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ASYMMETRY OF RECOIL NUCLEI IN CAPTURE OF POLARIZED MUONS

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ABSTRACT

We present a general theory of the asymmetry parameter for the recoil nucleus after the capture of polarized muons. This theory is applied to the study of the asymmetry parameter in the capture $\mu^- + {}^3\text{He} \rightarrow {}^3\text{H} + \nu_\mu$.

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1. INTRODUCTION

The asymmetry of neutrons emitted following capture of polarized muons has been studied both theoretically ¹⁾ and experimentally ^{2),3)}. There seems to be no agreement neither between the two available experimental results nor with the theoretical expected value obtained by invoking the usual C.V.C. hypothesis and the Goldberger-Treiman relation within the frame of first class weak currents. To reduce the uncertainties originated from the nuclear physics involved in the theoretical side it might be worth while to focus the attention on the study of the asymmetry parameters, α_{if} , of the recoil nuclei in a typical capture process

with no break-up of the daughter nuclear state.

Formulae of the asymmetry parameter, α_{if} , for the two cases i) $J_i = 0$, ii) $N_i = \text{hydrogen}$, have been given by Primakoff ⁴⁾. As several other cases actually exist, which may prove interesting from the experimental viewpoint, mainly because of the sensitiveness of their asymmetry parameter on g_p/g_A , it seems interesting to obtain general formulae for α_{if} , as well as for α_{if}^F associated to the capture in the two hyperfine states $F = J_i \pm \frac{1}{2}$.

The obtained formulae will be finally applied to the process

taking into account the fact that the ^3He and ^3H wave functions may be a superposition of several terms with different structure. For the usual values of the coupling constants, we obtain

$$\alpha_{\text{He}^3 - \text{H}^3} = 0.283 + 0.022 \quad (3)$$

which, in contradistinction to the hydrogen case, is large enough to be measured experimentally.

Section 2 is devoted to the derivation of α_{if} (with the inclusion of first order relativistic corrections), which is physically meaningful only when the hyperfine effects do not appear, i.e., $J_i = 0$. As a by-product of the formalism, we are able to prove that the $G_A - g_V$ cross term entering the expression of the capture rate, $\Gamma_M(N_i \rightarrow N_f)$ vanishes exactly when the usual approximation of neglecting the nucleus recoil is made. In Section 3, we deduce general formulae for α_{if}^F , without relativistic corrections, and Section 4 contains the discussion of the asymmetry parameter for the main channel of ${}^3\text{He}$. All through this paper, we will use the impulse approximation for the current $\times$ current interaction.

2. ASYMMETRY PARAMETER FOR SPINLESS INITIAL NUCLEI

The matrix element, M_{fi} , for process (1) is

$$\begin{aligned} M_{fi} &= \alpha_\nu^+ (1 - \hat{\nu} \cdot \vec{\sigma}) \alpha_M \left\{ G_V \left[1 + G_A \hat{\nu} \cdot \vec{\sigma} + \frac{g_A}{M_p} \int \vec{p} \cdot \vec{\sigma} \right] \right. \\ &\quad \left. + \alpha_\nu^+ (1 - \hat{\nu} \cdot \vec{\sigma}) \vec{\sigma} \alpha_M \left\{ G_A \int \vec{\sigma} + \frac{g_V}{M_p} \int \vec{p} \right\} \right\} = \quad (4) \\ &\equiv A_{fi} \alpha_\nu^+ (1 - \hat{\nu} \cdot \vec{\sigma}) \alpha_M + \vec{B}_{fi} \alpha_\nu^+ (1 - \hat{\nu} \cdot \vec{\sigma}) \vec{\sigma} \alpha_M \end{aligned}$$

with the usual notation ⁵⁾.

