Filamentational Instability of Partially Coherent Femtosecond Optical Pulses in Air

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The filamentational instability of spatially broadband femtosecond optical pulses in air is investigated by means of a kinetic wave equation for spatially incoherent photons. An explicit expression for the spatial amplification rate is derived and analyzed. It is found that the spatial spectral broadening of the pulse can lead to stabilization of the filamentation instability. Thus, optical smoothing techniques could optimize current applications of ultra-short laser pulses, such as atmospheric remote sensing.

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Recently, there has been a great deal of interest in investigating the nonlinear propagation of optical pulses in air. In order for the pulse propagation over a long distance, it is necessary to avoid filamentational instabilities that grow in space. Filamentation instabilities of optical pulses occur in nonlinear dispersive media, where the medium index of refraction depends on the pulse intensity. This happens in nonlinear optics (viz. a nonlinear Kerr medium) where a small modulation of the optical pulse amplitudes can grow in space due to the filamentation instability arising from the interplay between the medium nonlinearity and the pulse dispersion/diffraction. The filamentational instability is responsible for the break up of pulses into light pipes. It is, therefore, quite important to look for mechanisms that contribute to the nonlinear stability of optical pulses in nonlinear dispersive media. One possibility would be to use optical pulses that have finite spectral bandwidth, since the latter can significantly reduce the growth rate of the filamentation instability. Physically, this happens because of the distribution of the optical pulse intensity over a broad spectrum, which is unable to drive the filamentation instability with fuller efficiency, contrary to a coherent pulse which has a delta-function spectrum. In this Letter, we present for the first time a theoretical study of the filamentation instability of partially coherent optical pulses in air. We show that the spatial amplification rate of the filamentation instability is significantly reduced by means of spatial spectral broadening of optical pulses. The present results could be of significance in applications using ultra-short pulses for remote sensing of the near Earth atmosphere.

The dynamics of coherent femtosecond optical pulses with a weak group velocity dispersion in air is governed by the modified nonlinear Schrödinger equation

\[ i\partial_z \psi + \nabla_\perp^2 \psi + f(|\psi|^2)\psi + i\nu|\psi|^{2K-2}\psi = 0, \tag{1} \]

where \( \psi(z, \mathbf{r}_\perp) \) is the spatial wave envelope, \( \mathbf{r}_\perp = (x, y) \), and
\[ f(|\psi|^2) = \alpha|\psi|^2 - \varepsilon|\psi|^4 - \gamma|\psi|^{2K}. \]

Here \( \alpha = 0.466, \varepsilon = 7.3 \times 10^{-7} \text{ cm}^2/\text{w}_0^2, \gamma = 8.4 \times 10^{-40} \text{ cm}^{2(K-1)}/\text{w}_0^{2(K-1)}, \) and \( \nu = 1.2 \times 10^{-35} \text{ cm}^{2(K-2)}/\text{w}_0^{2(K-2)} \) for a pulse duration of 250 fs, and \( \text{w}_0 \) (in units of cm) is the beam waist \( ^{10} \) (for a discussion of the approximations leading to Eq. (1), we refer to \( ^{9} \)). We note that Eq. (1) has been used in Ref. \( ^{11} \) to analyze the multi-filamentation of optical beams.

Following Ref. \( ^{12} \), we can derive a wave kinetic equation that governs the nonlinear propagation intense optical pulses which have a spectral broadening in space. Accordingly, we apply the Wigner-Moyal transform method\(^ {13,14,15,16} \). The multi-dimensional Wigner-Moyal transform, including the Klimontovich statistical average, is defined as

\[ \rho(z, \mathbf{r}_\perp, \mathbf{p}) = \frac{1}{(2\pi)^2} \int d^2\xi \ e^{i\mathbf{p}\cdot\xi} \langle \psi^\dagger(z, \mathbf{r}_\perp + \xi/2) \psi(z, \mathbf{r}_\perp - \xi/2) \rangle, \tag{2} \]
where \( \mathbf{p} = (p_x, p_y) \) represents the momenta of the quasiparticles and the angular bracket denotes the ensemble average.\(^{17} \) The pulse intensity \( \langle |\psi|^2 \rangle \equiv I \) satisfies
\[
I = \int d^2p \rho(z, \mathbf{r}_\perp, \mathbf{p}).
\] (3)

Applying the transformation (2) on Eq. (2), we obtain the Wigner-Moyal kinetic equation\(^{14,15,16,18} \) for the evolution of the Wigner distribution function,
\[
\partial_z \rho + 2 \mathbf{p} \cdot \nabla_\perp \rho + 2f(I) \sin \left( \frac{1}{2} \nabla_\perp \cdot \nabla_\perp \right) \rho + 2viK^{-1} \cos \left( \frac{1}{2} \nabla_\perp \cdot \nabla_\perp \right) \rho = 0.
\] (4)

