Gravitino Propagator in anti de Sitter space

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Abstract

We construct the gauge invariant part of the propagator for the massless gravitino in $AdS_{d+1}$ by coupling it to a conserved current. We also derive the propagator for the massive gravitino.
1 Introduction

In order to formulate a theory of quantum gravity, it is necessary to understand the propagation of particles in curved space-times. Calculation of correlation functions to measure scattering amplitudes involves the propagators for various particles in this background geometry. So the calculation of propagators in curved space–times is a problem of physical interest. This also assumes significance following the AdS/CFT correspondence [1–3] which proposes a duality between a theory of quantum gravity on $AdS_{d+1}$ and a $d$–dimensional conformal field theory living on its boundary. In fact, this correspondence uses information about the theory of quantum gravity on $AdS_{d+1}$ to make predictions about the boundary conformal field theory. In order to do so, one needs a prescription that relates the correlators in these two theories. The precise map between correlators in the two theories is obtained from the relation schematically given by [2, 3]

$$e^{-S[\phi_0]} = \langle e^{\mathcal{O}[\phi_0]} \rangle. \quad (1)$$

Here $S[\phi_0]$ stands for the string/M theory action evaluated on fields $\phi$ that take boundary values $\phi_0$, which act as sources for the operators $\mathcal{O}$ in the boundary conformal field theory. This correspondence has been extensively studied by considering supergravity backgrounds with choices of metric and fluxes that preserve the $SO(d, 2)$ isometry of $AdS_{d+1}$ and analyzing the dual conformal field theory, and vice versa.

So the calculation of bulk–to–bulk as well as bulk–to–boundary propagators for various supergravity modes propagating in $AdS_{d+1}$ has been an active area of research (see [4–6], and [7] for further references.). Our aim is to calculate the bulk–to–bulk propagator for the massless as well as the massive gravitino. The kinetic term in the action for the gravitino in $AdS_{d+1}$ is given by

$$S = -\int d^{d+1}z \sqrt{|g|} \bar{\psi}_\mu \left( \Gamma^{\mu\nu\rho} D_\nu \psi_\rho + m \Gamma^{\mu\nu} \psi_\nu \right), \quad (2)$$

where

$$D_\mu \psi_\nu = \partial_\mu \psi_\nu + \frac{1}{8} \omega_{\mu}^{ab} [\gamma_a, \gamma_b] \psi_\nu - \Gamma^\rho_{\mu\nu} \psi_\rho. \quad (3)$$

Note that the parameter $m$ is not the physical mass of the gravitino, but is related to it. In fact the physically massless gravitino has non–zero $m$, as we shall discuss later. In this work, we are interested in constructing the propagator for the physically massless as well as the physically massive gravitino.

The massless gravitino in $AdS_{d+1}$ arises in theories that are obtained from compactifications of supergravity that preserve at least some supersymmetry. So this gravitino lies
in the massless BPS multiplet containing the graviton. In fact it is possible to construct a theory of gravity possessing local supersymmetry only when the gravitino is massless. This gravitino couples to the conserved supercurrent which follows by supersymmetry as the graviton couples to the conserved stress tensor. The parameter $m$ is fixed which results in a gauge invariant action which we shall discuss when we construct the massless gravitino propagator.

The massive gravitino arises in theories where supersymmetry is broken, and the gravitino gets mass by the super Higgs mechanism. The value of the parameter $m$ depends on the mechanism of supersymmetry breaking. Hence the massive gravitino propagator is useful in computations in theories with broken supersymmetry in $AdS_{d+1}$.

On the other hand, the physical part of the massive Kaluza–Klein gravitino propagator can be constructed along the lines of the massless gravitino propagator. This is because the massive KK gravitino in supersymmetric theories couples to a conserved current. Essentially it is due to the fact that the gravitino couples to a conserved current in the parent theory, and though the choice of the vacuum configuration specified by the metric and fluxes spontaneously breaks the diffeomorphism invariance, the lower dimensional gravitino continues to couple to a current, which satisfies a generalized conservation equation. This conservation equation can be obtained directly from the conservation equation of the supercurrent in the parent theory, by analyzing the details of the KK compactification.

We shall construct the generic massive gravitino propagator, as well as the massless gravitino propagator which couples to a conserved current. In order to calculate the propagator, we find it useful to proceed using the intrinsic geometric objects used in calculations in maximally symmetric spaces. The essential idea is to decompose the propagators given their tensor or spinor structures in a suitable basis defined by purely geometric objects with the coefficients given by functions of the geodesic distance between the two points, which we review below. This approach was originally used to construct the bulk–to–bulk vector propagator in [8] in maximally symmetric spaces by suitably decomposing bitensors in these spaces. For constructing the massless propagator, the number of physical structures gets reduced as we can neglect the pure gauge contributions. Using a different basis for the decomposition, this has been used in [9] to deduce the gauge invariant part of the gauge boson and graviton propagators. The spinor propagator in four dimensions has been constructed in [10] using geometric objects and has been shown to match with the result obtained previously [11]. More recently, the calculation of the spinor propagator has been generalized to arbitrary dimensions in [12]. We shall use this formalism to obtain the physical part of
the massless gravitino propagator in $AdS_{d+1}$ up to pure gauge contributions\(^3\).

We want to construct the gravitino propagator $\Theta_{\mu\nu'}(z, w)$ in $AdS_{d+1}$ defined by

$$
\Theta_{\mu\nu'}(z, w) = \langle \psi_\mu(z) \bar{\psi}_{\nu'}(w) \rangle, 
\tag{4}
$$

where $\mu$ and $\nu'$ refer to the points $z$ and $w$ respectively. Given the vector–spinor structure of the propagator, we need to define intrinsic objects in maximally symmetric spaces that carry vector and spinor indices. The vectorial objects are given by \cite{8}

$$
n_\mu = D_\mu \mu(z, w), \quad n_{\nu'} = D_{\nu'} \mu(z, w),
\tag{5}
$$

where $\mu(z, w)$ is the geodesic distance between $z$ and $w$. One also has the bitensor $g_{\mu\nu'}(z, w)$ under which vectors transform as

$$
V_{\mu}(z) = g_{\mu \nu'}(z, w)V_{\nu'}(w).
\tag{6}
$$

Also $n_\mu$ and $n_{\nu'}$ satisfy the relation

$$
n_\mu = -g_{\mu \nu'}n_{\nu'}.
\tag{7}
$$

To account for the spinor indices, we have the bi–spinor parallel propagator \cite{10}

$$
\Lambda^{\alpha \beta}(z, w),
\tag{8}
$$

under which spinors transform as

$$
\psi^{\alpha}(z) = \Lambda^{\alpha \beta}(z, w)\psi^{\beta}(w).
\tag{9}
$$

It is very convenient to decompose the gravitino propagator in terms of independent structures constructed out of $n_\mu, n_{\nu'}, g_{\mu\nu'}$ and $\Lambda^{\alpha \beta}$. Thus the propagator can be written as

$$
\Theta_{\mu\nu'}(z, w) = A_1(\mu)g_{\mu\nu'}\Lambda + A_2(\mu)n_\mu n_{\nu'}\Lambda + A_3(\mu)g_{\mu\nu'}n^\sigma \Gamma_\sigma \Lambda + A_4(\mu)n_\mu n_{\nu'}n^\sigma \Gamma_\sigma \Lambda
+ A_5(\mu)n_\mu \Lambda \Gamma_{\nu'} + A_6(\mu)\Gamma_\mu n_{\nu'}\Lambda + A_7(\mu)n_\mu n^\sigma \Gamma_\sigma \Lambda \Gamma_{\nu'}
+ A_8(\mu)n_{\nu'}n^\sigma \Gamma_\sigma \Gamma_\mu \Lambda + A_9(\mu)\Gamma_\mu \Lambda \Gamma_{\nu'} + A_{10}(\mu)n^\sigma \Gamma_\sigma \Gamma_\mu \Lambda \Gamma_{\nu'},
\tag{10}
$$

where the $A_i$’s are scalar functions of the geodesic distance $\mu$.