Therefore, the squared matrix element, summed over final polarizations and averaged over the initial ones is

$$\begin{aligned} \sum_i \sum_f |M_{fi}|^2 &= |A|^2 (1 - \vec{S}_M \cdot \hat{\nu}) + 2 \text{Re} A \vec{B}^* \cdot (\vec{S}_M - \hat{\nu} + i(\hat{\nu} \times \vec{S}_M)) + \\ &\quad + |\vec{B}|^2 (1 + \vec{S}_M \cdot \hat{\nu}) - 2 \text{Re} (\vec{B} \cdot \vec{S}_M) (\vec{B} \cdot \hat{\nu})^* - i(\vec{B} \times \vec{B}^*) \cdot (\hat{\nu} + \vec{S}_M) \quad (5) \end{aligned}$$

where $\vec{s}_M$ is the muon polarization vector, and

$$|A|^2 \equiv \frac{1}{2J_i+1} \sum_{M_i} \sum_{M_f} |A_{fi}|^2$$

$$AB_k^* \equiv \frac{1}{2J_i+1} \sum_{M_i} \sum_{M_f} A_{fi} (B_{fi}^*)_k$$

$$B_j B_k^* \equiv \frac{1}{2J_i+1} \sum_{M_i} \sum_{M_f} (B_{fi})_j (B_{fi}^*)_k$$
(6)

M_i, M_f being the third components of the initial and final nuclear spins J_i, J_f , respectively.

With the usual approximation of taking the nuclear wave function at rest, covariance under rotations implies that

$$|A|^2 = a^2$$

$$AB_k^* = c \hat{v}_k$$

$$B_j B_k^* = b^2 (\delta_{jk} - \hat{v}_j \hat{v}_k) + \beta^2 \hat{v}_j \hat{v}_k + i \epsilon_{jke} \gamma \hat{v}_e$$
(7)

where a^2, c, b^2, β^2 and γ are independent of the direction, $\hat{v}$, of the emitted neutrino. Furthermore, a^2, b^2 and β^2 are non-negative quantities (note that $B_j B_k^*$ is a positive semi-definite matrix). On the other hand, the matrix $B_j B_k^*$ is symmetric, since by using the time reversal operation, T , we have

$$\sum_{M_i} \sum_{M_f} \langle J_i M_i | \left(\int G_A \sigma_j + \frac{g_V}{M_P} p_j \right) | J_i M_i \rangle \langle J_i M_i | \left(\int G_A \sigma_k + \frac{g_V}{M_P} p_k \right)^+ | J_f M_f \rangle =$$

$$= \sum_{M_i} \sum_{M_f} \langle J_i - M_i | T \left(\int G_A \sigma_j + \frac{g_V}{M_P} p_j \right) T^{-1} | J_f - M_f \rangle \langle J_f - M_f | T \left(\int G_A \sigma_k + \frac{g_V}{M_P} p_k \right)^+ T^{-1} | J_i - M_i \rangle =$$

$$= \sum_{M_i} \sum_{M_f} \langle J_i - M_i | \left(\int G_A \sigma_j + \frac{g_V}{M_P} p_j \right)^+ | J_f - M_f \rangle \langle J_f - M_f | \left(\int G_A \sigma_k + \frac{g_V}{M_P} p_k \right) | J_i - M_i \rangle$$
(8)

Q.E.D. Consequently $\gamma = 0$, and hence

$$\sum_i \sum_f |M_{fi}|^2 = a^2 + 2b^2 + \beta^2 - 2\text{Rec} - \vec{S}_M \cdot \hat{\nu} \{a^2 - 2b^2 + \beta^2 - 2\text{Rec}\} \quad (9)$$

One simply verifies that

$$\begin{aligned} a^2 &= G_V^2 |\int 1|^2 + G_P^2 |\hat{\nu} \cdot \vec{\sigma}|^2 + \frac{2G_P G_A}{M_P} \text{Re}(\hat{\nu} \cdot \vec{\sigma})(\vec{p} \cdot \vec{\sigma})^* \\ b^2 &= \frac{1}{2} G_A^2 [|\int \vec{\sigma}|^2 - |\hat{\nu} \cdot \vec{\sigma}|^2] \\ \beta^2 &= G_A^2 |\hat{\nu} \cdot \vec{\sigma}|^2 \end{aligned} \quad (10)$$

$$c = G_A G_P |\hat{\nu} \cdot \vec{\sigma}|^2 + \frac{G_V G_V}{M_P} (\int 1)(\hat{\nu} \cdot \vec{p})^* + \frac{G_A G_A}{M_P} ((\vec{p} \cdot \vec{\sigma})^* (\hat{\nu} \cdot \vec{\sigma}))$$