Seeking the solution \( \hat{\rho} = \rho(z, \mathbf{p}) \) to Eq. (4), we may write \( \hat{\rho}(z, \mathbf{p}) = \rho_0(\mathbf{p})\bar{I}(z), \) where \( \rho_0 \) is an arbitrary function of \( \mathbf{p} \) satisfying \( \int d^2p \rho_0 = 1, \) and \( \bar{I}(z) = \bar{I}(0) = I_0(2K - 2)/[2vI_0^{2K-2}z + (2K - 2)^{2K-2}], \) with \( I_0 = \bar{I}(0). \) Thus, the effect of a small but non-zero \( v \) is to introduce a slow fall-off in the intensity along the \( z \)-direction when \( K \geq 1. \) Moreover, as \( v \to 0 \) this solution reduces to \( \bar{I} = I_0. \)

We now consider spatial filamentation of a well defined optical pulses against small perturbations having the parallel wavenumber \( k_\parallel \) and the perpendicular wavevector \( k_\perp, \) by assuming that \( v \) is small so that \( k_\parallel \gg |\partial_z| \) for the background distribution. We let \( \rho = \hat{\rho}(z, \mathbf{p}) = \rho_1(\mathbf{p}) \exp(ik_\parallel z + ik_\perp \cdot \mathbf{r}_\perp) + \text{c.c.} \) and \( I = \bar{I}(z) + I_1 \exp(ik_\parallel z + ik_\perp \cdot \mathbf{r}_\perp) + \text{c.c.}, \) where \( |\rho_1| \ll \hat{\rho}, |I_1| \ll \bar{I}, \) and c.c. stands for the complex conjugate. We linearize (4) with respect to the perturbation variables and readily obtain the nonlinear dispersion equation
\[
1 = \int d^2p \left[ f'(\bar{I}) + iv(K - 1)\bar{I}^{K-2} \right] \hat{\rho}(z, \mathbf{p} - k_\perp/2) - \left[ f'(\bar{I}) - iv(K - 1)\bar{I}^{K-2} \right] \hat{\rho}(z, \mathbf{p} + k_\perp/2),
\] (5)
which is valid for partially coherent femtosecond pulses in air. Here the prime denotes differentiation with respect to the background intensity \( \bar{I}. \)

We simplify the analysis by assuming that the perpendicular dependence in essence is one-dimensional. In the coherent case, i.e. \( \rho(z, p) = \bar{I}(z)\delta(p - p_0), \) Eq. (5) yields
\[
k_\parallel = -2kp_0 + iv(K + 1)\bar{I}^{K-1} \pm \sqrt{k^2[2f' \bar{I}^{K-1} - v^2(K - 1)^2\bar{I}^{2K-2}],}
\] (6)
where \( k \) represents the perpendicular wavenumber in the one-dimensional case. Letting \( k_\parallel = -2kp_0 - i\Gamma \) in (6), where \( \Gamma \) is the filamentation instability growth rate, we thus obtain\(^{19} \)
\[
\Gamma = -v(K + 1)\bar{I}^{K-1} + \sqrt{k^2[2f' \bar{I}^{K-1} - k^2] + v^2(K - 1)^2\bar{I}^{2K-2}},
\] (7)
which reduces to the well known filamentation instability growth rate in a Kerr medium (i.e. \(\nu = 0\) and \(f(I) = \alpha I\)). We note that a nonzero \(\nu\) gives rise to an overall reduction of the growth rate. In Fig. 1 we have plotted a number of different curves for the growth rate in the coherent case.

In the partially coherent case, we investigate the effects of spatial spectral broadening using the Lorentz distribution

\[
\bar{\rho}(z,p) = \frac{I(z)}{\pi} \frac{\Delta}{(p - p_0)^2 + \Delta^2},
\]

where \(\Delta\) denotes the width of the distribution around the quasiparticle momenta \(p_0\). Inserting (8) into (5) and carrying out the integration in a straightforward manner, we obtain

\[
k_{\parallel} = -2kp_0 + i\nu(K + 1)\bar{I}^{K-1} + 2ik\Delta \pm \sqrt{k^2[k^2 - 2f'(\bar{I})\bar{I} - \nu^2(K - 1)^2\bar{I}^{2K-2}].}
\]

