EQUILIBRIUM STAR CLUSTER FORMATION

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ABSTRACT

We argue that rich star clusters take at least several local dynamical times to form, and so are quasi-equilibrium structures during their assembly. Observations supporting this conclusion include morphologies of star-forming clumps, momentum flux of protostellar outflows from forming clusters, age spreads of stars in the Orion Nebula Cluster (ONC) and other clusters, and the age of a dynamical ejection event from the ONC. We show that these long formation timescales are consistent with the expected star formation rate in turbulent gas, as recently evaluated by Krumholz & McKee. Finally, we discuss the implications of these timescales for star formation efficiencies, the disruption of gas by stellar feedback, mass segregation of stars, and the longevity of turbulence in molecular clumps.

Subject headings: stars: formation — stars: kinematics — stars: winds, outflows

1. INTRODUCTION

Star clusters are born from the densest gas clumps in giant molecular clouds and are likely to be responsible for the majority of stars ever formed (Lada & Lada 2003). The timescale over which a clump transforms into a cluster is a basic constraint for theoretical models. If formation takes only \( \sim 1 \) dynamical time, then star formation is a result of the global collapse of the clump that internal sources of feedback are insufficient to impede. If it takes several dynamical times, then the clump gas must reach an approximate equilibrium, with self-gravity resisted by internal sources of pressure. In this case, star formation is a local process within the protocluster. Runaway gravitational instability occurs on scales much smaller than the overall cluster, involving only a small fraction of the mass at any given time. The cluster formation timescale also determines how much dynamical mass segregation occurs in the gas-rich phase, which may account for the observed central concentration of massive stars in young clusters (Hillenbrand & Hartmann 1998).

Elmegreen (2000) has argued that, over a wide range of scales, star formation occurs in \( \sim 1 \) crossing time. For star clusters the observational evidence cited for short formation times is the age spread of stars and substructure in their spatial distributions. We examine these arguments in detail in §2 and show that the evidence for rich clusters in fact points to a considerably slower formation process. Hartmann, Ballesteros-Paredes, & Bergin (2001) have argued for star formation in one crossing time for low mass, distributed star-forming regions like Taurus. Tassis & Mouschovias (2004) have presented counter arguments. However, neither of these analyses apply to individual rich star clusters, on which we focus.

We define here the important timescales for a protocluster gas clump of mass \( M \), radius \( R \), 1D velocity dispersion \( \sigma \), density \( \rho \), and column density \( \Sigma \). The free-fall time is \( t_{\text{ff}} = (3\pi/32G\rho)^{1/2} \), and the dynamical, or crossing, time is \( t_{\text{dyn}} = R/\sigma \). The virial parameter, \( \alpha_{\text{vir}} \equiv 5\sigma^2 R/(GM) \) (Bertoldi & McKee 1992), defines the relationship of these two; for \( \alpha_{\text{vir}} \sim 1 \), \( t_{\text{ff}} \approx 0.5 \) \( t_{\text{dyn}} \). Clumps are centrally concentrated (Mueller et al. 2002), so the free-fall and dynamical times vary by location within them. We therefore define \( t_{\text{ff}} \) and \( t_{\text{dyn}} \) as mass-weighted averages over the region containing 90\% of the mass, although in practice this may be difficult to determine due to confusion on the outskirts of clusters. Finally, we define the formation time \( t_{\text{form}} \) as the time over which 90\% of the stars in a cluster form. Note, this is 2.3 times the exponentiation time \( t_0 \) in the accelerating star formation model of Palla & Stahler (2000).

In §2 we discuss observational evidence that rich clusters take at least several dynamical times to form, drawing on various stages of the formation process. In §3 we use a theoretical estimate of the star formation rate to show that long formation times are to be expected. Finally, we discuss the implications of this result in §4.