Therefore, the partial capture rate and the asymmetry parameter for the recoil nucleus are

$$\begin{aligned} \Gamma_M(N_i \rightarrow N_f) &= (2\pi)^{-3} (Z_{N_i} \alpha m'_M)^3 \left(\frac{Z_{eff} N_i}{Z_{N_i}} \right)^4 \frac{v_{if}^2}{1 + \frac{v_{if}}{M_{N_f}}} \\ &\cdot \int d\hat{\nu} \{a^2 + 2b^2 + \beta^2 - 2\text{Rec}\} \end{aligned} \quad (11)$$

$$\alpha_{if} = \frac{a^2 - 2b^2 + \beta^2 - 2\text{Rec}}{a^2 + 2b^2 + \beta^2 - 2\text{Rec}} \quad (12)$$

which, neglecting the relativistic corrections and in the allowed approximation, becomes

$$\begin{aligned} (\alpha_{if})_{all} &= \frac{G_V^2 |\int 1|_{all}^2 + \frac{1}{3} [(G_P - G_A)^2 - 2G_A^2] |\int \vec{\sigma}|_{all}^2}{G_V^2 |\int 1|_{all}^2 + \frac{1}{3} [(G_P - G_A)^2 + 2G_A^2] |\int \vec{\sigma}|_{all}^2} \end{aligned} \quad (13)$$

It should be noted that in the general expression (11), the $G_A - g_V$ cross term does not appear as a consequence of the assumption that the final nucleus wave function is taken at rest.

3. GENERAL THEORY FOR THE ASYMMETRY PARAMETER

The asymmetry parameter α_{if} calculated in the previous Section is only meaningful when $J_i = 0$, but this is not the case in many interesting situations which makes it worth deriving an expression for the asymmetry parameter α_{if}^F corresponding to each of the two hyperfine states $F = J_i \pm \frac{1}{2}$.

For the derivation of α_{if}^F we have assumed ⁶⁾ the initial hyperfine density matrix to be of the type

$$\rho_F = \frac{1}{2F+1} \left\{ 1 + \frac{3\lambda_F}{F+1} \vec{F} \cdot \vec{S}_\mu \right\} P_F \quad (14)$$

where P_F is the projector on the initial states with total angular momentum F . When the population weights of the states with a given F are those corresponding to the formation of mesic atoms with the muon in the K orbit, and when completely polarized muons impinge on an unpolarized target, then

$$\lambda_{J_i + 1/2} = \frac{1}{3} \frac{2J_i + 3}{2J_i + 1} \quad \lambda_{J_i - 1/2} = -\frac{1}{3} \quad (15)$$

The probability of emission of ν_μ after μ capture in the initial state ρ_F is proportional to

$$|M_F|^2 = 2 \text{Tr} (M \rho_F M^\dagger) \quad (16)$$

where the matrix elements of M are given by (4), wherein the relativistic terms will be henceforth neglected.

The computation of (16) is considerably simplified by first rotating ρ_F so as to have $\hat{\nu}$ pointing along the quantization axis, $\vec{s}_M$. The axial symmetry of the problem allows us to take $\hat{\nu}$ in the plane Oxz , so that

$$\begin{aligned} \rho_F(\hat{\nu}) &\equiv e^{-i\theta F_2} \rho_F e^{+i\theta F_2} = \\ &= \frac{1}{2F+1} \left\{ 1 + \frac{3\lambda_F}{F+1} \cos\theta F_3 + \frac{3\lambda_F}{F+1} \sin\theta F_1 \right\} \end{aligned} \quad (17)$$

where $\cos\theta = \vec{s}_M \cdot \hat{\nu}$.

We are thus led to compute

$$|M_F|^2 = 2 \text{Tr} (M' \rho_F(\hat{\nu}) M'^{\dagger}) \quad (18)$$

with M'_{if} given by (4) with $\hat{\nu} \cdot \vec{\sigma} = \sigma_3$. The $\sin\theta$ term in (17) obviously gives a zero contribution to $|M_F|^2$.