Unlike the massive gravitino, for the massless gravitino, the structure in \cite{10} can be simplified using the gauge invariance of the theory, and one can write down the physical

\(^3\)See \cite{13} for a discussion

\(^4\)As before, unprimed and primed indices refer to $z$ and $w$ respectively.
part of the propagator using lesser number of independent structures, as we shall discuss later. The method of calculating the coefficients $A_i$ is the same as in [14] apart from the overall normalization, and so we shall be brief.

In fact for the massless gravitino, in section 3 we shall show that these coefficients are given by

$$A_9(y) = \lambda y^{-d/2}(y - 1)^{-(d+1)/2}, \quad A_{10} = \sqrt{\frac{y-1}{y}} A_9,$$

$$A_6 = \sqrt{y(y-1)} \left[ \frac{dA_9}{dy} - \frac{A_9}{2y} \right], \quad A_8 = \sqrt{y(y-1)} \left[ \frac{dA_{10}}{dy} - \frac{A_{10}}{2(y-1)} \right],$$

$$A_1 = -A_8 - (d+1)A_9, \quad A_2 = (d-1)A_8, \quad A_3 = -A_6 - (d+1)A_{10},$$

$$A_4 = -2(d+1)A_{10} - (d+3)A_6, \quad A_5 = -A_6 - 2A_{10}, \quad A_7 = A_8,$$

where $2y = 1 + \cosh \mu$, and $\lambda$ is a constant which we fix. And finally in section 4, we deduce the gauge invariant form of the gravitino propagator which is given by

$$\Theta_{\mu
u}(z, w) = A(u)\partial_\mu \partial_\nu u(\partial u. \Gamma)\Lambda + B(u)\partial_\mu u\partial_\nu u(\partial u. \Gamma)\Lambda$$

where

$$A(u) = -2^{d+3/2}\lambda(d+1)(u+1)u^{-(d+3)/2}(u+2)^{-d/2-2},$$

$$B(u) = 2^{d+3/2}\lambda(d+1)u^{-(d+5)/2}(u+2)^{-d/2-3}(d+2)(u+1)^2 + 1.$$  

2 The massive gravitino propagator

We first consider the massive gravitino propagator for which the analysis is simpler than the massless one. The analysis is similar to [14], to which we refer the reader for various details. The main strategy is to substitute (10) into the equation of motion obtained from (2) and solve for the coefficients. It is easier to solve for the coefficients by simplifying the equation of motion along the lines of [15] which leads to the equivalent set of equations

$$D.\psi = 0, \quad \Gamma.\psi = 0, \quad (\bar{\psi} - m)\psi_\mu = 0.$$  

Following the analysis of [14], we obtain a system of coupled differential equations involving only $A_9$ and $A_{10}$, which leads to a differential equation for $A_9$. Changing variables to

$$2y = 1 + \cosh \mu,$$  

5
and defining $A_9 = \sqrt{y} \tilde{A}_9$, we have that
\[
\left[ y(y-1)(y-a) \frac{d^2}{dy^2} + \left\{ (\alpha+\beta+1)y^2 - (\alpha+\beta+1-\gamma) \right\} \frac{d}{dy} + (\alpha\beta y - q) \right] \tilde{A}_9(y) = 0,
\]
where
\[
a = \frac{(d+1)(d-1) - 4m^2}{(d-1)^2 - 4m^2}, \quad \alpha = \frac{d+1}{2} \pm m, \quad \beta = \frac{d+1}{2} \mp m, \quad \gamma = \delta = \frac{d+3}{2},
\]
\[
q = \frac{(d-1)(d+1)(d^2 + 2d + 5) + 8m^2(2m^2 - d^2 - d - 2)}{4((d-1)^2 - 4m^2)}.
\]

Now the differential operator in (16) is the Heun operator, and so (16) is solved by the Heun function (see [16] for various details). The choice of the specific Heun function to obtain the propagator in $AdS_{d+1}$ is as discussed in the literature. The same issue arises, for example, in the choice of the hypergeometric function in the expression for the scalar or the vector propagator. We want a solution in $AdS_{d+1}$ whose singularity at $\mu = 0$ has the same strength as in flat space, and which has the fastest fall–off at spatial infinity corresponding to $\mu = \infty$ [8, 10]. While the former condition is natural as the two points approach each other, the later condition is necessary in specifying boundary conditions and for the stability of $AdS$ against small fluctuations [17, 18].

The two solutions $H$ of Heun equation (16) with the properties mentioned above are given by
\[
\tilde{A}_9(y) = \lambda_\alpha y^{-\alpha} H \left( \frac{1}{a}, \frac{q + \alpha[a(\alpha - \gamma - \delta + 1) - \beta + \delta]}{a}; \alpha, \alpha - \gamma + 1, \alpha - \beta + 1, \delta; \frac{1}{y} \right),
\]
with characteristic exponent $\alpha$ and
\[
\tilde{A}_9(y) = \lambda_\beta y^{-\beta} H \left( \frac{1}{a}, \frac{q + \beta[a(\beta - \gamma - \delta + 1) - \alpha + \delta]}{a}; \beta - \gamma + 1, \beta, \beta - \alpha + 1, \delta; \frac{1}{y} \right),
\]
with characteristic exponent $\beta$. Here $\lambda_\alpha$ and $\lambda_\beta$ are arbitrary coefficients. Demanding the fastest fall–off at spatial infinity, considering $\alpha, \beta$ with the upper signs from (17) we see that for $m > 0$, (18) is the required solution, while for $m < 0$, (19) is the required solution (with the situation reversed for $\alpha, \beta$ with the lower signs). These are the two local solutions of Heun equation at $y = \infty$ and have been obtained in [19] (see tables on pages 24 and 26).

Now in order to fix the coefficient $\lambda$, one has to match the singularity as $\mu \to 0$ (i.e., $y \to 1$) of the Heun function with the flat space answer. In order to do this, one has to express the Heun function at $1/y$ in terms of Heun functions at $1 - y$ and keep the leading term. However, unlike hypergeometric functions, for Heun functions such explicit
expressions are not known in closed form in literature and so it is not easy to get an explicit expression for $\lambda$. However, the coefficients relating two local solutions at two singular points have been implicitly computed in [20]. Using this, one can relate the local solution at $1/y$ to the ones at $1 - y$ and determine $\lambda$ implicitly.

