma_\mu (1 + \gamma_5) \quad \text{for} \quad (Q = - 1) \rightarrow (Q = 0)$

Such a model should also give rise to the well established part of the current algebra, i. e. the local chiral $SU(3) \otimes SU(3)$ ⁽⁸⁾.

Model such as those proposed by Maki and Hara⁽⁹⁾, and Bacry, Nuyts and Van Hove⁽¹⁰⁾, Han and Nambu⁽¹¹⁾ can be made to satisfy condition b), by a suitable definition of the octet of axial currents. However one could be reluctant to abandon, as we propose, the quarks in view of the many successful predictions of the simple picture according to which hadrons are to be considered as bound states of non-relativistic quarks. In order to dispel this reluctance we will show how the model of ref. (11) can be furtherly specified so as to obey conditions a) and b) and at the same time allow a composite picture of the hadrons completely parallel to the quark model.

The model is based on three fermion triplets which we will call S, U and B, where S and U are Sakata triplets, while B is displaced downward by one unit of charge; see Table I. We will call these particles SUB's.

Table I

	S_p, U_p	S_n, U_n	S_{λ}, U_{λ}	B_p	B_n	B_{λ}
Q	1	0	0	0	-1	-1
T	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0
Y	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$
C	1	1	1	-2	-2	-2

The charge can be expressed in terms of T_3 , Y and the quantum number C (charm)⁽¹¹⁾ according to:

$$Q = T_3 + \frac{1}{2} Y + \frac{1}{3} C \quad (6)$$

Although the algebra which can arise from three triplets is very large, we are only interested in its observable part, i. e. the e. m. and weak currents. These will be given by two octets:

$$j_{\mu}^i = \frac{i}{2} (\bar{S} \lambda_i \gamma_{\mu} S + \bar{U} \lambda_i \gamma_{\mu} U + \bar{B} \lambda_i \gamma_{\mu} B) \quad (7)$$

$$g_{\mu}^i = -\frac{i}{2} (\bar{S} \lambda_i \gamma_{\mu} \gamma_5 S + \bar{U} \lambda_i \gamma_{\mu} \gamma_5 U - \bar{B} \lambda_i \gamma_{\mu} \gamma_5 B) \quad (8)$$

The hadron currents in weak interactions are built from the octet $j_{\mu}^i + g_{\mu}^i$ according to the usual prescription⁽¹²⁾. The e. m. current differs from the usual form by a piece corresponding to the charm quantum number, so that

$$j_{\mu}^{e.m.} = j_{\mu}^3 + \frac{1}{\sqrt{3}} j_{\mu}^8 + \frac{1}{3} j_{\mu}^C \quad (9)$$

where

$$j_{\mu}^C = i (\bar{S} \gamma_{\mu} S + \bar{U} \gamma_{\mu} U - 2\bar{B} \gamma_{\mu} B) \quad (10)$$

Eqs. (7) and (8) give rise to the standard local chiral algebra (apart from Schwinger terms):

$$\begin{aligned} \delta(x_0) [j_0^i(x), j_{\mu}^j(0)] &= \delta(x_0) [g_0^i(x), g_{\mu}^j(0)] = \delta^4(x) i f_{ijk} j_{\mu}^k(0) \\ \delta(x_0) [j_0^i(x), g_{\mu}^j(0)] &= \delta(x_0) [g_0^i(x), j_{\mu}^j(0)] = \delta^4(x) i f_{ijk} g_{\mu}^k(0) \end{aligned} \quad (11)$$

Furthermore since (again apart from Schwinger terms) j_0^C commutes with j_{μ}^i and g_{μ}^i , the commutators of $j_0^{e.m.}$ with these currents are the same as in the quark model. The commutators involving the space components of j_{μ}^i and g_{μ}^i are not closed within these currents, and by iterative commutation these currents give rise to a $SU(6) \otimes SU(6) \otimes SU(6) \otimes SU(6)$ chiral algebra.

However the commutators of space components of the e.m. current with the charged components of the two octets (i.e., $i = 1, 2, 4, 5$) have the simple form:

$$\begin{aligned} \delta(x_0) [j_n^{e.m.}(x), j_m^i(0)] &= -\delta^4(x) i \epsilon_{nme} g_e^i(0) \\ \delta(x_0) [j_n^{e.m.}(x), g_m^i(0)] &= -\delta^4(x) i \epsilon_{nme} j_e^i(0) \end{aligned} \quad (12)$$

(for $n \neq m = 1, 2, 3$; and $i = 1, 2, 4, 5$).