A long but straightforward computation yields

$$\begin{aligned} |M_F|^2 &= 2 \sum_M a(M) \left\{ C(J_c, 1/2, F; M+1/2, -1/2) f(M+1/2) + \right. \\ &\quad \left. + C(J_c, 1/2, F; M-1/2, 1/2) g(M-1/2) \right\} \end{aligned} \quad (19)$$

with

$$a(M) = \frac{1}{2F+1} \left(1 + \frac{3\lambda_F}{F+1} M \cos\theta \right) \quad (20)$$

$$\begin{aligned} f(M+1/2) &= \sum_{p=0}^{\infty} (-i)^p \sqrt{4p(2p+1)} \left[C(J_c, p, J_f; M+1/2, 0) \bar{G}_V^{(e)} + \right. \\ &\quad \left. + \sum_j C(p, 1, j; 0, 0) C(J_c, j, J_f; M+1/2, 0) \left\{ \bar{G}_P^{(e)} [j] - \bar{G}_A^{(e)} [j] \right\} \right] \end{aligned} \quad (21)$$

$$g(M-1/2) = -\sqrt{2} \sum_{p=0}^{\infty} (-i)^p \sqrt{4p(2p+1)} \cdot \sum_j C(p, 1, j; 0, 1) C(J_L, J_F; M-1/2, 1) \bar{G}_A^{(e)}[j] \quad (22)$$

$$\bar{G}_V^{(e)} = \frac{C(T_L, 1, T_F; (T_L)_3 - 1, (T_F)_3)}{\sqrt{2(2J_F+1)(2T_F+1)}} \langle J_F || \sum_i c_i j_e(r_i) y_e || J_L \rangle G_V \quad (23)$$

$$\bar{G}_{A,E}^{(e)}[j] = \frac{C(T_L, 1, T_F; (T_L)_3 - 1, (T_F)_3)}{\sqrt{2(2J_F+1)(2T_F+1)}} \langle J_F || \sum_i c_i j_e(r_i) y_j^{(e)} || J_L \rangle G_{A,E}$$

where

$$y_{jm}^{(e)} = \sum_{\mu} C(l, 1, j; m-\mu, \mu) Y_{l, m-\mu} \sigma_{\mu} \quad (24)$$

For the Clebsch-Gordan and Racah coefficients, we will follow the notation in Rose's book ⁷⁾, but the reduced matrix elements are defined instead as

$$\langle J'' M'' T'' T_3'' | A_{JM} B_{TT_3} | J' M' T' T_3' \rangle = \frac{(-1)^{2J+2T}}{\sqrt{(2J''+1)(2T''+1)}} \cdot \quad (25)$$

$$\cdot C(J' J J''; M' M M'') C(T' T T''; T_3' T_3 T_3'')$$

$$\cdot \langle J'' T'' || A_J B_T || J' T' \rangle$$

Further elaboration of the r.h.s. of (19) leads to

$$\begin{aligned}
|M_F|^2 = & 8\pi (2J_f + 1) \sum_{\ell, \ell'} (i)^{\ell - \ell'} \left\{ \sum_r \frac{X_r^{(\ell)} X_r^{(\ell')*}}{2r + 1} + \right. \\
& + \lambda_F \cos \Theta \sqrt{\frac{3F(F+1)}{F+1}} (-1)^{F - J_f + 1} \sum_{r,s} (-1)^{r+s} \\
& \cdot W(F F r s; 1 J_f) C(r s 1; -\frac{1}{2}, \frac{1}{2}) X_r^{(\ell)} X_s^{(\ell')*} \left. \right\}
\end{aligned} \quad (26)$$