Though it is difficult to determine $\lambda$ in general, we can make a consistency check of (18) and (19). Note that at $y = 1$, the Heun function $H(a, q; \alpha, \beta, \gamma, \delta; y)$ has leading behaviour $(1 - y)^{1 - \delta}$ [16] and so (18) and (19) both have leading behaviour $\mu^{-(d+1)}$ as $\mu \to 0$ on using the expression for $\delta$ in (17), thus leading to $A_9 \sim \mu^{-(d+1)}$ as $\mu \to 0$. Later on, we shall calculate the various coefficients for the flat space case for the massless gravitino. Though the details of constructing the propagator are different for the massive and massless case, the scaling of the singularity is the same. In fact we shall see that in flat space $A_9 \sim \mu^{-(d+1)}$ as $\mu \to 0$ which agrees with the scaling deduced above. It turns out that one can obtain the expressions for the remaining eight coefficients in (10) in terms of $A_9$ and $A_{10}$, where the equations involved are purely algebraic [14]. Thus this gives us the complete expression for the massive gravitino propagator.

3 The massless gravitino propagator

In order to construct the massless gravitino propagator in $AdS_{d+1}$, we couple the gravitino to a conserved current leading to the action

$$\mathcal{S} = -\int d^{d+1}z \sqrt{g} \left[ \bar{\psi}_\mu \left( \Gamma^{\mu\rho} D_\nu \psi_\rho + m \Gamma^{\mu\nu} \psi_\nu \right) + \left( \bar{\psi}_\mu \mathcal{J}^\mu + \text{h.c.} \right) \right].$$

The action for the massless gravitino has a gauge invariance because of which the gravitino has the correct number of degrees of freedom [21, 22]. In fact the free part of the action (20) has a gauge symmetry given by [23]

$$\delta \psi_\mu \equiv D_\mu \eta = D_\mu \eta - \frac{m}{d - 1} \Gamma_\mu \eta,$$

for

$$m = \pm \frac{d - 1}{2}.$$  

Thus the free action is gauge invariant, and hence the gravitino is massless, only for the specific values of $m$ given by (22). Thus demanding the interaction to be gauge invariant leads to the covariant current conservation

$$D_\mu \mathcal{J}^\mu = \left( D_\mu - \frac{m}{d - 1} \Gamma_\mu \right) \mathcal{J}^\mu = 0.$$  

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5 We thank R. Maier for bringing this reference to our notice.

6 Local Weyl invariance has been used to identify a special mass value for the scalar field in [24].

7 Note that this is different from $D_\mu \mathcal{J}^\mu = 0$ as naively expected.
The general result for the massless gravitino (22) is consistent with the results in literature, when one considers the KK spectrum of compactification of $d = 11$ supergravity on $AdS_4 \times S^7$ [25], $AdS_7 \times S^4$ [26]; and type IIB supergravity on $AdS_5 \times S^5$ [27].

We note that this feature of appearance of gauge invariance for massless particles is generic in curved space–times. The analysis for the graviton has been performed in [28], where only for special values of the parameters in the action, the graviton is massless and the theory has diffeomorphism invariance. It is also worth mentioning that in the discussion above as well as in the literature, a particle in curved space–time is defined to be massless if it has the same number of degrees of freedom as the massless particle in flat space–time. It does not necessarily mean that the particle has null propagation, in fact, the propagation can have support inside the light–cone [22]. However, it should be noted that it has been argued that massless modes do have light–cone propagation in [29].

We now need to solve for the coefficients in (10) for the massless gravitino propagator

$$\psi_\mu(z) = \int d^{d+1}w \sqrt{g} \Theta_{\mu\nu'}(z,w) J^{\nu'}(w),$$

which we outline in some detail. We substitute (10) into the equation of motion obtained from (20) given by

$$\Gamma^\mu D_\Gamma \psi - D_\mu \Gamma \psi - \Gamma_\mu D_\psi \psi_\mu + m \Gamma_{\mu\nu} \psi_\nu = -J_\mu.$$  \hspace{1cm} (25)

As mentioned before, it is easier to obtain them after simplifying the equation of motion by obtaining expressions for the lower spin components in (25) which are $D_\psi$ and $\Gamma_\psi$. In order to do this, we contract (25) with $D_\mu$ and $\Gamma_\mu$ respectively, and use covariant current conservation (23) to obtain

$$(d-1) \varphi \Gamma_\psi - (d-1) D_\psi + dm \Gamma \psi = -\Gamma_\psi.$$  \hspace{1cm} (26)

Then one can easily generalize the calculations of [15] to obtain the relations

$$D_\psi = \frac{1}{d-1} \Gamma_\psi, \quad \Gamma_\psi = 0,$$

which when substituted in (25) leads to the expression

$$(\varphi - m) \psi_\mu = -J_\mu + \frac{1}{d-1} \Gamma_\mu \Gamma J.$$  \hspace{1cm} (28)

The system of equations (27) and (28) contain the same information as in (25). So substituting (10) into the expressions (27) and (28) gives a coupled system of eighteen
equations involving $A_1, \ldots, A_{10}$ and their first derivatives. These are given by equations (3.6)-(3.17) in [14], where $d = n - 1$, and $A_1, \ldots, A_{10} = \alpha, \ldots, \kappa$, which can be seen by comparing (10) and equation (3.5) in [14]. The homogeneous equations are obtained by setting the current to zero and have been solved in [14]. We shall first find a solution to these equations in the massless limit which involves all the coefficients. Then we shall obtain the gauge invariant part of the propagator.

Setting the contact terms to zero and solving the system of equations gives us the homogeneous solution which has an arbitrary coefficient which is independent of $\mu$, and the role of the contact terms is to fix this coefficient. As discussed before, we need the solution in flat space including the overall normalization to fix the $AdS_{d+1}$ normalization. This is discussed in detail in Appendix A.

Now we proceed to obtain the coefficients in $AdS_{d+1}$. Clearly the homogeneous system of equations is the same as in the massive case (the only difference is that $m$ is fixed), and so we consider the solution for $A_9$ which is given by (16). Now the Heun function has four singular points at $y = 0, 1, a$ and $\infty$ [16]. From the value of $a$ in (17), note that for the class of solutions given by $d - 1 = \pm 2m$, the singular point $a$ coincides with the singular point at $\infty$ and so the nature of the equation changes, as this is a degeneration limit. Now from (22) we see that (29) is precisely the case when the gravitino is massless.

One should note that the expressions for the various other coefficients in terms of $A_9$ are naively singular as they involve a factor of $(d - 1)^2 - 4m^2$ in the denominator, and the numerator also vanishes. So one has to take care to obtain the coefficients, which are of course finite, in this degenerate limit. The procedure essentially involves setting $d - 1 \pm 2m = \epsilon$ so that the singular points of the Heun equation are at $0, 1, O(1/\epsilon)$ and $\infty$ and then taking the limit $\epsilon \to 0$ smoothly at the end so that the singular point at $O(1/\epsilon)$ merges with the singular point at $\infty$. We will find the coefficients directly by considering the equations they satisfy in the massless limit. We now present the solutions for the coefficients below when $d - 1 = 2m$ is satisfied$^8$.