With \(k_{\parallel} = -2kp_0 - i\Gamma\) the filamentation instability growth rate is

\[
\Gamma = -\nu(K + 1)\bar{I}^{K-1} - 2k\Delta + \sqrt{k^2[2f'(\bar{I})\bar{I} - k^2] + \nu^2(K - 1)^2\bar{I}^{2K-2}].
\]

In the limit \(\Delta \to 0\), Eq. (9) reduces to the dispersion relation (6), while for \(\nu = 0\) the dispersion relation (9) reduces to the standard expression for the filamentation instability growth rate

\[
\Gamma = -2k\Delta + k\sqrt{2I_0f'(I_0) - k^2}.
\]

In Fig. 2 we have displayed the filamentation instability growth rate (10). The effect of the finite width \(\Delta\) of the quasiparticle distribution can clearly be seen. In particular, multi-photon absorption (here chosen to be a modest \(K = 3\)), determined by the coefficient \(\nu\), as well as multi-photon ionization, represented by the coefficient \(\gamma\), combined with finite spectral width of the optical pulse give rise to a significant reduction of the filamentation instability growth rate. This is evident from Fig. 2, where the plotted normalized growth rate is reduced by as much as a factor of six, compared to the case of full coherence.

In practice, optical smoothing techniques, such as the use of random phase plates\(^{20}\) or other random phase techniques well suited for the results in the present Letter, have been used in inertial confinement fusion studies for quite some time (see, e.g. Ref. 21). Such spatial partial coherence controls are reproducible and can be tailored as to give a suitable broadband spectrum (as in, e.g. 22, where optical vortices were generated). Thus, in the case of ultra-short pulse propagation in air, such random phase techniques can be used to experimentally prepare an ultra-short optical pulse for a long-distance propagation, and a large spatial bandwidth of optical pulses, in conjunction with multi-photon ionization and absorption, may drastically reduce (down to less than 20% of
the coherent value in the present study) the filamentation instability growth rate. This will lead to a greater long range stability, since the onset of strong optical pulse filamentation is delayed, resulting in several times longer stable propagation. A rough estimate based on the numbers found in the present Letter shows that an optical beam could propagate a distance as much as six times longer with proper random phasing.

To summarize, we have investigated the filamentation instability of partially coherent femtosecond optical pulses in air. For this purpose, we introduced the Wigner-Moyal representation on the modified nonlinear Schrödinger equation and obtained a kinetic wave equation for optical pulses that have a spectral bandwidth in wavevector space. A perturbation analysis of the kinetic wave equation gives a nonlinear dispersion relation, which describes the filamentation instability (spatial amplification) of broadband optical pulses. Our results reveal that the latter would not be subjected to filamentation due to spectral pulse broadening. Hence, using partial spatial coherence effects for controlling the filamentational instability, femtosecond optical pulse propagation in air can be improved significantly. The result presented here is also indicative that optical smoothing techniques, as used in inertial confinement studies, could be very useful for ultra-short pulse propagation in air. This can help to optimize current applications of ultra-short laser pulses for atmospheric remote sensing over a long distance.

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References
Fig. 1. The coherent filamentation instability growth rate, given by (7), plotted for different parameter values; all curves with $I_0 = 0.5$, $\alpha = 1$, and $K = 3$. The full thick line represents the standard filamentation instability growth rate for a nonlinear Schrödinger equation, i.e. $\nu = \varepsilon = \gamma = 0$; the thin dashed curve has $\nu = \gamma = 0$, while $\varepsilon = 0.5$; the thin dotted curve has $\nu = \varepsilon = 0$ and $\gamma = 0.5$; the thin dashed–dotted curve has $\nu = 0$ and $\varepsilon = \gamma = 0.5$; the thick dashed curve has $\nu = 0.1$ and $\varepsilon = \gamma = 0$; finally, the thick dashed–dotted curve has $\nu = 0.1$ and $\varepsilon = \gamma = 1/2$.

Fig. 2. The partially coherent filamentation instability growth rate, given by (10), plotted for different parameter values; all curves with $I_0 = 0.75$, $\alpha = 1$, and $K = 3$. The full thick line again represents the standard filamentation instability growth rate for a nonlinear Schrödinger equation, i.e. $\Delta = \nu = \varepsilon = \gamma = 0$; the thin full curve has $\nu = \varepsilon = \gamma = 0$, while $\Delta = 0.1$; the thin dashed curve has $\varepsilon = \gamma = 0$ while $\nu = 0.05$ and $\Delta = 0.1$; the thin dotted curve has $\nu = 0.05$ and $\gamma = 0.1$ while $\varepsilon = 0$. The effects finite width of the background intensity distribution of the optical pulse, as well as the influence of the higher order nonlinearity and losses are clearly seen here.
Fig. 1.

Fig. 2.