where the possible non-vanishing $X_r^{(\ell)}$, for a given ℓ , are

$$\begin{aligned}
X_{\ell-3/2}^{(\ell)} &= \bar{G}_p^{(\ell)} [\ell-1] (\ell-1) \sqrt{\frac{2\ell}{2\ell-1}} W(F J_\ell \ell-3/2 \ell-1; 1/2 J_f) \\
X_{\ell-1/2}^{(\ell)} &= \left\{ \sqrt{\ell} \bar{G}_v^{(\ell)} + \sqrt{\ell+1} \bar{G}_A^{(\ell)} [\ell] \right\} \sqrt{2\ell} W(F J_\ell \ell-1/2 \ell; 1/2 J_f) + \\
&+ \left\{ (2\ell-1) \bar{G}_A^{(\ell)} [\ell-1] - \ell \bar{G}_p^{(\ell)} [\ell-1] \right\} \sqrt{\frac{2\ell}{2\ell-1}} W(F J_\ell \ell-1/2 \ell-1; 1/2 J_f) \\
X_{\ell+1/2}^{(\ell)} &= - \left\{ \sqrt{\ell+1} \bar{G}_v^{(\ell)} - \sqrt{\ell} \bar{G}_A^{(\ell)} [\ell] \right\} \sqrt{2(\ell+1)} W(F J_\ell \ell+1/2 \ell; 1/2 J_f) - \\
&- \left\{ (\ell+1) \bar{G}_p^{(\ell)} [\ell+1] - (2\ell+3) \bar{G}_A^{(\ell)} [\ell+1] \right\} \sqrt{\frac{2(\ell+1)}{2\ell+3}} W(F J_\ell \ell+1/2 \ell+1; 1/2 J_f) \\
X_{\ell+3/2}^{(\ell)} &= \bar{G}_p^{(\ell)} [\ell+1] (\ell+2) \sqrt{\frac{2(\ell+1)}{2\ell+3}} W(F J_\ell \ell+3/2 \ell+1; 1/2 J_f)
\end{aligned} \quad (27)$$

As a consequence

$$d\Gamma_{\mu}^F(N_i \rightarrow N_f) = (2\pi)^{-3} (Z_{N_i} \alpha m_{\mu}^i)^3 \left(\frac{Z_{eH N_i}}{Z_{N_i}} \right)^4 \cdot \frac{\nu_{if}^2}{1 + \frac{\nu_{if}}{M_{N_i}}} |\bar{M}_F|^2 \{1 - \alpha_{if}^F \cos \theta\} d\hat{v} \quad (28)$$

with $|\bar{M}_F|^2$ obtained from (26) by putting $\cos \theta = 0$, and

$$\alpha_{if}^F = (-1)^{F-J_f+1/2} \lambda_F \sqrt{\frac{3F(2F+1)}{F+1}} \cdot \frac{\sum_{e,e'} i^{-e'+e} \sum_{r,s} (-1)^{r+s} W(F F r s; 1 J_f) C(r s 1, -1/2 1/2) \bar{X}_r^{(e)} \bar{X}_s^{(e')*}}{\sum_{e,e'} i^{-e'+e} \sum_r \frac{\bar{X}_r^{(e)} \bar{X}_r^{(e')*}}{2r+1}} \quad (29)$$

For light nuclei, where the allowed approximation is reasonably good, it may prove useful to have the limiting expressions for $|\bar{M}_F|^2$ and α_{if}^F and they are explicitly given in the Appendix.

4. ASYMMETRY PARAMETER FOR ${}^3\text{He}$

Since for light nuclei it is easier to measure the recoil of the daughter nuclei and hence the asymmetry parameter, we are going to consider in detail the reaction (2). We have considered this process since the capture in hydrogen presents a very small asymmetry parameter ⁴⁾ and the capture in deuterium does not present a channel of type (1). Therefore ${}^3\text{He}$ is the lightest nucleus for which the measurement of the asymmetry parameter can be performed using the existing experimental techniques.

For ${}^3\text{He}$ the energy difference between the two hyperfine states is so small that the Auger transition between them is not energetically possible and the capture takes place from a statistical mixture of the two hyperfine states. In this case the effective $|M|^2$ is given by

$$|M|^2 = \frac{3}{4} |M_L|^2 + \frac{1}{4} |M_0|^2 \quad (30)$$

and using the expressions in the Appendix, we get for the asymmetry parameter

$$\alpha_{\text{He}^3 - \text{H}^3} = \frac{|\bar{G}_V + \frac{1}{\sqrt{3}} (\bar{G}_A - \bar{G}_P)|^2}{2 \left[|\bar{G}_V|^2 + |\bar{G}_A|^2 + \frac{1}{3} |\bar{G}_P|^2 - \frac{2}{3} \text{Re } \bar{G}_A^* \bar{G}_P \right]} \quad (31)$$

where the allowed approximation has been used. It has been estimated ^{8),9)} that the higher order terms give a contribution smaller than 1% compared to the allowed ones in the capture rate of (2). From now on we are going to neglect the relativistic terms since their contribution to this capture rate is lesser than 1% of the allowed ones ¹⁰⁾.