One can directly deduce the equation satisfied by $\tilde{A}_9$ in this limit to get

$\left[ y(1 - y) \frac{d^2}{dy^2} + \left\{ \frac{d + 3}{2} - (d + 3)y \right\} \frac{d}{dy} - (d + 1) \right] \tilde{A}_9(y) = 0.$

(30)

Now the differential operator in (30) is the hypergeometric operator, and so the solution

$^8$The $d - 1 = -2m$ case works analogously.
is a hypergeometric function, which has three singular points at \( y = 0, 1 \) and \( \infty \) as expected. The solution of (30) for AdS\(_{d+1}\) is as discussed before [8,10], and so we need the local solution at \( y = \infty \) with the fastest fall–off at spatial infinity. There are two such solutions to the hypergeometric equation

\[
y(1 - y) \frac{d^2 u(y)}{dy^2} + [\gamma - (\alpha + \beta + 1)y] \frac{du(y)}{dy} - \alpha \beta u(y) = 0 \tag{31}
\]
given by [30]

\[
u(y) \sim y^{-\alpha} F\left(\alpha, \alpha - \gamma + 1, \alpha - \beta + 1, \frac{1}{y}\right), \quad u(y) \sim y^{-\beta} F\left(\beta, \beta - \gamma + 1, \beta - \alpha + 1, \frac{1}{y}\right). \tag{32}
\]

From (30) we see that \( \gamma = \frac{d + 3}{2} \), \( (\alpha, \beta) = (d + 1, 1) \) or \( (\alpha, \beta) = (1, d + 1) \). \( \tag{33} \)

Considering the solution with the fastest fall–off at spatial infinity we get that

\[
A_9(y) = \lambda y^{-d/2} (y - 1)^{-(d+1)/2}. \tag{34}
\]

To obtain the expression for \( \lambda \), we compare it to the flat space expression given by (97). Equating the leading singularity in (34) and (97) as \( \mu \to 0 \), we see that

\[
\lambda = -\frac{\Gamma\left(\frac{d+1}{2}\right)C}{(d-1)^{d+2}} = -\frac{\Gamma\left(\frac{d-1}{2}\right)}{(4\pi)^{(d+1)/2}(d+1)!}, \tag{35}
\]
on using (96).

Having obtained \( A_9 \), we note that \( (A_9, A_{10}) \) form a coupled system of equations [14]. In fact knowing \( A_9 \), we see that \( A_{10} \) can be solved algebraically using the equation

\[
m\left[4m^2 - (d - 1)^2 + \frac{2(d - 1)}{y - 1}\right] A_{10} = [4m^2 - (d - 1)^2]\sqrt{y(y - 1)} \frac{dA_9}{dy}
\]

\[
+ \left[ \sqrt{\frac{y - 1}{y}} \left( \frac{(d - 2)(4m^2 - (d - 1)^2)}{2} - (d - 1)^2 \right) + 4m^2 \sqrt{\frac{y}{y - 1}} \right] A_9. \tag{36}
\]

For the case \( 2m = d - 1 \), (36) simplifies and gives

\[
A_{10} = \sqrt{\frac{y - 1}{y}} A_9 = \lambda y^{-(d+1)/2}(y - 1)^{-d/2}. \tag{37}
\]
We can now write down the expressions for $A_6$ and $A_8$ in terms of $A_9$ and $A_{10}$. One can show that $A_6$ satisfies the equation [4]

$$\frac{A_6}{y} = \frac{dA_{10}}{dy} + \frac{(d-2)A_{10}}{2(y-1)} - \frac{mA_9}{\sqrt{y(y-1)}}.$$  \tag{38}

Expressing $A_{10}$ in terms of $A_9$ and working in this limit gives

$$A_6 = \sqrt{y(y-1)} \left[ \frac{dA_9}{dy} - \frac{A_9}{2y} \right].$$ \tag{39}

Similarly using the equation for $A_8$ given by

$$\frac{A_8}{y-1} = \frac{dA_9}{dy} + \frac{(d-2)A_9}{2y} - \frac{mA_{10}}{\sqrt{y(y-1)}}$$  \tag{40}

gives

$$A_8 = \sqrt{y(y-1)} \left[ \frac{dA_{10}}{dy} - \frac{A_{10}}{2(y-1)} \right].$$ \tag{41}

Having obtained $(A_6, A_8, A_9, A_{10})$, the other coefficients are given by

$$A_1 = -A_8 - (d+1)A_9, \quad A_2 = (d-1)A_8, \quad A_3 = -A_6 - (d+1)A_{10},$$

$$A_4 = -2(d+1)A_{10} - (d+3)A_6, \quad A_5 = -A_6 - 2A_{10}, \quad A_7 = A_8,$$ \tag{42}

thus completing the construction of the propagator.

As mentioned before, the propagator constructed above can be rewritten using the gauge invariance in the theory. This simplifies the calculation of Witten diagrams involving the massless gravitino exchange in $AdS_{d+1}$. We should mention that in calculating the gravitino propagator we have worked on the subspace of conserved currents and did not use any gauge–fixing to obtain the propagator. We now proceed to obtain the gauge invariant part of the propagator.

4 The gauge invariant massless gravitino propagator

The propagator in any theory which has a gauge invariance is ambiguous. One always has the freedom to add pure gauge terms to the propagator, which does not affect physical gauge–invariant amplitudes. So using this freedom, one can write the propagator in a convenient form. The propagator \[10\] has ten independent structures which follows from its vector–spinor structure, while the coefficients are determined by the equation of motion. Now one can use the gauge invariance of the theory to add appropriately chosen pure
gauge terms so that one has less than the ten independent structures in the expression for the propagator. We shall construct the propagator which has the least number of the independent structures in \((10)\), which we call the gauge invariant part of the propagator. Any other expression for the propagator will differ from the one we construct by pure gauge terms. The pure gauge terms we shall add to \((10)\) are of the form \(\mathcal{D}_\mu(z)P_\nu(z,w)\) or \(\mathcal{D}_\nu(w)Q_\mu(z,w)\) with suitable choices for \(P_\nu\) and \(Q_\mu\). This is because \(\mathcal{D}_\mu(z)P_\nu(z,w)\) yields a pure gauge term of the form

\[
\psi_\mu^\text{pure gauge}(z) = \mathcal{D}_\mu(z) \int d^{d+1}w \sqrt{g} P_\nu(z,w) \mathcal{J}^\nu(w), \tag{43}
\]

which can be removed by a gauge transformation using \((21)\), and \(\mathcal{D}_\nu(w)Q_\mu(z,w)\) yields

\[
\psi_\mu^\text{pure gauge}(z) = - \int d^{d+1}w \sqrt{g} Q_\mu(z,w) \mathcal{D}_\nu(w) \mathcal{J}^\nu(w), \tag{44}
\]

which vanishes due to current conservation using \((23)\). We will consider only those \(P_\nu\) and \(Q_\mu\) which vanish as \(u \to \infty\), so that the boundary contributions can be neglected.