2. OBSERVATIONAL EVIDENCE FOR SLOW FORMATION

2.1. Morphologies of Gas Clumps

Shirley et al. (2003) observed CS \( (J = 5 \rightarrow 4) \) emission from about 60 dense gas clumps containing high-mass star formation. Their surface densities and masses are very similar to more revealed rich, young star clusters, so they are likely to be similar objects at an early stage of formation. Shirley et al. determined clump aspect ratios, defined as the ratio of major and minor axes of the 20\% of peak contour, which is well detected and resolved. The distribution of aspect ratios peaks at 1.26 \( \pm \) 0.22, consistent with most clumps being circularly symmetric. This morphology contrasts with simulations of rapid
cluster formation in which one or two gas filaments tend to dominate (e.g. Bate, Bonnell, & Bromm 2003). Non-periodic simulations with stellar feedback that run for longer times need to be performed to see how long it takes to establish spherical morphologies. From the CS data it is not possible to tell if the clumps of gas relaxed before star formation started or after. Studies of earlier stages of star cluster formation, as traced by the Infrared Dark Clouds (Egan et al. 1998), can address this.

2.2. Morphologies of Embedded Stars

Since molecular clumps are turbulent, stars should form in substructures, which will dissolve as the stars orbit. Thus, the amount of substructure can constrain the formation time. The dissipation time of a substructure depends on whether it is bound (Scally & Clarke 2002) and the nature of its velocity dispersion (Goodwin & Whitworth 2004). If unbound, it dissipates in a time \( t_{\text{dyn}} \). Assuming all stars form in substructures at constant rate, this implies \( t_{\text{form}} \sim (M/M_{\text{sub}}) t_{\text{dyn}} \), where \( M_{\text{sub}} \) is the mass in substructures. If a subcluster is bound, rather than dissolving it will sink to the cluster center due to dynamical friction. The sinking time from radius \( r_{\text{form}} \) is \( t_{\text{sink}} = 0.68(r_{\text{form}}/R)^2(\Lambda/\ln \Lambda) t_{\text{dyn}} \) (Binney & Tremaine 1987), where \( \Lambda \) is the cluster to most-stellar mass ratio. This assumes the cluster density varies as \( r^{-2} \), but a different exponent would not substantially change the result. Assuming a constant star formation rate, so the last subcluster formed at least a time \( t_{\text{form}}/\Lambda \) ago, that all stars form in subclusters, and that on average subclusters form at \( r_{\text{form}} = R/2 \), the formation time and the mass of the largest visible subcluster are related by \( t_{\text{form}} \leq 0.17(2r_{\text{form}}/R)^2(\Lambda^2/\ln \Lambda) t_{\text{dyn}} \). To improve upon these approximate analytic estimates requires global numerical simulations of stars forming from turbulent gas (e.g. Schneja & Klessen 2006).

Comparing to observations, we find that, contrary to Elmegreen (2000), they are consistent with long formation times. The ONC has quite smooth contours of projected stellar surface density with no significant substructure (Hillenbrand & Hartmann 1998), giving no limit on \( t_{\text{form}} \) (see also Scally & Clarke 2002). In IC 348, there are 8 small subclusters, but these contain only \( \sim 10 - 20 \) stars each, roughly 20% of the total 345 stars in the cluster (Lada & Lada 1995), giving \( t_{\text{form}} \sim 5t_{\text{dyn}} \) even if all subclusters are unbound. The largest subcluster has 18 members, so \( \Lambda \approx 19 \) and \( t_{\text{form}} \leq 21t_{\text{dyn}} \) if subclusters are bound. Note we have neglected the artificial enhancement of apparent substructure by patchy extinction. If this effect is significant, the true ages may be even larger. In no case do we find rich clusters with all or most of their mass in substructures, which would imply \( t_{\text{form}} \approx t_{\text{dyn}} \).