In our case

$$\begin{aligned} \bar{G}_V &= \frac{1}{2\sqrt{3}} \langle H^3 \| \sum_{i=1}^3 \tau_i j_0(\nu r_i) Y_0 \| \text{He}^3 \rangle G_V \\ \bar{G}_{A,P} &= \frac{1}{2\sqrt{3}} \langle H^3 \| \sum_{i=1}^3 \tau_i j_0(\nu r_i) Y_1^{(0)} \| \text{He}^3 \rangle G_{A,P} \end{aligned} \quad (32)$$

To compute these reduced matrix elements, we use the wave functions given in Eq. (3.1) of Ref. 10), with the assumption that the amplitudes for the isospin $\frac{3}{2}$ terms are zero. Furthermore, using isospin invariance, we will assume that the amplitudes of the different states are equal in ${}^3\text{He}$ and ${}^3\text{H}$. Then, in the notation of Ref. 10), we get

$$\bar{G}_V = -\frac{A}{6\sqrt{\pi}} G_V \quad \bar{G}_{A,P} = +\frac{B}{6\sqrt{\pi}} \sqrt{3} G_{A,P} \quad (33)$$

where

$$\begin{aligned} A = & \langle 1^s 1 0 1 1^s \rangle |S_1|^2 + \frac{1}{2} \langle 1_1^m 1 0 1 1_1^m \rangle (|S_3|^2 + 2|D_3|^2) + \\ & + \frac{1}{2} \langle 1_2^m 1 0 1 1_2^m \rangle (|S_3|^2 + 2|D_1|^2 + 2|D_2|^2) + \sqrt{3} \langle 1_1^m 1 0 1 1_2^m \rangle \cdot \\ & \cdot (|D_1|^2 + |D_2|^2 - |D_3|^2) + \frac{1}{\sqrt{2}} [\langle 1^s 1 0 1 1_1^m \rangle + \sqrt{3} \langle 1^s 1 0 1 1_2^m \rangle] \cdot \\ & \cdot (S_1^* S_3 + S_3^* S_1) \end{aligned} \quad (34)$$

$$\begin{aligned} B = & \langle 1^s 1 0 1 1^s \rangle |S_1|^2 - \frac{1}{2} \langle 1_1^m 1 0 1 1_1^m \rangle (|S_3|^2 - \frac{2}{3} |D_3|^2) + \\ & + \frac{1}{6} \langle 1_2^m 1 0 1 1_2^m \rangle (|S_3|^2 + 2|D_1|^2 + 2|D_2|^2) + \\ & + \frac{1}{\sqrt{3}} \langle 1_1^m 1 0 1 1_2^m \rangle (2|S_3|^2 + |D_1|^2 + |D_2|^2 - |D_3|^2) \end{aligned}$$

and therefore

$$\alpha_{\text{He}^3 - \text{H}^3} = \frac{[G_V - (G_A - G_P) \frac{B}{A}]^2}{2 [G_V^2 + (3G_A^2 + G_P^2 - 2G_A G_P) \frac{B^2}{A^2}]} \quad (35)$$

where

$$\left. \begin{aligned} G_V &= 1.025 g_V^{(\beta)} \\ G_A &= - (0.251 + \lambda) g_V^{(\beta)} \\ G_P &= - (0.251 - 0.0549 (\lambda - \kappa)) g_V^{(\beta)} \end{aligned} \right\} \quad (36)$$

and

$$g_A^{(\mu)} = -\lambda g_V^{(\beta)} \quad g_P^{(\mu)} = -\kappa g_V^{(\beta)} = +\frac{\kappa}{\lambda} g_A^{(\mu)}$$