Given the index structure of the propagator, the only non–trivial possibilities that are pure gauge contributions are

\[
\mathcal{D}_\mu \left( F_1(u) n_\nu \Lambda \right), \quad \mathcal{D}_\mu \left( F_2(u) \Lambda \Gamma_\nu \right), \quad \mathcal{D}_\mu \left( F_3(u) n_\nu n^\sigma \Gamma_\sigma \Lambda \right), \\
\mathcal{D}_\mu \left( F_4(u) n^\sigma \Gamma_\sigma \Lambda \Gamma_\nu \right), \quad \mathcal{D}_\nu \left( F_5(u) n_\mu \Lambda \right), \quad \mathcal{D}_\nu \left( F_6(u) \Gamma_\mu \Lambda \right), \\
\mathcal{D}_\nu \left( F_7(u) n_\mu \Lambda n^\sigma \Gamma_\sigma \Gamma_\nu \right), \quad \mathcal{D}_\nu \left( F_8(u) \Gamma_\mu \Lambda n^\sigma \Gamma_\sigma \Gamma_\nu \right), \tag{45}
\]

where the \(F_i\)'s are arbitrary functions of \(u\). Note that naively one might also want to add four terms \(\mathcal{D}_\mu \left( F_9(u) g_{\nu\sigma} \Gamma_\sigma \Lambda \right), \mathcal{D}_\mu \left( F_{10}(u) g_{\nu\sigma} n^\lambda \Gamma_\lambda \Lambda \right), \mathcal{D}_\nu \left( F_{11}(u) g_{\mu\sigma} \Lambda \Gamma_\sigma \right)\) and \(\mathcal{D}_\nu \left( F_{12}(u) g_{\mu\sigma} \Lambda n^\lambda \Gamma_\lambda \Gamma_\sigma \right)\) to \((45)\). However using the relations

\[
n_\mu = -g_\mu^\nu n_\nu, \quad \Gamma_\mu \Lambda = g_\mu^\nu \Lambda \Gamma_\nu, \tag{46}
\]

we see that they give back terms in \((45)\) and thus are not linearly independent. We find it convenient to express the scalar functions in terms of the chordal distance \(u\) defined by \(u = \cosh \mu - 1\). In fact, the basic vectorial and bitensor structures are given in terms of the chordal distance by

\[
n_\mu = \frac{\partial_\mu u}{\sqrt{u(u+2)}}, \quad n_\nu = \frac{\partial_\nu u}{\sqrt{u(u+2)}}, \quad g_{\mu\nu} = -\partial_\mu \partial_\nu u + \frac{\partial_\mu u \partial_\nu u}{u+2}. \tag{47}
\]

\(^9\)We consider the case \(2m = d - 1\).
We now obtain expressions for the pure gauge terms given by (45) in terms of the various structures in (10). To do so, we use the relations

\[ D_{\mu} \eta_{\nu} = - \frac{1}{\sqrt{u(u+2)}} (g_{\mu\nu} + \eta_{\mu}\eta_{\nu}), \]
\[ D_{\mu} \eta_{\nu} = - \frac{1}{\sqrt{u(u+2)}} (g_{\mu\nu'} + \eta_{\mu}\eta_{\nu'}), \]
\[ D_{\mu} \eta_{\nu} = \frac{u+1}{\sqrt{u(u+2)}} (g_{\mu\nu} - \eta_{\mu}\eta_{\nu}), \]
\[ D_{\mu} \eta_{\nu} = \frac{u+1}{\sqrt{u(u+2)}} (g_{\mu\nu'} - \eta_{\mu}\eta_{\nu'}), \]
\[ D_{\mu} \Lambda = \frac{1}{2} \sqrt{\frac{u}{u+2}} (\Gamma_{\mu}\Gamma_{\nu} \eta_{\nu} - \eta_{\mu}), \]
\[ D_{\mu} \Lambda = - \frac{1}{2} \sqrt{\frac{u}{u+2}} (\Gamma_{\mu}\Gamma_{\nu} \eta_{\nu} - \eta_{\mu}). \quad (48) \]

Straightforward calculation gives

\[ D_{\mu} (F_{1}(u)\eta_{\nu} \Lambda) = - \frac{F_{1}}{\sqrt{u(u+2)}} \left[ (u(u+2) \frac{F_{1}'}{F_{1}} - 1 - \frac{u}{2}) \eta_{\mu}\eta_{\nu} \Lambda - g_{\mu\nu} \Lambda + \frac{u}{2} n_{\nu} \Gamma_{\mu} \eta \Lambda \right] \]
\[ - \frac{F_{1}}{2} n_{\nu} \Gamma_{\mu} \Lambda, \]
\[ D_{\nu} (F_{5}(u)n_{\mu} \Lambda) = - \frac{F_{5}}{\sqrt{u(u+2)}} \left[ (u(u+2) \frac{F_{5}'}{F_{5}} - 1 - \frac{u}{2}) n_{\mu}\eta_{\nu} \Lambda - g_{\mu\nu} \Lambda - \frac{u}{2} n_{\mu} \Gamma_{\mu} \eta \nu \Lambda \right] \]
\[ - \frac{F_{5}}{2} n_{\mu} \Lambda \Gamma_{\nu}, \]
\[ D_{\mu} (F_{4}(u)n_{\Lambda} \Gamma_{\nu}) = F_{4} \sqrt{\frac{u+2}{u}} \left[ \left( \frac{uF_{4}'}{F_{4}} - \frac{1}{2} \right) n_{\mu} \eta n_{\Lambda} \Gamma_{\nu} + \frac{1}{2} \Gamma_{\mu} \Lambda \Gamma_{\nu} \right] \]
\[ - \frac{F_{4}}{2} \Gamma_{\mu} n_{\Lambda} \Gamma_{\nu}, \]
\[ D_{\nu} (F_{8}(u)\Gamma_{\mu} n' \Gamma_{\nu}) = - F_{8} \sqrt{\frac{u+2}{u}} \left[ \left( \frac{uF_{8}'}{F_{8}} - \frac{1}{2} \right) n_{\nu} \eta n_{\Lambda} \Gamma_{\nu} + \frac{1}{2} \Gamma_{\mu} \Lambda \Gamma_{\nu} \right] \]
\[ + \frac{F_{8}}{2} \Gamma_{\mu} n_{\Lambda} \Gamma_{\nu}, \]
\[ D_{\mu} (F_{2}(u)\Lambda \Gamma_{\nu}) = \frac{F_{2}}{\sqrt{u(u+2)}} \left[ (u(u+2) \frac{F_{2}'}{F_{2}} - \frac{u}{2}) \eta_{\mu} \Lambda \Gamma_{\nu} + \frac{u}{2} \Gamma_{\mu} n_{\Lambda} \Gamma_{\nu} \right] \]
\[ - \frac{F_{2}}{2} \Gamma_{\mu} \Lambda \Gamma_{\nu}, \]
Similarly using the relations involving $F$ where

$$D_{\nu'}(F_6(u)\Gamma \Lambda) = \frac{F_6}{\sqrt{u(u+2)}} \left[ \left( u(u+2) \frac{F_6'}{F_6} - \frac{u}{2} \right) n_{\nu'} \Gamma \mu \Lambda - \frac{u}{2} \Gamma_{\mu n} \Gamma \Lambda_{\nu'} \right]$$

$$- \frac{F_6}{2} \Gamma_{\mu \Lambda \Gamma_{\nu'}}$$,

$$D_{\mu}(F_3(u)n_{\nu'} n. \Gamma \Lambda) = \frac{F_3}{\sqrt{u(u+2)}} \left[ \left( u(u+2) \frac{F_3'}{F_3} - \frac{u+4}{2} \right) n_{\mu} n_{\nu'} n. \Gamma \Lambda \right]$$

$$- g_{\mu' n.} \Gamma \Lambda_{\nu'} + \frac{u+2}{2} n_{\nu'} \Gamma_{\mu \Lambda} - \frac{F_3}{2} \Gamma_{\mu n} n. \Gamma \Lambda,$$

$$D_{\nu'}(F_7(u)n_{\mu} \Lambda n'. \Gamma') = - \frac{F_7}{\sqrt{u(u+2)}} \left[ \left( u(u+2) \frac{F_7'}{F_7} - \frac{u+4}{2} \right) n_{\mu} n_{\nu'} n. \Gamma \Lambda \right]$$

$$- g_{\mu' n.} \Gamma \Lambda_{\nu'} - \frac{u+2}{2} n_{\mu \Lambda \Gamma_{\nu'}} + \frac{F_7}{2} n_{\mu} n. \Gamma \Lambda_{\Gamma_{\nu'}},$$

where $F'_i$ denotes derivative with respect to $u$.