Looking back at eqs. (2) and (3), one can immediately verify that these commutators guarantee the cancelling of the divergences of the radiative corrections to β -decay.

We shall now investigate the possible construction of hadrons as bound states of SUB's; it is only in this context that a justification will be found for the duplication of the S and U triplets.

All known hadrons are required to have zero charm; the baryon octet and decuplet can be built as bound states of S + U + B all in S-wave and completely symmetric in the spin and unitary spin indices. Higher baryons can be built in the usual way by allowing L-excitation and/or non-symmetric configurations in spin and unitary indices.

The quark-model results for the magnetic moments of the baryons are reproduced if one assumes that the magnetic moment of the SUB's is given by

$$\vec{\mu} = \mu_0 \vec{Q} \quad (13)$$

for each SUB, and μ_0 is an overall constant, which is, as in the quark-model, equal to μ_p . Predictions on e.m. mass splittings, including the Coleman-Glashow relation, remain unchanged. The axial coupling constant in the neutron β -decay can be computed by assuming that

$$-\frac{G_A}{G_V} (\vec{\sigma} \cdot \vec{\tau}^+) = - \left(\frac{g_A}{g_V} \right)_{\text{SUB}} \left[- (\vec{\sigma} \cdot \vec{\tau}^+)_S - (\vec{\sigma} \cdot \vec{\tau}^+)_U + (\vec{\sigma} \cdot \vec{\tau}^+)_B \right]; \quad (14)$$

this gives:

$$-\frac{G_A}{G_V} = -\frac{5}{9} \left(-\frac{g_A}{g_V} \right)_{\text{SUB}}$$

implying a sizeable renormalization from the bare value $(-g_A/g_v)_{\text{SUB}} = +1$; a similar renormalization but of smaller size is also required in the quark model.

Meson states can be built from SUB-antiSUB pairs. In order to specify the admixture of S, U and B pairs we require that vector mesons do not couple with the charm current, or more generally that the vector mesons reproduce the structure of the j_μ^i currents. This insures that the vector meson dominance is implemented in the usual way. So, e. g.

$$\begin{aligned}\rho_0^\dagger &= \frac{1}{\sqrt{6}} \left[(\bar{S}_p^\dagger S_p^\dagger - \bar{S}_n^\dagger S_n^\dagger) + (\bar{U}_p^\dagger U_p^\dagger - \bar{U}_n^\dagger U_n^\dagger) + (\bar{B}_p^\dagger B_p^\dagger - \bar{B}_n^\dagger B_n^\dagger) \right] \\ \omega^\dagger &= \frac{1}{\sqrt{6}} \left[(\bar{S}_p^\dagger S_p^\dagger + \bar{S}_n^\dagger S_n^\dagger) + (\bar{U}_p^\dagger U_p^\dagger + \bar{U}_n^\dagger U_n^\dagger) + (\bar{B}_p^\dagger B_p^\dagger + \bar{B}_n^\dagger B_n^\dagger) \right] \\ \phi^\dagger &= \frac{1}{\sqrt{3}} (\bar{S}_\lambda^\dagger S_\lambda^\dagger + \bar{U}_\lambda^\dagger U_\lambda^\dagger + \bar{B}_\lambda^\dagger B_\lambda^\dagger)\end{aligned}$$

etc.

With these assignments the usual $\omega - \phi$ mixing angle is recovered as well as the quark model prediction for the ratios $(\rho^0 \rightarrow e^+ e^-) : (\omega^0 \rightarrow e^+ e^-) : (\phi \rightarrow e^+ e^-)$.

At this point we note that if we had only two triplets S and B, the requirement that ω and ϕ do not couple to the charm-current implies that the vector mesons have the structure $2\bar{S}S + \bar{B}B$, which does not allow to consider them as being universally coupled to the nonet of currents j_μ^i . This we consider as a major advantage⁽¹³⁾ of this model over that of ref. (10).