It is worth noticing that for hydrogen $B/A = -1$ while in our case $B/A \approx -1$ (see below). This fact can be intuitively understood by the following argument: the initial and final nuclear states, when the three nucleons are in an s subshell, are respectively

$$|He^3\rangle = a_{p\uparrow}^+ a_{p\downarrow}^+ a_{n\uparrow}^+ |0\rangle, \quad |H^3\rangle = a_{p\uparrow}^+ a_{n\downarrow}^+ a_{n\uparrow}^+ |0\rangle$$

where we have taken, for the sake of definiteness, the total spin pointing along $+Oz$. It is now clear that the relative sign of the matrix elements

$$\langle H^3 | \sum_i \tau_i^- | He^3 \rangle \quad \langle H^3 | \sum_i \sigma_{i3} \tau_i^- | He^3 \rangle$$

is minus while a similar argument for muon capture by protons yields a plus relative sign for $\langle n\uparrow | \tau^- | p\uparrow \rangle$, $\langle n\uparrow | \tau^- \sigma_3 | p\uparrow \rangle$.

Furthermore there exists theoretical and experimental evidence ¹⁰⁾ pointing to the fact that the state ψ_1 exists with a probability higher than 94%, then B/A can be written

$$B/A = 1 - \delta$$

$$\delta \approx \frac{1}{|S_1|^2} \left\{ 1.26 |S_3|^2 + 0.633 (|D_1|^2 + |D_2|^2 + |D_3|^2) + 0.088 (S_1^* S_3 + S_3^* S_1) \right\} \quad (37)$$

To compute the numerical coefficients, we have used the Gaussian wave function of Ref. 10). Taking into account the bounds for the amplitudes as given in Section 7 of Ref. 10), we get

$$0 \leq \delta \leq 0.09 \quad (38)$$

[Notice that the wave function used is important only to fix the range of variation of δ . The same limitation on δ appears if Irving-type wave functions are used.] Since the asymmetry parameter is quite insensitive to the value of λ we take $\lambda = 1.18$ and then we get for the asymmetry parameter the results given in the Figure, where the two boundaries of the dashed region correspond to the limiting cases $\delta = 0$ and $\delta = 0.09$. For the value of κ corresponding to the Goldberger-Treiman relation, $g_P^{(\mu)}/g_A^{(\mu)} = +7.2$, i.e., $\kappa = 8.5$, we get the value (3) where the errors are due to the incertitude in the wave function.

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A P P E N D I X

If the allowed approximation is done, then $|\bar{M}_F|^2$ and α_{cf}^F (see Section 3) become, after writing $\bar{G}_V \equiv \bar{G}_V^{(0)}$ and $\bar{G}_{A,P} = \bar{G}_{A,P}^{(0)} [1]$:

$$i) \quad F = J_i + \frac{1}{2}, \quad J_F = J_i + 1$$

$$|\bar{M}_F|^2 = 8\pi \frac{2J_i+3}{18(J_i+1)} \left\{ |\bar{G}_P - 3\bar{G}_A|^2 + \frac{J_i+2}{2J_i+1} |\bar{G}_P|^2 \right\}$$

$$\alpha_{cf}^F |\bar{M}_F|^2 = -8\pi \lambda_F \frac{2J_i+1}{2(J_i+1)} \left\{ \left| \bar{G}_A + \frac{\bar{G}_P}{2J_i+1} \right|^2 - \frac{(J_i+2)(2J_i+5)}{5(2J_i+1)^2} |\bar{G}_P|^2 \right\}$$

$$ii) \quad F = J_i + \frac{1}{2}, \quad J_F = J_i$$

$$|\bar{M}_F|^2 = 8\pi \frac{1}{2} \left\{ \left| \bar{G}_V + \sqrt{\frac{J_i}{J_i+1}} \left(\bar{G}_A - \frac{1}{3} \bar{G}_P \right) \right|^2 + \frac{2J_i+3}{9(J_i+1)} |\bar{G}_P|^2 \right\}$$