2.3. Momentum Flux of Protostellar Outflows

Tan & McKee (2002) estimated the star formation rate in 8 clusters using the observed momentum flux of protostellar outflows. Outflows are magnetocentrifugally driven from the star and inner accretion disk (e.g. Shu et al. 2000), expelling a fraction \( f_p \) of the mass flux \( \dot{m}_* \) reaching the star. Outflow models find that the ratio \( f_p \equiv f_wv_w/v_K \) is constant to within \( \sim 30\% \) (Najita & Shu 1994), where \( v_w \) is the outflow velocity and \( v_K \) is the Keplerian velocity at the equatorial radius of the star. Thus, the total momentum flux \( \dot{p}_w = f_w\dot{m}_*v_K \) is determined by the star formation rate and the evolution of protostellar radii, with only a weak dependence on the latter. Tan & McKee (2002) find that, for clusters with a Salpeter IMF from \( 0.1 - 120 \, M_\odot \), \( \dot{p}_w/\dot{M}_* \approx 87 \, \text{km} \, \text{s}^{-1} \), where \( \dot{M}_* \) is the total star formation rate. Loss of momentum in outflow-outflow interactions would lower this estimate, but this effect is probably small as outflow jets are well-collimated. Thus, observations of \( \dot{p}_w \) give a measurement of \( \dot{M}_* \) and hence the cluster formation timescale. The measurement of \( \dot{p}_w \) is quite uncertain, but the data suggest that it would take \( \sim 3 - 5 \) dynamical times to transform \( \sim 30\% \) of the mass into stars (Tan & McKee 2002). Although this is currently one of the more uncertain methods of estimating \( t_{\text{form}} \), its accuracy should improve as models and observations of individual protostellar outflows are refined.

2.4. Age Spreads of Pre-Main-Sequence Stars

The age spread of stars in a cluster is a direct measure of the formation time. Ages can be difficult to determine because they require fitting observed luminosities and temperatures to pre-main-sequence models that have quite large systematic uncertainties, particularly for low (sub-solar) mass stars. The models also depend on uncertain parameters such as the deuterium abundance, the accretion rate, and accretion geometry (Stahler 1988; Pallà & Stahler 1999, hereafter PS99; Hartmann 2003), which influence the position of the “birthline”, where stars first appear on the HR diagram. Fortunately, the initial stellar contraction is quite rapid, so the birthline position mostly affects ages \( \lesssim 1 \, \text{Myr} \). Patchy extinction also introduces systematic errors into age estimates, as do unresolved binaries, although for these one may make approximate statistical corrections (PS99). Photometric variability is another potential source of error, but recent observations by Burningham et al. (2005) find that variability cannot mimic age spreads of several Myr. In summary, age estimates for intermediate mass (\( M \sim M_\odot \)) stars are reasonably robust.

In the ONC, PS99 estimate ages of 258 stars with masses \( 0.4 < m_*/M_\odot < 6.0 \) from the sample of Hillenbrand (1997). PS99 found 82 stars aged 0-1 Myr, 57 aged 1-2 Myr, 34 aged 2-3 Myr, 17 aged 3-4 Myr, 8 aged 4-5 Myr, 8 aged 5-6 Myr, 8 aged 6-7 Myr, and 6 aged 7-10 Myr. Hartmann (2003) has argued that the oldest ages (\( \sim 10 \, \text{Myr} \)) may be due to a problem of foreground contamination. We conclude that a significant fraction of the stars are 3 Myr old. This is a lower limit to \( t_{\text{form}} \) since star formation is still continuing in the cluster, and because the sample is potentially incomplete for the oldest low mass stars. This result is broadly consistent with ONC age determinations based on Li abundances in pre-main-sequence stars (Palla et al. 2005), a few of which imply ages as large as 10 Myr. Elmegreen (2000) estimates a density of \( n_\text{H} = 1.2 \times 10^4 \, \text{cm}^{-3} \) for the gas clump from which the ONC formed, giving a dynamical time of \( 2.5 \times 10^5 \, \text{yr} \) and an age \( \gtrsim 12 \) crossing times. Note that Elmegreen argued star formation was rapid in the ONC, adopting an age spread of only 1 Myr.