We now show using (49) that it is possible to express the propagator (10) in terms of only two of the ten independent structures. We begin by rewriting (10) as

$$\Theta_{\mu \nu'}(z, w) = A_1 g_{\mu' \nu'} \Lambda + (A_2 + 2A_8) n_{\mu} n_{\nu'} \Lambda + A_3 g_{\mu' \nu'} n. \Gamma \Lambda + A_4 n_{\mu} n_{\nu'} n. \Gamma \Lambda$$

$$+ A_6 (\Gamma_{\mu n} n_{\nu'} \Lambda - n_{\mu \Lambda \Gamma_{\nu'}}) + A_8 (n_{\mu} n. \Gamma \Lambda_{\Gamma_{\nu'}} - n_{\nu'} \Gamma_{\mu \Lambda})$$

$$+ A_9 \Gamma_{\mu \Lambda \Gamma_{\nu'}} - A_{10} \Gamma_{\mu n.} \Gamma \Lambda_{\Gamma_{\nu'}}$$,

where we have used the relations among the $A_i$'s. Now using the relations involving $F_2$ and $F_6$ from (49), we see that

$$A_6 (\Gamma_{\mu n} n_{\nu'} \Lambda - n_{\mu \Lambda \Gamma_{\nu'}}) = D_{\nu'} \left( \frac{\sqrt{u+2} F_{\nu} \Lambda}{u \sqrt{u+2}} \right)$$

$$- D_{\mu} \left( \frac{\sqrt{u+2} F \Lambda_{\Gamma_{\nu'}}}{u \sqrt{u+2}} \right) \Gamma_{\nu'}$$

$$+ \sqrt{u} F_{\nu} n. \Gamma \Lambda_{\Gamma_{\nu'}}$$,

where

$$F' = \frac{A_6}{\sqrt{u(u+2)}}$$.

Similarly using the relations involving $F_4$ and $F_8$ from (49), we see that

$$A_8 (n_{\mu} n. \Gamma \Lambda_{\Gamma_{\nu'}} - n_{\nu'} \Gamma_{\mu n.} \Gamma \Lambda) = D_{\mu} \left( \sqrt{u} G n. \Gamma \Lambda_{\Gamma_{\nu'}} \right)$$

$$+ D_{\nu'} \left( \sqrt{u} G \Gamma_{\mu \Lambda} n'. \Gamma' \right) - \sqrt{u+2} F \Gamma_{\mu \Lambda} \Gamma_{\nu'},$$

where

$$G' = \frac{A_8}{u \sqrt{u+2}}$$.

Thus neglecting the $D_{\mu}(...)$ and $D_{\nu'}(...) \Gamma_{\nu'}$ terms, we get that

$$\Theta_{\mu \nu'}(z, w) = A_1 g_{\mu' \nu'} \Lambda + (A_2 + 2A_8) n_{\mu} n_{\nu'} \Lambda + A_3 g_{\mu' \nu'} n. \Gamma \Lambda + A_4 n_{\mu} n_{\nu'} n. \Gamma \Lambda$$

(55)
where we have used the relation\(^\text{10}\)
\[
F = G = \int_\infty^u \frac{1}{\sqrt{u' + 2}} \left( A'_9 - \frac{A_9}{2(u' + 2)} \right) = \frac{A_9}{\sqrt{u + 2}}.
\] (56)

Thus using four of the relations in (49), we see that the propagator depends on only
four of the ten independent structures. Finally using the relations involving \(F_1\) and \(F_3\) in
(49), we see that
\[
A_1 g_{\mu\nu} \Lambda = -D_\mu \left( \sqrt{u(u + 2)} A_1 n_{\nu} \Lambda + u A_1 n_{\nu} n. \Gamma \Lambda \right)
+ u \sqrt{u + 2} H n_{\mu} n_{\nu} \Lambda + u^{3/2} H n_{\mu} n_{\nu} n. \Gamma \Lambda - \frac{\sqrt{u}}{u + 2} H g_{\mu\nu} n. \Gamma \Lambda,
\] (57)

where
\[
H = \sqrt{u + 2} A_1.
\] (58)

Again neglecting the \(D_\mu(\ldots)\) and \(D_{\nu}(\ldots)\) terms, this leads to
\[
\Theta_{\mu\nu}(z, w) = \left( (d + 1) A_8 + u(u + 2) A'_1 + \frac{u A_1}{2} \right) n_{\mu} n_{\nu} \Lambda + \left( A_3 - \frac{\sqrt{u}}{u + 2} A_1 \right) g_{\mu\nu} n. \Gamma \Lambda
+ \left( A_4 + u^{3/2} \sqrt{u + 2} A'_1 + \frac{u^{3/2} A_1}{2 \sqrt{u + 2}} \right) n_{\mu} n_{\nu} n. \Gamma \Lambda
\] (59)

Note that using the remaining relations involving \(F_5\) and \(F_7\) in (49), we cannot remove
any other structure, thus showing that three terms remain. In fact, it is easy to check
that the coefficient of the \(n_{\mu} n_{\nu} \Lambda\) term in (59) identically vanishes, thus leaving only two
different structures. This gives the gauge-invariant part of the propagator, and any other
propagator is related to it by pure gauge terms. Note that we could have chosen to keep any
subset of the structures in (10), however it seems to us that the choice in (59) is convenient
for calculating correlators involving a bulk gravitino exchange in \(AdS_{d+1}\).

Thus using the relations (47), the gauge invariant part of the massless gravitino propa-
gator is given by
\[
\Theta_{\mu\nu}(z, w) = A(u) \partial_\mu \partial_\nu u(\partial u. \Gamma) \Lambda + B(u) \partial_\mu u \partial_\nu u(\partial u. \Gamma) \Lambda
\] (60)

where
\[
A(u) = -2^{d+3/2} \lambda(d + 1)(u + 1) u^{-(d+3)/2} (u + 2)^{-d/2-2},
B(u) = 2^{d+3/2} \lambda(d + 1) u^{-(d+5)/2} (u + 2)^{-d/2-3} \left( (d + 2)(u + 1)^2 + 1 \right)
\] (61)

and \(\lambda\) is given by (35).