As for the pseudoscalar mesons, we define their structure in

such a way that the charm-part of the e. m. current does not contribute to $V \rightarrow PS + \gamma$ decays, i. e.

$$\pi^+ = \frac{1}{\sqrt{3}} \left[\bar{S}_n S_p + \bar{U}_n U_p + \bar{B}_n B_p \right]_{\text{singlet state}} \quad (16)$$

etc.

With this choice, we have $\Gamma(\omega \rightarrow \pi\gamma) / \Gamma(\rho \rightarrow \pi\gamma) = 9$, and using eq. (13) and repeating the arguments made in the non-relativistic quark-model⁽¹⁴⁾ we find the same results, e. g. $A(\omega \rightarrow \pi\gamma) = \mu_p$.

The structure of the pions eq. (16), can reproduce that of $\partial^\mu g_\mu^i$ as required by PCAC, if the B's obey Dirac's equation with a reversed sign for the mass, which can be interpreted as the B's having an opposite parity with respect to the S's and U's.

We think that the SUB model is able to reproduce most of the other results obtained in the realistic quark model, perhaps with some simple additional hypotheses.

Summing up we have shown that the requirement of finite radiative corrections to β -decay compels us to abandon the commutation relation (4), predicted by the quark-model, in favour of eq. (12)⁽¹⁵⁾. This requirement has a fundamental role in a rigorous definition of the concept of universality in weak interactions. Eq. (12) requires a composite theory of hadrons to be based on integrally charged particles. Among models of this type the most appealing seems to be one based on the original proposal of Han and Nambu. The SUB-model is able to reproduce in a natural way most of the successes of the non-relativistic quark model, with the added advantage of not requiring parastatistics, or complicated space wave functions for the lowest baryon states.

References and footnotes

- 1) N.Cabibbo, L.Maiani, G.Preparata - Finiteness of Radiative Corrections to π β -decay. To appear in Phys.Letters. We have extended the result of this paper to the radiative corrections to $\pi^- \rightarrow \ell^- \bar{\nu}$ decay. The graphs which describe this decay with its radiative corrections can be deduced from those for the decay $\pi^- \rightarrow \pi^0 \ell^- \bar{\nu}$ by using PCAC on the neutral pion. Assuming minimality, the equation $\partial^\mu A_\mu^3 = c \Phi_{\pi^0}$ is expected to hold even in presence of e.m. interactions. Details will be given elsewhere.
- 2) J.D.Bjorken - Phys.Rev., 148, 1467 (1966)
- 3) E.S.Abers, R.E.Norton, D.A.Dicus - Phys.Rev.Letters, 18, 676 (1967)
- 4) The contribution of an axial vector term has also been considered by Bjorken in an Erratum to ref.(2) and in his talk at the IV Coral Gables Conference on Symmetries at High Energy. February 1967.
- 5) Eq.(2) appeared with a wrong sign in ref.(1), however without any consequence for the conclusions reached in that paper.
- 6) The divergent terms for pointlike quarks are explicitly given in ref.(1), footnote (5). This interpretation of eq.(3) has been discussed previously by Bjorken in ref.(2) and (4).
- 7) See ref.(1), footnote (5).
- 8) A discussion on the validity of the local chiral algebra can be found in M.Gell-Mann - Proc. of the 1966 Int.School "E.Majorana". (ed. by A.Zichichi). Academic Press N.Y.
- 9) Z.Maki - Progr.Theor.Phys. (Kyoto) 31, 331 (1964). Y.Hara -Phys.Rev., 134, B701 (1964)
- 10) H.Bacry, J.Nuyts, L.Van Hove - Phys.Letters, 9, 279 (1964).
- 11) M.Y.Han and Y.Nambu - Phys.Rev., 139, B1006 (1965).
- 12) N.Cabibbo - Phys.Rev.Letters, 10, 531 (1963)
- 13) A second advantage is that with three different triplets we have no troubles with Fermi statistics in our construction of the low-lying baryon states. See also ref.(11).

- 14) C.Becchi, G.Morpurgo - Phys.Letters, 17, 352 (1965). A detailed survey on this argument as well as a complete list of references may be found in R.H.Dalitz, Proc. of the 1965 Les Houches Summ. School, (ed. by C. De Witt and M.Jacob) Gordon and Breach, N.Y.
- 15) M.P.Khanna, S.Okubo - Ann. of Phys., 40, 153 (1966) have also remarked that eq.(4) is model dependent, and proposed some tests for its validity.