Elmegreen’s dynamical time is probably somewhat low; a better estimate is \( t_{\text{dyn}} = 0.95(\nu_\text{vir} G)^{-1/2}(M/M_\odot)^{1/4} \). We use this rather than relying on a measured velocity dispersion because, for star clusters, this is often hard to
determine due to incompleteness, confusion, and variation in the velocity dispersion with location in the cluster. The mass and surface density are easier to measure. For gas systems the typical virial parameter is $\alpha_{\text{vir}} \approx 1.3$ (McKee & Tan 2003). For stellar systems we adopt a King model, which implies $\alpha_{\text{vir}} \geq 2.0$ if we take $\sigma$ to be the dispersion of the Maxwellian velocity distribution. For the ONC, Hillenbrand & Hartmann (1998) find $M = 4600 M_\odot$, $\Sigma = 0.12$ g cm$^{-2}$, giving $t_{\text{dyn}} = 7 \times 10^5$ yr and an implied cluster age of 4 dynamical times.

In addition to the ONC, which has the best observational data, we can make estimates of the formation and dynamical times for only a few other rich, forming star clusters. Palla & Stahler (2000) find age spreads of $t_{\text{form}} \approx 2.3$ Myr for both $\rho$ Ophiuchi and IC 348. For $\rho$ Oph, the central cluster is embedded in the L1688 dark cloud. The cluster has a radius of $\approx 1$ pc, and the gas mass within this radius (which dominates the total mass) is roughly $1500 M_\odot$ (Loren 1989). The inferred dynamical time is $7.6 \times 10^5$ yr, implying a formation time of 3 dynamical times – in a system that is still gas-dominated. The central cluster in IC 348, which contains roughly half the stars and for which Palla & Stahler make their age estimate, has a radius of 0.5 pc and contains $\approx 200 M_\odot$ of stars (Lada & Lada 1995). Stars probably dominate the mass (Herbig 1998), so we infer a dynamical time of $6 \times 10^5$ yr, giving a formation time of 4 dynamical times.

It is more difficult to determine both formation and dynamical times for other systems, so, contrary to Elmegreen (2000), few conclusions can be drawn. For example, Forbes (1996) did not find evidence for an age spread in NGC 6531, but the analysis was insensitive to timescales shorter than $\approx 3$ Myr. Hodapp & Deane (1993) determined ages up to 6 Myr for stars in L1641, but studied only 12 objects, did not correct for binarity, and used relatively old pre-main sequence tracks. Palla & Stahler (2000) found formation times for Taurus-Auriga, Lupus, Chamaeleon, Upper Scorpius, and NGC 2264 that are $\approx 3$ Myr, but the first three of these are not rich clusters, Upper Scorpius has undergone too much dynamical spreading to reliably estimate its dynamical time at formation, and NGC 2264 is too distant for a complete census to reliably determine its mass and column density, and thus its dynamical time. In NGC 3603, Eisenhauer et al. (1998) find both young stars ($\lesssim 0.5$ Myr) and Wolf-Rayet stars 2 – 3 Myr old, but as with NGC 2264 it is too distant to reliably determine a dynamical time. Elmegreen (2000) comments that star clusters with age spreads $\sim 10$ Myr, e.g. NGC 1850, NGC 2004, NGC 4755, NGC 6611 and the Pleiades, could be the result of “multiple and independent star formation events”, which then form a single cluster by merging or only appear to be a single cluster because of projection. We would argue that in the former case, the age spread is a true indication of the cluster formation time, although, as Elmegreen points out, the relevant dynamical timescale may need to be evaluated over a larger region that hosted the initial subclusters. However, in all these systems, there is no evidence to favor merging of independent subclusters over continuous formation in situ.