\(^{10}\)We drop constants of integration as the coefficients have to vanish as \(u \to \infty\).
5 The spinor parallel propagator

Now in explicit calculations of correlators involving the bulk gravitino exchange in Euclidean $AdS_{d+1}$ we also need the expression for the spinor parallel propagator $\Lambda^\alpha_{\beta'}(z, w)$, which we now deduce. We work in Euclidean $AdS_{d+1}$ defined as the upper half space in $z^\mu \in \mathbb{R}^{d+1}$, with $z^0 > 0$, and metric given by

$$ds^2 = \sum_{\mu, \nu=0}^{d} g_{\mu \nu} dz^\mu dz^\nu = \frac{1}{z_0^2} \left( dz_0^2 + \sum_{i=1}^{d} dz_i^2 \right).$$

The coordinates $z_\mu$ will be raised and lowered with the flat space metric unless otherwise mentioned. We choose the vielbein to be given by

$$e^a_\mu = \frac{1}{z_0} \delta^a_\mu,$$

so that the spin connection is given by

$$w_{ab}^\mu = \frac{1}{z_0} \left( \delta^a_0 \delta^b_\mu - \delta^b_0 \delta^a_\mu \right),$$

where $a, b$ are tangent space indices. Thus the Dirac matrices in curved space $\Gamma^\mu$ will be related to those in the tangent space $\gamma^a$ by the relation $\Gamma^\mu = e^a_\mu \gamma^a$, where $\{\gamma^a, \gamma^b\} = 2 \delta^{ab}$. Note that in this coordinate system the chordal distance $u$ is given by

$$u = \frac{(z - w)^2}{2z_0 w_0}.$$ (65)

To deduce the expression for $\Lambda^\alpha_{\beta'}$ that is useful for explicit calculations, we shall use the known expression for the bulk--to--bulk spinor propagator. We shall equate the expressions for the propagator given in [31] and [12] to obtain an expression for $\Lambda^\alpha_{\beta'}$. It should be noted that the two expressions involve non--trivial factors of the mass $m$ of the spinor, and these factors cancel in the expression for $\Lambda^\alpha_{\beta'}$, as they should. The overall numerical factor of $\Lambda^\alpha_{\beta'}$ is fixed because $\Lambda^\alpha_{\beta'}(z, z) = \delta^\alpha_{\beta'}$. We now briefly outline the steps of our analysis, where we denote the spinor propagator by $S(z, w)$.

From [31], we see that the expression for the spinor propagator is given by

$$S(z, w) = \frac{1}{\sqrt{z_0 w_0}} \left[ (\gamma^\mu z_\mu P_- - P_+ \gamma^\mu w_\mu) G_{\Delta-}(u) + (\gamma^\mu z_\mu P_+ - P_- \gamma^\mu w_\mu) G_{\Delta+}(u) \right],$$

where

$$G_{\Delta}(u) = \frac{\Delta \Gamma(\Delta/2) \Gamma(\Delta+1/2)}{4\pi^{(d+1)/2} \Gamma(1 + \Delta - d/2)(1 + u)^{\Delta+1}} F \left( \frac{\Delta}{2} + 1, \frac{\Delta + 1}{2}; \frac{1}{(1 + u)^2} \right),$$

(66)

(67)
\[ \Delta_\pm = \frac{d}{2} + m \pm \frac{1}{2}, \]  
and 
\[ P_\pm = \frac{1 \pm \gamma_0}{2}. \]

Using the relations among hypergeometric functions \[ [30] \]
\[ F(\alpha, \beta; \gamma; x) = (1 - x)^{\gamma - \alpha - \beta} F(\gamma - \alpha, \gamma - \beta; \gamma; x), \]
\[ F\left(\alpha, \alpha + \frac{1}{2}; \gamma; x\right) = (1 \pm \sqrt{x})^{-2\alpha} F\left(2\alpha, \gamma - \frac{1}{2}; 2\gamma - 1; \frac{\pm 2\sqrt{x}}{1 \pm \sqrt{x}}\right), \]

we see that \[ (66) \] becomes
\[ S(z, w) = \frac{1}{2^{m+(d+1)/2} \pi^{d/2}} \frac{\Gamma(m + \frac{d+1}{2})}{\Gamma(m + \frac{1}{2})} \frac{1}{\sqrt{z_0 w_0 (u + 2)^{m+(d+1)/2}}} \]
\[ \times \left[ (\gamma^\mu z_\mu P_- - P_+ \gamma^\mu w_\mu) F\left(m + \frac{d+1}{2}, m; 2m; \frac{2}{u+2}\right) \right. \]
\[ + \left. \frac{m + \frac{d+1}{2}}{(2m + 1)(u + 2)} (\gamma^\mu z_\mu P_- - P_+ \gamma^\mu w_\mu) F\left(m + \frac{d+3}{2}, m + 1; 2m + 2; \frac{2}{u+2}\right) \right]. \]

On using the relations
\[ \gamma F(\alpha, \beta; \gamma; x) - \gamma F(\alpha, \beta + 1; \gamma; x) + \alpha x F(\alpha + 1, \beta + 1; \gamma + 1; x) = 0, \]
\[ \gamma F(\alpha, \beta; \gamma; x) - (\gamma - \beta) F(\alpha, \beta; \gamma + 1; x) - \beta F(\alpha, \beta + 1; \gamma + 1; x) = 0, \]

we finally get from \[ (71) \]
\[ S(z, w) = -\frac{1}{2^{m+(d+3)/2} \pi^{d/2}} \frac{\Gamma(m + \frac{d+1}{2})}{\Gamma(m + \frac{1}{2})} \frac{1}{\sqrt{z_0 w_0 (u + 2)^{m+(d+1)/2}}} \]
\[ \times \left[ (\gamma^\mu z_\mu \gamma_0 + \gamma_0 \gamma^\mu w_\mu) F\left(m + \frac{d+1}{2}, m; 2m + 1; \frac{2}{u+2}\right) \right. \]
\[ - \gamma^\mu (z - w)_\mu F\left(m + \frac{d+1}{2}, m + 1; 2m + 1; \frac{2}{u+2}\right) \right]. \]

We now consider the expression for the spinor propagator given in \[ [12] \] constructed using the geometric objects we have mentioned before. There are two expressions for the propagator depending on the sign of \( m \), and for our purpose of matching the answer we choose the positive sign which is allowed for generic choices of \( m \), which we also take to be positive.
This gives
\[
S(z, w) = -\frac{1}{2^{2m+d+1}\pi^{d/2}} \frac{\Gamma(m + \frac{d+1}{2})}{\Gamma(m + \frac{1}{2})} \left( \frac{2}{u+2} \right)^{m+d/2} \times \left[ F \left( m + \frac{d+1}{2}, m; 2m + 1; \frac{2}{u+2} \right) \right. \\
\left. - \frac{\partial_\mu u}{u+2} \Gamma^\mu F \left( m + \frac{d+1}{2}, m + 1; 2m + 1; \frac{2}{u+2} \right) \right] \Lambda(z, w).
\] (74)

Equating (73) and (74) we get
\[
\Lambda(z, w) = \frac{\gamma^\mu z_\mu \gamma_0 + \gamma_0 \gamma^\mu w_\mu}{\sqrt{2w_0 z_0 (u+2)}},
\] (75)
which satisfies \( \Lambda(z, z) = 1 \). Note that (75) has a very simple form and one could have directly guessed the answer.\(^{11}\) As a consistency check, we note that
\[
D_\mu \Lambda(z, w) = 0,
\] (76)
which leads to \( n^\mu D_\mu \Lambda = 0 \), which is one of the defining properties of \( \Lambda \).

So the gauge invariant part of the massless gravitino propagator in \( AdS_{d+1} \) is given by (60), (61) and (75). It would be interesting to derive the propagator using other approaches and check the gauge invariant part with the results we have obtained.