$$\alpha_{cf}^F |\bar{M}_F|^2 = 8\pi \lambda_F \frac{1}{2} \left\{ \left| \bar{G}_V + \sqrt{\frac{J_i}{J_i+1}} (\bar{G}_A - \bar{G}_P) \right|^2 - \frac{2J_i-1}{5(J_i+1)} |\bar{G}_P|^2 \right\}$$

$$iii) \quad F = J_i + \frac{1}{2}, \quad J_F = J_i - 1$$

$$|\bar{M}_F|^2 = 8\pi \frac{2J_i-1}{6(2J_i+1)} |\bar{G}_P|^2$$

$$\alpha_{cf}^F = \frac{3}{5} \lambda_F$$

$$\text{iv)} \quad F = J_i - \frac{1}{2}, \quad J_F = J_i + 1$$

$$|\bar{M}_F|^2 = 8\pi \frac{2J_i + 3}{6(2J_i + 4)} |\bar{G}_P|^2$$

$$\alpha_{if}^F = -\frac{3}{5} \lambda_F \frac{2J_i - 1}{2J_i + 1}$$

$$\text{v)} \quad F = J_i - \frac{1}{2}, \quad J_F = J_i$$

$$|\bar{M}_F|^2 = 8\pi \frac{1}{2} \left\{ |\bar{G}_V + \sqrt{\frac{J_i + 1}{J_i}} \left(\frac{1}{3} \bar{G}_P - \bar{G}_A \right)|^2 + \frac{2J_i - 1}{9J_i} |\bar{G}_P|^2 \right\}$$

$$\alpha_{if}^F |\bar{M}_F|^2 = -8\pi \lambda_F \frac{1}{2} \frac{2J_i - 1}{2J_i + 1} \left\{ |\bar{G}_V + \sqrt{\frac{J_i + 1}{J_i}} (\bar{G}_P - \bar{G}_A)|^2 - \frac{2J_i + 3}{5J_i} |\bar{G}_P|^2 \right\}$$

$$\text{vi)} \quad F = J_i - \frac{1}{2}, \quad J_F = J_i - 1$$

$$|\bar{M}_F|^2 = 8\pi \frac{2J_i - 1}{18J_i} \left\{ |\bar{G}_P - 3\bar{G}_A|^2 + \frac{J_i - 1}{2J_i + 1} |\bar{G}_P|^2 \right\}$$

$$\alpha_{if}^F |\bar{M}_F|^2 = 8\pi \lambda_F \frac{2J_i - 1}{2J_i} \left\{ \left| \bar{G}_A - \frac{1}{2J_i + 1} \bar{G}_P \right|^2 - \frac{(2J_i - 3)(J_i - 1)}{5(2J_i + 1)^2} |\bar{G}_P|^2 \right\}$$

R E F E R E N C E S

- 1) M.K. Akimova, L.D. Blokhintsev and E.I. Dolinskii, Soviet Phys. - JETP 12, 1260 (1961), where further references may be found.
 - 2) V.S. Evseev et al., Phys. Letters 6, 332 (1963).
 - 3) R.M. Sundenlin, CAR-882-22 (1967).
 - 4) H. Primakoff, Revs. Modern Phys. 31, 802 (1959).
 - 5) J.R. Luyten, H.P.C. Rood and H.A. Tolhoek, Nuclear Phys. 41, 236 (1963).
 - 6) I.M. Shmushkevich, Soviet Phys. - JETP 9, 673 (1959).
 - 7) M.E. Rose, "Elementary Theory of Angular Momentum", J. Wiley, 1957.
 - 8) C. Werntz, Nuclear Phys. 16, 59 (1960).
 - 9) A. Fujii and H. Primakoff, Nuovo Cimento 12, 327 (1959).
 - 10) R. Pascual and P. Pascual, Nuovo Cimento 44B, 434 (1966).
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FIGURE CAPTION

The asymmetry parameter $\alpha_{^3\text{He}-^3\text{H}}$ is plotted against the value of $\chi \equiv -g_p^{(\mu)}/g_v^{(\beta)}$.

The asymmetry parameter $\alpha_{\text{He}^3\text{-H}^3}$ is plotted against the value of

$$\chi \equiv -g_p^{(\mu)}/g_v^{(\beta)}$$

FIG.1