2.5. Age of a Dynamical Ejection Event in Orion

Dynamical ejection events provide another method of age estimation. One such event involving 4 massive stars (a binary and two singles) that appear to have come from the ONC has been dated to $\approx 2.5$ Myr ago (Hoogerwerf, de Bruijne & de Zeeuw 2001). The central value of the time since this ejection event is $2.3 \pm 0.2$ Myr in Hoogerwerf et al.’s analysis; however, if the cluster’s distance of about 450 pc is adopted, then the best estimate is 2.5 Myr. The identification with the ONC is based upon the extrapolation of the motion of the center of mass of the four stars from the ejection event to the present day, leading to a predicted position coincident with the ONC (uncertainties are a couple of pc). This result implies that 2.5 Myr ago the ONC was already a rich cluster containing at least four stars of spectral type earlier than O9/B0. Before this the stars had to form and have enough time to find and eject each other in a close interaction. Thus the estimate of 2.5 Myr is again a lower limit to $t_{\text{form}}$ for the ONC, so that $t_{\text{form}} \geq 4t_{\text{dyn}}$.

3. THEORETICAL FORMATION TIMESCALE

Krumholz & McKee (2005, hereafter KM05) estimate the star formation rate in supersonically turbulent gas, and we use this result to compute how long star formation in a clump must continue to reach $\approx 30$% efficiency. Consider a clump with density and pressure profiles $\rho \propto r^{-k_\rho}$ and $P \propto r^{-k_P}$. Hydrostatic equilibrium requires that its mass, radius, and effective sound speed $c \equiv (P/\rho)^{1/2}$ be related by $M = k_P c^2 R/G$. The effective sound speed is related to the 1D velocity dispersion $\sigma_{cl}$ by $c = \phi_V^{1/2} \sigma_{cl}$, where $\phi_V$ is a factor accounting for magnetic support. The clump velocity dispersion in terms of clump mass and surface density is thus

$$\sigma_{cl} = \left( \frac{G^2 \pi M \Sigma}{k_\rho^2 \phi_B} \right)^{1/4} \longrightarrow 2.4 M_3^{1/4} \Sigma_0^{1/4} \text{ km s}^{-1},$$

where $M_3$ is the clump mass in units of $10^3 M_\odot$, $\Sigma_0$ is column density in g cm$^{-2}$, and the numerical evaluation is for the fiducial parameters of McKee & Tan (2003) ($k_P = 1$, $\phi_B = 2.8$, and $\alpha_{vir} = 1.3$). Based on analysis of the structure of turbulent media and comparison to numerical simulations, KM05 find that the fraction of the mass forming stars every free-fall time is $\text{SFR}_{ff} \approx 0.073 \alpha_{\text{vir}}^{-0.52} M^{-0.32}$, where $M \equiv \sigma_{cl}/c_s$ is the Mach number, and $c_s$ is the thermal sound speed. For our clump, this gives $\text{SFR}_{ff\rightarrow cl} \approx 0.027 T_1^{0.16} (M_3 \Sigma_0)^{-0.08}$, where $T_1$ is temperature in units of 10 K. At this rate, turning 30% of the gas into stars takes $\approx 5$ – 6 dynamical times for a 1000 $M_\odot$ clump, with a very weak dependence on temperature, mass, or surface density.

KM05 modeled clouds that were not centrally concentrated, so we can improve this estimate by considering density variation. In turbulent media that are not centrally condensed, the velocity dispersion increases with length scale $\ell$ as $\sigma \propto \ell^{1/2}$ regardless of position in the medium, but the tidal field of a clump may introduce a dependence on the distance $r$ from the clump center as well. The expected variation is $\sigma \propto r^{-k_\sigma}$, and observations show $k_\sigma \approx 1.5$ (Mueller et al. 2002). On length scales $\ell \ll r$, the tidal field of the clump is negligible and we should find $\sigma \propto \ell^{1/2}$, as for a uniform medium, while for $\ell \gg r$ the tidal field will dominate. Since the size scale of a star-forming core is much less than $r$ over the vast majority of a star-forming clump, we can approximate this behavior by $\sigma \approx \sigma_{cl}^\ell (1-k_\sigma)/2 R_{k_\sigma}^{2-1}\ell^{1/2}$ for $\ell \ll r$. 