### A The flat space propagator

One needs to construct the flat space propagator in order to fix the overall normalization of the coefficients in the propagator in \( AdS_{d+1} \). In flat space, substituting (10) into (28) yields the coupled set of equations
\[
A_4' + \frac{d+2}{\mu} A_4 - mA_2 = 0, \\
A_2' - \frac{2}{\mu} A_2 - mA_4 = 0,
\] (77)
where ' denotes derivative with respect to \( \mu \), which has the solution
\[
A_2(\mu) = C(m\mu)^{(1-d)/2} K_{(d+3)/2}(m\mu), \quad A_4(\mu) = -C(m\mu)^{(1-d)/2} K_{(d+5)/2}(m\mu),
\] (78)
\(^{11}\)The other obvious choice \( \tilde{\Lambda}(z, w) = \frac{1}{\sqrt{2w_0 z_0 (u+2)}} (\gamma^\mu w_\mu \gamma_0 + \gamma_0 \gamma^\mu z_\mu) \) does not satisfy \( n^\mu D_\mu \tilde{\Lambda}(z, w) = 0 \). In fact \( \tilde{\Lambda}(z, w) = \Lambda(w, z) \).
on demanding singular behaviour at $\mu = 0$, where we have used the recursion relation

$$\frac{dK_\nu(z)}{dz} = \frac{\nu}{z}K_\nu(z) - K_{\nu+1}(z).$$  (79)

Now the set of equations (77) does not receive any contact term contributions and so $C$ remains undetermined. In order to determine $C$, we want to consider those equations which have delta functions on the right hand side. Generally one would expect to obtain two coupled differential equations in two variables which one can solve easily, but it turns out not to be the case.

So to determine $C$ we consider one of the equations coming from inserting (10) into (28) given by

$$A'_3 + \frac{d}{\mu}A_3 - 2\frac{d}{\mu}A_6 - mA_1 = -\delta^{d+1}(z-w).$$  (80)

In order to determine $C$ the standard technique is to integrate both sides and pick up the contribution of the left–hand side at the origin. The main motivation for considering (80) is that among all the equations this is the only equation where the derivative acts on only one function, there is a non–vanishing delta function contribution and the first two terms on the left–hand side are of the form

$$f' + \frac{d}{\mu}f.$$  (81)

Now for $f = g'$, we know that $g'' + \frac{d}{\mu}g' = \Box g$, and this fact will be useful to us in determining $C$. So first we need to obtain the expressions for the coefficients in (80) which are solutions of the homogeneous equations for flat space. Using the relation

$$A_3 = \frac{1}{2}\left(A_4 + (d+1)A_6\right),$$  (82)

which follows from inserting (10) into (27)\textsuperscript{12}, we see that (80) reduces to

$$\left(A'_4 + (d+1)A'_6\right) + \frac{d}{\mu}\left(A_4 + (d+1)A_6\right) - \frac{4}{\mu}A_6 - 2mA_1 = -2\delta^{d+1}(z-w).$$  (83)

In order to analyze (83), apart from $A_4$, we also need the expressions for the coefficients $A_1$ and $A_6$ which are the solutions of the homogeneous system of equations. In fact $A_1$ and $A_6$ are given by

$$A_1 = \frac{d}{d-1}A_2 + \frac{(d+1)(d+3)}{m^2\mu^2}A_2 + \frac{d+1}{m\mu}A_4$$

$$= \frac{d}{d-1}C(m\mu)^{(1-d)/2}K_{(d+3)/2}(m\mu) - (d+1)C(m\mu)^{-(d+1)/2}K_{(d+1)/2}(m\mu),$$  (84)

\textsuperscript{12}In fact there are no contact term contributions to (82).
and

\[ A_6 = \frac{1}{d-1} \left( A_4 + \frac{2(d+1)}{m\mu} A_2 \right) \]
\[ = \frac{C}{d-1} \left( (m\mu)^{(1-d)/2} \left( \frac{2(d+1)}{m\mu} K_{(d+3)/2}(m\mu) - K_{(d+5)/2}(m\mu) \right) \right), \]

where we have used the recursion relation

\[ K_\nu(z) = K_{\nu-2}(z) + \frac{2(\nu-1)}{z} K_{\nu-1}(z). \]

Now consider integrating both sides of (83) over a small sphere centered at the origin \( \mu = 0 \), which produces a factor of \( \int d\mu d\) in the integrand. Using the expression at small \( z \) for \( \nu \) positive,

\[ K_\nu(z) = \frac{1}{2} \Gamma(\nu) \left( \frac{z}{2} \right)^{-\nu}, \]

it is easy to see that \( A_1, A_4 \) and \( A_6 \) all contribute at the origin individually to the integral. We want to group the various contributions together in such a way that the constant \( C \) can be determined easily. To do that we use the combination

\[ \tilde{A} = A_4 + (d+3)A_6, \]

and it is easy to see that

\[ \tilde{A} = -\frac{2(d+1)C}{(d-1)} (m\mu)^{(1-d)/2} K_{(d+1)/2}(m\mu). \]

Then we rewrite (88) as

\[ \left( \frac{\tilde{A}'}{\mu} + \tilde{A} \right) - 2 \left( A_6' + \frac{d+2}{\mu} A_6 + mA_1 \right) = -2\delta^{d+1}(z-w). \]

Now consider the second set of terms in (90) which is proportional to

\[ \frac{dA_6(z)}{dz} + \frac{d+2}{z} A_6(z) + A_1(z), \]

where \( z = m\mu \). Now when integrating (90) over the small sphere centered at the origin and looking at the contribution due to (91), we see that the individual terms \( A_6' \) and \( A_6/z \) both contribute to the integral at \( O(z^{-d-3}) \) and \( O(z^{-d-1}) \), while the \( A_1 \) term contributes at \( O(z^{-d-1}) \). In order to see this, we use the expressions (84) and (85), and the expansion of the Bessel function at small \( z \) given by

\[ K_\nu(z) = \frac{1}{2} \Gamma(\nu) \left( \frac{z}{2} \right)^{-\nu} \left[ 1 + \frac{z^2}{4(1-\nu)} + \frac{z^4}{32(1-\nu)(2-\nu)} \right]. \]
for $\nu = (d + 1)/2, (d + 3)/2$ and $(d + 5)/2$. However the total contribution due to
\[
\frac{dA_6(z)}{dz} + \frac{d + 2}{z}A_6(z)
\]
vanishes at $O(z^{-d-3})$, and gives
\[ -\frac{2^{(d+1)/2}C}{d-1} \Gamma\left(\frac{d+3}{2}\right) \]
at $O(z^{-d-1})$. Now (94) is exactly cancelled by the contribution due to $A_1$ at $O(z^{-d-1})$, and so (94) does not contribute to the integral at the origin. Thus writing $\tilde{A} = d\Sigma/d\mu$ and integrating over the small sphere, (90) gives
\[
\Omega_{d+1} \lim_{\mu \to 0} \mu^{d} \frac{d\Sigma}{d\mu} = \Omega_{d+1} \lim_{\mu \to 0} \mu^{d} \tilde{A} = -2,
\]
where $\Omega_{d+1} = 2\pi^{(d+1)/2}/\Gamma((d + 1)/2)$. So using (89) we get that
\[
C = \frac{(d - 1)m^d}{(2\pi)^{(d+1)/2}(d + 1)},
\]
thus giving the flat space propagator. The expression for $A_9$ is given by
\[
A_9 = C(m\mu)^{-(d+1)/2} K_{(d+1)/2}(m\mu) - \frac{C'}{d-1} (m\mu)^{(1-d)/2} K_{(d+3)/2}(m\mu),^{13}
\]
which is also needed.

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**References**


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13Note that $A_9 \sim \mu^{-(d+1)}$ as $\mu \to 0$, which is consistent with the massive propagator.