Thus, the star formation rate varies within the clump as $\text{SFR}_{\text{ff}} = \text{SFR}_{\text{ff}-\text{cl}} k_{\text{ff}}/2$. Integrating over radial shells, the total star formation rate is

$$M_\star = \int_0^R 4\pi r^2 \frac{\text{SFR}_{\text{ff}}}{t_{\text{ff}}} dr = \left(\frac{3 - k_{\text{ff}}}{2.3 (2 - k_{\text{ff}})}\right) M_\star \frac{t_{\text{ff}}}{t_{\text{ff}-\text{cl}}}.$$ 

Thus, central condensation increases the star formation rate by a factor of $(3 - k_{\text{ff}})/(2.3(2 - k_{\text{ff}}))$ relative to a uniform medium. For our fiducial $k_{\text{ff}} = 1.5$, this means that the time required to reach 30% star formation efficiency is reduced by a factor of 1.6 relative to our previous estimate, giving $3 - 4$ dynamical times.

4. DISCUSSION

We have presented observational and theoretical evidence that rich star clusters require at least several dynamical timescales to form. This is significant, because it implies that star clusters cannot form by a process of freely decaying turbulence leading to global collapse, which could not possibly take so long. Instead, something must impede or entirely prevent global collapse, so that rich star clusters are in approximate equilibrium during their assembly. For the ONC, $\rho$ Oph, and IC 348, where we can reasonably estimate both formation and dynamical times, formation typically requires $\gtrsim 3 - 4$ dynamical times, consistent with theoretical predictions for the time required to turn $\sim 30\%$ of the gas into stars. The central regions have much smaller dynamical times than the cluster average, so we predict that in the center a larger fraction of the gas will form stars.

Another implication of this work is that massive star feedback may not be as effective as once assumed in dispersing gas in young clusters. This is consistent with observations that massive stars are not always the last to form in their clusters (e.g. Eisenhauer et al. 1998; Hoogerwerf et al. 2001), and theoretical work showing that clumpiness greatly inhibits gas dispersal (Tan & McKee 2004; Dale et al. 2005). Tan & McKee find that, in a clump like the proto-ONC, if $\sim 3\%$ of the mass forms stars per dynamical time, feedback requires $\sim 2$ Myr (about $3 t_{\text{dyn}}$) to disperse the gas. Dynamical ejection of massive stars, as observed in the ONC (Hoogerwerf et al. 2001; Tan 2004), would increase this time.

Long-term timescales are also important for mass segregation. For example, a 3 Myr formation time for the ONC corresponds to about 8 diameter crossing times at the half mass radius of 0.5 pc, which is the unit of time used in the study of Bonnell & Davies (1998). If the 30 most massive stars are born at the half-mass radius then after 3 Myr the median location of the 6 most massive stars migrates to only 0.075 pc, suggesting that some of the observed mass segregation (Hillenbrand & Hartmann 1998) could be dynamical rather than primordial. Gas drag will likely enhance the segregation beyond the purely N-body effects explored by Bonnell & Davies.

Although undriven supersonic turbulence decays in $\sim 1$ dynamical time (Stone, Ostriker, & Gammie 1998), the observed turbulence in molecular clumps does not damp so quickly. Driving by protostellar feedback is a possible explanation. A virialized clump that radiates away half its kinetic energy per dynamical time has a luminosity $L_{\text{diss}} \approx 4M_\odot^3/4\Sigma^3/2 L_\odot$, but the IMF-averaged energy release associated with accretion is $L_{\text{acc}} \approx 2000M_\odot\Sigma^3/4 (\eta/10)^{-1} L_\odot$, where $\eta$ is the number of dynamical times required to transform 50% of the gas into stars. Outflows should eject about half this energy back into the clump (Shu et al. 2000), so even if only $\sim 1\%$ of this goes into driving turbulence, that is sufficient to offset the decay. Recent observations that find outflows inject enough energy to maintain turbulence (Williams, Plambeck, & Heyer 2003; Quillen et al. 2005) support this idea, as do the numerical simulations of Li & Nakamura (2006). However, this work is preliminary and has not yet shown that feedback can maintain turbulence over a cluster lifetime of $4 t_{\text{dyn}}$ that observations seem to require.

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